Geared neutral transmissions

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Published: 01/01/2002
Geared Neutral Transmissions

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Report DCT 2002.34
Geared Neutral Transmissions
driving off without clutching

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May 21, 2002
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Introduction

A geared neutral transmission is a special solution for building a geared neutral driveline in a vehicle. With this driveline a vehicle is able to drive off without coupling or decoupling a clutch. In a standard driveline, this is not possible.

Over the last decades many researchers have tried to develop a layout for building a geared neutral transmission. So far, no transmission that is developed for the automotive industry has made it to mass production. At this moment Torotrak inc. claims to have a geared neutral transmission that works properly. The key in this gearbox is the toroid variator that Torotrak developed for torque control.

This report, that is based on literature studies, discusses the advantages of geared neutral drivelines and the different options to built such a driveline. Afterwards, it focuses on geared neutral transmissions, for which only one basic layout (also used by Torotrak) is feasible. The analysis of both static and dynamic analysis of the Torotrak transmission are used as a basis to identify the feasibility and controllability of a geared neutral transmission with a pushbelt variator. This variator is developed by Van Doorne's Transmissie (VDT), on whose behalf this research has been performed. Constrains are given for the control of such a variator when it is used in a geared neutral driveline.
Chapter 1

Outline and solution algorithm

The purpose of this research is to compose a list of conditions, necessary for the implementation of geared neutral transmissions with pushbelt variator in a vehicle’s drive line. In order to compose this list of constraints, a literature study is combined with both static and dynamic analysis of two types of geared neutral transmissions. The first part of this report focuses merely on the literature that is available on geared neutral drivelines and geared neutral transmissions respectively. The following structure is used:

1. Geared neutral drivelines (chapter 2).
2. Geared neutral transmissions (chapter 2).
3. The design and behaviour of components in a geared neutral transmission (chapter 3).
4. Kinematics of variators in general and in a geared neutral transmission (chapter 4).

The second part discusses the feasibility and constraints for control of the geared neutral transmission with pushbelt variator. This is done by analyzing the dynamics of two geared neutral transmissions for their mechanical behaviour. The first transmission is developed by Torotrak Inc. and includes a toroid variator. The second is merely the same, but now a pushbelt variator is used. The second part has the following layout:

1. Analyzing methods and modeling (chapter 5)
2. Dynamic analysis of Torotrak transmission with toroid variator (chapter 6)
3. Static and dynamic analysis of transmission with pushbelt variator (chapter 7)
4. Conclusions and recommendations according to feasibility and constraints for the control of a geared neutral transmission with pushbelt variator (chapter 8)

Note that all results are based on theoretical analysis and numerical simulations. No practical tests have been performed on real build transmissions. However, test results of other practical researches are used to improve the theoretical models.
Chapter 2

Literature: geared neutral

2.1 Vehicle drivelines

A geared neutral transmission as discussed in this report is a sub part of a vehicle driveline. Figure 2.1 shows such a driveline. It's major parts are:

- The drive unit
  transposes gasoline or electricity into mechanical power

- The transmission
  transmits this power, while changing the ratio between torque and speed.

- Differential gear
  a component that distributes the power to the wheels. This part is not discussed here.

![Fig. 2.1: Traditional driveline](image)

2.2 Geared neutral drivelines

A geared neutral driveline has the same main components as shown in fig. 2.1. In order to get a specific behaviour, the layout of these components differs from a traditional driveline (fig. 2.2). This behaviour leads to the following definition:

Definition geared neutral driveline:
Driveline that is fully engaged, also when the vehicle has zero speed \( u_{veh} = \)
Without changing this kinematic connection between the drive unit and the wheels, the vehicle is able to start moving.

The definition for a geared neutral driveline directly implies a mechanical boundary condition: A geared neutral driveline has the ability to generate torque $T \neq 0$ at vehicle speed $\omega_{\text{wheels}} = 0$.

This boundary condition is not as obvious as it seems: A traditional driveline is only fully engaged when the clutch in the driveline is closed (fig. 2.2) and a traditional transmission has the kinematic constraint, that, if the output has zero speed ($\omega_{\text{vehicle}} = 0$), then the input also has zero speed. With closed clutch, fig. 2.2 shows that: $\omega_{\text{engine}} = 0$.

It is now possible to define that, in order to generate torque at vehicle speed $\omega_{\text{vehicle}} = 0[\text{rpm}]$, the driveline requires an additional component that allows the combustion engine to run at $\omega_{\text{engine}} \neq 0$. In traditional drivelines, this component is a manual clutch, which disengages the wheels from the engine.

![Traditional driveline with manual clutch](image)

**Fig. 2.2:** Traditional driveline with manual clutch according to Lechner[1]. With closed clutch, the engine's shaft has the same speed as the shaft left of the transmission.

The manual clutch has the following disadvantages:

1. **Inefficiency**

   *Between the states of total engagement and total disengagement of the driveline, the clutch suffers from severe power loss.*

2. **Uneasy or uncomfortable**

   *The manual clutch is controlled by the driver via a pedal. Smooth control is quite hard and rough handling decreases driving comfort.*

3. **Maximum power loss at drive-off, where maximum acceleration is required.**

   *At drive-off slip in the clutch is very large thus much power is lost.*

Because a geared neutral driveline does not have a clutch, it hasn’t got all these clutch related disadvantages. The extra advantage is that all components related to the clutch can be eliminated.
2.3 Design of geared neutral driveline

Based on the driveline layout of fig. 2.1, there are 3 possible ways to design a geared neutral driveline:

1. Application of different drive unit: \( T_e \neq 0 \) as \( \omega_{\text{engine}} = 0 \text{[rpm]} \).
   
   *This is done with electric devices. A simple electric motor supplies nonzero torque at zero speed. It has its advantages towards simplicity. The major problem with electric devices is the limited range of the vehicle and the heavy weight of fuel cells.*

2. Other type of transmission: \( \omega_{\text{vehicle}} = 0 \) as \( \omega_{\text{engine}} = 0 \).
   
   *This option is chosen in this research. With the use of a continuously variable transmission (CVT) it is possible to make a transmission with a speed ratio*

   \[
   I_{\text{transmission}} = \frac{\omega_{\text{vehicle}}}{\omega_{\text{engine}}} = 0
   \]

   *and to be able to smoothly shift to another speed ratio.*

3. Combination of one and two

   *Various solutions are available, e.g. hybrid drivelines. In this type of drivelines, there are two engines and their power is combined in the transmission. Apart from the extra engine, the transmission needs an extra input shaft. These extra components cause extra unwanted weight.*

This research focuses on geared neutral transmissions.

2.4 Geared neutral transmissions

When discussing geared neutral transmissions, in the driveline of fig. 2.1 it is the transmission that makes the driveline a geared neutral driveline. If the combustion engine and differential gear set are the same as shown in fig. 2.2, then the following definition holds:

Definition **geared neutral transmission (GNT):**
A transmission that is able to have a rotating input shaft and a non-rotating output shaft at the same time. This geared neutral point is achieved without making use of clutches. Without changing this state, the transmission is able to transmit power from the input shaft to the output shaft.

In which:

Definition **geared neutral point:**
The state of the driveline where: \( \omega_{\text{vehicle}} = 0 \) and \( \omega_{\text{engine}} \neq 0 \).
(or \( I_{\text{transmission}} = 0 \)).
There are several options available to design a GNT. Transmission designers already engineered the torque converter (TC) (Lechner[1]) as a solution to this problem. In this layout, the manual clutch (par. 2.2) is replaced by an "hydraulic clutch". The TC increases engine torque and does not require manual handling. The downside is that the TC suffers from even more energy loss in the geared neutral point than a simple, disengaged manual clutch. This is because of the fact that the hydraulic fluid in the TC works as a damper between the rotating shaft from the engine and the non-rotating shaft from the transmission (see fig. 2.2 and replace clutch by the TC). The lost damping-energy in the TC is more than the lost power in a clutch that is fully disengaged.

Taken into account that energy losses are unwanted, the following constrains are defined for the design of a geared neutral transmission:

1. No startup clutch or torque converter.
2. The rotation of the transmission's input shaft must be eliminated within the transmission in such way, that the output shaft, which is connected to the wheels, does not rotate.

Based on these constrains, several researchers (e.g. Vahabzadeh[2], Jurgen[3]), concluded that the GNT must basically have a layout as shown in the flowchart of fig. 2.3.

![Fig. 2.3: Basic flowchart of a geared neutral transmission](image)

The GNT must split the power from the input shaft over two parallel shafts: one with a variable ratio gear and one with a fixed ratio gear. Combining these two shafts at their ends in an epicyclic gearing set (EGS), gives the ability to eliminate their rotational speed towards the exit of the transmission. This is an input coupled system (Vahabzadeh[4]).

The three components that provide geared neutral behaviour are the variable ratio gear, the fixed ratio gear and the EGS.

As mentioned earlier, two concepts are analyzed, a GNT with a toroid variator and a GNT with a pushbelt variator. Chapter 4 and Appendix A give specific details on the behaviour of these variators.

In addition to variators and CVTs, the following definitions are introduced to prevent misunderstandings:

**Definition Continuous Variable Transmission (CVT):**
The whole transmission that is placed in a driveline between the engine and the
differential gear. This transmission can vary its ratio (input-output speed or input-output torque) continuously, between a minimum and maximum ratio.

**Definition Variator:**
The component in the CVT that can change its ratio (input-output speed or input-output torque) continuously, between its own minimum and maximum ratio.

**Remark 1:**
Analyzing fig. 2.3 it seems feasible to use the same layout, but instead place the EGS at the engine side (=output coupled system). Vahabzadeh[4] concluded, correctly, that this is not intrinsic a geared neutral transmission. However, the firm Fendt Gmbh currently produces a transmission with geared neutral ability (for tractors), which uses this output coupled structure. This is not in contradiction with Vahabzadeh, since Fendt uses a hydrostatic variator which itself has geared neutral ability. This type of CVT c.q. variator is not further discussed here and details can be found in Fendt[5].

**Remark 2:**
Vahabzadeh [4] defines that only an input coupled GNT in which the wheels of the vehicle are attached to the sun or the annular of the EGS provide geared neutral abilities. In this research only the attachment to the annular is analyzed.

### 2.5 Practical Implementation of GNT

The flowchart of fig. 2.3 shows that at least three components are necessary for building a geared neutral transmission.

Figure 2.4 and 2.5 show two practical lay-outs that can be used in a vehicle driveline. These two transmissions are the same as later used for static and dynamic analysis (chapter 5-8). The figures show that both transmission lay-outs split the power over two sides: one is going directly into the variator and one over a fixed gear. The two shafts are again combined in the EGS. The end gear has the same behaviour as the fixed ratio gear of fig. 2.3.

It is misleading that in the Torotrak transmission (Torotrak[6]) of fig. 2.4, there are still clutches involved. These clutches are only used to increase ratio coverage. Only in low gear, in which the left clutch is closed and the right clutch is open, a geared neutral state is possible. The GNT of fig. 2.5, only provides low gear, but since this research analyses especially the geared neutral state, this is not a problem. From this point on, also for the Torotrak transmission, only the low gear is represented.
Fig. 2.4: Schematic view of engine and GNT layout, developed by TOROTRAK inc. Notations according to Lechner[1].

Fig. 2.5: Schematic view of engine and GNT with pushbelt, modification of Jurgen[3]
Chapter 3

Kinematics GNT

The previous chapter showed that a geared neutral transmission basically consists of three components. In this context, the kinematics of the EGS and the fixed gear are well known. Based on their behaviour it is possible to define constrains for the kinematics of the variator in a geared neutral system. The next sections show what behaviour is required from the variator to make a GNT in a driveline feasible. Chapter 4 discusses whether a variator has this kinematic behaviour.

3.1 Kinematics of fixed ratio gear

Lechner [1] describes the kinematics of a fixed ratio gear with power losses:

\[
\begin{align*}
\omega_{sec} &= I_{fg} \omega_{pri} \\
T_{sec} &= \frac{\eta_{fg} T_{pri}}{I_{fg}}
\end{align*}
\]

(3.1) (3.2)

In which \( I_{fg} \) is a fixed ratio value and \( \eta_{fg} \) is a efficiency factor. Review fig. 2.3-2.5 to see that \( \omega_{pri} \) can be replaced by \( \omega_{engine} \).

Remark:
This report uses the same definition of ratio as Torotrak[6] does. This is the inverse notation of VDT and is the same as used in equation 3.1. Thus "pri" indicates "closest to the engine" and "sec" indicates the other shaft.

3.2 Kinematics of EGS

Polder[8] describes the kinematics of the epicyclic gearing set. Fig. 3.1a shows a figure of this set that consists of 5 gearing wheels and one planet carrier. Figure 3.1b shows the implantation of the set in the Torotrak transmission layout. Review fig. 2.4 and 2.5 to see that the sun is connected to the variator, the planet carrier to the fixed ratio gear and the annular to the output shaft (via the end gear \( I_d \)).
The kinematic relations are based upon the kinematic constrains of the planet. Fig. 3.2a and 3.2b show that constrains are identified by defining speed and torque of the planet in a static situation.

Based on fig 3.2a the following linear relation is true for all speeds in the variator set:

\[ \omega_a = \omega_p + \frac{1}{E}(\omega_p - \omega_s) \]

with \( E = \frac{R_s}{R_a} \) (3.3)

Based on fig. 3.2b, the following equations hold for one planet in the EGS, if it is supposed to be in (quasi-)stationary state (acceleration forces/torques are supposed to be zero):

\[ M_{\text{planet}}R_p\omega_p = \frac{1}{3} \left( \frac{T_p}{R_p} + \frac{R_sT_s}{R_pR_s} + \frac{R_aT_a}{R_pR_a} \right) = 0 \Rightarrow T_p = -(T_s + T_a) \]  

(3.4)

\[ J_{\text{planet}}\ddot{\omega}_{\text{planet}} = \frac{1}{3} \left( \frac{T_s}{R_s} - \frac{T_a}{R_a} \right) = 0 \Rightarrow T_a = ET_s \]  

(3.5)

Introduction of power losses as efficiency loss is not trivial, since there are 3 axles and it is necessary to identify what is input and what is output. In the transmissions of fig. 2.4 and 2.5 the output is the annular shaft. Therefore the following definition is used:

\[ \frac{1}{\eta_{\text{egs}}} T_a + T_s + T_p = 0 \]  

(3.6)

in which \( \eta_{\text{var}} \) is a factor for efficiency and power losses

The combination of eq. 3.3, 3.5 and 3.6 defines the kinematics of the EGS. Based on these equations, the following theorem holds for the epicyclic gearing set:
Theorem EGS:
The kinematics of a simple EGS with three shafts are defined by one torque and two speeds

This theorem will prove to be very useful in the definition of the required behaviour of the variator.

3.3 Kinematics of GNT

Based on the kinematics of the EGS and the fixed ratio gear the kinematics of the GNT can be defined. These kinematics are split in two parts. The first is speed and the second is torque. The next two subsections discuss each part. The result is a definition of the required kinematics of the variator to make the GNT feasible in a vehicle’s driveline.

3.3.1 Speed in GNT

Reviewing fig. 2.3, it is possible to identify all speeds in the shafts of the transmission by combining equations 3.1-3.6 and introducing eq. 3.7 for the definition of speed ratio over the variator:

$$\omega_{\text{var,sec}} = I_{\text{var}} \omega_{\text{var,pri}}$$  \hspace{1cm} (3.7)

The fact that an extra index $\omega$ is added to $I_{\text{var}}$ is because it will later appear that in a variator, the speed ratio $I_{\text{var,}\omega}$ is not necessarily the inverse of the ratio in torque ($I_{\text{var,}T}$).

To prevent from misunderstandings the following definition is introduced:

$$I_{\text{part,} \omega} = \frac{\omega_{\text{part,sec}}}{\omega_{\text{part,pri}}}$$  \hspace{1cm} (3.8)

$$I_{\text{part,}T} = \frac{T_{\text{part,pri}}}{T_{\text{part,sec}}}$$  \hspace{1cm} (3.9)

Fig 3.3: Basic flowchart of a geared neutral transmission: speed

$$\omega_{\text{in}} = \omega_{\text{var,pri}} = \omega_{\text{f,g,pri}} = \omega_{\text{engine}}$$  \hspace{1cm} (3.10)

$$\omega_{\text{f,g,sec}} = I_{\text{f,g}} \omega_{\text{engine}}$$  \hspace{1cm} (3.11)

$$\omega_{\text{var,sec}} = I_{\text{var,}\omega} \omega_{\text{engine}}$$  \hspace{1cm} (3.12)
The combination of eq. 3.3 with eq 3.8-3.10 and the fact that the EGS is connected as shown in fig. 2.4, gives the following relation between $\omega_{\text{engine}}$ and $\omega_{\text{vehicle}}$ (eq. 3.15):

$$ E = \frac{R_a}{R_s} $$

$$ \omega_{\text{vehicle}} = \frac{I_{\text{cut}}}{I_{\text{engine}}} \omega_{\text{engine}} = \omega_{fg,sec} + \frac{1}{E} (\omega_{fg,sec} - \omega_{\text{var,sec}}) $$

$$ \omega_{\text{vehicle}} = \omega_{\text{engine}} \left( I_{fg} + \frac{1}{E} (I_{fg} - I_{\text{var,\omega}}) \right) $$

The variator ratio $I_{\text{var}}$ for which the geared neutral ratio $I_{\text{cut}} = \frac{\omega_{\text{vehicle}}}{\omega_{\text{engine}}} = 0$ is obtained, can be found by rewriting eq. 3.15:

$$ \omega_{\text{engine}} \left( I_{fg} + \frac{1}{E} (I_{fg} + I_{\text{var,\omega}}) \right) = 0 \Rightarrow I_{\text{var,\omega}} = I_{fg} (E + 1) $$

Eq. 3.16 clearly shows that in the geared neutral point eq. 3.16 always equals zero, regardless what $\omega_{\text{engine}}$ is. This is equal to a neutral state in a traditional transmission, showing the "geared neutral state" of the GNT.

### 3.3.2 Torque in GNT

Torque in the transmission cannot be defined in the same way as speeds, because power is split over two shafts in the GNT. If the power of the engine is chosen as basis, this power split gives a singular solution for the definition of torque in the geared neutral point.

Definition of torque with $T_{\text{cut}2\text{vehicle}}$ as basis, does not give singular solutions. Fig. 3.4 shows a schematic view of a whole driveline, in which the engine and the vehicle are represented as 2 individual inertia masses.

---

**Fig. 3.4: Schematic view of driveline**

14
In addition to fig. 3.4, choosing \( T_{\text{cvt2vehicle}} \) as basis and using eq. 3.1-3.16 gives:

\[
\begin{align*}
T_{\text{annular}} &= T_{\text{vehicle2cvt}} = -T_{\text{cvt2vehicle}} \quad (3.17) \\
T_{\text{var, sec}} &= -\frac{1}{E}T_{\text{cvt2vehicle}} \quad (3.18) \\
T_{\text{var, pri}} &= -\frac{I_{\text{var,T}}}{E}T_{\text{cvt2vehicle}} \quad (3.19) \\
T_{f_\text{g, sec}} &= \frac{1+E}{E}T_{\text{cvt2vehicle}} \quad (3.20) \\
T_{f_\text{g, pri}} &= \frac{I_{f_\text{g}}(1+E)}{E}T_{\text{cvt2vehicle}} \quad (3.21) \\
T_{\text{cvt2engine}} &= -(T_{\text{var, pri}} + T_{f_\text{g, pri}}) = -T_{\text{cvt2vehicle}} \left( \frac{I_{f_\text{g}}(1+E)}{E} - \frac{I_{\text{var,T}}}{E} \right) \quad (3.22)
\end{align*}
\]

Review eq 3.16 to see that in the geared neutral point eq. 3.22 equals zero and there is no relation between \( T_{\text{cvt2vehicle}} \) and \( T_{\text{cvt2engine}} \). This lack of relationship shows that in the geared neutral point torques in the GNT cannot be defined via a relation to \( T_{\text{engine}} \). This proofs the statement at the beginning of this subsection, that calculations of torque from engine to vehicle gives singular solutions.

### 3.3.3 Variator: Torque control

In the previous subsection is stated that all torques can be accounted by introducing a given value for \( T_{\text{cvt2vehicle}} \). The problem that remains is which input in the driveline defines the amount of \( T_{\text{cvt2vehicle}} \). In a traditional driveline the engine defines this, but the previous subsection showed that there is no relation between \( T_{\text{cvt2vehicle}} \) and \( T_{\text{cvt2engine}} \) in the geared neutral point. Thus another part in the transmission must define the amount of \( T_{\text{cvt2vehicle}} \). This is done by the variator. Underneath here, proof will be given that this is indeed true, because if not, than the GNT is not feasible.

**Statement:**  
The variator must control a specific torque ratio between input and output shaft and not a specific speed ratio between the same shafts.

**Proof:**  
The theorem of par 3.2 stated that the kinematics of the EGS are described by 1 torque and 2 speeds. Review fig. 3.4 to see that the EGS has 3 inputs, of which two are connected to inertia masses, so their speed cannot change infinitely fast.

The variator has two ways to influence the connection between the inertia of the engine and the inertia of the vehicle: it can define a speed ratio, such as a fixed ratio gear, or it can define a torque ratio between it’s input and output shafts.

First, suppose the variator defines a speed ratio, than the following algorithm is true:  
1. Suppose e.g. that the EGS is in balance and \( \omega_{\text{engine}} = A \) and \( \omega_{\text{vehicle}} = B \). This results in fig. 3.5a
2. Now the ratio $I_{\text{var}}$ is decreased. Directly after this decrease fig. 3.5b is true.

Fig. 3.5b clearly shows that immediately after the change in ratio $I_{\text{var}}$ the kinematics of the EGS are violated. The only way out of this is immediate increase or decrease in $\omega_{\text{vehicle}}$ and/or $\omega_{\text{engine}}$. Since these speed values are subject to the inertia masses of the engine and the vehicle, this is not possible unless torques are (infinite) high. That however is not possible, since infinite torques will break down all shafts in the transmission. Thus control of defined speed ratio at the variator results in a non feasible system.

Second suppose the variator controls not a speed ratio, but a defined secondary torque, then torques will not become infinite high (they are controlled!) and speed of the two inertia masses is only changed according to the torques that is applied by the transmission onto the inertia masses.

**Conclusion:** control of torque at the variator results in a feasible system.

In the next chapter is shown that in practical situations the variator indeed controls torque. In order to define all torques in the GNT, it is supposed that $T_{\text{var,sec}}$ is controlled to a specific value.

If power losses are neglected then the following equations are true:

\[
\begin{align*}
T_{\text{sun}} &= T_{\text{var,sec}} \\
T_{\text{var, pri}} &= I_{\text{var,T}} T_{\text{var, sec}}
\end{align*}
\tag{3.23}
\tag{3.24}
\]

and based on the kinematics of the EGS and the fixed ratio gear:

\[
\begin{align*}
T_{\text{cvt2vehicle}} &= -T_{\text{annular}} = -ET_{\text{var, sec}} \\
T_{f_g,sec} &= -(E + 1) T_{\text{var,sec}} \\
T_{f_g, pri} &= -I_{f_g} (E + 1) T_{\text{var, sec}}
\end{align*}
\tag{3.25}
\tag{3.26}
\tag{3.27}
\]

Introducing $T_{\text{cvt2engine}}$ as torque that works on inertia $J_{\text{engine}}$:

\[
T_{\text{cvt2engine}} = T_{\text{var,pri}} + T_{f_g, pri} = T_{\text{var, sec}} (-I_{f_g} (E + 1) + I_{\text{var,T}})
\tag{3.28}
\]
The combination of eq. 3.16 and 3.22 shows that if in the geared neutral point $I_{var,\omega}$ equals $I_{var,T}$ then $T_{cut2engine} = 0$. This directly means that in this state no power is transmitted from the engine into the transmission and that all power produced by the engine via external combustion torque $T_{engine}$ must be eliminated by the inertia of the engine. It also means that loss of power in the transmission cannot be compensated by energy from the combustion engine. View chapter 6 and 7 for the dynamic consequences.
Chapter 4

Variators

Chapter 3 showed that the variator must control torque rather than speed ratio.

In this research the GNTs of fig. 2.4 and 2.5 are analyzed. The first has a toroid variator, the second a pushbelt variator. Both variators are typical "force closure systems", which directly implies control of torque. For more information about the difference between force and form closure systems, see appendix B.

The next two subsections discuss the kinematic behaviour of each variator according to torque control. Specific information on the toroid is given in appendix A for the toroid. The pushbelt variator is well described in literature such as SAE[7]. Only relevant information is given in section 4.2.

Remark:
In this research, the hydrostatic variator of Fendt Gmbh, is not further analyzed, because this is a variator with intrinsic geared neutral ability.

4.1 Toroid variator

The toroid variator is physically displayed in fig. 4.1a and schematic in side view in fig. 4.1b. When only one roller is analyzed, figure 4.2a and 4.2b can be drawn. The rollers are enclosed between the primary and secondary discs. The enclosure is pretensioned via pressurized fluid in the pressure chamber. The angle of the roller is a measure for the speed ratio over the variator.

The variator has 2 inputs: the pressure in the fluid of the pressure chamber resulting in a force $F_{\text{pretension}}$ and the difference in pressure on both sides of the piston in the cylinder (see fig. 4.2a), which results in a force $F_{\text{trig}}$.

View appendix A to see that the variator automatically shifts according to its primary and secondary speed, purely based on friction balances. Therefore, within the minimum and maximim ratio of the variator($I_{\text{low}} < I_{\text{var,\omega}} < I_{\text{OD}}$):

$$I_{\text{var,\omega}} = \frac{\omega_{\text{sec}}}{\omega_{\text{pri}}} = \frac{R_{\text{pri}}}{R_{\text{sec}}}$$  \hspace{1cm} (4.1)
Fig. 4.1a: Toroid variator with components

Fig. 4.1b: schematic view of 2 rollers between discs

Fig. 4.2a: isometric view of roller between discs.

Fig. 4.2b side view of roller between discs

Also view appendix A to see that torque is controlled by the piston/cylinder construction. Thus, within the boundaries of the variator ($I_{low} < I_{var,\omega} < I_{OD}$) hold per roller:

$$\frac{1}{6} T_{sec} = \frac{1}{6I_{var,T}} T_{sec} = \frac{1}{6I_{var,\omega}} T_{sec} = \frac{1}{2} R_{sec} F_{ctrl}$$

(4.2)

and for 6 rollers:

$$T_{sec} = 3R_{sec} F_{ctrl}$$

(4.3)

In which $F_{ctrl}$ is defined by controlling the pressures in the piston/cylinder construction. The maximum amount of transmitted torque is defined by the maximum friction force between the roller and the discs. This value is controlled by the pretension caused by the pressure in the pressure chamber. Suppose simple coulomb friction then:

$$T_{sec,max} = \max (3R_{sec} F_{ctrl}) = \max (3R_{sec} \mu F_{pretension})$$

(4.4)

in which $\mu$ is a fixed value. Thus $T_{sec}$ is limited by the maximum value for $F_{pretension}$.

Remark 1:
By controlling an amount of torque, power is transmitted over the variator. This causes an increase or decrease of speed in the primary and/or secondary shaft. The roller of the variator
will automatically rotate in such way that this change in speed ratio is followed by a change in $I_{\text{var},T}$, while eliminating macro slip.

**Remark 2:**
Friction is supposed to be purely coulomb friction. This is never true for friction between moving, non flat surfaces, however, for a first impression this is sufficient.

**Remark 3:**
All equations are only true if the variator is supposed to have negligible inertia. This holds, because the inertia of the engine and the vehicle are much higher.

Special attention must be given to $I_{\text{var},\omega}$ and $I_{\text{var},T}$. It occurs that these are equal to each other in case of the toroid variator. This is caused by the fact that the toroid variator automatically shifts according to a balance in friction forces. This balance is also used for torque control. Appendix A discusses this friction balance.

### 4.2 Pushbelt variator

Figure 4.3a shows the pushbelt variator and figure 4.3b gives an exploded view with forces drawn in it. In general the variator transmits power from the primary to the secondary pulley via a belt that is enclosed between the primary pulley and the secondary pulley. The controlled clamping forces $F_{\text{pri}}$ and $F_{\text{sec}}$ are a basis for the transmitted power over the variator. Note that $F_{\text{pri}}$ and $F_{\text{sec}}$ are the only 2 controlled quantities and they are generated by pressurized fluid that works on the pulleys.

![Fig 4.3a: schematic view of pushbelt between pulleys.](image)

![Fig. 4.3b: exploded view of variator with forces.](image)

Shifting is introduced by two translational sheaves that leave more or less space between the pulley. This translation is caused by the controlled forces $F_{\text{sec}}$ and $F_{\text{pri}}$ and the acceleration/deceleration of inertia’s on both sides of the variator. VDT has described this behaviour via the Ide-model. Between the boundaries of the variator: $I_{\text{low}} < I_{\text{var},T} < I_{OD}$ this Ide model is defined as:

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\[ \frac{d(I_{\text{var},T})}{dt} = k_i \omega_{\text{pri}} (F_{\text{pri}} - k_{\text{pk}s} F_{\text{sec}}) \quad I_{\text{var},T} = \frac{R_{\text{pri}}}{R_{\text{sec}}} \]  

(4.5)

in which \( k_i = k_i(I_{\text{var},T}) \) and \( k_{\text{pk}s} = k_{\text{pk}s}(I_{\text{var},T}, S_f) \).

\( S_f \) is a safety factor that prevents from macro slip in variator. Both \( k_i(I_{\text{var},T}) \) and \( k_{\text{pk}s}(I_{\text{var},T}, S_f) \) are determined experimentally.

\( R_{\text{pri}}, R_{\text{sec}} \) are an approximation of the radius over which the belt is moving.

Note that in equation 4.5 the ratio is specifically defined as \( I_{\text{var},T} \), which refers to the ratio between input and output torque. It is based on an empirically defined "geometrical ratio" of the pushbelt variator. The ratio between input and output speed is always defined as:

\[ I_{\text{var},\omega} = \frac{\omega_{\text{sec}}}{\omega_{\text{pri}}} \]  

(4.6)

\( T_{\text{var},T} \) and \( T_{\text{var},\omega} \) are not necessarily equal to each other, but the ratio between \( T_{\text{pri}} \) and \( T_{\text{sec}} \) is always defined as the geometrical ratio that results from the Ide-model.

The amount of torque at the primary and secondary side of the variator is defined by eq. 4.7:

\[ T_{\text{sec}} = \frac{1}{I_{\text{var},T}} T_{\text{pri}} = \max \left( \frac{2 F_{\text{pri}} R_{\text{pri}} \mu}{I_{\text{var},T} \cos(\lambda)}, \frac{2 F_{\text{sec}} R_{\text{sec}} \mu}{\cos(\lambda)} \right) \]  

(4.7)

In which \( \lambda \) is the nose angle of the pulley. The friction coefficient \( \mu \), is described in eq. 4.8 and it's behaviour is determined experimentally by VDT:

\[ \mu = \mu(\text{slip}) = \mu \left( \frac{\omega_{\text{sec}}}{\omega_{\text{pri}}} - I_{\text{var},T} \right) \cdot 100 \]  

(4.8)

Figure 4.4 shows the relation between slip and \( \mu \):

![Figure 4.4: connection between slip(%) and \( \mu \)](image)
Special in fig. 4.4 is that when slip is zero, then also $\mu$ is zero. Combination with eq. 4.7 shows that torque can only be controlled if $I_{\text{var}, T} \neq I_{\text{var}, \omega}$.

This directly implies one of the problems with torque control of the variator: It cannot generate power, but only transmit it. Reviewing the equations, it occurs that the power is always transmitted in such way, that slip is eliminated. Since no slip equals no control of torque (eq. 4.7-4.8 and fig. 4.4), the only way to control torque is introduction of slip. This can be done in two ways if in the driveline the variator and the engine can be controlled (see also fig 3.4):

- Differ the controlled forces $F_{\text{pri}}$ and $F_{\text{sec}}$ in order to differ the geometrical ratio $I_{\text{var}, T}$.
- Applying an engine torque $T_{\text{engine}}$ that changes the speed of the engine’s inertia and introduces a different speed ratio over the variator (not possible in a GNT in the geared neutral point!).

It is easy to see that on a short term basis the first option is possible, but then only the potential energy of speed of the inertia masses on both sides of the GNT is used for torque control. Unfortunately, this is only possible, until the minimum/maximum geometrical ratio ($I_{\text{var}, T}$) in the variator is reached.

The second option can be used on a long term basis, since extra power is added to or taken out of the driveline via combustion power. Eq. 3.16 showed that this option is not possible in a GNT in the geared neutral point, since in that point the speed ratio over the variator is not changed by a change of $\omega_{\text{engine}}$. 

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Chapter 5

Analyzing method and modeling

This chapter describes how the geared neutral transmissions that are displayed in fig. 2.4-2.5 are analyzed for their dynamic behavior. In the previous chapters only algebraic equations are used, which directly implies a static analysis. Appendix D gives full description of this static analysis and also introduces power losses in the GNT. This chapter introduces the steps that are taken for a dynamic analysis that is carried out with numerical methods in Matlab-Simulink.

The dynamic model is built after processing the following three steps:

1. What is interesting to analyze?
   The goal of this research is to define feasibility of a geared neutral transmission with pushbelt variator and to define constrains for control of this GNT. These two definitions require an analysis on the stability of the GNT in the geared neutral point. Thus the stability must be analyzed. Particularly the system response to a change in the controllable inputs in the driveline (2 inputs for the variator and $T_{engine}$) is interesting.

2. Which analyzing method provides the right information?
   To define stability and system response, the driveline must be analyzed for its dynamic behavior and not only its static behavior.

3. Which assumption in the analysis can be made to simplify the modeling of the driveline?
   This is discussed in the following subsection.

5.1 Basic assumptions

Appendix A and chapter 4 describe that in a GNT, the variator describes what the amounts of torque in the drivelines are. The major inertia masses prescribe speed in the driveline, since they are most inert. The driveline of a vehicle is therefore modeled as shown in fig. 5.1.

The engine and the vehicle are both modeled as simple inertia masses that are subjected to external torques, which is a common modeling method for these parts. Their resulting speeds are input for the CVT. Just as in the static analysis. The GNT combines the two speeds in the variator and together with its control inputs, the variator defines torque in the GNT. This is calculated back through the transmission to the two inertia parts. The driveline can now mathematically be described by time-dependent, differential equations, which can be solved with numerical methods. In this research, the computer program...
MATLAB and its toolbox Simulink are used to perform this numerical analysis. The used integration method is ode23tb (see Matlab help files [10] for details).

In order to simplify the simulation model, the following assumptions are made about the dynamics of the driveline and the CVT:

1. **The transmission itself has no inertia.**
   
   This is based on the fact that all parts of the transmission have very little inertia in comparison to especially the vehicle. It is supposed to be sufficient to define an engine inertia. The advantage of this non-inert interpretation of the GNT is that the algebraic equations of chapter 3 can be used to calculate speed and torque in the transmission. It also decreases the simulation time for the whole model of the driveline.

2. **The combustion power of the engine is represented by a prescribed torque that works on a crankshaft.**

   The crankshaft is supposed to represent all the inertia of the engine ($J_{\text{engine}} = 0.1\,[kgm^2]$). Stiffness in the engine is neglected, because the crankshaft is much stiffer than the (much longer) driveshaft at the other side of the GNT (beware of sign definitions):

   $$J_{\text{engine}}\dot{\omega}_{\text{engine}} + b\omega_{\text{engine}} = T_{\text{engine}} + T_{\text{external}}$$  \hspace{1cm} (5.1)

3. **The vehicle is modeled as an inertia mass of $M_{\text{vehicle}} = 1000\,[kg]$. In order to accelerate it has to overcome three types of energy losses, apart from its own inertia:**

   - Air friction.
   - Coulomb friction: causes a minimum amount of torque necessary to make the car start moving (e.g. brake force).
   - External Torque: can be anything from a slope in the road, to a barrier. It is prescribed in time.

   All constants arbitrarily chosen, see appendix C for values.

   Stiffness in the vehicle is neglected since this gives no extra information for this research:

   $$Mr_{\text{wheel}}^2\dot{v}_{\text{vehicle}} = \frac{T_{\text{coulomb}} + T_{\text{external}}}{r_{\text{wheel}}} + F_{\text{air friction}}$$  \hspace{1cm} (5.2)

4. **Power losses in the transmission are introduced to all parts and by:**

   ```plaintext
   \begin{align*}
   &\text{5.3. Power losses in the transmission are introduced to all parts and by:} \\
   \end{align*}
   ```
The variator also has power losses because of percent slip \((C_3)\). This is introduced as:

\[
\omega_{\text{out}} = I_{\text{variator}} (1 - C_3) \omega_{\text{in}}
\]  

(5.4)

All values for \(C_1\), \(C_2\) and \(C_3\) are chosen in such a way that at an input speed of 5000rpm, the losses per part are equal to the standard values for efficiency losses over that part. These standard values are based on Dubbel[9].

5. No stiffness is defined for the transmission.

Since the variator partly functions as a force closure system (e.g. clutch, see appendix B), it is not necessary to define stiffness. Some simulations are done with stiffness but the differences in the results are less than 1% of the simulations without stiffness.

6. The variator controls the amount of torque, via its control parameters and both primary and secondary of itself and is therefore decisive for all torques in the transmission.

As a result the GNT itself is calculated by accounting all speeds towards the variator and all torques from the variator to the outside. This is shown in fig. 5.2

![Diagram of CVT](image)

Fig. 5.2: Calculation method in dynamic simulation through the CVT

The dynamic analyses of Torotrak (fig. 2.4) is described in chapter 6.
The dynamic analyses of the pushbelt-GNT (fig. 2.6) is described in chapter 7.
Extra specific information on the modeling of the variators is described in appendix A.
Chapter 6

Torotrak GNT

This chapter gives details about the GNT as it is developed by Torotrak. Fig. 2.4 already showed this transmission. Fig. 6.1 shows a new figure of the Torotrak layout, in which only this low gear state is represented. Standard values are given for the behaviour of each part. The transmission consists of a toroid variator (var), a chain gear (c), a fixed gear (g), an EGS (egs) and an end gear (d).

![Diagram of Torotrak transmission in low gear]

Fig. 6.1: Torotrak transmission in low gear. Values for all components are based on Torotrak[6]

The static analysis is described in appendix D. Necessary assumptions for inertia masses, friction forces, etc. are discussed in chapter 5 and appendix C. Here only the results of the dynamic analysis with and without power losses are discussed.

6.1 No power losses in GNT

The system is modeled as a system where $T_{\text{ct2veh}} (= T_{\text{dreh}})$ (see fig. 5.1) is controlled by adjusting the $T_{\text{sec}}$ of the toroid variator. A setpoint is defined for $T_{\text{dreh}}$ and this is calculated backwards through the transmission to define the required value for $T_{\text{sec}}$ and thus a value for $F_{\text{ctrl}}$. See appendix D for the relation:
\[ F_{\text{ctrl}} = 2 \frac{T_{\text{out}}}{R_{\text{sec}}} = SET(T_{\text{cvt2vehicle}}) I_d I_e (1 + I_{\text{var,T}}) \frac{ER_{r2prI_{\text{var,T}}}}{E_{\text{R}}} \]  

Eq. 6.1 shows that feedback of the variator ratio is necessary to define the required \( F_{\text{ctrl}} \). This is not a problem since chapter 4 already showed that for the toroid variator \( I_{\text{var,T}} = I_{\text{var,\omega}} \) and simple measurement of the speed of the engine shaft and the vehicle shaft gives information on this variator ratio. (eq. D.10-13).

The simulation is started in the geared neutral point. The following external inputs are introduced. They are constant during the simulation:

- \( T_{\text{coulomb}} = 5 \text{Nm} \) 
  This is a very low value, however sufficient to makes the variator very stable. Especially in the simulation of the next section, this proves to be useful for a stable situation in the geared neutral point.

- torque \( T_e = 3 \text{ Nm} \)

- torque \( T_{\text{earth}} = 0 \text{ Nm} \)

The setpoint for \( T_{\text{drsh}} \) is shown in fig. 6.2 together with the realized torque:

![Torque in simulation without power losses for Torotrak GNT](image)

Fig. 6.2: torque in simulation without power losses for Torotrak GNT.

In fig. 6.2 it is shown that \( T_{\text{cvt2engine}} \) remains below \( T_e \) until approx. \( t = 7 \text{[s]} \). Until that moment the engine is speeding up. In fig. 6.3 this is shown. In this figure the corresponding
speeds of the inertia masses and $\omega_{sec}$ of the variator are plotted. The variator ratio is defined as the difference between $\omega_e$ and $\omega_{sec}$ (view app. D).

<table>
<thead>
<tr>
<th>$\omega_e$</th>
<th>$\omega_{sec}$</th>
<th>$\omega_{drsh}$</th>
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<tbody>
<tr>
<td>0</td>
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</tbody>
</table>

Fig. 6.3: speed in simulation without power losses for Torotrak GNT.

Fig. 6.4a and 6.4b shows $F_{ctrl}$ and $I_{var,T}$, $I_{var,\omega}$ and $I_{gnt}$. The first two are the same for the toroid variator (see app. A).

Fig. 6.4a: $F_{ctrl}$

Fig. 6.4b: ratios variator and GNT

In appendix E some extra figures for this simulation (e.g. $F_{ctrl}$ are plotted. The combination of fig. 6.2-6.4 gives the following information on the behaviour of the Torotrak GNT.

1. The setpoint $SET(T_{drsh})$ is well followed with the simple control method for $F_{ctrl}$. The introduction of time delay and a first order filter for the generation of $F_{ctrl}$ has no particular influence.
2. The amount of necessary $F_{ctrl}$ is about 5 times of the resulting $T_{sec}$. This is a sensitivity of lower than one. A disturbance in the pressure in the control cylinder is not a problem. This makes the system easier to control with inferior pump equipment.

3. The variator exclusively determines the amount of $T_{drsh}$. If the engine generates power and the variator does not react, then the system remains in geared neutral ($0 \text{ at } t=5[s]$). All power generated by combustion is used to speed up the crankshaft of the engine. This is also true out of the geared neutral point, as long as the force $F_{ctrl} = 0$ (after $t=9[s]$).

4. The system shifts out of geared neutral in a stable way. Torque $T_{drsh}$ has no overshoot in comparison to the its setpoint.

5. If the engine is itself not generating enough power from combustion, then the variator uses he inertia power of the engine for accelerating the vehicle.

### 6.2 Power losses in GNT

In the simulation with power losses, the same values are used as in the previous section. This means that the power losses are not accounted in the definition for $F_{ctrl}$. This gives the following results:

![Graph showing torques of various components](image)

Fig. 6.5: speed in simulation without power losses for Torotrak GNT. All other figures are placed in appendix E.

Fig. 6.5 shows that the setpoint is well followed. The actual $T_{drsh}$ is nonzero for the first 5 seconds, as the setpoint is zero. This is due to the power losses in the transmission that
causes a torque on the annular shaft of the EGS. With the very low braking torque of 5 Nm (see chapter 5) this does not result in an acceleration of the vehicle.

The following results are extracted from figure 6.5 (and the appendix):

1. The engine speed is decelerated because the potential energy of its inertia mass is used to compensate the power losses in the transmission.

2. The setpoint for $T_{d_{rsh}}$ is still followed well, however the power losses introduce a torque on the wheels is negative because of the losses in the GNT. Later the maximum amount of the setpoint($T_{d_{rsh}}$) is never reached by the actual $T_{d_{rsh}}$.

The simulation shows that the setpoint of $T_{d_{rsh}}$ is well followed, even though a time delay and filter is used for $F_{ctrl}$.

### 6.3 Conclusions: Torotrak GNT

The GNT can be controlled very well by using only the $\omega_{pri}$ and $\omega_{sec}$ of the variator as feedback (or, as used in the simulation model, $R_{sec}$, which is actually the same (see app. A)). The Torotrak GNT is stable and does not suffer from infinit torques. The GNT is also well controllable.

**Remark:**
The results for the torotrak gnt are based on the literature that is available on toroid variators and of Torotrak[6]. This information is not checked by real experiments. This type of verification is recommended.
Chapter 7

Pushbelt GNT

Figure 2.5 already showed the GNT that is used in this research. Figure 7.1 also shows this GNT, however, now specific values are given for the behaviour of the various components. In this case, the GNT exists of a variator (var), a fixed gear (fg1), another fixed gear (fg2), an EGS (egs) and an end gear (d):

\[ 0.570 < I_{\text{in}} < 1.775 \]

Engine side

\[ I_{e1} = - \frac{\omega_e}{\omega_{\text{in}}} \]

Vehicle side

\[ I_{g1} = - \frac{\omega_{g1}}{\omega_{\text{in}}} \]

\[ E_{\omega} = \frac{\omega_{\text{in}}}{\omega_{\text{in}}} = \frac{\omega_{\text{in}}}{\omega_{\text{in}}} \]

Fig. 7.1: Pushbelt GNT. Values chosen as described in appendix C

The static analysis is described in appendix D. Chapter 5 describes how the GNT is simulated in a vehicle driveline. The specific flowchart of the variator is shown in appendix A. Necessary values for inertia masses, friction forces, etc. are given in appendix C. Here only the results of the dynamic analysis are discussed. More results can be found in appendix F.

7.1 No power losses in GNT

Chapter 4 showed that the variator can only transmit torque in a slipping condition. This slip always introduces an amount of lost power. It is assumed in this analysis that all parts except the variator work without suffering from power losses.

Just as in chapter 6 the controllable inputs \( F_{\text{pri}} \) and \( F_{\text{sec}} \) are used to follow a setpoint \( T_{\text{end2vehicle}}(= T_{\text{desh}}) \). However, in this case \( I_{\text{var,T}} \neq I_{\text{var,\omega}} \). Equation 4.7 and D.34 describe
the relations for control:

\[
T_{sec} = \frac{1}{I_{var,T}} T_{pri} = \max \left( \frac{2 F_{pri} R_{pri} \mu}{I_{var,T} \cos (\lambda)}, \frac{2 F_{sec} R_{sec} \mu}{\cos (\lambda)} \right)
\] (7.1)

\[
T_{sec} = \frac{I_g I_g}{E \eta_d \eta_g \eta_g} T_{drsh}
\] (7.2)

An the Ide-model for \( I_{var,T} \):

\[
\frac{d(I_{var,T})}{dt} = k_i \omega_{pri} (F_{pri} - k p k s F_{sec}) \quad I_{var,T} = \frac{R_{pri}}{R_{sec}}
\] (7.3)

At this moment, it is in practical situation not possible to use \( I_{var,T} \) as feedback. \( I_{var,\omega} \) can be used, which is done here.

Only having \( I_{var,\omega} \) as feedback is a major problem, because the difference between \( I_{var,T} \) and \( I_{var,\omega} \) defines the amount of slip. At this moment the amount of slip can not be reproduced.

And this result in more unknown values:

Because \( I_{var,T} \) is unknown, in eq. 7.1 the following quantities are unknown: \( I_{var,T}, R_{pri}, R_{sec} \) and \( \mu \).

In order to be able to calculate torque, VDT always assumes that slip is small, which (see eq. 4.8) means that \( I_{var,T} \approx I_{var,\omega} \) and thus \( I_{var,\omega} \) is used in eq. 7.1-7.3 to replace \( I_{var,T} \).

Furthermore \( R_{pri} \) and \( R_{sec} \) are a function of \( I_{var,T} \) and thus now a function of \( I_{var,\omega} \).

The factor \( \mu \) is still unknown, but for definition of the controlled force \( F_{pri} \) and \( F_{sec} \) this factor is assumed to be a fixed value of 0.06 that only changes in sign, depending on the direction in which the power is assumed to flow. This method seems vague, but is caused by the fact that \( \mu \) is depending on slip, which cannot be measured (yet). In simulations, this definition works quite well.

In general, torque control can now be done in two ways the following subsections describes each of them.

### 7.1.1 Control of minimum macroslip

The method in which the variator is prevented from macroslip is the classical method that is currently used by VDT. In this method it is assumed that slip of more than 4% severely damages the variator and must be avoided. It works as follows:

1. \( \omega_{pri}, \omega_{sec} \) of the variator are measured together with \( T_e \).

2. Based on this information the current \( I_{var,T} \) is defined and, via the Ide-model, this again defines the relation between \( F_{pri} \) and \( F_{sec} \) for which slip is minimized.

3. Based on the amount of \( T_e \) and the safety factor, the required amount of \( F_{pri} \) is defined and via "2" this leads back to \( F_{sec} \).

4. The safety factor is a value used in the empirically defined kpks-diagram and ensures that slip remains within its boundaries.

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This method has the advantage that slip is never too high and torque in the transmission is only controlled by $T_e$. The disadvantage is that in a GNT, the state of the GNT is independent of $T_e$ in the geared neutral point.

In the next simulation, the value for $T_e$ is therefore replaced by the required torque $T_{sec}$, which is calculated via eq. 7.2.

With the same input parameters as used in chapter 6, this method gives the following results:

![Graph showing torques of various components](image)

Fig. 7.2: Classical method without power losses: Torque

Fig. 7.2 shows that the setpoint is not followed. Torques remain (almost) zero. This is quite obvious. The whole control method is based on preventing slip. Since slip is already zero in the geared neutral point, the clamping forces will clamp the pulleys in such way that this state is maintained. Fig. 7.3 shows that slip is indeed very small.

![Graph showing relative slip of belt over pulleys](image)

Fig. 7.3: Classical method without power losses.
The following conclusion follow from this control method:

1. In the geared neutral point $T_e$ cannot control $T_{dreh}$ and the primary and secondary speed of the variator change proportionally to the variator ratio and according to $\omega_{engine}$. Clamping $F_{pri}$ and $F_{sec}$ according to the corresponding kpks in the geared neutral point has therefore no effect.

2. Contradiction in control method: slip is always minimized, while it is only possible to control torque in slipping condition.

In conclusion it is quite obvious that this method does not work in the geared neutral point. A simple simulation with a starting point outside the geared neutral point however shows that in that situation the slip is very well minimized. See appendix F for an example of this.

### 7.1.2 Control of required $I_{var,T}$

In this second method, the same measurements are done, but now the combination of the measured $T_e$ and the setpoint $T_{dreh}$ is used to define the required ratio $I_{gnt}$. This required $I_{gnt}$ is calculated to a required value for $I_{var,T}$:

1. $T_e$ is measured.

2. The preferred $I_{gnt}$ is defined by: ratio is defined as:
   $$ setpoint(I_{cut}) = \frac{setpoint(T_{dreh})}{T_{engine}} $$

3. The preferred variator ratio is calculated according to (see app. D):
   $$ Setpoint(I_{var}) = Setpoint(I_{cut}) + \frac{F_{t}}{I_{g2}l_{g2}} - \frac{(1+E)I_{g2}}{I_{g2}} $$

4. $F_{pri}$ and $F_{sec}$ are changed so that their relation is based on
   $$ kpks(I_{var}, Sf) = kpks(SET(I_{var}), 1.3) $$
   and the quantity of $F_{sec}$ is defined by the
   $$ F_{sec} = \frac{setpoint(T_{sec}) \cos(\lambda)}{\mu h_{sec}} = \frac{I_{d}setpoint(T_{sec}, \text{side}) \cos(\lambda)}{E \mu h_{sec}} $$

Implementing this control method in the GNT with the same conditions as in the previous subsection results in:
Fig. 7.4: new control method without power losses.
Fig. 7.4 shows much overshoot, while later in the trajectory, the system has an actual $T_{\text{drsh}}$ that is too low. Fig. 7.5 shows the corresponding slip curve which is still well beneath the value of 4% at which VDT calculated that damage would occur.

Fig. 7.5: new control method without power losses.
This control method reacts too sensitive around the geared neutral point, resulting in much overshoot.

Two remarks must be made in addition to the results:

1. The clamping forces only have a minimum value in this model. Their practical maximum values are not considered here. Normally, this maximum prevents the variator from torques of more than approx. $T=300[Nm]$ at the variator. This point is not reached in this simulation.
2. The pushbelt variator is very insensitive towards a change in the clamping forces. To achieve a torque of $T_{pri} = 80\text{Nm}$ a clamping force of $F_{pri} = 50000[N]$. This insensitivity is also caused by the very low amount of slip in the pushbelt variator. If this slip is forced to higher values, then (see fig. 4.4 and eq. 4.7-4.8), the sensitivity is rises.

7.2 Power losses in GNT

The same control strategies can be used in case of a system with losses in the GNT. Just as in chapter 6, the controller itself does not implement these losses.

7.2.1 Control of minimum slip

With power losses the classical method to minimize slip results in equal plots to the first time. The lost power in the GNT is minimized, if slip is minimal. The result is not plotted. In this simulation the power losses are less than for the torotrak system. This is because (see appendix C) no extra efficiency factors are introduced to define losses in the variator. This is because under normal working conditions, the losses caused by slip in the pushbelt variator already lower down the efficiency of this component to 96% just as Dubbel[9] defined.

7.2.2 Control of required $I_{var,T}$

Only the result for torques is plotted. This figure shows that maximum overshoot is lowered from $370[Nm]$ (fig. 7.4) to $300[Nm]$.

7.3 Conclusions: pushbelt GNT

This conclusion must be split in two: First the feasibility is discussed, than the controllability.

The pushbelt GNT is definitely a feasible system. With the relatively easy control methods that are introduced in this chapter, slip can be kept at a satisfying level (well beneath 4%). This also means that torques in the GNT are always finite.

The controllability of the pushbelt GNT is not that easy to define In comparison to the easy control of the Torotrak GNt, the pushbelt GNT is very hard to control. However, the two control methods that are introduced in this chapter show some opportunities: The classical method, currently used by VDT gives bad results around the geared neutral point, but proves to be very useful outside the geared neutral point. VDT has already built a CVT with power split where this control method is used. The new method, in which the required ratio is defined, is too sensitive in the geared neutral point, however is able to use the kinetic energy of the engine to accelerate the vehicle. This is the same as for the Torotrak GNT and is very useful for fast acceleration.

Since both control methods have their own advantages and have the opposite reaction around the geared neutral point (insensitive vs. too sensitive). A combination of the two controllers should be able to give satisfying results.
Some extra possibilities are available (not analyzed in this research):

1. Combined Control of engine and variator
   A first introduction is given by the second method in which the required ratio is coupled to $T_e$. This relation can be expanded by introducing e.g. a specific trajectory for acceleration of the vehicle out of the geared neutral point.
   Note that this method does not work in the specific geared neutral point!

2. reproduction of slip and/or torque. This would simplify the controller severely. If slip can be reproduced, than eq. 7.1-7.3 can simply be calculated algebraically and the required controller forces $F_{pri}$ and $F_{sec}$ can be defined.

### 7.4 Constrains for control

Specifically, the constrains for control of the pushbelt GNT follow by combining the information in this chapter. This results in the following constrains:

1. A control method that is based on control of torque in the GNT

2. In the geared neutral point the controller must minimize slip in the variator to improve the efficiency of the driveline

3. Outside the geared neutral point the controller must act so that the GNT transmits all power that is generated at the engine to the wheels to improve the performance of the car

4. Close to geared neutral the controller must avoid high torque in the transmission by very gentle shifting and clamping.

In order to build such a controller, the following recommendations for the controller are given:

1. Find an algorithm or measurement to reproduce slip and/or torque in the GNT.

2. Define combined control methods for the variator and the engine:
   - In the geared neutral point pure control of the variator to minimize slip.
   - Close to the geared neutral point combined control to avoid overshoot in the required $T_{dreh}$.
   - Outside geared neutral point: combined control as used in the classical methods of VDT, so that almost all power is transmitted from the engine to the wheels or vice versa.

One thing should always be taken into account in the analysis of a geared neutral transmission: The goal of a GNT is to eliminate the clutch and build a system that provides higher efficiency. If this clutch is replaced by very inefficient systems or very complicated controllers, then the GNT becomes infeasible in an economic sense.
Chapter 8

Conclusions and recommendations

In a vehicles driveline, a geared neutral transmission (GNT) is a component that does not require clutches for driving of a vehicle. A geared neutral transmission always exists of a power split configuration with a variator. The system is only feasible if this variator works as a force closure system and thus controls torque in the transmission.

In order to define feasibility and constrains for control two GNTs are analyzed: a GNT with Toroid variator and a GNT with pushbelt variator. The results from there analysis show that a GNT with toroid variator has advantages over the other GNT when it comes to controllability. This is due to the kinematics of the toroid variator that an easy way of torque control.

The GNT with pushbelt variator that is analyzed, is a feasible solution for building a GNT. However, in order to achieve the same results as the GNT with toroid variator, a new controller must be build.

The controller that makes the GNT with pushbelt variator controllable, has the following constrains:

1. A control method that is based on control of torque in the GNT.
2. In the geared neutral point the controller must minimize slip in the variator to improve the efficiency of the driveline.
3. Outside the geared neutral point the controller must act so that the GNT transmits all power that is generated at the engine to the wheels to improve the performance of the car.
4. Close to geared neutral the controller must avoid high torque in the transmission by very gentle shifting and clamping.

In order to build such a controller, the following recommendations for further research can be given:

1. Find an algorithm or measurement to reproduce slip and/or torque in the GNT and/or:
2. Find combined control methods for the variator and the engine:
   - In the geared neutral point pure control of the variator to minimize slip.
   - Close to the geared neutral point combined control to avoid overshoot in the required \( T_{dref} \).
   - Outside geared neutral point: combined control as used in the classical methods of VDT, so that almost all power is transmitted from the engine to the wheels or vice versa.
Chapter 9

Bibliography


The following Patents are used as background (found at www.espacenet.com):


US 6122984, "Shaft phase control mechanism", 2000
Appendix A

Variators

A.1 Toroid variator

The toroid variator is physically displayed in fig. A.1.

In figure A.1 is shown that the variator exists of 6 rollers that are held between resp. 2 primary discs and 2 secondary discs. The whole construction is pretensioned by the pressure in the pressure chamber. At the bottom of the picture a piston/cylinder house is shown. This is the control module of the variator.

In figure A.2 a schematic view of the front of the variator is given:
Fig. A.2 shows that the rollers are forced onto the discs by the pressure in the pressure chamber. This pretension results in friction forces between the discs and the roller and this is the basis for power transfer from the primary to the secondary shaft. In fig. A.3 only one roller and one disc per shaft is displayed. It shows that a roller is placed between the primary disc and the secondary disc.

The roller has 3 degrees of freedom (DOF): 2 rotations around its principle axes and one translation. The translation is the only DOF that is controlled. Control takes place by varying the pressures in the cylinder, that work on both sides of the piston. As shown in fig. A.2, the roller is held between the primary and secondary disc by the pressure in the "pressure chamber". At the end of the discs, there is a pressure chamber that pretensions the whole construction (fig. A.2.).

Fig. A.3 shows when e.g. the primary disc is rotating clockwise, then the rotation of the secondary disc will be counterclockwise. The combination of fig. A.2b and A.3 show that the ratio of the toroid variator is depending on the roll of the roller over the surfaces of respectively the primary and secondary disc:
\[ \omega_{\text{sec}} R_{\text{sec}} = -\omega_{\text{pri}} R_{\text{pri}} \]  
(A.1)

\[ \frac{\omega_{\text{sec}}}{\omega_{\text{pri}}} = \frac{R_{\text{pri}}}{R_{\text{sec}}} = I_{\text{var}} \]  
(A.2)

Since the roller rotates around its 2\(^{nd}\) axis, a relation between the \( R_{\text{sec}} \) and the ratio \( I_{\text{var}} \) can be given:

\[ R_{\text{sec}} = \frac{2R_{r2\text{pri}} I_{\text{var}}}{1 + I_{\text{v}}} \]  
(A.3)

In which \( R_{r2\text{pri}} \) is the distance between the rollers 2\(^{nd}\) principle axis and the primary disc axis.

### A.1.1 Shifting of toroid variator

The shifting procedure in a variator is different from a normal transmission, since the transmission can shift continuously. In order to show how the variator shifts the following example is used.

**Example A1:**

Suppose that the toroid variator has a certain ratio \( I_{\text{var}} = 1 \), that allows the primary and secondary axis to run at certain speed \( \omega_{\text{pri}} = \omega_{\text{sec}} \). Suppose that this is a stable situation, so without any slip between roller and discs, this state is maintained. At a certain moment, the speed of the primary axis is increased.

The variator now has two options:

1. Do nothing and let the roller slip over one or both of the disc surfaces
2. Rotate the roller around his second principle axis (fig. A.3), in order to find a new, lower ratio (eq. A.2). In that new situation there is no slip between the discs and the rollers.

The question is now, which of the options is true. The answer lies in the amount of friction between the rollers and the disc. If there is little friction, the roller will start to slip, but if the friction is large enough, than the roller is going to shift towards low. The amount of friction can be controlled by the pressure chamber.

Suppose that there is sufficient friction, then the shifting follows as described underneath:

**Situation 1:** The primary disc and the secondary disc run at the same speed
A.4.1: $\frac{\omega_{\text{sec}}}{\omega_{\text{pri}}}$ = $I_{\text{var}}$

Situation 2: The primary disc runs faster than the secondary disc:

A.4.2 $\frac{\omega_{\text{sec}}}{\omega_{\text{pri,new}}}$ < $I_{\text{var}}$ ⇒ roller is rolling out of the center at secondary disc

a. The roller starts rolling faster at primary side and therefore it moves out of the center at the secondary side (see top view).

b. The secondary side of the roller ran out of center in b. and changed the vector of the friction force at that side (fig. A.4.3). This friction force consists of a vertical and a horizontal part. The vertical part results in a momentum over the 2nd axis of the roller. The horizontal part pushes the roller back to the center (fig. A.4.3):

c. As a result of the forces in c., the roller moves to a smaller radius at the primary disc and a higher radius at the secondary disc. This means (eq. A.2) that the ratio changes to low:
A.4.4. The variator shifts to low, which means that $R_{pri}$ increases and $R_{sec}$ decreases.

d The final result for the new variator ratio is: $\frac{\omega_{sec, new}}{\omega_{pri, new}} = \frac{R_{pri, new}}{R_{sec, new}} = I_{var, new}$

The previous example shows that the toroid variator shifts purely because of the mechanical balance of friction forces. The only control action that is taken is that the pressure in the pressure chamber is held at a sufficient level to prevent macro slip. It is very clear that the control unit "piston/cylinder" that is shown in fig. A.3 has no active part in this.

A.1.2 Transmission of torque

The transmission of torque is controlled by the piston/cylinder. Figure A.5 shows an exploded top view of the roller between the discs and the piston/cylinder that is attached to the roller via an axle. The forces that work on the roller due to the pressure in the piston/cylinder are plotted.

\[ \text{Fig. A.5a: Top view: Forces due to } F_{ctrl} \]

\[ \text{Fig. A.5b: Isometric view of forces} \]

Fig. A.5a,b visualize that the force from the piston on the center of the axle, causes a reaction force on both the primary and secondary side of the roller. Since the roller is held by the discs, it cannot accelerate or decelerate. Therefore Newton’s second law of motion is applied to calculate the mechanical balance at the center of the roller.
So it occurs that $F_{pri}(=\text{input side})$ is always equal to $F_{sec}(=\text{output side})$. See fig.A.5b for the connection between force and torque:

\begin{align}
T_{pri} & = F_{pri}R_{pri} \\
T_{sec} & = F_{sec}R_{sec}
\end{align}

(A.6) (A.7)

Since there is a direct connection between the forces $F_{pri}, F_{sec}$ and the controlled pressures from the piston, The following is equation is true:

\begin{align}
T_{sec} & = \frac{A_a}{2} \left( \frac{p_bA_i}{A_a} - p_a \right) R_{sec} \\
\text{and:} \\
T_{pri} & = I_{var}T_{sec}
\end{align}

(A.8) (A.9)

If necessary, than $R_{sec}$ can also be written as a function of $I_{var}$ and the center distance $R_{r2pri}$, which is the distance of the center of the roller to the center of the primary axle:

\begin{align}
I_{var} & = frac{R_{pri}R_{sec}}{R_{r2pri} x} \\
\Rightarrow \dot{r}_{sec} & = \frac{2R_{r2pri}I_{var}}{1 + I_{var}}
\end{align}

(A.10) (A.11)

Since it occurs that the variator is controlling the torque that it transmitting through it's roller, the variator is also referred to as a "torque ratio controlling " element, rather than a "speed ratio controlling " element, such as a fixed ratio gear is.

In the simulation model for the toroid variator which is used, is introduced according to the following flowchart:

Fig. A.6: Flowchart of toroid variator as used in simulation models
A.2 Pushbelt variator

The pushbelt variator works as described in chapter 5. In the computer model, this is implemented according to the flow chart of fig. A.7

In fig. A.7 is a internal feedback of $T_{pri}$ is plotted. This seems strange, because in chapter 4, this feedback has not been mentioned. VDT use this feedback in their own models, to define the safety factor that is used for definition of $k_{pks}$ (see chapter 4).

The forces $F_{pri}$ and $F_{sec}$ are minimized to 3000 [N] because the actuator pump that controls these forces has a minimum working pressure.
Appendix B

Form vs. Force closure

Components in a GNT differ from each other since the variator is a force closure system, that controls a specific ratio of force. The EGS and the fixed ratio gears behave like form closure systems. The difference between these systems has consequences for the modeling of a component. The next subsections describe each sort of system

B.1 Force closure

Definition force closure system:
The system has a fixed torque-ratio between input force and output force or input and output torque:

\[ I_{\text{torque}} = \frac{T_{\text{out}}}{T_{\text{in}}} \] (B.1)

With an additional control parameter \( F_{\text{ctrl}} \). This type of systems controls the amount of power that is transmitted from input to output.

The most simple example of a torque closure system is a clutch. A simple manual clutch has \( I_{\text{torque}} = 1 \).

A force closure system between inertia masses acts corresponding to the flowchart of fig. B.1 (compare fig. B.1 to fig. 5.1). A minimum of one extra input \( F_{\text{ctrl}} \) (eg. clamping force in a clutch) is necessary to define the relationship between the amount of torque and the difference between \( I_{\text{torque}} \) and \( I_{\text{speed}} \). In uncontrolled systems (eg. torque converter), this input \( F_{\text{ctrl}} \) is a constant.

In addition to figure B.1, the following analysis holds for the “force closure CVT”, if power losses are set to zero:

\[ P_{\text{out}} = P_{\text{in}} \]
\[ \omega_{\text{engine}} \cdot T_{\text{engine}} = \omega_{\text{vehicle}} \cdot T_{\text{vehicle}} \] (B.3)
\[ \frac{\omega_{\text{vehicle}}}{\omega_{\text{engine}}} = \frac{T_{\text{out}}}{T_{\text{in}}} \] (B.4)

Eq. B.4 shows that \( I_{\text{out}}(t) \) is a result of the speed ratio and is therefore not fixed.
Fig. B.1: Schematic view of driveline with CVT transmission. The transmission has force closure.

This leaves the function of $F_{ctrl}$ to be described as:

$$T_{cvt2engine} = f(F_{ctrl})$$  \hspace{1cm} (B.5)

Combination of eq. B.2-B.5 leads to eq. B.6-B.7, in which is shown that in a force closure system the amount of transmitted power is controlled by the controlled parameter $F_{ctrl}$:

$$P_{cvt,in} = \omega_{engine} T_{cvt2engine} = \omega_{engine} f(F_{ctrl})$$  \hspace{1cm} (B.6)

$$P_{cvt,out} = \omega_{vehicle} T_{cvt2vehicle} = \omega_{vehicle} \frac{f(F_{ctrl})}{I_{cvt}(t)}$$  \hspace{1cm} (B.7)

A dynamic analysis shows the same behaviour:

The dynamic behaviour of the system can be defined for both sides of the CVT individually:

$$J_{engine} \omega_{engine} = T_{engine} - T_{cvt2engine}$$  \hspace{1cm} (B.8)

$$J_{vehicle} \omega_{vehicle} = T_{earth} - T_{cvt2vehicle}$$  \hspace{1cm} (B.9)

And if power losses are set to zero, the combination of eq. B.8,B.9 and B.4 show that both sides can be lumped into one:

$$J_{engine} \omega_{engine} = T_{engine} - I_{cvt} T_{cvt2vehicle}$$  \hspace{1cm} (B.10)

$$J_{engine} \omega_{engine} = T_{engine} - I_{cvt} \left(-J_{vehicle} I_{cvt} \omega_{engine} + T_{earth}\right)$$  \hspace{1cm} (B.11)

$$J_{engine} + (I_{cvt})^2 J_{vehicle} \omega_{engine} = T_{engine} - I_{cvt} T_{earth}$$  \hspace{1cm} (B.12)

In eq. B.12 $I_{cvt} = I_{cvt}(t)$, thus not a fixed parameter.

This directly shows that the speed $\omega_{engine}$ can be increased without increasing $\omega_{vehicle}$, which is the same as setting $T_{cvt2vehicle}$ to match $-T_{earth}$. Furthermore, this means that $F_{ctrl}$ (which is the input for $T_{cvt2vehicle}$ (eq. B.5)) defines the amount of power that is transmitted through the transmission to the vehicle's inertia.

**Conclusion:** In a force closure system, the amount of transmitted power is defined by a controlled parameter $F_{ctrl}$ and not by the external torques $T_{engine}$ and $T_{earth}$.  

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Remark 1: The same behaviour is also true when power losses are taken into account. In that case, Eq. B.2 requires an extra efficiency factor. The rest follows naturally.

Remark 2: If at least one of the components in the driveline has force closure, then the whole system is described by force closure.

Remark 3: all CVTs that are currently built, have a force closure layout.

B.2 Form closure

Definition form closure system:

The system has a fixed ratio between the input speed and output speed:

\[ I_{\text{speed}} = \frac{\omega_{\text{out}}}{\omega_{\text{in}}} \]  \hspace{1cm} (B.13)

A form closure system cannot control the amount of transmitted power from input to output.

A fixed ratio gear with gearing wheels is a common example of a form closure system. A system with form closure can always be modeled as shown in figure B.2:

\[ W_{\text{vehicle}} = \frac{T_{\text{cvt2engine}}}{I_{\text{speed}}} \]  \hspace{1cm} (B.14)

Equation B.2 and B.3 are also true for a form closure system. However eq. B.4 must be rearranged, because the ratio of the fixed ratio gear is always a fixed value:

\[ J_{\text{engine}}\omega_{\text{engine}} = T_{\text{engine}} - I_{\text{cvt}}T_{\text{cvt2vehicle}} \]  \hspace{1cm} (B.15)

\[ J_{\text{engine}}\omega_{\text{engine}} = T_{\text{engine}} - I_{\text{cvt}} (-J_{\text{vehicle}}I_{\text{cvt}}\omega_{\text{engine}} + T_{\text{earth}}) \]  \hspace{1cm} (B.16)

\[ (J_{\text{engine}} + (I_{\text{cvt}})^2 J_{\text{vehicle}}) \omega_{\text{engine}} = T_{\text{engine}} - I_{\text{cvt}}T_{\text{earth}} \]  \hspace{1cm} (B.17)

In this case, \( I_{\text{cvt}} \) is a fixed value. Therefore the amount of power that is transmitted through the transmission is defined by \( T_{\text{engine}} \) and \( T_{\text{earth}} \).
Conclusion: In a form closure system, the amount of power that is transmitted through the transmission is defined by $T_{\text{engine}}$ and $T_{\text{earth}}$.

Remark 1: When power losses are set to zero, than the power that is transmitted through the transmission is not the same for both a form closure and a force closure system. The force closure system is able to decrease $I_{\text{out}}$ and prevent that the power from the engine is used to accelerate the vehicle. Then, all power from the engine is used to increase the speed of inertia $J_{\text{engine}}$ (potential energy).
Appendix C

Symbols and common values

first section gives central table of quantities. Following section describes specific values for each component in the simulated driveline.

C.1 Quantities and Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Unity (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>angular speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$J$</td>
<td>inertia mass (2nd moment)</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>$M$</td>
<td>inertia mass (1st moment)</td>
<td>kg</td>
</tr>
<tr>
<td>$T$</td>
<td>torque</td>
<td>N m</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
<td>N</td>
</tr>
<tr>
<td>$R$</td>
<td>radius</td>
<td>m</td>
</tr>
<tr>
<td>$I_T$</td>
<td>gear ratio based on torque</td>
<td>-</td>
</tr>
<tr>
<td>$I_\omega$</td>
<td>gear ratio based on speed</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency, 0 &lt; $\eta$ &lt; 1</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>N m$^2$/s</td>
</tr>
<tr>
<td>$b$</td>
<td>damping</td>
<td>N m s</td>
</tr>
<tr>
<td>$k$</td>
<td>stiffness</td>
<td>N m</td>
</tr>
<tr>
<td>$P$</td>
<td>power</td>
<td>W</td>
</tr>
<tr>
<td>$\mu$</td>
<td>coulomb friction coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$C_1$</td>
<td>torque depending loss of torque</td>
<td>-</td>
</tr>
<tr>
<td>$C_2$</td>
<td>speed depending loss of torque</td>
<td>N m s</td>
</tr>
</tbody>
</table>

C.2 specific values

The following tables describe specific values for each component. If no value is given, then this value is not fixed. Values for efficiency are based on Dubbel[9]. values $C_1$ and $C_2$ are calculated to be proportional with torque and speed respectively. Their values are chosen in such way that the efficiency of the part equals Dubbel[9] at 5000 [rpm] and 100Nm on the primary shaft.
Engine

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_e$</td>
<td>engine speed</td>
<td></td>
<td>rad</td>
</tr>
<tr>
<td>$T_e$</td>
<td>torque caused by combustion</td>
<td></td>
<td>Nm</td>
</tr>
<tr>
<td>$J_e$</td>
<td>inertia mass (2nd moment)</td>
<td>0.1</td>
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</tr>
<tr>
<td>$b_e$</td>
<td>damping</td>
<td>0</td>
<td>Nms</td>
</tr>
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Vehicle

The coulomb friction force is based on the sign of $\omega_{veh}$ and is maximized outside the model by $T_{veh,stick}$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{veh}$</td>
<td>vehicle speed</td>
<td></td>
<td>rad</td>
</tr>
<tr>
<td>$T_{external}$ or $T_{earth}$</td>
<td>torque caused by road obstacles</td>
<td></td>
<td>Nm</td>
</tr>
<tr>
<td>$F_{airfriction}$</td>
<td>Force caused by air friction</td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>$r_{wheel}$</td>
<td>wheel radius</td>
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<td>m</td>
</tr>
<tr>
<td>$\rho_{air}$</td>
<td>mass density air</td>
<td>1.2</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$A$</td>
<td>frontal area of vehicle</td>
<td>3</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$C_w$</td>
<td>air friction coefficient of vehicle</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>$M_{veh}$</td>
<td>inertia mass (1st moment)</td>
<td>1000</td>
<td>kg</td>
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Fixed ratio gear 1

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{fg}$ or $I_{g1}$</td>
<td>ratio based on speed</td>
<td>$\frac{62}{97}$</td>
<td></td>
</tr>
<tr>
<td>$\eta_{fg}$</td>
<td>efficiency</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>$C1_{g1}$</td>
<td>torque depending loss of torque</td>
<td>0 or 0.005</td>
<td></td>
</tr>
<tr>
<td>$C2_{g1}$</td>
<td>speed depending loss of torque</td>
<td>0 or $\frac{1}{1000}$</td>
<td>Nms</td>
</tr>
</tbody>
</table>

Epicyclic gearing set

The factors C1 and C2 are calculated towards the exit of the driveline, therefore on $T_a$. C2 is split in two parts: C2s for the sun and C2p for the planet.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{egs}$ or $E$</td>
<td>ratio based on speed</td>
<td>$\frac{67}{97}$</td>
<td></td>
</tr>
<tr>
<td>$\eta_{egs}$</td>
<td>efficiency</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>$C1_{egs}$</td>
<td>torque depending loss of torque</td>
<td>0 or 0.001</td>
<td></td>
</tr>
<tr>
<td>$C2s_{egs}$</td>
<td>speed depending loss of torque</td>
<td>0 or $\frac{1}{1000}$</td>
<td>Nms</td>
</tr>
<tr>
<td>$C2p_{egs}$</td>
<td>speed depending loss of torque</td>
<td>0 or $\frac{1}{1000}$</td>
<td>Nms</td>
</tr>
</tbody>
</table>
End gear

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_d$</td>
<td>ratio based on speed</td>
<td>$-\frac{14}{37}$</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_d$</td>
<td>efficiency</td>
<td>0.99</td>
<td>–</td>
</tr>
<tr>
<td>$C_{1d}$</td>
<td>torque depending loss of torque</td>
<td>0 or 0.005</td>
<td>–</td>
</tr>
<tr>
<td>$C_{2d}$</td>
<td>speed depending loss of torque</td>
<td>0 or $\frac{1}{1000}$</td>
<td>Nms</td>
</tr>
</tbody>
</table>

Torotrak GNT: chain gear

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_c$</td>
<td>ratio based on speed</td>
<td>$\frac{45}{37}$</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>efficiency</td>
<td>0.985</td>
<td>–</td>
</tr>
<tr>
<td>$C_{1c}$</td>
<td>torque depending loss of torque</td>
<td>0 or 0.0075</td>
<td>–</td>
</tr>
<tr>
<td>$C_{2c}$</td>
<td>speed depending loss of torque</td>
<td>0 or $\frac{2}{1000}$</td>
<td>Nms</td>
</tr>
</tbody>
</table>

Torotrak GNT: variator

Index "pri" or "ti" refers to side of toroid variator that is closest to the engine. "sec" or "to" refers to the other side.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{var,T}$</td>
<td>ratio based on torque</td>
<td>$-1.902 &lt; I_{var,T} &lt; -0.502$</td>
<td>–</td>
</tr>
<tr>
<td>$I_{var,u}$</td>
<td>ratio based on speed</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$R_{\var2pri}$</td>
<td>distance center roller to center primary axle</td>
<td>0.075</td>
<td>m</td>
</tr>
<tr>
<td>$\eta_{var}$</td>
<td>efficiency</td>
<td>0.96</td>
<td>–</td>
</tr>
<tr>
<td>$C_{1var}$</td>
<td>torque depending loss of torque</td>
<td>0 or 0.01</td>
<td>–</td>
</tr>
<tr>
<td>$C_{2var}$</td>
<td>speed depending loss of torque</td>
<td>0 or $\frac{4}{175}$</td>
<td>Nms</td>
</tr>
</tbody>
</table>

Pushbelt GNT: fixed ratio gear 2

Ratio $I_{fg2}$ in chosen in such way that the minimum ratio of the pushbelt-GNT equals the minimum ratio of the Torotrak GNT.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{fg2}$ or $I_{g2}$</td>
<td>ratio based on speed</td>
<td>$-\frac{24}{37}$</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_{fg}$</td>
<td>efficiency</td>
<td>0.99</td>
<td>–</td>
</tr>
<tr>
<td>$C_{1g1}$</td>
<td>torque depending loss of torque</td>
<td>0.005</td>
<td>–</td>
</tr>
<tr>
<td>$C_{2g1}$</td>
<td>speed depending loss of torque</td>
<td>$\frac{1}{1000}$</td>
<td>Nms</td>
</tr>
</tbody>
</table>

Pushbelt GNT: variator

The tables of $ki$ and $kpk$s are not discussed here. All indices "pri" refer to the side of the pushbelt variator that is closest to the engine. "sec" refers to the other side.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{var,T}$</td>
<td>ratio based on torque</td>
<td>$0.57 &lt; I_{var,T} &lt; 1.775$</td>
<td>–</td>
</tr>
<tr>
<td>$I_{var,w}$</td>
<td>ratio based on speed</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_{var}$</td>
<td>efficiency</td>
<td>0.96</td>
<td>–</td>
</tr>
<tr>
<td>$ta$ or $\lambda$</td>
<td>nose angle of pulley</td>
<td>7</td>
<td>degree</td>
</tr>
<tr>
<td>$\cos \phi$</td>
<td>cosine of $ta$</td>
<td>0.992</td>
<td>–</td>
</tr>
<tr>
<td>$\mu$</td>
<td>coulomb friction coefficient</td>
<td>fig. 4.4 or 0.06 (in 1de-model)</td>
<td>–</td>
</tr>
<tr>
<td>$S_f$</td>
<td>Safety factor (VDT)</td>
<td>1.3</td>
<td>–</td>
</tr>
<tr>
<td>$C1_{var}$</td>
<td>torque depending loss of torque</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>$C2_{var}$</td>
<td>speed depending loss of torque</td>
<td>0</td>
<td>$Nms$</td>
</tr>
</tbody>
</table>

**Extra values**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{transmission}$</td>
<td>stiffness GNT</td>
<td>1e6</td>
<td>–</td>
</tr>
<tr>
<td>$dt$</td>
<td>time delay in control pressure in variators</td>
<td>0.02</td>
<td>$s$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time constant in 1st order filter</td>
<td>0.01</td>
<td>$\frac{1}{s}$</td>
</tr>
<tr>
<td>$T_{cw}$</td>
<td>torque from CVT to vehicle</td>
<td></td>
<td>$Nms$</td>
</tr>
<tr>
<td>$T_{cvt}$</td>
<td>torque from CVT to engine</td>
<td></td>
<td>$Nms$</td>
</tr>
</tbody>
</table>
Appendix D

Static analysis GNT

This appendix discusses the Static analysis of the GNT of Torotrak and the Pushbelt GNT. These are almost the same, therefore Torotrak is fully discussed (section D.1) and for the pushbelt GNT only the differences are discussed (section D.2)

D.1 Low regime TOROTRAK

In this appendix, only the low regime of the Torotrak GNT is analyzed. High regime is not further analyzed. Details on this can be found in Torotak[6]

![Diagram of Torotrak GNT in low regime clutch]

Fig. D.1: Torotrak GNT, in low regime clutch C1 is closed and C2 open

When in figure D.1 the low regime clutch C1 is closed and the high regime clutch C_h is open, then the kinematic behaviour for a stationary running transmission can be calculated.

Speed in the transmission (from engine(e) to wheels(w)):

\[
\begin{align*}
\omega_1 &= \omega_\gamma = \omega_e \\
\omega_3 &= \frac{R_1}{R_3} = I_g \omega_e
\end{align*}
\]  

(D.1)  

(D.2)
In the toroid variator slip can occur. See also section 4.1 to see that this can be described according to equation D.3:

$$\omega_{T_o} = \omega_T \frac{T_R}{T_{T_o}} (1 - f(T_{T_o})) = I_{\text{var, w}} \omega_e$$  \hspace{1cm} (D.3)

$$\omega_2 = \omega_{T_o}$$  \hspace{1cm} (D.4)

$$\omega_4 = \frac{R_2}{R_4} \omega_2 = I_c I_{\text{var, w}} \omega_e$$  \hspace{1cm} (D.5)

Resulting: speed in all other parts in the GNT

$$\omega_p = I_g \omega_e$$  \hspace{1cm} (D.6)

$$\omega_s = I_c I_{\text{var, w}} \omega_e$$  \hspace{1cm} (D.7)

$$\omega_a = \omega_p + \frac{R_4}{R_2} (\omega_p - \omega_s)$$  \hspace{1cm} (D.8)

$$\omega_n = \omega_{\text{out}} = \omega_e I_g + \frac{1}{E} (I_g - I_{\text{var, w}} I_c)$$  \hspace{1cm} (D.9)

$$\omega_w = \omega_{\text{out}} I_d = I_d \omega_e (I_g + \frac{1}{E} (I_g - I_{\text{var, w}} I_c))$$  \hspace{1cm} (D.10)

The resulting $I_{\text{gnt}}$ is now defined as:

$$I_{\text{gnt}} = \frac{\omega_w}{\omega_e} = I_d (I_g + \frac{1}{E} (I_g - I_{\text{var, w}} I_c))$$  \hspace{1cm} (D.11)

Based on the standard values of appendix C, the following relations for speed can be drawn (condition without slip is assumed):

Fig. D.2: relation in speed for various $I_{\text{gnt}}$

Fig. D.2 shows that all relations are linear and that for $I_{\text{gnt}} = 0$ $\omega_w = 0$.

Since the toroid transmission has a self-aligning ratio, eq. D.10 can be rewritten to define this ratio:

$$I_{\text{var, w}} = -\frac{E \omega_w}{I_d \omega_e} + \frac{(1 + E) I_g}{I_c}$$  \hspace{1cm} (D.12)

$$I_{\text{var, w}} = -\frac{E I_{\text{gnt}}}{I_d I_c} + \frac{(1 + E) I_g}{I_c}$$  \hspace{1cm} (D.13)
Introducing the standard values of the Torotrak GNT for ratios, based on the previous equations the linear relation between $I_{gnt}$ and $I_{var,w}$ is plotted in fig. D.3.

![Fig. D.3: relation $I_{gnt} \leftrightarrow I_{var,w}$](image)

**Fig. D.3**: relation $I_{gnt} \leftrightarrow I_{var,w}$

Fig D.3 shows that the geared neutral ratio where $I_{gnt} = 0$ is related to:

$$I_{var,w} = \frac{E \times 0}{I_s I_c} + \frac{(1 + E) I_g}{I_c} = -1.477.$$  \hfill (D.14)

If slip is minimal, then it is possible to calculate all possible solutions for $\omega_w$ as function of $\omega_e$. The area is a measure for the range of control of the GNT. This is shown in fig. D.4 by the area between the 4 plotted lines. Within this area, $\omega_e$ and $\omega_w$ can be changed continuously.

For definitions of torques in the GNT, power losses are introduced by an efficiency factor $\eta$.

Review chapter 3 and appendix A, all torques are accounted by starting at the variator.

$$T_{T0} = 3F_{ctrl} R_{sec} = \frac{6 R_{2p} I_{var,T} F_{ctrl}}{1 + I_{var,T} I_c}$$ \hfill (D.15)

$$T_{Tf} = i_{var,T} T_{T0}$$ \hfill (D.16)

$$T_s = T_4 = \frac{I_s}{I_c} I_c T_{T0}$$ \hfill (D.17)

$$T_p = T_3 = -(1 + E) T_8$$ \hfill (D.18)

$$T_q = -\eta_{egs} (T_p + T_3) = -\eta_{egs} E T_8$$ \hfill (D.19)

$$T_{out} = -T_a$$ \hfill (D.20)

$$T_w = \frac{\eta_d T_{out}}{I_d}$$ \hfill (D.21)

In static situation, torque $T_e$ is a reaction torque to the controlled torque at the variator. Thus:

$$T_e = T_1 = \frac{I_g}{I_g} T_3 + T_{Ti}$$ \hfill (D.22)

**Remark 1**: If torque $T_e$ is not a reaction to the controlled torque at the variator, but another value, then there is no static situation and the analysis in this appendix is incorrect. This solution is discussed in the dynamic analysis (chapter 5,6), where $T_e \neq T_{cedin}$.
Combination of eq.D.11-22 leads to:

\[
T_w = \frac{\eta_n \eta_{egs} \eta_d E}{I_c I_d} T_{To}
\]  
(D.23)

\[
T_c = \left( -\frac{\eta_f (1+E)}{\eta_g I_c} + \eta_{var} I_{var,T} \right) T_{To}
\]  
(D.24)

Without power losses these results lead to figure D.5

Figure D.5 shows:

1. The torques in the transmission increase as \( I_w \) increases.

2. If \( T_{ti} > 0 \), then \( T_w < 0 \). Thus in order to accelerate the car in forward direction, \( T_{ti} \) must be negative.

3. If \( T_{ti} > 0 \), then \( T_{fgin} < 0 \). This represents circulating power in the torque-split configuration.

4. If \( I_{gmi} > 0 \), then \( \text{abs}(T_{ti}) < \text{abs}(T_{fgin}) \). Thus in the torque split configuration the power over the fixed ratio gear is higher than the power over the variable ratio gear.

5. If \( I_{gmi} = 0 \), then \( T_c = 0 \) and \( T_w \neq 0 \). Thus in geared neutral, there is no power going into the transmission from the engine side. However, there is a torque \( T_w \neq 0 \) measured at the drive shaft.

**Remark:** This behaviour can be explained by reviewing appendix A. If \( F_{ctrl} \neq 0 \), the piston/cylinder tries to pull the roller out between the discs. This is not possible, so in reaction (eq. 4.1) \( T_{ti} \neq 0 \) and \( T_{to} \neq 0 \). This pretensions the primary and secondary shaft of the variator. Since the transmission has a self-locking behaviour in the geared neutral point (eq. A??). This pretension in the variator's shafts pretensions the whole transmission. Thus \( T_w \neq 0 \)

6. Absolute values of torques are highest in the EGS and the end gear.
D.1.1 TOROTRAK: controlling torque.

In appendix A is shown that the toroid CVT is controlled with a pressured cylinder. By changing the pressures $P_a$ and $P_b$, it is possible to change the magnitude of force $F_{ctrl}$. In most situations it is required that $T_w$ can be controlled. The relation between $F_{ctrl}$ and $T_w$ can be found by rewriting eq. D.23 and introducing eq. A.7:

$$T_w = \frac{\eta_c \eta_{egs} \eta_d ER_{sec}}{I_c I_d} F_{sec}$$

(D.25)

D.1.2 Torotrak: Power losses

If power losses are calculated as efficiency losses of torque and slip is neglected, the following equations gives the overall efficiency loss over the GNT (C shows all standard values, based on Dubbel[9]). Efficiency loss from $T_{ti}$ to $T_w$ can be calculated as:

$$\eta_{T_{ti} \rightarrow T_w} = \eta_{egs} \eta_c \eta_d = 0.96$$

(D.26)

And from $T_{to}$ to input $T_e$:

$$\eta_{T_{to} \rightarrow T_e} = \frac{\eta_c \eta_{var}}{\eta_g} = 0.95$$

(D.27)

Finally from $T_e$ to $T_w$:

$$\eta_{gnt} = \left( \eta_{T_{ti} \rightarrow T_e} \right) \cdot \left( \eta_{T_{ti} \rightarrow T_w} \right) = 0.893$$

(D.28)

Eq. D.28 shows that for static analysis, the loss of torque from input to output is 10.7%. Because eq. D.28 is not related to $I_{gnt}$, this is also true in the geared neutral point. Thus, in the geared neutral point, power from the engine or the wheels is necessary to overcome these efficiency loss in the transmission. The amount of necessary power of the engine is related to $T_w$.

If in the geared neutral point $T_w$ is supposed to stay at a given level then eq. D.29 holds: $T_e$ must be 1% of $T_w$.

$$T_e = T_w \frac{2I_d}{\eta_d (1 + \eta_{egs})} \left( \frac{I_g (-1 - E)}{E \eta_g} + \frac{I_c I_{var}}{E \eta_c \eta_{var}} \right) = (0.01) * T_w$$

(D.29)

And the loss of power in the transmission is:

$$P_{loss} = P_c - P_w = \omega_e T_e - \omega_w T_w$$

(D.30)

$$P_{loss} = T_w (0.01 \omega_e - \omega_w)$$

(D.31)

And in the geared neutral point, where $\omega_{vehicle} = 0$, this leads to:

$$P_{loss} = \omega_e T_e = \omega_e * (0.01) * T_w$$

(D.32)
D.2 Pushbelt-GNT

This analyzes is almost the same as the Torotrak GNT. All equations are equal, however, $I_c$ must be replaced by $I_g2$ and $\eta_c$ by $\eta_g2$. This is necessary because a toroid variator always has a negative ratio, while a pushbelt variator has always a positive ratio.

Figure D.6 shows the layout that is used for this static analysis. Implementation of eq. D.1-D.13 lead to the following plot for speed in the variator:

In addition to eq. D.14 the geared neutral ratio for the pushbelt variator is (see appendix C for ratio values):

$$I_{\text{var,w}} = \frac{-E \times 0}{I_d I_g^2} \frac{(1+E)I_g}{I_g^2} = 1.600$$  \hfill (D.33)

Fig. D.7 shows the relation between $I_{\text{gmt}}$ and $I_{\text{var}}$. Fig. D.8 shows the area in which $\omega_c$ and $\omega_w$ can be changed continuously. Because the ratio coverage of the pushbelt variator is slightly smaller than the toroid variator, the control area is also slightly smaller (see appendix C and fig. D.4).
When all torques are plotted, almost the same figure as fig. D.5 is found. However, because of other values for $I_{\text{var}}$, some changes in sign can be found:

\[ T_{\text{sec}} = \frac{I_d I_g^2}{E \eta_d \eta_{\text{legs}} \eta_g} T_w \]  \hspace{1cm} (D.34)

and the relation between $T_{\text{sec}}$ and $F_{\text{pri}}, F_{\text{sec}}$ is given in eq. 4.7.

**D.2.1 Pushbelt GNT: Power losses**

Power losses in the pushbelt GNT is almost the same as for the Torotrak GNT. In the pushbelt GNT, the chain gear ($\eta = 0.98$) is replaced by a slightly more efficient fixed ratio gear ($\eta = 0.99$). However, the rollers are replaced by a pushbelt of which the efficiency will be lower, because of its internal friction.

In conclusion it is assumed that power losses are merely the same and therefore eq. D.26-32 also hold for the pushbelt GNT.
Appendix E

Dynamic Analysis Torotrak

Two simulations are represented: one without power losses in the transmission (all damping, \( C_1 \) and \( C_2 \) are set to zero) and with power losses. Only figures are given, see chapter 6 for more information and appendix C for specific values of parameters.

E.1 Without power losses

Simulation is carried out with the file "torotrak.md1" and the parameters for "no losses"
E.2 With power losses

Extra figures in addition to the simulation as described in section 6.2. Simulations are carried out with "torotrak.mdl" and the parameters with "losses".
speed of various components [rad/s]

ratios of variator and CVT, based on speed ratio [-]

controlled force on rollers [N]
relative slip of belt over pulleys [%]

velocity of vehicle \( (\dot{\mathbf{x}}, \dot{\mathbf{v}}) \)

power [W]
Appendix F

Dynamic analysis pushbelt GNT

Simulation are done for two types of controllers. They are described in chapter 7. The first section gives the result when simulating without power losses (except for slip in the variator). The second gives the simulation when power losses are taken into account.

F.1 Without power losses

F.1.1 Control to minimize slip

Simulations are done with the file "duwband.mdl" and the parameters "no losses".

![Graph showing torques of various components](image1)

![Graph showing speed of various components](image2)
ratios of variator and CVT, based on speed ratio [\cdot]

clamp forces on pulleys [N]

relative slip of belt over pulleys [%]
The same method, however, now starting outside the geared neutral point:

\[ \omega_{\text{tech}}(t_0) = 0.5 [\text{rad/s}] \]

The clamping force makes sure that slip does not rise. (because of the low value of \( T_e \) the minimum controlled clamping force does this already).
speed of various components [rad/s]

clamp forces [N]

ratios of variator and CVT, based on speed ratio [-]

clamp forces on pulleys [N]
F.1.2 Control of required \( I_{\text{var,}T} \)

The method is described in chapter 7. The simulations are done with the file "duwband2.mdl" and the parameters for "no losses"
Clamp forces on pulleys (N):

Relative slip of belt over pulleys [%]:

Velocity of vehicle ($m^2$):
F.2 including power losses

F.2.1 Control to minimize slip

Simulations are done with file "duwband.mdl" and parameters for "losses". The results are almost equal to the results of subsection F.1.1. Chapter 7 explains why: Due to the definition of power losses in the pushbelt variator, the variator has (in this model) zero power loss in a nonslipping condition. This is not corresponding with the real variator, however, under normal working conditions the slip introduces the efficiency loss of 4Dubbel[9].

F.2.2 Control of required $I_{\text{var,T}}$

Simulations are carried out with "duwband2.mdl" and the parameters for "losses". Just as in the previous section, the results are merely the same as for "no losses". The next figure shows the major difference: less overshoot than in the simulation without losses: