Sheet flow modeling in a printer paper path

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Table of contents

Table of contents..........................................................................................................................................1
List of symbols.............................................................................................................................................2
1. Introduction..............................................................................................................................................3
2. Case description.......................................................................................................................................4
  2.1 The printer paper path........................................................................................................................4
  2.2 Initial assumptions.............................................................................................................................6
  2.3 Describing relations...........................................................................................................................6
3. The piecewise affine modeling formalism..............................................................................................7
4. Model descriptions...................................................................................................................................8
  4.1 Simplex loop......................................................................................................................................8
  4.2 Duplex loop: nominal sheet movement.............................................................................................8
  4.3 Duplex loop: tracking error of the motors ........................................................................................11
    4.3.1 Option 1 ....................................................................................................................................11
    4.3.1.1 Mathematical model:.........................................................................................................12
    4.3.1.2 Global representation of the variables ..............................................................................16
    4.3.2 Option 2 ....................................................................................................................................17
    4.3.2.1 Mathematical model:.........................................................................................................17
    4.3.2.2 Global representation of the variables ..............................................................................20
    4.3.3 Comparison...............................................................................................................................20
  4.4 State-space form ..............................................................................................................................21
    4.4.1 Simplex loop.............................................................................................................................21
    4.4.2 Duplex loop ..............................................................................................................................22
5. Simulations and results..........................................................................................................................25
  5.1 M-file ...............................................................................................................................................25
  5.2 Simulink model for the simplex loop ...............................................................................................25
  5.3 Simulink model for the duplex loop...............................................................................................26
6. Conclusions............................................................................................................................................28
References..................................................................................................................................................29
List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Rad/s</td>
<td>Angular velocity</td>
</tr>
<tr>
<td>$n$</td>
<td>-</td>
<td>Transmission ratio</td>
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<tr>
<td>$v$</td>
<td>m/s</td>
<td>Velocity</td>
</tr>
<tr>
<td>$r$</td>
<td>m</td>
<td>Radius</td>
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<tr>
<td>$x$</td>
<td>m</td>
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</tr>
<tr>
<td>$d$</td>
<td>m</td>
<td>Alternative transported distance</td>
</tr>
<tr>
<td>$q$</td>
<td>-</td>
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</tr>
</tbody>
</table>

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<th>Description</th>
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</thead>
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<td>-</td>
<td>Pinch</td>
</tr>
<tr>
<td>$(..)_m$</td>
<td>-</td>
<td>Motor</td>
</tr>
<tr>
<td>$(..)_s$</td>
<td>-</td>
<td>Sheet</td>
</tr>
<tr>
<td>$(..)_0$</td>
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</table>

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<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_s$</td>
<td>0.297 m</td>
<td>Length of an A4 sheet</td>
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1. Introduction

In mechanical engineering, the modeling of dynamical systems is very important. A model is an abstraction of the reality, so a model has a certain precision. In the case of a high volume printer, a model can be used for designing a controller that controls the angular velocity of the pinches that drive the sheet through the path. Because the sheet flow through the entire path is non-linear, due to a finite number of pinches, the modeling and simulation of the sheet flow can become very complex. In order to reduce computation times and complexity of the model, a linear approach is preferred.

Part of current research activities at the Technische Universiteit Eindhoven focuses on the sheet control in a simplex paper path (for single-sided printing). In order to design these controllers, the sheet flow is modeled using the piecewise affine formalism. The future focus in this research is the modeling and control of the sheet flow in the duplex paper path.

The problem that is discussed in this report is if the sheet flow in the paper path of a high volume printer can be modeled using the piecewise affine modeling formalism. Therefore, the goal will be to find a model of the sheet flow and to simulate it for both a simplex loop (for single-sided printing) and for a duplex loop (for double-sided printing).

The remainder of this report is organized as follows: in chapter 2, the case description and accompanying assumptions will be given, after which the piecewise affine formalism will be explained in chapter 3. In chapter 4, the models will be explained and chapter 5 discusses the implementation and simulation of the models that are designed for both a simplex and a duplex sheet flow. The final chapter contains the conclusions.
2. Case description

2.1 The printer paper path

The paper path in the printer that will be discussed in this report is schematically depicted in Figure 2.1. The sheets are entering the path via the PIM and are transported by pinches controlled by motors. After the first time printing at the FUSE, the sheet can go 2 ways: leaving the path by going to the FIN or re-entering the path for double-sided printing (duplex loop). The pinches are numbered from P0 for the first pinch till P12 for the last pinch. The motors are numbered from M1 for the first motor till M6 for the last motor.

![Figure 2.1: Schematic representation of the printer paper path](image)

As can be seen in Figure 2.1, a pinch is driven by a motor. The pinch consists of two parts: the driven roller and the non-driven roller. Figure 2.2 gives a more detailed schematic representation of this. The motor drives the driven roller via a rubber belt with a certain transmission ratio. The non-driven roller takes care of the prestress which causes the sheet not to slip between both parts of the pinch.
Figure 2.2: Schematic representation of a connection between motor and pinch
2.2 Initial assumptions
Before deriving the sheet flow model, first some assumptions regarding the sheet flow are presented:

- The sheet movement will only be affected by one pinch at a time. The sheet can be in two pinches, but when the leading edge of the sheet enters the next pinch, the previous pinch does not affect the movement of the sheet anymore.
- The position of the sheet, defined as the position of the leading edge of the sheet, is always known.
- There is no slip between the pinch and the sheet.
- There is an infinitely stiff connection between the motors and pinches.
- The sheet is assumed to be massless.

2.3 Describing relations
Given the assumptions, the sheet flow modeling can be carried out. First, the relations between a single motor, a pinch and a sheet are derived. The angular velocity of the pinch $\dot{\phi}_p$ is linearly dependent on the angular velocity of the motor $\dot{\phi}_m$. This dependency is described via a holonomic kinematic constraint relation. Furthermore, the velocity of the sheet $\dot{x}_s$ is calculated by multiplying the angular velocity of the pinch with the radius of the pinch $r_p$, i.e. the relation between the sheet velocity and the pinch velocity is also described using a holonomic kinematic constraint relation.

$$\dot{\phi}_p = n \cdot \dot{\phi}_m \quad (2.1)$$
$$\dot{x}_s = \dot{\phi}_p \cdot r_p \quad (2.2)$$

with $n$ the transmission ratio between motor and pinch.

Substitution of (2.1) into (2.2) yields a relation between the angular velocity of the motor and the velocity of the sheet:

$$\dot{x}_s = n \cdot r_p \cdot \dot{\phi}_m \quad (2.3)$$
3. The piecewise affine modeling formalism

Piecewise affine modeling is used for modeling systems with different dynamics in different parts of the state space. The system is divided in parts where linear equations hold. Piecewise affine systems can be modeled using the following equations:

\[
\begin{align*}
\dot{x} &= A_i x + B_i u + a_i \\
y &= C_i x + D_i u
\end{align*}
\]  

(3.1)

where \( x \) is the state of the model, \( u \) the model input and \( y \) the model output. Furthermore, \( A_i, B_i, C_i \) and \( D_i \) represent the system matrices, input matrices, output matrices and connection matrices respectively [4], whereas \( a_i \) represents the affine term.

An example of a piecewise affine system is a so-called bi-modal piecewise affine system [1], [2]:

\[
\begin{align*}
\dot{x} = \begin{cases} 
A_i x + B_i u + a_i & \text{for } H^T x < 0 \\
A_2 x + B_2 u + a_2 & \text{for } H^T x \geq 0
\end{cases}
\end{align*}
\]

(3.2)

The state in this example is divided into 2 linear regimes.

The basic idea of modeling a printer paper path using the piecewise affine formalism is to divide the entire paper path in small parts in which linear equations hold. Since the sheet movement is assumed to be linearly dependent on the rotational speed of a pinch and the rotational speed of a pinch is assumed to be linearly dependent on the rotational speed of the motor that is driving it (see Section 2.3) the sheet flow can be written in a piecewise affine form if the paper path is divided in different linear regimes where one pinch is controlling the sheet movement.
4. Model descriptions

4.1 Simplex loop

The first model that will be discussed is that of the sheet flow in the simplex loop [3]. This model is straightforward, because the sheet will go through pinches P0 until P7, ‘skips’ pinch P8 and goes through pinches P9 until P12 before it reaches the finisher. The angular velocity of the motors is taken as the input in the system and the transported distance of the sheet is taken as the output.

Combining formula 2.3 with the piecewise affine formalism yields the following model:

\[
\begin{align*}
\dot{x}_s &= \begin{cases} 
\frac{v_0}{n_4 \cdot \phi_{m4} \cdot r_{p5}} & \text{if } x_s \geq x_{p5} \wedge x_s < x_{p6} \\
\frac{n_4 \cdot \phi_{m4} \cdot r_{p5}}{r_{p5}} & \text{if } x_s \geq x_{p6} \wedge x_s < x_{p7} \\
\frac{n_5 \cdot \phi_{m5} \cdot r_{p6}}{r_{p6}} & \text{if } x_s \geq x_{p6} \wedge x_s < x_{p7} \\
\frac{n_5 \cdot \phi_{m5} \cdot r_{p7}}{r_{p7}} & \text{if } x_s \geq x_{p7} \wedge x_s < x_{p8} \\
\frac{n_6 \cdot \phi_{m6} \cdot r_{p9}}{r_{p9}} & \text{if } x_s \geq x_{p9} \wedge x_s < x_{p10} \\
\frac{n_6 \cdot \phi_{m6} \cdot r_{p10}}{r_{p10}} & \text{if } x_s \geq x_{p10} \wedge x_s < x_{p11} \\
\frac{n_6 \cdot \phi_{m6} \cdot r_{p11}}{r_{p11}} & \text{if } x_s \geq x_{p11} \wedge x_s < x_{p12} \\
\frac{n_6 \cdot \phi_{m6} \cdot r_{p12}}{r_{p12}} & \text{if } x_s \geq x_{p12} \wedge x_s < x_{p12} + L_s 
\end{cases}
\end{align*}
\]

with \(v_0\) representing the initial speed while the sheet is in the PIM, \(x_s\) is the transported distance of the sheet. Furthermore, \(x_{pj}\) and \(r_{pj}\) are the position and the radius of pinch \(Pj\), \(j \in \{0,1,\ldots,12\}\) respectively, and \(n_i\) and \(\phi_{mi}\) are the transmission ratio and the angular velocity of motor \(i\), \(i \in \{1,2,\ldots,6\}\) respectively. \(L_s\) is the length of the sheet.

4.2 Duplex loop: nominal sheet movement

Since the sheet will re-enter the first part of the paper path in the duplex loop, it is not sufficient to use only one state in the model. An extra state has to be introduced that will determine if the sheet passes through a point for the 1st or 2nd time. First, the case of nominal sheet movement will be discussed. Nominal sheet movement is the movement according to the schedule of the sheet flow.

Before modeling the duplex loop some more assumptions have to be made:
• The sheet is assumed to be a rigid body. As a result the sheet does not blouse nor stretch. So we can use $x_s - L_s$ as a definition of the position of the trailing edge of the sheet.

• The sheet never enters pinch P2 for the 2nd time after it has left the duplex loop.

• The sheets that are used in this case are sheets with A4 paper format. For sheets with a larger size, the model needs to be slightly adjusted.
The model for this case will be as follows:

\[
\begin{align*}
\dot{x}_s &= \begin{cases} 
0 & \text{if } x_s < x_{p0} \\
1 & \text{in PIM (regime 1)} \\
n_1 \cdot \phi_{m1} \cdot r_{p0} & \text{if } x_s \geq x_{p0} \land x_s < x_{p1} \\
in pinch P0 (regime 2) \\
n_1 \cdot \phi_{m1} \cdot r_{p1} & \text{if } x_s \geq x_{p1} \land x_s < x_{p2} \\
in pinch P1 (regime 3) \\
n_1 \cdot \phi_{m1} \cdot r_{p2} & \text{if } x_s \geq x_{p2} \land x_s < x_{p3} \\
in pinch P2 (regime 4) \\
n_2 \cdot \phi_{m2} \cdot r_{p3} & \text{if } x_s \geq x_{p3} \land x_s < x_{p4} \\
in pinch P3 (regime 5) \\
n_3 \cdot \phi_{m3} \cdot r_{p4} & \text{if } x_s \geq x_{p4} \land x_s < x_{p5} \\
in pinch P4 (regime 6) \\
n_4 \cdot \phi_{m4} \cdot r_{p5} & \text{if } x_s \geq x_{p5} \land x_s < x_{p6} \\
in pinch P5 (regime 7) \\
n_5 \cdot \phi_{m5} \cdot r_{p6} & \text{if } x_s \geq x_{p6} \land x_s < x_{p7} \\
in pinch P6 (regime 8) \\
n_5 \cdot \phi_{m5} \cdot r_{p7} & \text{if } x_s \geq x_{p7} \land x_s < x_{p8} \\
in pinch P7 (regime 9) \\
n_5 \cdot \phi_{m5} \cdot r_{p8} & \text{if } x_s \geq x_{p8} \land x_s < L \\
in pinch P8 (regime 10) \\
n_2 \cdot \phi_{m2} \cdot r_{p3} & \text{if } x_s \geq L \land x_s - L_s < L - (x_{stop} - x_{p3}) \\
in pinch P3 (regime 11) \\
n_3 \cdot \phi_{m3} \cdot r_{p4} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p4} - x_{stop}) \\
in pinch P4 (regime 13) \\
n_3 \cdot \phi_{m3} \cdot r_{p4} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p5} - x_{stop}) \\
in pinch P5 (regime 14) \\
n_4 \cdot \phi_{m4} \cdot r_{p5} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p6} - x_{stop}) \\
in pinch P6 (regime 15) \\
n_5 \cdot \phi_{m5} \cdot r_{p6} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p7} - x_{stop}) \\
in pinch P7 (regime 16) \\
n_5 \cdot \phi_{m5} \cdot r_{p7} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p9} - x_{stop}) \\
in pinch P9 (regime 17) \\
n_6 \cdot \phi_{m6} \cdot r_{p9} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p10} - x_{stop}) \\
in pinch P10 (regime 18) \\
n_6 \cdot \phi_{m6} \cdot r_{p10} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p11} - x_{stop}) \\
in pinch P11 (regime 19) \\
n_6 \cdot \phi_{m6} \cdot r_{p11} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p12} - x_{stop}) \\
in pinch P12 (regime 20) \\
n_6 \cdot \phi_{m6} \cdot r_{p12} & \text{if } x_s - L_s \geq L - (x_{stop} - x_{p3}) \land x_s < L_s - (x_{stop} - x_{p3}) + (x_{p12} - x_{stop}) + L_s
\end{cases}
\end{align*}
\]

with \( L \) representing the distance from the PIM to pinch P3 via pinches P0-P8 and \( x_{stop} \) the position where the trailing edge of the sheet stops before the sheet starts moving towards the FUSE pinch again.

In the 11th regime, a minus sign is used. This is because the sheet is oriented differently than the first time it passes this point. As a result, the transported distance
will continue to increase, even when the angular velocity of the motor becomes negative.

4.3 Duplex loop: tracking error of the motors

It is possible that the trailing edge of the sheet does not stop exactly at $x_{stop}$ due to a tracking error of the motors. The model in Section 4.2 needs to be adjusted for this case, since the actual transported distance is not equal to the nominal desired distance anymore. Since the sheet can stop anywhere (randomly), the use of only one variable (the transported distance) in the switch conditions is not enough anymore for describing the sheet flow behavior. In Sections 4.3.1 and 4.3.2, two options will be discussed that solve these problems.

4.3.1 Option 1

The first option is to augment the state space by two variables: the angular position of motor two, $\phi_{M2}$, and a variable $d$. This variable $d$ is defined as the transported distance, starting when the trailing edge passes point $S$ (Figure 2.1) when it enters the duplex loop and stops when the trailing edge passes point $S$ on its way to pinch P4. The angular position of motor M2 itself is not very important in the model. What matters is the derivative of the angular position (angular velocity) of motor M2 which can be found in the model in a state-space notation. The angular velocity as well as variable $d$ will be used in the switch conditions.

An additional assumption is the following:
When the trailing edge of the sheet goes past $S$, the stiffness of the sheet will cause the sheet to flip down so the sheet can’t go back to pinch P8 anymore.
4.3.1.1 Mathematical model:
The first part is the same as in the previous model when it comes to the transported distance. Although the state has three variables now:

\[
\begin{bmatrix}
V_0 \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s < x_{p0} \) in pinch PIM (regime 1)

\[
\begin{bmatrix}
n_1 \cdot \dot{\phi}_{m1} \cdot r_{p0} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p0} \land x_s < x_{p1} \) in pinch P0 (regime 2)

\[
\begin{bmatrix}
n_1 \cdot \dot{\phi}_{m1} \cdot r_{p1} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p1} \land x_s < x_{p2} \) in pinch P1 (regime 3)

\[
\begin{bmatrix}
n_1 \cdot \dot{\phi}_{m1} \cdot r_{p2} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p2} \land x_s < x_{p3} \) in pinch P2 (regime 4)

\[
\begin{bmatrix}
n_2 \cdot \dot{\phi}_{m2} \cdot r_{p3} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p3} \land x_s < x_{p4} \) in pinch P3 (regime 5)

\[
\begin{bmatrix}
n_3 \cdot \dot{\phi}_{m3} \cdot r_{p4} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p4} \land x_s < x_{p5} \) in pinch P4 (regime 6)

\[
\begin{bmatrix}
n_4 \cdot \dot{\phi}_{m4} \cdot r_{p5} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p5} \land x_s < x_{p6} \) in pinch P5 (regime 7)

\[
\begin{bmatrix}
n_5 \cdot \dot{\phi}_{m5} \cdot r_{p6} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p6} \land x_s < x_{p7} \) in pinch P6 (regime 8)

\[
\begin{bmatrix}
n_6 \cdot \dot{\phi}_{m5} \cdot r_{p7} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p7} \land x_s < x_{p8} \) in pinch P7 (regime 9)

\[
\begin{bmatrix}
n_7 \cdot \dot{\phi}_{m5} \cdot r_{p8} \\
\dot{\phi}_{m2} \\
0
\end{bmatrix}
\]

if \( x_s \geq x_{p8} \land x_s < L \) in pinch P8 (regime 10)
However, when the sheet reaches pinch P3 for the second time we introduce a mechanism which holds a possible error (overshoot) into account:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} -n_2 \cdot \dot{\phi}_{m2} \cdot r_{p3} \\ \dot{\phi}_{m2} \\ 0 \end{bmatrix} \\
\dot{d} &= \begin{bmatrix} -n_2 \cdot \dot{\phi}_{m2} \cdot r_{p3} \\ \dot{\phi}_{m2} \\ 0 \end{bmatrix} \\
\end{align*}
\]

if \( \dot{\phi}_{m2} < 0 \wedge \)

in pinch P3 (regime 11)

\[
\begin{align*}
x_s &\geq L \\
x_s - L_s < S
\end{align*}
\]

if \( \dot{\phi}_{m2} < 0 \wedge \)

in pinch P3 (regime 12)

\[
\begin{align*}
x_s - L_s = S
\end{align*}
\]

if \( \dot{\phi}_{m2} \geq 0 \wedge \)

in pinch P3 (regime 13)

\[
\begin{align*}
x_s - L_s \geq S \\
x_s - L_s < S + l \wedge \\
d \leq 0
\end{align*}
\]

if \( \dot{\phi}_{m2} \geq 0 \wedge \)

in pinch P3 (regime 14)

\[
\begin{align*}
x_s - L_s > S \\
x_s - L_s < S + l
\end{align*}
\]

if \( \dot{\phi}_{m2} < 0 \wedge \)

in pinch P3 (regime 15)

The stop position \( x_{stop} \) is not used here anymore. However, this model also takes the case of stopping at \( x_{stop} \) into account (see below). So an advantage of this model is that the stopping position can vary.

We have to check that this model holds for a few possible/realistic situations:

1. The sheet stops before the trailing edge reaches S and goes back to pinch P8:
   - The sheet remains in the 11th regime until the trailing edge goes past S, so \( x_s \) decreases again till the sheet enters the 10th regime again.

2. The trailing edge of the sheet stops exactly on S and after that it moves towards pinch P4:
   - The sheet goes through the 11th and 14th regime.

3. The trailing edge of the sheet stops past S and after that it moves towards pinch P4:
- The sheet goes through the 11th, 12th, 13th and 14th regime.

Besides the normal stops, it is also possible that the sheet will not stop exactly at a certain position, but it starts to vibrate around that position. This can be caused by a lack of sufficient damping in the controllers. In this case, there can be four more situations:

4. The sheet will vibrate before the trailing edge reaches S:
   - The sheet remains in the 11th regime until the trailing edge goes past S.

5. The sheet will vibrate around S:
   - This situation is similar to the 2nd situation. However, after regime 14 it will reach regime 15 instead of entering pinch P4. After that the sheet will enter the 12th regime again and so on. The number of cycles is determined by the amount of vibrations the sheet makes. The order of regimes in case of one vibration is: regime 11 - regime 12 - regime 13 - regime 14 - regime 15 - regime 12 - regime 13 - regime 14.

6. The sheet will vibrate on the right side of S:
   - This situation is similar to the 3rd situation. Only this time it will switch a few times between regime 12 and 13 before it reaches regime 14. In case of one vibration, the order of regimes becomes: regime 11 - regime 12 - regime 13 - regime 12 - regime 13 - regime 14.

7. The sheet will vibrate on the left side of S:
   - After regime 5, the sheet will enter regime 4 again. The order of regimes in case of one vibration becomes: regime 11 - regime 12 - regime 13 - regime 14 - regime 15 - regime 14.

The model described in this section covers all these seven situations.
The last part of the model becomes:

\[
\begin{bmatrix}
\dot{n}_3 \cdot \Phi_{m3} \cdot r_{p4} \\
\dot{n}_4 \cdot \Phi_{m4} \cdot r_{p5} \\
\dot{n}_5 \cdot \Phi_{m5} \cdot r_{p6} \\
\dot{n}_6 \cdot \Phi_{m6} \cdot r_{p7} \\
\dot{n}_7 \cdot \Phi_{m7} \cdot r_{p8} \\
\dot{n}_8 \cdot \Phi_{m8} \cdot r_{p9} \\
\dot{n}_9 \cdot \Phi_{m9} \cdot r_{p10} \\
\dot{n}_10 \cdot \Phi_{m10} \cdot r_{p11} \\
\dot{n}_11 \cdot \Phi_{m11} \cdot r_{p12}
\end{bmatrix}
\begin{bmatrix}
\phi_{m2} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P4 (regime 16)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P5 (regime 17)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P6 (regime 18)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P7 (regime 19)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P9 (regime 20)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P10 (regime 21)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P11 (regime 22)} \\
x_s - L_s < S + l + (x_{p4} - x_{p4}) \quad \text{in pinch P12 (regime 23)}
\end{bmatrix} \]
with \( x \) being the transported distance with a small area where it does not increase or decrease. \( l \) is the distance from point S to pinch P4, \( S \) is the distance from the PIM to the spot where the sheet enters the path between pinch P3 and P4 for the 2\(^{nd} \) time via pinches P0-P8. Finally, \( d \) is the transported distance that only increases and decreases in the area where \( x \) does not change.

### 4.3.1.2 Global representation of the variables

Figure 4.1 sketches a possible sheet flow for the model described in Section 4.3.1.1. Here, the magnitude of the variables is plotted as a function of time.

---

**Figure 4.1: Variables as function of time**
4.3.2 Option 2
The 2\sup{nd} option has only two states. The 2\sup{nd} state in this model is that variable $q$. This variable will be equal to 0 when the sheet is in the first loop and unequal to 0 when the sheet enters the duplex loop.

4.3.2.1 Mathematical model:
The first part is pretty obvious when starting from the first part of the model in Section 4.3.1.1:

\[
\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{q} \end{bmatrix} = \begin{bmatrix} v_0 \\ 0 \end{bmatrix} \quad \text{if } x_s < x_{p_0} \land q = 0 \quad \text{in PIM (regime 1)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_0} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_0} \land x_s < x_{p_1} \land q = 0 \quad \text{in pinch P0 (regime 2)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_1} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_1} \land x_s < x_{p_2} \land q = 0 \quad \text{in pinch P1 (regime 3)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_2} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_2} \land x_s < x_{p_3} \land q = 0 \quad \text{in pinch P2 (regime 4)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_3} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_3} \land x_s < x_{p_4} \land q = 0 \quad \text{in pinch P3 (regime 5)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_4} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_4} \land x_s < x_{p_5} \land q = 0 \quad \text{in pinch P4 (regime 6)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_5} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_5} \land x_s < x_{p_6} \land q = 0 \quad \text{in pinch P5 (regime 7)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_6} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_6} \land x_s < x_{p_7} \land q = 0 \quad \text{in pinch P6 (regime 8)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_7} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_7} \land x_s < x_{p_8} \land q = 0 \quad \text{in pinch P7 (regime 9)}
\]

\[
\begin{bmatrix} n_1 \cdot \hat{\phi}_m \cdot r_{p_8} \\ 0 \end{bmatrix} \quad \text{if } x_s \geq x_{p_8} \land x_s < L \land q = 0 \quad \text{in pinch P8 (regime 10)}
\]
The part in which pinch P3 is controlling the movement of the sheet becomes:

\[
\dot{x} = \begin{bmatrix}
-x_i \cdot \phi \cdot r_p \\
0 \\
1
\end{bmatrix}
\begin{cases}
-x_i \cdot \phi \cdot r_p & \text{if } x_i \geq L \land x_i - L_s < S \land q = 0 \\
0 & \text{in pinch P3 (regime 11)} \\
-x_i \cdot \phi \cdot r_p & \text{if } x_i - L_s \geq S \land q \geq 0 \\
1 & \text{in pinch P3 (regime 12)} \\
-x_i \cdot \phi \cdot r_p & \text{if } x_i - L_s < S \land x_i - L_s \geq S - l \land q > 0 \\
1 & \text{in pinch P3 (regime 13)}
\end{cases}
\]

We have to check again that this model holds for the seven situations:

1. The sheet stops before the trailing edge reaches S and goes back to pinch P8:
   - The sheet remains in the 11th regime until the trailing edge goes past S, so \( x_i \) decreases again till the sheet enters the 10th regime again.

2. The trailing edge of the sheet stops exactly on S and after that it moves towards pinch P4:
   - The sheet goes through the 11th, 12th and 13th regime. The sheet will stay in the 12th regime for an infinite small amount of time before it can reach the 13th regime.

3. The trailing edge of the sheet stops past S and after that it moves towards pinch P4:
   - The sheet goes through the 11th, 12th and 13th regime.

The situations where vibration occurs:

4. The sheet will vibrate before the trailing edge reaches S:
   - The sheet remains in the 11th regime until the trailing edge goes past S.

5. The sheet will vibrate around S:
   - After leaving the 11th regime, the sheet will switch between the 12th and 13th regime, depending on the amount of vibrations the sheet makes. The order of regimes in case of one vibration is: regime 11 - regime 12 - regime 13 - regime 12 - regime 13.

6. The sheet will vibrate on the right side of S:
   - The sheet will stay in the 12th regime till it passes position S on its way to pinch P4. In case of one vibration, the order of regime becomes: regime 11 - regime 12 - regime 13.

7. The sheet will vibrate on the left side of S:
   - The sheet will stay in the 13th regime till it reaches pinch P4. In case of one vibration, the order of regime becomes: regime 11 - regime 12 - regime 13.

This model also covers all these seven situations.
The last part of the model becomes:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} = \\
&= \begin{bmatrix}
-\frac{\dot{n}_3 \cdot \Phi_{m3} \cdot r_{p4}}{1} \\
-\frac{\dot{n}_4 \cdot \Phi_{m4} \cdot r_{p5}}{1} \\
-\frac{\dot{n}_5 \cdot \Phi_{m5} \cdot r_{p6}}{1} \\
-\frac{\dot{n}_6 \cdot \Phi_{m5} \cdot r_{p7}}{1} \\
-\frac{\dot{n}_6 \cdot \Phi_{m6} \cdot r_{p9}}{1} \\
-\frac{\dot{n}_6 \cdot \Phi_{m6} \cdot r_{p10}}{1} \\
-\frac{\dot{n}_6 \cdot \Phi_{m6} \cdot r_{p11}}{1} \\
-\frac{\dot{n}_6 \cdot \Phi_{m6} \cdot r_{p12}}{1}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\text{if } x_s - L_s < S - l \land \\
x_s - L_s \geq S - l - (x_{p5} - x_{p4}) \land \\
q > 0 & \text{ in pinch P4 (regime 14)} \\
\text{if } x_s - L_s < S - l - (x_{p5} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p6} - x_{p4}) \land \\
q > 0 & \text{ in pinch P5 (regime 15)} \\
\text{if } x_s - L_s < S - l - (x_{p6} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p7} - x_{p4}) \land \\
q > 0 & \text{ in pinch P6 (regime 16)} \\
\text{if } x_s - L_s < S - l - (x_{p7} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p9} - x_{p4}) \land \\
q > 0 & \text{ in pinch P7 (regime 17)} \\
\text{if } x_s - L_s < S - l - (x_{p9} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p10} - x_{p4}) \land \\
q > 0 & \text{ in pinch P9 (regime 18)} \\
\text{if } x_s - L_s < S - l - (x_{p10} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p11} - x_{p4}) \land \\
q > 0 & \text{ in pinch P10 (regime 19)} \\
\text{if } x_s - L_s < S - l - (x_{p11} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p12} - x_{p4}) \land \\
q > 0 & \text{ in pinch P11 (regime 20)} \\
\text{if } x_s - L_s < S - l - (x_{p12} - x_{p4}) \land \\
x_s - L_s \geq S - l - (x_{p12} - x_{p4}) - L_s \land \\
q > 0 & \text{ in pinch P12 (regime 21)}
\end{align*}
\]
with \( x \) being the transported distance in the 1\(^{st} \) loop and minus the transported distance in the duplex loop. The variable \( q \) determines if the sheet is in the 1\(^{st} \) loop or in the duplex loop.

### 4.3.2.2 Global representation of the variables

Figure 4.2 sketches a possible sheet flow for the model described in Section 4.3.2.1. Here, the magnitude of the variables is plotted as a function of time.

![Figure 4.2: Variables as function of time](image)

### 4.3.3 Comparison

- The state space in option 1 has three variables, whereas option 2 has only two variables. Option 2 reduces the complexity of the model.
- The switching conditions are not only based on the states in option 1. An input variable, which is also a time derivative of one of the states, is used in the switching conditions of option 1, which is not common in piecewise affine models. Whether or not this is allowed in a piecewise affine formalism is not sure.
- In the switching conditions of option 1, an ‘equal to-sign’ is used in regime 12. In theory this is possible, but due to the finite precision of computers, this condition will never be met.
- The 1\(^{st} \) option is easier to expand to a so-called triplex loop, since in option 2 you have to introduce an extra variable, which is not needed in the 1\(^{st} \) option.
• The way $x_i$ varies in time in option 1 is closer to what is expected, since in option 2, $x_i$ decreases again after a certain amount of time.

Option 1 has two advantages over option 2. But because modeling the sheet flow for a triplex loop was not one of the goals in this report, this leaves only one advantage. This advantage is only based on an expectation and has no real mathematical background. The conclusion is that option 2 is the best option in this case and will therefore be used in the remaining topics in this report.

### 4.4 State-space form

#### 4.4.1 Simplex loop

The model described in Section 4.1 leads to the following state-space model:

$$
\begin{align*}
\dot{x}_i &= A_i x_i + B_i u, \quad \text{for } x_i \in \mathcal{X}_i, \ i = 0,1,\ldots,12 \\
y &= C_i x_i + D_i u
\end{align*}
$$

with:

- $\mathcal{X}_0 = \{x_i \mid x_i \in [0, x_{p0})\}$
- $\mathcal{X}_1 = \{x_i \mid x_i \in [x_{p0}, x_{p1})\}$
- $\mathcal{X}_2 = \{x_i \mid x_i \in [x_{p1}, x_{p2})\}$
- $\mathcal{X}_3 = \{x_i \mid x_i \in [x_{p2}, x_{p3})\}$
- $\mathcal{X}_4 = \{x_i \mid x_i \in [x_{p3}, x_{p4})\}$
- $\mathcal{X}_5 = \{x_i \mid x_i \in [x_{p4}, x_{p5})\}$
- $\mathcal{X}_6 = \{x_i \mid x_i \in [x_{p5}, x_{p6})\}$
- $\mathcal{X}_7 = \{x_i \mid x_i \in [x_{p6}, x_{p7})\}$
- $\mathcal{X}_8 = \{x_i \mid x_i \in [x_{p7}, x_{p8})\}$
- $\mathcal{X}_9 = \{x_i \mid x_i \in [x_{p8}, x_{p9})\}$
- $\mathcal{X}_{10} = \{x_i \mid x_i \in [x_{p9}, x_{p10})\}$
- $\mathcal{X}_{11} = \{x_i \mid x_i \in [x_{p10}, x_{p11})\}$
- $\mathcal{X}_{12} = \{x_i \mid x_i \in [x_{p11}, x_{p12} + L]\}$
and:

$$A_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (0 \leq i \leq 12)$$

$$B_i = \begin{bmatrix} v_0 & 0 & 0 & 0 & 0 \\ n_i r_{p_0} & 0 & 0 & 0 & 0 \end{bmatrix} \quad \quad B_i = \begin{bmatrix} n_i r_{p_0} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} n_i r_{p_1} & 0 & 0 & 0 & 0 \\ 0 & n_2 r_{p_3} & 0 & 0 & 0 \end{bmatrix} \quad \quad B_i = \begin{bmatrix} n_i r_{p_2} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & n_2 r_{p_3} & 0 & 0 & 0 \\ 0 & n_3 r_{p_5} & 0 & 0 \end{bmatrix} \quad \quad B_i = \begin{bmatrix} 0 & n_3 r_{p_4} & 0 & 0 & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 & n_4 r_{p_5} & 0 & 0 \\ 0 & 0 & n_5 r_{p_7} & 0 \end{bmatrix} \quad \quad B_i = \begin{bmatrix} 0 & n_4 r_{p_6} & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 & 0 & n_5 r_{p_7} & 0 \\ 0 & 0 & 0 & n_6 r_{p_{10}} \end{bmatrix} \quad \quad B_i = \begin{bmatrix} 0 & 0 & 0 & n_6 r_{p_9} \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 & 0 & 0 & n_6 r_{p_{10}} \end{bmatrix} \quad \quad B_i = \begin{bmatrix} 0 & 0 & 0 & n_6 r_{p_{11}} \end{bmatrix}$$

$$C_i = [1] \quad (0 \leq i \leq 12)$$

$$D_i = [0 \ 0 \ 0 \ 0 \ 0] \quad (0 \leq i \leq 12)$$

### 4.4.2 Duplex loop

For the duplex loop, the model described in Section 4.3.2.1 will be used since this is the best option. This option leads to the following state-space model:
\[
\begin{align*}
\dot{x} &= A x + B u + a_i \\
y &= C x + D u
\end{align*}
\]
for \( x = \begin{bmatrix} x_i \\ q \end{bmatrix} \in \mathcal{X}_i, \ i = 0,1,\ldots,20
\]
with:
\[
\begin{align*}
\mathcal{X}_0 &= \{ x_i \mid x_i \in [0, x_{p0}), q \mid q = 0 \} \\
\mathcal{X}_1 &= \{ x_i \mid x_i \in [x_{p0}, x_{p1}), q \mid q = 0 \} \\
\mathcal{X}_2 &= \{ x_i \mid x_i \in [x_{p1}, x_{p2}), q \mid q = 0 \} \\
\mathcal{X}_3 &= \{ x_i \mid x_i \in [x_{p2}, x_{p3}), q \mid q = 0 \} \\
\mathcal{X}_4 &= \{ x_i \mid x_i \in [x_{p3}, x_{p4}), q \mid q = 0 \} \\
\mathcal{X}_5 &= \{ x_i \mid x_i \in [x_{p4}, x_{p5}), q \mid q = 0 \} \\
\mathcal{X}_6 &= \{ x_i \mid x_i \in [x_{p5}, x_{p6}), q \mid q = 0 \} \\
\mathcal{X}_7 &= \{ x_i \mid x_i \in [x_{p6}, x_{p7}), q \mid q = 0 \} \\
\mathcal{X}_8 &= \{ x_i \mid x_i \in [x_{p7}, x_{p8}), q \mid q = 0 \} \\
\mathcal{X}_9 &= \{ x_i \mid x_i \in [x_{p8}, L), q \mid q = 0 \} \\
\mathcal{X}_{10} &= \{ x_i \mid x_i \in [L, S + L_s), q \mid q = 0 \} \\
\mathcal{X}_{11} &= \{ x_i \mid x_i \in [S + L_s, \infty), q \mid q \in [0, \infty) \} \\
\mathcal{X}_{12} &= \{ x_i \mid x_i \in [S - l + L_s, S + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{13} &= \{ x_i \mid x_i \in [S - l - (x_{p5} - x_{p4}) + L_s, S - l + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{14} &= \{ x_i \mid x_i \in [S - l - (x_{p6} - x_{p4}) + L_s, S - l - (x_{p5} - x_{p4}) + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{15} &= \{ x_i \mid x_i \in [S - l - (x_{p7} - x_{p4}) + L_s, S - l - (x_{p6} - x_{p4}) + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{16} &= \{ x_i \mid x_i \in [S - l - (x_{p9} - x_{p4}) + L_s, S - l - (x_{p7} - x_{p4}) + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{17} &= \{ x_i \mid x_i \in [S - l - (x_{p10} - x_{p4}) + L_s, S - l - (x_{p9} - x_{p4}) + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{18} &= \{ x_i \mid x_i \in [S - l - (x_{p11} - x_{p4}) + L_s, S - l - (x_{p10} - x_{p4}) + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{19} &= \{ x_i \mid x_i \in [S - l - (x_{p12} - x_{p4}) + L_s, S - l - (x_{p11} - x_{p4}) + L_s), q \mid q \in (0, \infty) \} \\
\mathcal{X}_{20} &= \{ x_i \mid x_i \in [S - l - (x_{p13} - x_{p4}), S - l - (x_{p12} - x_{p4}) + L_s), q \mid q \in (0, \infty) \}
\]
and:

\[
A_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (0 \leq i \leq 20)
\]

\[
B_0 = \begin{bmatrix} v_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_2 = \begin{bmatrix} n_i r_{p1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_4 = \begin{bmatrix} 0 & n_2 r_{p3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_6 = \begin{bmatrix} 0 & 0 & n_4 r_{p5} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_8 = \begin{bmatrix} 0 & 0 & 0 & n_5 r_{p7} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_{10} = \begin{bmatrix} 0 & -n_2 r_{p3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_{12} = \begin{bmatrix} 0 & -n_2 r_{p3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_{14} = \begin{bmatrix} 0 & 0 & 0 & -n_4 r_{p5} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_{16} = \begin{bmatrix} 0 & 0 & 0 & -n_4 r_{p5} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_{18} = \begin{bmatrix} 0 & 0 & 0 & -n_6 r_{p10} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
B_{20} = \begin{bmatrix} 0 & 0 & 0 & -n_6 r_{p12} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
C_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (0 \leq i \leq 20)
\]

\[
D_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (0 \leq i \leq 20)
\]

\[
a = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (0 \leq i \leq 11)
\]

\[
a = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (12 \leq i \leq 20)
\]
5. Simulations and results

Once the models have been made, they can be used to simulate a possible sheet flow. The models in Sections 4.1 and 4.3.2.1 will be simulated for respectively the simplex loop and the duplex loop. This will be done using Matlab/Simulink. The model can be found on the disk attached to this report.

5.1 M-file

To reduce the complexity of the simulation, a matlab-file is also included on the disk (filename: duplexwaarden.m). It asks you whether the standard parameters (angular velocity of the motors, transmission ratios and radii of the pinches) can be used. If this is not the case, you have to enter these values manually. Once finished, the m-file will run the simulink models (filenames: singlemodel.mdl and duplex.mdl) and writes the data to a file. For the simplex loop, this file is named xt.mat and for the duplex loop, this file is named xt1.mat for the transported distance and xt2.mat for the value of $q$.

5.2 Simulink model for the simplex loop

The inputs in the systems are the angular velocities of the motors. These are represented by the symbols $w_1$ for motor M1 until $w_6$ for motor M6. These are multiplied by the corresponding transmission ratio, represented by $n_1$ for motor M1 until $n_6$ for motor M6. The output of this multiplication is then multiplied with the radius of the corresponding pinch, represented by $r_0$ for pinch P0 until $r_{12}$ for pinch P12. This is shown in figure 5.1 for the case of motor five with the pinches it is driving.

![Motor with transmission ratio and 3 pinches](image)

*Figure 5.1: Motor with transmission ratio and 3 pinches*

The output of this subsystem is the sheet velocity. Combined with an integrator, the output can be changed into the transported distance. For the switching conditions, a switch in simulink is used. These switches have three inputs, numbered from top to bottom. There are two inputs (1 and 3) that can pass through to the output and one input (2) that decides which one of these two actually passes through to the output. The $2^{nd}$ input is compared with a certain entered value. Once the $2^{nd}$ input becomes greater than the entered reference value, the switch will switch from the $3^{rd}$ input to
the 1\textsuperscript{st} input. This way, we can switch between two pinches when the reference is set to the pinch position and the 2\textsuperscript{nd} input is the transported distance.

5.3 Simulink model for the duplex loop

For the duplex loop, the same principles can be used as in the simplex loop. However, the duplex loop has some additional features:

After the sheet enters pinch P3 for the 2\textsuperscript{nd} time, a minus sign is used (see the mathematical model). For the simulation model, a gain of minus 1 is used for this.

The mechanism that determines the value of the variable $q$ is shown in figure 5.2.

![Figure 5.2: Determination of the value of variable $q$](image)

The input in this subsystem is the transported distance and the output is the value of $q$. The switch determines if the sheet passes point S. Once past point S, the derivative of $q$ will become equal to one. Combined with the integrator, this subsystem has output $q$.

Instead of using a constant for motor M2, a signal generator has been used in order to simulate a vibration of the sheet. Figure 5.3 shows a possible way how motor M2 changes its angular velocity.

![Figure 5.3: Signal generator](image)

In this case, motor M2 changes its velocity between 7 and 9.5 seconds (of simulation time) from -10 rad/s to +10 rad/s.
The entire model is divided in two parts: one for the first loop and one for the second loop. Once the variable $q$ starts to increase, the model switches to the part of the second loop. Since motor M2 is active in both parts, the signal generator, representing motor M2, is connected to both parts of the model.

Figure 5.4 and 5.5 show possible sheet flows for a simplex and duplex loop respectively.

Figure 5.4: Possible sheet flow for the simplex loop

Figure 5.5: Possible sheet flow for the duplex loop
6. Conclusions

The main goal in this report was the modeling of the sheet flow behavior in a paper path of a printer using the piecewise affine modeling formalism. Besides modeling the sheet flow for one-sided printing (simplex loop), also the sheet flow for two-sided printing (duplex loop) has been modeled. To realize this goal, the linear equations of motion were derived which described the relation between the angular velocity of the motors and the velocity of the paper. These equations alone were not enough to set up a model for the sheet flow. Therefore, some assumptions were made:

- The sheet movement will only be affected by one pinch at the time. The sheet can be in two pinches, but when the leading edge of the sheet enters the next pinch, the previous pinch does not affect the movement of the sheet anymore.
- The position of the sheet, defined as the position of the leading edge of the sheet is always known.
- There is no slip between the pinch and the sheet.
- There is an infinitely stiff connection between the motors and pinches.
- The sheet is assumed to be massless.

Once these steps were finished, a model of the simplex loop was made. After that, the duplex loop was modeled via an iterative way, each model with its own additional assumptions:

- The sheet is assumed to be a rigid body. As a result the sheet does not blouse nor stretch. So we can use $x_s - L_s$ as a definition of the position of the trailing edge of the sheet.
- The sheet never enters pinch P2 for the 2nd time after it has left the duplex loop.
- The sheets that are used in this case are sheets with A4 paper format. For sheets with a larger size, the model needs to be slightly adjusted.
- The model described below is only for the duplex loop, i.e. the sheet will always be printed on both sides. For the simplex loop, the model in Section 4.1 can be used.
- When the trailing edge of the sheet goes past S, the stiffness of the sheet will cause the sheet to flip down so the sheet can’t go back to pinch P8 anymore.

Finally, the model described in Section 4.3.2.1 was used to perform a simulation in Matlab/Simulink. The output of this simulation matched our expectations, based on the mathematical model.
References


