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van Dongen, L.A.M.; Visscher, W.H.M.

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Theoretical Prediction of Electric Vehicle Energy Consumption and Battery State-of-Charge During Arbitrary Driving Cycles

by L. A. M. van Dongen (Eindhoven University of Technology), R. van der Graaf (Eindhoven University of Technology), W. H. M. Visscher (Eindhoven University of Technology) and H. C. J. Zeegers (Eindhoven University of Technology).

Battery State of Charge Model for Driving Cycle Operation

by W. Visscher and L. A. M. van Dongen

1. INTRODUCTION
The actual performance of an E.V. depends on the capability of the battery to meet the power requirements of the drive train. Operating modes differ widely and the vehicle characteristics vary with each model. Therefore, normalized drive cycles were proposed based on analysis of traffic patterns and several duty cycles are now in use to test battery performance for a given type of vehicle. Several attempts have been made to give also a model for the battery. Due to the chemical and physical processes that occur in the battery, its behaviour is more difficult to describe by an accurate model; moreover the battery characteristics change with time. With models for the components of the drive train and the battery, computer simulation can be carried out to study the interaction of battery and drive train and to predict energy use, vehicle performance and operating range.

This paper will describe a battery model and compare calculated state of charge values with experimental data.

2. THE STATE OF CHARGE
The amount of energy that the battery can deliver is determined by its current-voltage characteristic which in turn depends upon the state of charge. The state of charge of a fully charged battery is well defined; the concept of complete discharge depends on the discharge current due to the fact that in a battery the available capacity decreases with higher current. Hence the state of charge (S) at time t during discharge with current I must be related to the capacity \( C_t \) at current I [1].

\[ S = 1 - \frac{I_t}{C_t} \]  \hspace{1cm} (1)

With the Peukert relation

\[ I^n \tau = \text{constant} \]  \hspace{1cm} (2)

where \( \tau \) =time required for complete discharge at current I

\( n = \text{number, depending on the battery type,} \)

\( 1.2 < n < 1.4 \)

\( C_t \) can be expressed in the capacity \( (C_N) \) at standard rate \( (I_s) \)

\[ C_t = C_N \left( \frac{I_s}{I} \right)^{n-1} \]  \hspace{1cm} (3)

The state of charge at any current is then related to the standard capacity with

\[ S = 1 - \frac{I}{C_N} \left( \frac{1}{I_s} \right)^{n-1} \]  \hspace{1cm} (4)

3. BATTERY DISCHARGE MODEL
Mathematical models for porous electrodes have been developed to describe the extent of utilization of a battery plate as a function of rate of discharge, involving structural changes during the discharge process [2, 3, 4, 5]. These are derived from the kinetic relationship between current density and electrode potential, taking into account mass transfer processes. The complete battery behaviour is often described by the current-voltage relationship of Shephard [6].
\[
E = E_0 - k \frac{Q I}{Q - \eta} - R_E I
\]

(5)
in which:
- \(K\) = polarization parameter
- \(Q\) = amount of available active material
- \(R_E\) = electrolyte resistance
- \(E_s\) = constant voltage

This equation has been derived assuming a linear relationship between current and potential at both electrodes. However, such a behaviour is a priori restricted to very low polarization conditions. Moreover, to fit the experimental data with eq. (5) a negative value of the resistance had to be chosen. This inconsistency was recognized by Shephard and attributed to the empirical nature of the equation.

During discharge with electric vehicle duty cycles high polarization conditions will prevail. Therefore a current-voltage relation will be derived which is applicable to high polarization conditions and hence is not valid at very low current or at \(I > 0\).

At the two electrodes 1 and 2 of the battery the discharge process takes place via cathodic reaction at electrode 1 and anodic reaction at electrode 2; this can be expressed by the general electrochemical rate equations.

at electrode 1: \(p \, \text{OX}_1 + ne^- \rightarrow p' \, \text{RED}_1\)

at electrode 2: \(q \, \text{OX}_2 + ne^- \rightarrow q' \, \text{RED}_2\)

where \(\text{OX}_1\) and \(\text{RED}_1\), stand for the concentration of dissolved species at electrode 1 and 2 respectively.

\(p, p', q, q'\) = stoichiometric coefficients

\(n\) = number of electrons

At the two electrodes of the Pb acid battery these processes are:

at electrode 1: \(\text{PbO}_2 + 3 \, \text{H}^+ + \text{H}_2\text{SO}_4 + 2e^- \rightarrow \text{PbSO}_4 + 2 \, \text{H}_2\text{O}\)

at electrode 2: \(\text{Pb} + \text{HSO}_4^- \rightarrow \text{PbSO}_4 + \text{H}^+ + 2e^-\)

Under the conditions that the electron transfer occurs rapidly and that mass transfer to and in the pores of the electrode limits the rate of the reaction, the overpotential \(\eta\) for the reaction at electrode 1 is given by

\[
\eta = \frac{RT}{nF} \ln \left( \frac{C_{\text{OX},1}}{C_{\text{OX},1,t=0}} \right)^p
\]

(6)

and similarly at electrode 2

\[
\eta = -\frac{RT}{nF} \ln \left( \frac{C_{\text{RED},2}}{C_{\text{RED},2,t=0}} \right)^q
\]

(7)

where:
- \(\eta\) = overpotential \([V]\)
- \(R\) = gas constant \([J\text{ mol}^{-1} \text{ K}^{-1}]\)
- \(T\) = absolute temperature \([K]\)
- \(F\) = Faraday constant \([C\text{ mol}^{-1}]\)
- \(n\) = number of electrons

For a battery plate the concentration term \(c_{\text{OX},1,t=0}\) can be considered to be equivalent to the total amount of charge that is available at the fully charged plate 1, whereas \(c_{\text{OX},1}\) is the charge remaining after discharge with current \(I\) during time \(t\), so

\[
c_{\text{OX},1,t} = c_{\text{OX},1,t=0} - I t
\]

(8)

With \(C_{\text{OX},1}\) = capacity of electrode 1 at current \(I\) we have

\[
\frac{c_{\text{OX},1,t}}{c_{\text{OX},1,t=0}} = 1 - \frac{I t}{C_{\text{OX},1}}
\]

(9)

i.e. \(\frac{c_{\text{OX},1,t}}{c_{\text{OX},1,t=0}}\) represents the state of charge \(S_{\text{OX},1}\) of electrode 1.

Similarly \(\frac{C_{\text{RED},2}}{C_{\text{RED},2,t=0}}\) = \(S_{\text{RED},2}\) of electrode 2.

The total cell voltage \(E\) during discharge is given by the algebraic sum of the two electrode polarizations:

\[
E = E_{\text{eq},1} + \eta_1 - (E_{\text{eq},2} + \eta_2) - I R_E
\]

(10)

in which \(E_{\text{eq},1,2}\) = equilibrium potential of the electrode reaction 1, respectively 2

\(R_E\) = electrolyte resistance.

If both electrodes have the same capacity \(C_{\text{OX},1}\) = \(C_{\text{RED},2}\), then \(S_1 = S_2 = S\) and we can write for eq. (10) with substitution of (6), (7), (9)

\[
E = E_{\text{eq},1} + \frac{(p + q) \, R T}{2F} \ln S - I R_E
\]

(11)

i.e. the cell voltage of a fully charged battery and determined by the \(\text{HSO}_4^-\) concentration.

The electrolyte resistance \(R_E\) is in principle a function of the state of charge.

Eq. (11) describes the cell voltage during discharge with current \(I\) in dependence of the state of charge. It should be noted that this equation is restricted to high polarization conditions and hence is not valid at very low current or at \(I > 0\).

To establish the parameters of eq. (11) discharge curves were recorded at a Pb acid battery at various voltage. The battery was a Varta electric vehicle battery, 6 V, type 240-15 with nominal capacity \(C_0\) = 180 Ah. Capacity measurements as function of I gave a value of \(n = 1.26\) for the Peuckert relation (2).

After each discharge the battery was charged with 20 A and finally with 6 A until the specific gravity was constant. From the data E-I plots were constructed at constant S, with S calculated according to eq. (4). This is represented in Fig. 1.

Fig. 2 shows the electrolyte resistance as a function of the state of charge measured by discharging the battery at \(C_0\) rate to decreasing states of charge.

When the results of Fig. 1 are represented as a plot of E vs. \(S\) (Fig. 3) a linear relationship is obtained and the slope of the curves is independent of the current. This is in agreement with eq. (11). The observed slope was found to be 0.26 V.

About the same value was found when voltage - state of discharge plots given by Schleuter [7] for a tubular battery were replotted.

Fig. 2 shows that \(R_E\) does not vary significantly for \(1 > S > 0.6\), so from eq. (11) it would follow that the slope of the E-I plot for high S is independent of S and is equal to \(R_E\).

Though the experimental lines are indeed parallel, the slope is about \(2 \times R_E\) (At \(S = 1\) \(R_E = 1.37\, m\Omega\)). This can be explained by the resistance of the electrolyte in the pores which...
is not measured during steady state experiments of fig. 3 but will contribute during actual discharge.

The above results show that the discharge behaviour at high polarization can indeed be described by a rather simple relationship.

4. STATE OF CHARGE OF THE BATTERY DURING DUTY CYCLE OPERATION

4.1. Model
When the discharge of a battery takes place along the pattern of a duty cycle, the current changes rapidly, moreover regenerative braking is involved. To account for this the state of charge must be calculated for small time intervals \( \Delta t \) during which \( I \) is considered to be constant, hence during discharge:

\[
S = 1 - \frac{I \Delta t}{C_n} \left( \frac{I}{I_n} \right)^{-1}
\]  

(12)

whereas during charge the incremental change of state of charge \( \Delta S \) during a period \( \Delta t \) is given by [1]

\[
\Delta S = \frac{I_g \Delta t}{C_{ld}}
\]

(13)

or with (2)

\[
\Delta S = \frac{I_g \Delta t}{I_d} (1 - S)
\]

(14)

(Subscript c, d refers to charge respectively discharge).

A computer program was written to calculate the state of charge with eq. (11) and (13) after discharge with a given duty cycle, using the experimentally established \( E-I \) curves at constant \( S \). The program calculates also the cell voltage and current during the duty cycle.

4.2. Battery power schedules
These simulation results were compared with the actual battery performance during duty cycle operation. The load cycle experiments were carried out at a 6 V battery with a machine convertor, consisting of an induction machine coupled to a 20 V - 400 A DC machine [8]. The total battery requirement of a vehicle following a velocity profile was calculated for the vehicle being built by the Eindhoven Electric car group. The main drag forces to be overcome are given by

\[
F_v = f_1 g M + \frac{1}{2} \rho C_A v^2
\]

(15)

(Zie verder pag. 100)
The results, given where the parameters have the following meaning and specific value:

\[ F_{st} = \text{drag force due to tire hysteresis and wind resistance} \]

\[ f_r = \text{coefficient of rolling resistance} \]

\[ g = \text{gravitational acceleration} \]

\[ M = \text{vehicle mass} \]

\[ Q = \text{air density} \]

\[ C_d = \text{aerodynamic drag coefficient} \]

\[ A = \text{frontal surface area of the vehicle} \]

\[ v = \text{vehicle speed} \]

Substitution of these values in eq. (15) gives

\[ F_{st} = 264.87 + 0.488 v^2 \]

The total tractive effort of the vehicle \((F_t)\) is equal to:

\[ F_t = F_a + F_s = 264.87 + 0.488 v^2 + M a \]

The wheel power requirement (in Watt) can be represented as

\[ P = [264.87 + 0.488 v^2 + M a] v \]

Starting from this equation, the battery power has been determined assuming the average motor and gearbox efficiency to be 80 and 90% respectively. The battery behaviour was investigated during three types of duty cycles viz. the European cycle, the SAE J227 aD cycle and the THE cycle. The first two cycles are standard velocity versus time profiles; fig 4 and 5 show the power profiles, calculated for the total battery pack (144 V).

The THE cycle was chosen as a representative of actual duty cycles, which have been recorded in typical Dutch cities with the aid of a DAF 31 outfitted with speed sensors and torque transducers at the rear wheel axles. Conversion of the results with respect to the estimated mass and drive train efficiency of the Eindhoven Electric Vehicle resulted in the battery power for the total battery pack profile indicated in fig. 6. The duration of one cycle is 20 minutes in contrast with the usually shorter cycle time of the standard duty cycles. Table 1 summarizes some duty cycle specifications.

<table>
<thead>
<tr>
<th>Duty cycle specifications</th>
<th>EUR.</th>
<th>SAE J227 aD</th>
<th>THE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average vehicle energy [Wh/km]</td>
<td>145</td>
<td>189</td>
<td>140</td>
</tr>
<tr>
<td>Average vehicle speed [km/h]</td>
<td>18.3</td>
<td>44.7</td>
<td>24.3</td>
</tr>
<tr>
<td>Distance covered per cycle [km]</td>
<td>1010</td>
<td>1516</td>
<td>8.1</td>
</tr>
<tr>
<td>Duration of 1 cycle [s]</td>
<td>198</td>
<td>122</td>
<td>1200</td>
</tr>
<tr>
<td>Stop/s [km]</td>
<td>3</td>
<td>0.66</td>
<td>9</td>
</tr>
<tr>
<td>Idling time [%]</td>
<td>29.3</td>
<td>20.49</td>
<td>10.54</td>
</tr>
<tr>
<td>Charge recuperation [%]</td>
<td>19.4</td>
<td>10</td>
<td>22.2</td>
</tr>
</tbody>
</table>

4.3. Voltage-current characteristics

E-I diagrams at constant \(S\) for the charging process were obtained from constant charging curves at various \(I\) starting with a battery discharged to \(S = 0\) with \(I_a\). The results, given in fig. 7, represent only the E-I curves for which the charge efficiency is 100%. Due to concurrent water electrolysis, the charge efficiency becomes less than 100% for \(E \geq 2.35\) V per cell. To account for this in eq. (13) the charge efficiency factor must be introduced and E-I plots for \(S > 0.6\) will be presented later.

4.4. Comparison of test- and simulation results

The European and SAE J227 aD cycle tests

To avoid the voltage range where charging might be inefficient, the calculation of the battery performance during the European and the SAE J227 aD duty cycle was started at \(S = 0.6\). This was experimentally realized by discharging the battery (Varta electric vehicle battery 6 V, 240.15 nominal capacity \(C_s = 180\) Ah), during 2 hr at \(C_s\) rate. The battery was then subjected to a number of cycles (European or SAE) and thereafter the rest capacity \((C_R)\) was measured at the \(C_s\) rate.

**Results:**

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Experimental</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>European cycle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netto discharge during 60 cycles (Ah)</td>
<td>59.8</td>
<td>63.1</td>
</tr>
<tr>
<td>(C_{Ref}) (Ah)</td>
<td>36.0</td>
<td>35.1</td>
</tr>
</tbody>
</table>

Fig. 4 Battery power and vehicle speed for European duty cycle

Fig. 5 Battery power and vehicle speed for SAE J227 aD duty cycle
SAE J 227 aD cycle
Netto discharge during 30 cycles (Ah) 65.5 66.5
C\textsubscript{Rest} 24.0 13.8

The experimental and calculated voltage and profiles during the 60th European cycle and during the 30th SAE cycle are given in fig. 8 and 9 respectively.

The battery voltage during discharge agrees within 0.1 V with the computed value but the experimental data during charging are lower, indicating a retarded battery response such that very rapid current changes are less effective.

Calculation of the state of charge shows that S = 0 will be reached after 55 SAE cycles, i.e. an operating range of 83 km. This is in agreement with the experimental observation that the discharge could be continued during about 53 cycles before the power delivered by the battery at the highest discharge peak was 10% less than demanded by the duty cycle.

THE cycle tests
The average current of the THE cycle is ca. 33 A, the maximum current during discharge is ca. 200 A, during charge 160 A.

The battery could meet the duty cycle demands during 13 cycles (i.e. operating range 105.3 km).

After the battery had been discharged with 13 cycles C\textsubscript{R} was

![Picture 1](image1)

![Picture 2](image2)

![Picture 3](image3)
determined at \( C_s \) rate. The experimental \( \text{CR} \) was found to be \( 31 \pm 5 \text{ Ah} \), while the calculated \( \text{CR} \) was 19.2 Ah.

Fig. 10 shows the cell voltage \( (E_D) \) at the highest discharge peak and the cell voltage \( (E_C) \) at the highest charge current peak during 13 cycles and the cell voltage \( E_R \) at \( I = 0 \) at the end of each cycle. In the figure the computed data are given from 8th to 13th cycle.

5. CONCLUSIONS

The effect of state of charge of a battery upon the current voltage characteristics was described by a simple relation. Calculation of the state of charge during duty cycle discharge was found to agree within 7% with experimental results and the actual battery voltage during electric vehicle operation agrees with the simulated performance. With this model matching of power train and battery can be evaluated (9) and energy use and operation range can be predicted.

REFERENCES

[1] K. E. White, Society of Automotive Engineers, paper 78216

The Eindhoven Experimental Electric Vehicle: Vehicle Design and Drive Train

SUMMARY

At Eindhoven University of Technology a multidisciplinary team of chemical, electrical and mechanical engineers is collaborating on construction of an electric commuter car/van. A VW-Golf which concept appears to be very suitable for this purpose, has been electrified. Car-body and rear suspension were modified thus that a rapidly exchangeable battery pack could be placed in a central box. Various ways of controlling the powerflow from the 15/33 kW Siemens dc-motor to the wheels is tested in this vehicle. Three systems, which are under construction, are described:
- battery switching, field weakening and a fixed ratio transmission
- battery switching, field weakening and automatic gear-shifting
- fully electronic control by means of choppers.

by L. A. M. van Dongen and R. van der Graaf

1. Introduction

During the last decade the importance of the development of electric road vehicles has widely been recognized. In the beginning much effort has been displayed on the construction of electrically driven buses and vans for a variety of reasons. A group of interested persons at the Eindhoven University of Technology discerned the challenge which was put in this field by the passenger car as a replenishment of those activities. Especially in this application some features of the electric drive, such as battery weight, energy-efficiency of the drive line, selection and construction of components to be

1) Paper gepresenteerd tijdens 'Drive Electric 1982' te Amsterdam