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STRUCTURAL OPTIMIZATION HISTORY AND STATE-OF-THE-ART

A.J.G. Schoofs

Eindhoven University of Technology, Eindhoven, The Netherlands

1. INTRODUCTION

The main purpose of structural analysis is to predict the behaviour of designs. The results of structural analyses are used to judge the adequacy of alternative trial designs with respect to established design criteria. The existence of general and reliable structural analysis capabilities coupled with the continuing growth of digital computing power, has led rather naturally to a remarkable increase in structural optimization research, and applications.

Development of the finite element method for structural analysis began in the mid-1950's with the availability of the "high speed" digital computer. For the first ten or more years, large sums of money were invested in research aimed at making this a practical tool for engineering analysis, so that by 1970 the method could be considered reasonably well developed and a variety of software was becoming commercially available. It was now possible to model very large and complex structures which before could not be considered by analytic methods.

The statement of Schmit [1], made in 1984, still holds. He posed: "Historically, the desire to reduce structural weight while preserving structural integrity, particularly in aerospace applications, has been a strong driving force behind the development of structural optimization methods. Today, the need for energy conservation in transportation systems via weight reduction provides further motivation for the application of structural optimization methods. The growing use of fibre composite materials in structures is likely to increase the demand for modern analytical tools that will make it possible to fully exploit the design potential offered by these new materials".

The optimum seeking methods are also known as mathematical programming techniques and are generally studied as a part of operations research. Operations research is a branch of mathematics which is concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solutions. Besides mathematical programming, well-defined other areas of operations research comprise stochastic process techniques and statistical methods.

The mathematical programming techniques are useful in finding the minimum of a function of several variables under a prescribed set of constraints. The stochastic process techniques can be used to analyse problems which are described by a set of

random variables having known probability distributions. The statistical methods enable one to analyse the experimental data and build empirical models to obtain the most accurate representation of the physical situation. There is a growing use of techniques from this last area to build efficient approximating numerical analyses models based on detailed finite element analyses of structures, (Schoofs, [2]).

Structural optimization methods began their development in roughly the same timeframe as the finite element method (linear programming was actually developed in the late 1940's). However, this technology and its development followed a much different path. First, optimization is a design tool which requires the repetitive analysis of candidate designs. Thus it is natural that this technology would lag somewhat behind analysis. Secondly, optimization methods themselves do not follow from the study of basic mechanics or even from the study of traditional design methods. Finally, the funding that has been spent on development of design methods is only a fraction of that which has gone to development of the analysis sub-problems.

The general state-of-the-art then is that finite element analysis is well in hand, while considerable work remains before optimization can be considered fully developed. It is, however, mature enough to begin incorporating the methodology into commercially available software. In the particular case of structural optimization, this can be done with some confidence because this is the one area of engineering design which has been consistently pursued for over thirty years. Structural optimization can be applied to solve many engineering problems. To indicate the wide scope of the subject, some typical applications are given below.

- Design of aircraft and aerospace structures for minimum weight.
- Design of civil engineering structures like frames, foundations, bridges towers, chimneys and dams for minimum cost.
- Minimum weight design of structures for earthquake, wind and other types of random loading.
- Optimal plastic design of structures.
- Optimum design of linkages, cams, gears, machine tools and other mechanical components, exposed to dynamic loading.
- Optimum design of musical instruments for best performance.
- Optimum design of prostheses, used in human surgery.
- Design of material handling equipment like conveyors, trucks and cranes for minimum cost.

2. THE OPTIMAL DESIGN PROBLEM

The design process can be described in many ways, but there are certain elements in the process that any description must contain: a recognition of a need, an act of creation, and a selection of alternatives. Traditionally, the selection of the "best" alternative is the phase of design optimization. In a traditional description of the design phases, recognition of the original need is followed by a technical statement of the problem (problem definition), the creation of one or more physical configurations (synthesis), the study of the configuration's performance using engineering science (analysis), and the selection of "best" alternative (optimization). The process concludes with testing of the prototype against the original need.

Such sequential description, though perhaps useful for educational purposes, cannot describe reality adequately since the question of how a "best" design is selected within the available means is influencing all phases where decisions are made.

An operationally useful definition of "design optimization" can be reached if we answer the questions:

1. How do we describe different designs?
2. What is our criterion for the "best" design?
3. What are the "available means"?

The first question is addressed in the discussion on design models, where a design is described as a system defined by design variables, parameters, and constants. The second question concerns decision-making models where the idea of "best" design is introduced and the criterion for an optimal design is called an objective. The objective function is sometimes called a "cost" function since minimum cost often is taken to characterize the "best" design. In general, the criterion for selection of the optimal design is a function of the design variables in the model.

We are left with the last question on the "available means". Designing obviously imposes limitations on what we may achieve. It will be clear that any design decision will be subjected to limitations imposed by the natural laws, availability of material properties, and geometric compatibility. On a more practical level, the usual engineering specifications imposed by the clients or the codes must be observed. Thus, the "available means" build a set of requirements, so-called design constraints, that must be satisfied by any acceptable design.

Design constraints are closely related to the choice of an objective function and design variables.

Summarizing we can say that design optimization involves:

1. The selection of a set of variables to describe the design alternatives.
2. The selection of an objective (criterion), as function of the design variables, which we seek to minimize or maximize.
3. The determination of a set of constraints, as function of the design variables, which must be satisfied by any acceptable design.
4. The determination of a set of values for the design variables, which minimize (or maximize) the objective, while satisfying all the constraints.

3. HISTORICAL DEVELOPMENT

The existence of optimization methods can be traced to the days of Newton, Lagrange and Cauchy. The development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus. The foundations of calculus of variations were laid by Bernoulli, Euler, Lagrange and Weierstrass. The method of optimization for constrained problems, which involves the addition of unknown multipliers became known by the name of its inventor, Lagrange. Cauchy made the first application of the steepest descent method to solve unconstrained minimization problems. In spite of these early contributions, very little progress was made until the middle of the twentieth century, when high-speed digital computers made the implementation of the optimization procedures possible and stimulated further research on new methods.

Schmit [3], in 1960 proposed a rather general new approach to structural optimization, which has served as a conceptual foundation for the development of many modern structural optimization methods. It introduced the idea and indicated the feasibility of coupling finite element structural analysis and non-linear mathematical programming to create automated optimum design capabilities for a rather broad class of structural systems.

It is not surprising that many researchers considered these structural synthesis concepts to be a revolutionary change in our approach to design. Because this promising tool was so new, with so much development required to establish the methodology, it was enthusiastically pursued.

By the late 1960's, however, it was becoming apparent that structural synthesis was not being embraced by the professional community, as many people expected it would be. Some plausible explanations can be offered. First, design is far more complex than analysis, and at that time the finite element method was just becoming generally accepted after approximately fifteen years of development. A new design methodology takes longer to gain general acceptance. Second, structural synthesis represents an integration of engineering and operations research disciplines. Because mathematical programming methods were unknown to the vast majority of engineering researchers, educators and practitioners, it was unreasonable to expect immediate and widespread acceptance.

Each explanation was reasonable and required only time and patience to overcome. It was also becoming recognized, however, that there might be a fundamental limit of this new technology. The simplest problem often needed to be analyzed hundreds of times during optimization. If this analyses were time-consuming, as is often the case for large finite element models, the cost of optimization quickly became prohibitive. Although the generality of mathematical programming offered a most attractive design tool, practicality dictated that it was limited to problems defined by only a few design variables.

Furthermore, an alternative approach was offered, and was presented in analytical form by Prager, et al, (1968, [4]) and in numerical form by Venkayya, et al. (1968, [5]). This concept became popularly known as the "Optimality criteria" approach.

The optimality criteria approach begins with the same general statement of the design problem; however, rather than working directly to minimize the objective function (weight, for example), one specifies a criterion such that if it is satisfied, subject to the constraints, then the design is defined as optimum. The essence of the optimality criteria approach is first to establish the criterion that defines the optimum and then devise an efficient recursive formula that leads, iteratively, to the desired solution.

A special case is the design of statically determinate structures subject to stress limits only. Here it is required that each member will be fully stressed under at least one loading condition (or will be at its minimum gage).

Although the optimality criteria approach was largely intuitive, it was shown to be quite effective as a design tool. Its principal attraction was that the method was easily programmed for the computer, was relatively independent of problem size, and usually provided a near-optimum design with as few as 15 detailed structural analyses. This last feature represented a remarkable improvement over the number of analyses required for mathematical programming methods to reach a solution.

The strengths of the two methods suggested a natural separation of the design problem, where optimality criteria would deal with a large number of design variables and mathematical programming would solve the component-design problem. This approach was pursued with success by Sobieszcanski et al (1972, [6]) in the design of fuselage structures. This work is important, not only because it combined both methods, but also because it seemed to build a common ground where researchers in both areas began to look more closely for a fundamental relationship between the methods.

In that same time period a start was made to investigate "approximation concepts" as a mechanism to improve the efficiency of using mathematical programming techniques.

Schmit, et al. (1973, [7]) presented a concise statement of the approximation concept approach to structural synthesis using mathematical programming.

The approximation to the original problem having been created, the approximating functions are used in the optimization. Once the optimum solution to the approximate problem is found, a precise finite element analysis is performed and a new approximation is created. In this fashion, the final optimum is obtained iteratively. Thus a technique is provided in which all the features of the original problem are retained in such a way that a sequence of approximate optimizations leads to a precise solution. Because the approximate problem requires little effort for function and gradient evaluation, mathematical programming techniques can be used for this subproblem.

During the late 1970's development continued in both optimality criteria and mathematical programming approaches to structural optimization. In terms of understanding the automated design process, perhaps the most significant work was in the area of reconciling the mathematics of the two basic concepts. The work of Fleury, et al. (1977, [8]) offers fundamental insight into the mathematical basis of both approaches and, in fact, shows a common basis in the duality of the original problem statement. This work shows that optimality criteria are valid for a mathematically separable problem and, as such, may be viewed as a special case of mathematical programming.

4. STATE-OF-THE-ART

For practical design, only two criteria are meaningful. First, does the optimization program reliably achieve a near-optimum design (a precise optimum is seldom meaningful) and second, does the program use few enough function (and gradient) evaluations to be economically usable for the design task at hand?

The state-of-the-art in mathematical programming is such that engineers should no longer find it necessary to develop their own programs. It should be expected, however, that with experience, they may wish to tailor an existing code to solve their particular design problem more efficiently.

The second essential ingredient to structural synthesis is the analysis capability. It is assumed that the engineer is familiar with and will use a finite element program in design, although for specific cases other analysis techniques may be employed. The state-of-the-art in finite element analysis is becoming quite advanced. However, there is a missing ingredient, namely, the ability to obtain gradient (sensitivity) information as part of the analysis, even though the technology for calculating this information is well in hand.

The final and most critical ingredient to structural synthesis is the mechanism by which the analysis and optimization programs communicate. The most direct approach is to simply couple the optimization code to the analysis code. Any gradient information would be calculated by finite difference, and each function evaluation required by the optimization program would be a completely new analysis of the structure. This "black box" approach is only acceptable if the analysis stages requires very little computing time.

For the design of more complex structural systems, the cost of repeated analyses usually precludes the black box approach. Here the first step is to incorporate gradient computations into the finite-element-analysis code. It is conceptually straightforward to provide this information for stress, displacement, and frequency constraints. Using the gradient information approximations of the objective and constraint functions can be

defined. By generating the approximate problem, one drastically reduces the computational resources needed to reach the solution.

Although it is unreasonable to predict the precise form of the structural synthesis discipline of the future, it is possible to identify some needs, recognizing that this prediction, too, is speculative and incomplete.

In the mathematical programming discipline, there is a need for development of algorithms that are efficient for the solution of large-scale nonlinear programming problems. This will emphasize the need for high-quality approximations in structural synthesis. Especially the problems where the structural analysis problem in itself is non-linear ask for dedicated algorithms to efficiently solve the optimization problem. Recently, Haftka (1989, [9]) presented a paper in this research field.

In the area of finite element analysis, there is a lot of work to do in providing gradients for shape design variables in large-scale finite element codes. This is, however, primarily an economic issue rather than a technology question, and the capability is already coming available in some large-scale finite element codes.

Regarding development of methodology, analysis of composite materials (particularly failure criteria) remains an important issue. Both time-dependent and large-deformation nonlinear analysis, for use within the structural synthesis framework, may well become another important research area. As another example, the questions of stochastic loading and time-parametric constraints need to be addressed within the structural analysis/synthesis context far more than in the past.

The means by which mathematical programming and analysis codes are integrated into a structural synthesis capability will surely continue to be an area of intense research and development. Several areas can be identified where significant progress may be expected. First is the development of well defined and standardized databases which can serve as interfaces between analysis and optimization codes. Databases for finite element analyses are well developed. In such databases the requirements for the synthesis tasks should be incorporated, too. Here, an important task is the geometry definition of the structure. At the moment magnificent geometric modeling programs are available. In general, however, there is just a little missing link between the parameters describing the geometry and design variables as needed for shape optimization. Such links should be established and incorporated in the database.

The combinatorial, or topological, problem is another intriguing research area, where, in addition to treating member sizes and the shape of the structure as variable, one determines the actual topology of the structure. There are recent interesting developments in this area (Bendsøe, 1989, [10]).

Finally, there has been a dramatic increase in the past decade in the use of active control systems to improve structural performance. Now, at several places research is in progress on developing methods for the simultaneous (integrated) design of the structure and the control system. Recent papers in this field came from Rao et al. (1990 [11]) and Manning et al. (1990, [12]).

This short list of possible future developments in structural synthesis is by no means complete, but it does indicate the large amount of effort that lies ahead. Contrary to the sometimes expressed view that the computer will eliminate engineering jobs, it appears that there is ample work for the future!

As pointed out in the introduction, the concept of structural synthesis using mathematical programming offered a new design philosophy that only today is beginning to be broadly applied.

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