Linear identification of nonlinear systems

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Linear Identification of Nonlinear Systems

W.A.P. van den Bremer,
T.M.A. Marechal, R.P.A. Vugts

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Chapter 1

Introduction

In centrifugal compressors a type of unwanted dynamic behavior that can occur is surge. Surge is a large amplitude periodic oscillation of the total annulus averaged mass flow through the compressor. During surge, the compressor experiences a series of rapid flow reversals. This can cause damage to the compressor, can decrease the efficiency of the system and it makes the stable operating range smaller, \cite{AH93}. Surge is of course undesired, so one wants to have a model of the system that is as accurate as possible. With this model one can predict whether undesired behavior will occur at certain circumstances. This model can also be used to design a stabilizing controller.

A widely used model of the dynamic behavior of a compression system is the so-called Greitzer model\cite{Gre76}. The behavior of this model resembles the behavior of the real compression system very well. One of the problems of the Greitzer lumped parameter model is the fact that its local dynamic behavior cannot always be determined uniquely from measurement data. \cite{HJSS04}. Therefore it is desired to find an accurate model of the local dynamic behavior of the centrifugal compression system.

Several linear identification methods will be used to find a model that resembles the nonlinear system as accurately as possible. Because it is very expensive and time-consuming to do many identification measurements on the real compression system, the simulation model of Greitzer lumped parameter system will be used to test and evaluate the identification methods.

In the future, the identification methods discussed in this report can be applied to measurement data from the actual compression system. Hopefully at least one of the identification methods will result in a model of the real centrifugal compressor system that gives more accurate local dynamic behavior.

Chapter 2 starts with a description of the centrifugal system and the dynamic Greitzer model will be derived for this system. In Chapter 3 several aspects of system identification are discussed. In Section 3.1 some general aspects and limitations of identification of the compressor are presented. Section 3.2 proposes criteria, which an identification method for this system should fulfill. In Section 3.3 several interesting identification methods are described. In Section 3.4 an inventory of identification methods is presented. In Section 3.4 the methods are evaluated and compared on the basis of the proposed criteria. After that, four methods are selected for further evaluation by applying them to the dynamic model of the compression system under study.
In Chapter 4 the selected identification methods will be tested and the results will be discussed. This chapter is especially useful for finding what the influence is of adjusting parameters of the identification methods. In Chapter 5 the identification methods will be tested on the system again, but this time both noise and the slow dynamics of the control valve are taken into account, to get more realistic results. Finally, conclusions are drawn from the evaluation of identification methods and directions for further work are given.
Chapter 2

The Greitzer model

In this chapter some general aspects of the centrifugal compressor will be described. The Greitzer lumped parameter model will be derived in full dimensional form. Moreover it will be given in dimensionless form, because it often appears in literature in this form. For more details on the derivation of this Greitzer lumped parameter model, see [HJSS04], [Gre76].

The behavior of a compressor can be described by a compressor characteristic. An example of a compressor characteristic is given in Figure 2.1. The compressor characteristic or compressor curve (dotted line) describes the resulting pressure difference for a given mass flow. The load line (solid line) represents the demand of the system behind the compressor. This can for instance be an oilfield or a complete factory. The operating point is the intersection point between the compressor curve and the load curve. In Figure 2.1 the operating point is stable, as can be seen by the constant pressure in the right figure.

When the operating point is shifted to the left in Figure 2.1, the derivatives of both curves become positive. This results in surge, an unstable operational mode of a compressor, that can occur at low mass flows. When surge occurs, both the mass flow and the pressure difference start oscillating. The mass flow can even become negative. This behavior is of course undesired. It can cause damage to the compressor or the system behind the compressor. An example of a
system in surge is shown in Figure 2.2. The goal is to stabilize this unstable operating range by means of active control.

The compression system that will be studied in this report exists of a single stage compressor, driven by an electric motor. It is a large compressor that could be used for example in the oil and gas industry. The compressor is throttled by a relatively large butterfly valve, which represents the load. In parallel with this large valve, a smaller control valve is used to make more precise adjustments in the mass flow rate. To operate the compressor in closed circuit, return piping couples the throttle outlet and the compressor inlet. This is depicted in Figure 2.3.

In order to describe the dynamic behavior of the compressor, a lumped parameter model has been used, following from the geometry of Figure 2.3. The model is analogous to [Gre76] and the same assumptions hold. However, the inertial effects in the throttle and the time lag associated with rotating stall development have been neglected, in accordance with [Wil00][Maz81]. Therefore, the impulse balance for the throttle and the relaxation equation of the original Greitzer model
are not presented here. By applying the principles of mass and momentum conservation the following equations can be obtained:

\[
\dot{m}_c = \alpha (\Delta p_c(m_c, N) - \Delta p) \quad (2.1)
\]

\[
\frac{d\Delta p}{dt} = \beta (m_c - m_l(\Delta p, u_l) - m_s(\Delta p, u_s)) \quad (2.2)
\]

Where \( \dot{m}_c \) represents the mass flow, \( \Delta p = p_2 - p_1 \) represents the pressure difference over the compressor, parameter \( \alpha \) is defined as

\[
\alpha = \frac{A_c}{L_c}
\]

and \( \beta \) is defined as

\[
\beta = \frac{a_1^2}{V_1} + \frac{a_2^2}{V_2}
\]

The symbols used in all equations and Figure 2.3 are explained in Table 2.1 on page 9.

In order to be able to compare this report with other literature, it is useful to state the equations in dimensionless form. Therefore, the dimensionless form of the Greitzer lumped parameter equations will be given. Consider the dimensionless flow coefficient \( \phi_c \) and dimensionless pressure coefficient \( \psi \) given by

\[
\phi_c = \frac{\dot{m}_c}{\rho_1 A_c U} \quad (2.3)
\]

\[
\psi = \frac{\Delta p}{\frac{1}{2} \rho_1 U^2} \quad (2.4)
\]

Time is scaled with the Helmholtz frequency \( \omega_H \):

\[
\tilde{t} = t \omega_H = t a_2 \sqrt{\frac{A_c}{V_2 L_c}} \quad (2.5)
\]

The Greitzer parameter \( B \) is given by:

\[
B = \frac{U}{2 \omega_H L_c} = \frac{U}{2 a_2 \sqrt{V_2 A_c L_c}} \quad (2.6)
\]

This parameter can be regarded as the ratio between resistive and inertial forces. So in case \( B \) is large, the time scale of surge oscillation is set by a balance between resistive and restoring forces. In case \( B \) is small, the time scale is set by a balance between inertial and restoring forces.

Because the return piping couples the throttle outlet and the compressor inlet, the principle of mass conservation are applied to volumes \( V_1 \) and \( V_2 \) separately. Because the total mass in the system is constant, the two mass balances can be combined. This provides a single differential equation for the pressure difference \( \Delta p \). The effect of the coupling by the return piping is described by Greitzer parameter \( F \) and is given by:
\[ F = 1 + \frac{Z_1 T_1 V_2}{Z_2 T_2 V_1} \]  

(2.7)

For a decoupled system, \( V_1 \) becomes infinitely large, yielding \( F = 1 \). This results into the original Greitzer model, derived in [Gre76]. These parameters and formulas can be used to derive a dimensionless Greitzer lumped parameter model, [Gre76]:

\[
\frac{d\phi_c}{dt} = B (\psi_c(\phi_c) - \psi) \\
\frac{d\psi}{dt} = \frac{F}{B} (\phi_c - \phi_l(\psi, u_l) - \phi_s(\psi, u_s))
\]  

(2.8) (2.9)

The behavior of this model resembles the behavior of the real compression system very well. The Greitzer model is able to describe surge behavior, however, the local dynamics cannot be determined uniquely from measurement data [HJSS04]. The local dynamic behavior is needed in order to stabilize the compressor in the unstable operating range. Therefore, in this report identification is used in order to obtain a model which accurately describes the local dynamic behavior.

The compressor which is regarded in this report has the following parameter values: \( \text{Ac} = 0.0186 \text{ m}^2 \), \( \text{Lc} = 1.73 \text{ m} \), \( V_1 = 3.28 \text{ m}^3 \), \( V_2 = 0.270 \text{ m}^3 \). The rotational speed is \( N = 8992 \text{ rpm} \). The sonic velocities \( a_1 \) and \( a_2 \) at compressor inlet and outlet have been determined from initial pressure and temperature conditions. This model, but with slightly different parameters, has been proven to work very well as can be seen in Figure 2.4.

![Figure 2.4: Simulation of a surge limit cycle.](image)
Table 2.1: Additional nomenclature.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>speed of sound</td>
<td>m/s</td>
</tr>
<tr>
<td>$A$</td>
<td>area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$L$</td>
<td>length</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>mass flow rate</td>
<td>kg/s</td>
</tr>
<tr>
<td>$N$</td>
<td>rotational speed</td>
<td>rpm</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>$\tilde{t}$</td>
<td>scaled time</td>
<td>-</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
<td>K</td>
</tr>
<tr>
<td>$u$</td>
<td>normalized valve position</td>
<td>-</td>
</tr>
<tr>
<td>$U$</td>
<td>impeller tip velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$Z$</td>
<td>compressibility factor</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>pressure difference $p_2 - p_1$</td>
<td>Pa</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>flow coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$\psi$</td>
<td>pressure coefficient</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>piping, suction side</td>
</tr>
<tr>
<td>2</td>
<td>plenum, discharge side</td>
</tr>
<tr>
<td>$c$</td>
<td>compressor</td>
</tr>
<tr>
<td>$t$</td>
<td>large throttle</td>
</tr>
<tr>
<td>$s$</td>
<td>small throttle</td>
</tr>
</tbody>
</table>
CHAPTER 2. THE GREITZER MODEL
Chapter 3

System identification methods

In order to start identification, first some limitations and general aspects of identification are discussed in Section 3.1. From these limitations and aspects, some criteria are proposed in Section 3.2. These criteria are used, in order to evaluate identification methods. A global inventory of linear identification methods is presented in Section 3.3. In Section 3.4, the methods are evaluated, based on the proposed criteria. From this evaluation, a selection of methods will follow. This selection of methods will be used in the next chapter to identify the compressor.

3.1 Limitations for system identification

The goal is to obtain an accurate model for the local dynamic behavior of the compressor, especially in the unstable region, because it is desired to stabilize the system with a controller based on this model. Control action in the stable region is only needed if it is desired to improve the dynamics (i.e. fasten the response), obviously not for stabilization. However, a controller can only be designed if an accurate model is available. Therefore, we start with identification in open loop, in the stable region. In further research, closed loop identification could be used in the unstable region.

The operating point must lie at the right hand side of the top of the surge line of Figure 3.1. In a sufficiently small neighborhood around this operating point, the nonlinear dynamics can be approximated by a linearized model of the system. Because the nonlinear system is stable in this operating point, the linear system will also be stable. By opening or closing the small valve, the mass flow and the pressure difference will alter only a little bit. It is assumed that when the small valve is switched from closed to fully open, the shifted operation point will stay close enough to the original operating point, so that this condition for linearization holds.

When the control valve is fully opened, the mass flow slightly increases and the pressure slightly decreases (Figure 3.1). When zooming in on the phase plot, the compressor curve is approximately linear. The linear approximation will be more accurate when the step size becomes smaller, thus when the valve is opened partially. However, the measured pressure difference will decrease and measurement noise will make it harder to identify the system due to a decreasing signal-to-noise ratio. This is a typical problem in linear system identification.

Due to the fact that costs of experimenting on the real compressor are too high, a simulation model of the compressor is used for this report. This simulation model is based on the Greitzer
equations of Chapter 2. The simulation model is able to correctly incorporate a lot of the realistic properties of the compressor, like valve dynamics and measurement noise. Moreover, it is also possible to turn off valve dynamics and noise, to test the compressor in an ideal situation. At first, identification is done for the simplest case when there are no valve dynamics and measurement noise (perfect actuator and perfect sensor). After that, measurement noise and valve dynamics are taken into account, to check whether the results hold for the more realistic situation.

On the real compressor using an impulse input signal would be useless. The valve is not capable of producing such a fast changing signal. However, an impulse is an interesting signal because it contains infinitely many frequencies. In the Greitzer model based simulation it is possible to turn off the valve dynamics, so in that case, an impulse can be used. Therefore an impulse signal is used for identification, to check how the results are effected by valve dynamics.

In the realistic situation, a step signal is easier realizable, simply by opening or closing the valve as fast as possible.

A step and an impulse signal only excite the system once; hence they are suitable for transient analysis. However for identification, one would like to have a signal which is constantly exciting the system - preferably with a large frequency content - to obtain more information about the system. One of such signals is a Pseudo-Random Binary Signal (PRBS). The PRBS is a periodic, deterministic signal with noise-like properties, an example can be seen in Figure 3.2. A PRBS is built up from pulses with different lengths.

First, identification in the ideal situation (perfect actuator and sensor) is done, using impulse, step and PRBS input signals. After that, the same input signals are used to identify the compressor in the realistic situation, with noise and valve dynamics. This is done to check whether the previous obtained results still hold in the realistic situation. This approach will develop a certain level of trust for each identification method. Moreover, it might indicate that a faster control valve or pressure sensor should be acquired.
3.2 Criteria for evaluation

There exists a vast range of identification methods. Which method is best depends on several aspects. Before a selection of identification methods can be made, it is important to determine criteria on which a selection can be made. From the previous we have seen that there are some practical restrictions and requirements involved with identification of a centrifugal compression system. From these limitations some criteria have been defined. In Section 3.4, identification methods will be compared using these criteria.

Practical restrictions

The methods will be applied to identify the lumped Greitzer model. But how well can the method deal with the restrictions of the real world? One could think of slow valve dynamics, which restricts the possible input signals. Another aspect of the real world is measurement noise. The compressor system contains nonlinear dynamics. However the methods that will be used are linear identification methods. Some problems might occur here. Therefore, the first criterion is "Practical restrictions".

Accuracy of the resulting model

How well does the identified model approximate the lumped Greitzer model? How do disturbances during measuring affect the outcome of the resulting model for this method? The accuracy can be expressed as the difference between identified model and the Greitzer model, which can be expressed in a quadratic error criterion: $y^2 - \hat{y}^2$. This (lack of) accuracy might be a result of the practical problems described in the previous criterion. Therefore, the next criterion is "Accuracy of the resulting model".

Costs

How cost effective is the method? The total costs of the method can be expressed in the total measuring time and the total calculation time required for this method. Included
in these costs are setup time for measurements, pre-processing and post-processing of measurement data. The total measuring and calculation time are described in the criterion "costs".

+ means that the costs are low

Numerical restrictions

Does the method contain numerical restrictions? There might be a chance of ending up in local optima. Does the method provide a unique solution, or a set of models? Is the method stable? The method might consume a lot of computer memory or disk space. These questions lead to the criterion "Numerical restrictions"

+ means that the method has little numerical restrictions.

A priori knowledge

Does one need a lot of information about the system to use the identification method well? Does the method require the user to set the order of the resulting model? Does it need to know values of model parameters? Does it need to know the structure of the model? If it needs particular information, is this information easy obtainable or not? These questions lead to the criterion "a priori knowledge".

+ means that little a priori knowledge about the system needs to be known.

Others

Are there any properties of the method that have not been mentioned in the previous criteria? An example can be the question whether the method is implemented ready-to-use in MATLAB or not.

3.3 Inventory of identification methods

For the purpose of this research an inventory of linear identification methods is made. There is a wide range of linear methods, so this inventory is not a complete overview of all existing methods. Each method is discussed in a subsection. A short description is given and the method is evaluated, based on the criteria from Section 3.2. A more detailed description of the selected methods, that will be applied to identify the compressor, will be given in Chapter 4.

Based on this evaluation a small number of methods is selected for further testing. This selection is presented in Section 3.4.

3.3.1 Sine wave testing

Sine wave testing methods find a frequency response function by applying a series of sinusoidal input signals to the system. The output signal of a linear dynamic system will have the same frequency as that of a sinusoidal input signal, but its amplitude and phase can differ from that of the input. The output signal will have a different amplitude than the input signal. The ratio between the amplitudes of the input signal and the output signal is the gain at that particular frequency. The phase shift for that particular frequency can be determined as well. By repeating this for a range of input frequencies, a frequency response function can be constructed. Such a frequency response function provides the gain and phase shift of the output for a given input frequency. [Lju99],[FPEN02].
3.3. INVENTORY OF IDENTIFICATION METHODS

Practical restrictions
Sine wave testing is for this compression system only possible for low frequent input signals (< 10 Hz), because of slow valve dynamics of the system. This means the method will only provide information up to 10 Hz. This restriction is caused by the investigated system, not by the method.

Accuracy of the resulting model
Using a sine with a large number of periods as an input signal, provides the possibility to average the responses. This will decrease the influence of disturbances. The larger the disturbances, the more periods the input signal should have to have a reasonable accuracy. So the accuracy of the resulting model can be very large, however this will cost a lot of measuring time, especially when the disturbances are high.

Costs
For each frequency a separate measurement must be done. This means the measurement costs will be high. After measuring, the gain and the phase shift must be determined for each frequency. This is also not very cost effective.

Numerical restrictions
There is a lot of measurement data. They require a lot of hard disk space. A frequency response function is not a very convenient model. It has to be converted to for instance a state space model. This requires more calculations and fitting. This might affect the accuracy.

A priori knowledge
The result of the sine wave testing method is a vector of complex numbers, or the frequency response function. This is not a suitable model to experiment with, or to design a controller. Therefor, a model has to be fitted to the frequency response function. In order to fit a model, some knowledge of the order of the system is necessary.

Others
No remarks.

3.3.2 Frequency response analysis
A frequency response function can also be constructed using one input signal, in stead of a set of a set of sinusoidal inputs. The Fourier transformations of the input and output signal are divided to yield the frequency response function. An impulse signal is the perfect input signal, because it contains an infinite range of frequencies. Also white noise is a suitable input signal, because it contains a range of frequencies. [Lju99]

Practical restrictions
For an ideal frequency response analysis, an ideal impulse is required as input signal. An ideal impulse can not be realized in practice with the compression system, because the valve is not fast enough. White noise as input signal yields quite good results, but the valve can not follow a white noise signal in practice either. A step as input signal, which can also not accurately be followed by the valve, gives bad results too. So again the practical restriction is caused by the investigated system, not by the method.
Accuracy of the resulting model
   Beforehand not much can be said about accuracy of this method.

Costs
   The costs are low, because the input signal (in practice a not ideal impulse) is simple and
   because calculations are simple.

Numerical restrictions
   No iterative algorithms are necessary. The resulting model is a frequency response function
   and not a state space model. This means calculations or fitting is needed to generate a state
   space model. This might affect the accuracy in a negative sense.

A priori knowledge
   Similar to sine wave testing, again fitting is required in order to get a state space model.
   This requires a priori knowledge of the system that has to be identified.

Others
   The command tfestimate, which calculates the Fourier transformations of input and output
   and estimates the frf, is implemented in MATLAB.

3.3.3 Subspace methods

Subspace methods try to find a minimal subspace of the state-space model, which still describes
the dynamics of the identified system. The basic idea is to first find the minimal state space,
which describes the main dynamics. Then the system matrices are adapted so that they describe
the relation between the input and output signal. Several algorithms exist, based on different
input signals.

Practical restrictions
   The N4SID and MOESP algorithms can use any kind of input and output signal.

Accuracy of the resulting model
   In advance, from literature nothing can be said about the accuracy of the model. There was
   too little time to do in depth research on this.

Costs
   The calculation costs are low. (see Numerical restrictions)

Numerical restrictions
   The subspace methods do not use complex non-linear iterative algorithms. Instead they
   use numerical well-developed linear algebraic operations (matrix calculations).

A priori knowledge
   The N4SID method finds a minimal subspace itself. The user does not have to supply
   information about the system that has to be identified. Therefor, it looks like little a priori
   knowledge is required.

Others
   N4SID and MOESP are implemented in the system identification toolbox of MATLAB,
   complete with graphical user interface.
3.3.4 Approximate realization methods

Approximate realization is concerned with finding a state-space description, given data about an LTI system. Most approximate realization methods, such as the Ho/Kalman method use an impulse as input signal. Methods that use a step input signal are the modified Kung method and the ‘Hankel step method’. In the latter method, from step response data the finite block Hankel matrices are computed. Then, the system matrices are computed using singular value decomposition (SVD). For reference, [HWA80] and [Bos04].

Practical restrictions
Most approximate realization methods use an impulse as input signal. An ideal impulse can not be realized in practice. The modified Kung algorithm and the Hankel step method make it possible to use a step as input signal.

Accuracy of the resulting model
In advance, from literature nothing can be said about the accuracy of the model. There was too little time to do in depth research on this.

Costs
The calculation costs are strongly dependent on the size of measurements data. The calculation costs grow very fast when the sizes of measurement data increase. For small sizes of measurement data, the calculation costs are low.

Numerical restrictions
Approximate realization methods do not use complex non-linear iterative algorithms. Instead they use numerical well-developed linear algebraic operations (matrix calculations).

A priori knowledge
The order of the resulting model is determined by the singular value decomposition. The user does not have to supply this. Therefore, it looks like little a priori knowledge is required.

Others
For using the Hankel step method a MATLAB file is available.

3.3.5 Prediction error methods

Prediction error methods start with choosing a specific model structure. Then, the parameters of the model are obtained by minimizing the prediction error. The prediction error is the difference between measured output and predicted output. Hence, these methods solve an optimization problem to find the optimal set of model parameters. These parameters can be found using optimization algorithms.

Practical restrictions
The method can be applied for each arbitrary input signal.

Accuracy of the resulting model
The method is very accurate, unless it ends in a local optimum.
Costs
Because of the fact that the method can work for any input signal, one can use an input signal for which the measurement costs are low. The calculation costs are high.

Numerical restrictions
A non-linear optimization problem has to be solved. An iterative algorithm is used. This means computationally hard calculations have to be done. There is a chance of ending up in a local optimum.

A priori knowledge
In the short description it is already noted that a prediction error method start with choosing a specific model structure. The user has to make this choose. The choice of the structure influences the result of the model. In order to choose a suitable model, the user has to have quite a bit of knowledge of te system.

Others
Because of the fact that research on PEM is done for many years already, there are very easy to used MATLAB routines at hand already. A problem is that the user should have some knowledge of the system. The user must choose the structure of the resulting model.

3.3.6 Parametric estimation from transient response
The basic idea of parametric estimation from transient response is to use a transient, non periodic signal. An evaluation from the response can be made to derive the parameters of the response function. To evaluate the response, the plot of the output versus time is used. [Gra72], [Rak80]

Practical restrictions
The method is preferably applied using an ideal step or impulse response. On the real compressor it is only possible to determine an "approximate" step response.

Accuracy of the resulting model
The problem with parametric estimation from transient response is that the plot of the output is used to obtain parameters. The evaluation of the output plot becomes difficult when measurement noise is present in the data. The influence of noise can be reduced by averaging the measurement data. Therefore a lot of measurements have to be done.

Costs
The costs for measuring are high, because a lot of measurements must be done to decrease the influence of disturbances.

Numerical restrictions
It might be possible to implement the evaluation of the output plot in MATLAB, in stead of doing this by hand. The resulting model is a transfer function and not a state space model. This means calculations or fitting is needed to generate a state space model. This might affect the accuracy in a negative sense.

A priori knowledge
In order to estimate parameters from a transient response, the order of the system has to be known. Therefor the user has to have a priori knowledge of the system.
3.4 Evaluation and selection

In Table 3.1 an overview is given on how the various methods meet the selection criteria. In this table a ‘+’ means that the method has a positive score in the criteria, where positive is defined in Section 3.2. Obviously the ‘−’ represents a negative score for the given criterium. A ‘±’ means that the methods has both negative and positive points for the criterium. A ‘0’ means that the score for the criterium is unknown.

Table 3.1: Evaluation matrix for identification methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Practical restrictions</th>
<th>Accuracy</th>
<th>Costs</th>
<th>Numerical restrictions</th>
<th>A priori knowledge</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine wave testing</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Freq. response analysis</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Subspace methods</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>App. realization methods</td>
<td>+</td>
<td>0</td>
<td>±</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Prediction error methods</td>
<td>+</td>
<td>+</td>
<td>±</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Par. est. from transient resp.</td>
<td>−</td>
<td>±</td>
<td>−</td>
<td>±</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

After this first inventory of identification methods, some methods can be rejected. First of all sine wave testing will be rejected. The method is time consuming and only gives low frequent information.

The second method that will be rejected is parametric estimation methods from transient response. Also this method is highly time consuming and involves evaluation by hand. Although the method is rejected for this research, the method can be useful as a first identification step. The plot of the output usually reveals some first rough information on the system dynamics. However, in this case, it is not useful as a method to get detailed information on the system dynamics.

From here on, the frequency response analysis, the subspace methods, approximate realization theory and the prediction error methods will be regarded. In the next chapters a more in depth look will be taken into these methods.
Chapter 4

Evaluation of methods

In this chapter, methods selected in Chapter 3 will be evaluated, using a simulation model of the compressor. In Section 4.1, the evaluation approach is explained. At this stage, identification is done in the ideal situation (without noise and valve dynamics), using impulse, step and PRBS input signals. In Section 4.2, the frequency response approach is evaluated. Section 4.3 deals with an approximate realization approach. Section 4.4 presents the evaluation of prediction error methods. In Section 4.5, a subspace method is evaluated.

In Chapter 5, the same input signals are used to identify the compressor in the realistic situation, with noise and valve dynamics. This is done to check whether the results obtained in the ideal situation still hold in the realistic situation.

4.1 Evaluation approach

All data is generated with a MATLAB simulation. All simulations are executed with the following settings, unless indicated otherwise.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nr_in</td>
<td>1</td>
<td>Number of initialization file</td>
</tr>
<tr>
<td>Gas</td>
<td>‘N2’</td>
<td>Gas type</td>
</tr>
<tr>
<td>Fs</td>
<td>500</td>
<td>Sample frequency (Hz)</td>
</tr>
<tr>
<td>Tend</td>
<td>10</td>
<td>Simulation time (s)</td>
</tr>
<tr>
<td>Tc_on</td>
<td>0</td>
<td>Time controller/valve switch (s)</td>
</tr>
<tr>
<td>us_i</td>
<td>0</td>
<td>Initial control valve opening (0 or 1)</td>
</tr>
<tr>
<td>us_e</td>
<td>1</td>
<td>Final control valve position</td>
</tr>
<tr>
<td>us_dt</td>
<td>0</td>
<td>Slope of control valve change</td>
</tr>
<tr>
<td>Fqs</td>
<td>1.5</td>
<td>Shift of the operating point with respect to the surge mass flow&lt;br/&gt;(&lt; 1 unstable, &gt; 1 stable region)</td>
</tr>
</tbody>
</table>
The input/output (I/O) data for a pulse, step and PRBS signal are shown in the Figure 4.1. For now, valve dynamics are not regarded and measurement noise is omitted. The sample frequency, as well as the simulation time, influence the quality of the identified system and will be discussed for every method. The position of the valve \( u_s \) is equal to zero when the valve is closed and equal to one when the valve is open. Note that the settling time is about 0.1 seconds (Figure 4.1). Also note that the measured pressure difference \( \Delta p \) for a pulse input is very small (<100 Pa) due to the slow compressor dynamics. In practice, measurement noise will cause the peak to be unobservable but in a computer simulation this problem can be overcome by scaling. The measured pressure difference for a step and a PRBS is a factor 10 larger: 1 kPa (0.01 bar). Most identification methods imply that the initial state of the system is zero. Therefore the initial value is subtracted from the data during the analysis.

The quality of the fit is checked qualitatively as well as quantitatively. First, the data are compared visually. The fitted data should at least roughly follow the same track in time. This includes no negative responses (minimum phase behavior) and so on. Secondly, two error quantities are computed:

- The maximum absolute error: \( \max |e(t)| = \max |y(t) - \hat{y}(t)| \), where \( y(t) \) is the measured quantity and \( \hat{y}(t) \) the fitted quantity. The error is defined as \( e(t) = y(t) - \hat{y}(t) \).

- An integral quantity \( \int_0^1 e^2(t)dt \), from now on called ‘integrated error’.

Figure 4.1: I/O data without valve dynamics/compensation and measurement noise (Fs = 500 Hz).
Both quantities can be used to indicate the quality of the fit. In case of a poor fit, both quantities are large and of course, in case of a good fit, both quantities are small. The maximum absolute error indicates that at some specific time, there is a large difference between the measurement data and the fitted data. The so-called integral quantity is in fact a summation of the squared error in a certain time interval. The error is squared to make sure that both negative and positive values are ‘penalized’ and do not cancel out each other. Alternatively, the integral of the absolute value \(|e(t)| = \sqrt{e^2(t)}|\) could be used, but by squaring, large errors are more heavily penalized. The time interval over which the integral is computed is from 0 to 1 second, because we want to penalize static gain errors, but not too heavily. Numerically the integral changes to a summation: 

\[
    ts \sum_{i=1}^{N} e^2(t_i), \quad \text{where } ts \text{ is the sample time in seconds.}
\]

During validation of the identification methods, the output of the identified model is compared to the measured compressor data. The ‘best fit’ is defined as the data which visually coincides with the measured data the most and for which the two mentioned error quantities are as small as possible. Because only a limited amount of parameters has been tuned to fit the data, the best fit is only best of a limited amount of data sets. Obviously, it is not the best possible fit in a strict sense. Also note that the two error quantities are just two out of an infinite number of choices; the quality of the fit cannot be defined uniquely. They just help to show the difference between different sets of data in a quantitative way.

Both quantities are only computed for a step signal during validation, because it is the only signal that does not change when valve dynamics are included. Both the pulse and PRBS signal are changed to deal with the slow valve dynamics, which makes them unsuitable for comparison.

### 4.2 Frequency response analysis

#### 4.2.1 Introduction

One of the easiest identification methods is the frequency analysis method. In open loop, the system’s transfer function is equal to

\[
    G(s) = \frac{Y(s)}{U(s)}
\]

where \(Y(s)\) and \(U(s)\) are the Fourier transforms of the output and the input, respectively. Note that the computed transfer function \(G(s)\) becomes inaccurate when \(U(s)\) approaches zero. A step signal \((U(s) = 0)\) is therefore not suitable for frequency response analysis. On the contrary, when \(U(s) = 1\), the system’s transfer function equals the Fourier transform of the output. An impulse is therefore a perfect signal for frequency response analysis! An impulse is a signal that has an infinite value in an infinitely small time period. This is of course not possible in practice. The best approximation is a pulse as shown in Figure 4.1. Valve dynamics will cause this approximation of an impulse to deteriorate, but for now, valve dynamics are disregarded. The large frequency content of the PRBS signal makes it also well suited for frequency response analysis. However, because \(G(s)\) is defined as being the Laplace transform of the impulse response, only the pulse signal is used to identify the compressor with frequency response analysis. The algorithm used for computing the frequency response measurement is TFESTIMATE [Lju]. The result of TFESTIMATE is a vector \(G(j\omega)\) containing frequencies and corresponding imaginary numbers.
The angle and length of these imaginary numbers represent the phase and gain of the system at the corresponding frequency:

\[ |G(j\omega_i)| = \sqrt{Re(G(j\omega_i))^2 + Im(G(j\omega_i))^2} \]

and

\[ \angle G(j\omega_i) = \text{atan}\left(\frac{Im(G(j\omega_i))}{Re(G(j\omega_i))}\right) \]

4.2.2 Algorithm

\( G(j\omega_i) \) is a non-parametric model. To obtain a parametric model, an algorithm has to be used to fit the frequency response measurement with a parametric (i.e. state-space) model. In MATLAB, there are two algorithms for fitting the frequency response data: FITFRD and INVFREQS [Lju]. To check if one of these algorithms is cheaper, more robust and/or more accurate, they are both tested.

\[
B = \text{FITFRD}(A,N,\text{RD},\text{WT}) \text{ yields a state-space object } B \text{ with dimension } N \text{ and relative degree } RD \text{ whose frequency response closely matches the frequency response data in } A. \text{ WT is a weighting function.}
\]

\[
[B,A]=\text{INVFREQS}(H,\tilde{\omega},\text{NB},\text{NA},\text{WT},\text{ITER},\text{TOL}) \text{ gives real numerator and denominator coefficients } B \text{ and } A \text{ of orders } NB \text{ and } NA \text{ respectively, where } H \text{ is the desired complex frequency response of the system at frequency points } \tilde{\omega}. \text{ Again, WT is a weighting function. ITER is the maximum number of iterations and TOL is the tolerance.}
\]

There are only a few differences between the two algorithms. The most important difference is that FITFRD returns the state space system matrices and INVFREQS returns an I/O transfer function. It is beneficial to have a state-space representation as output (state-space systems are better for numerical simulations than transfer functions). Furthermore, state-space systems contain more information of the internal dynamics of the system. However, INVFREQS has two additional properties compared to FITFRD, namely ITER and TOL. This makes the algorithm somewhat more attractive.

In both algorithms, the model structure has to be specified. The frequency response measurement showed that the slope of the gain plot changes from 0 to -1 and a phase drop from 180 to 90 degrees. The phase starts at 180 degrees instead of 0 degree because of the fact that the pressure drops (negative change) when the valve is opened (positive change). From this result it can be concluded that the compressor behaves as a first order system, or a higher order system with relative degree one. The best fit was obtained using a second order system with relative degree one:

FITFRD: N = 2, RD = 1

INVFREQS: NA = 1, NB = 2
4.2.3 Simulation parameters

Sample frequency ($f_s$) and simulation time ($T_{end}$) have a large influence on the results. If the sample frequency is lower than the highest frequencies in the signal, these frequencies are folded back around half the sample frequency (Nyquist frequency) in the frequency domain. This is called aliasing. To avoid aliasing, the sample frequency should be as high as possible. The simulation time also influences the result. Low-frequent components in the signal will not appear in the frequency response function when the simulation time is shorter than the wavelength of the low-frequent component. A large simulation time and a high sample frequency requires large data storage and calculation time so the sample frequency and simulation time cannot be chosen arbitrarily large. A trade-off has to be made. Many combinations of sample frequency (100, 500 and 1000 Hz) and simulation time (0.05, 0.1, 1, 10 and 100 s) have been tested. Sample frequencies smaller than 500 Hz result in a poor fit at high frequencies. Increasing the sample frequency results in a slightly better fit at high frequencies, but the improvement of the resulting model is very small. Simulation times smaller than 1 second result in a poor fit at low frequencies. Increasing the simulation times beyond 10 seconds will not improve the fit. Finally, a sample frequency of 500 Hz and a simulation time of 10 seconds are used for identification.

4.2.4 Results

![Graph showing frequency response analysis](image)

Figure 4.2: Comparison of frequency response functions of linear model, measurement, FITFRD model ($N = 2$, $RD = 1$) and INVFREQS model ($NA = 1$, $NB = 2$).
Figure 4.2 shows four frequency responses. The first is from the original linear model, obtained by linearization of the nonlinear Greitzer model. The second is the frf measurement; the non-parametric model \( G(j\omega) \) which is computed by TFESTIMATE. The other two are the identified parametric models. It is shown that both identified models almost perfectly fit the frf measurement and the linear model. When taking a closer look at the fit at low frequencies, a small offset (0.11 dB) is observed between the measured frequency response function and the remaining frequency response functions. This offset is due to the fact that the true system is nonlinear. Therefore, the offset can probably be decreased by applying a smaller pulse (so the linear approximation becomes more accurate). Note that the sample frequency is 500 Hz, which causes the behavior at the Nyquist frequency at 250 Hz.

The continuous time transfer functions of the two identified models are of the form

\[
G(s) = \frac{a_1 s + a_0}{b_2 s^2 + b_1 s + b_0}
\]

where \( a_0, a_1, b_0, b_1 \) and \( b_2 \) are constant parameters, which are given in Table 4.2. A transfer function description of the system is chosen for comparison instead of a state-space description, because two identical systems must have an identical transfer function unlike a state-space description, where the system matrices depend on the choice of state variables.

<table>
<thead>
<tr>
<th></th>
<th>( a_1 \times 10^4 )</th>
<th>( a_0 \times 10^6 )</th>
<th>( b_2 \times 10^2 )</th>
<th>( b_1 \times 10^3 )</th>
<th>( b_0 \times 10^3 )</th>
<th>( 20 \log(\text{abs}(\frac{a_0}{b_0})) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear model</td>
<td>-4.55</td>
<td>-7.48</td>
<td>1</td>
<td>1.78</td>
<td>7.91</td>
<td>59.51</td>
</tr>
<tr>
<td>FITFRD model</td>
<td>-4.75</td>
<td>-7.54</td>
<td>1</td>
<td>1.80</td>
<td>8.08</td>
<td>59.40</td>
</tr>
<tr>
<td>INVFREQU model</td>
<td>-4.75</td>
<td>-7.54</td>
<td>1</td>
<td>1.80</td>
<td>8.06</td>
<td>59.40</td>
</tr>
</tbody>
</table>

Table 4.2: Transfer function parameters.

Table 4.2 shows the identified models do not differ much from the linear model and there is almost no difference between the performance of the two algorithms so far. Note that \( 20 \log(\text{abs}(\frac{a_0}{b_0})) \) is equal to the static gain (in dB) and the difference between the linear model and the identified models is equal to 0.11 dB, as was earlier observed in Figure 4.2. The difference in static gain of the frequency response measurement and the identified models is almost zero.

### 4.2.5 Model validation

To see whether the identified systems are a good approximation of the nonlinear system, they are validated with a pulse, a step and a PRBS as input and the output is compared as is shown in Figure 4.3.

In the top figures the output signals are shown and in the bottom figures the error compared to the measurements is shown. It can be seen that the pulse is almost perfectly reproduced by the identified models. This is not surprising, since a pulse signal was used to identify the system. Looking at the step output, it can be seen that the output has a pretty large static error. This can be explained by the nonlinear behavior of the compressor. When a step is applied, the operation point is shifted. Linearization about this new operation point will result in a different transfer function (or: system matrices). The advantage of a pulse signal is that the system goes back to the original operation point. The same thing holds for the PRBS output. The system switches
back and forth between two different operation points, which causes the relatively large error compared to the pulse output. A maximum absolute error of about 145 Pa on a measured pressure difference of approximately 1 kPa is 15% so this is a serious drawback of linear identification. The output of the original linear model is also shown in figure 4. It can be seen that the output of the identified models almost perfectly fits the output of the linear model. When taking a closer look at the step and PRBS output, the maximum pressure difference compared to the output of the original linear model is approximately 12 Pa. 12 Pa is negligible compared to the measured pressure difference of 1 kPa. So despite the fact that the frequency response fit differed about 0.11 dB at low frequencies, the difference in static gain is negligible. The integrated error for the step data is $2.04 \times 10^4$ for both identified models. This value can be used later on to compare it with different methods.

4.2.6 Conclusion

Two algorithms, FITFRD and INVFRQS have been used to fit the frequency response measurement using I/O signals of a pulse. It is shown that when omitting measurement noise and valve dynamics, both algorithms perform almost equally well. The resulting identified models are a good approximation of the linear system obtained by linearization of the nonlinear system in the specified operation point (from now on referred to as: original linear model). When a pulse is applied to the identified models, the outputs are almost identical with the output of the original
linear system and the measured output of the nonlinear system. However, when applying a step or a PRBS, there is a relatively large error between the outputs of the identified models and the measured output due to the limitation of linear identification. Thus, this is not considered to be a drawback of the frequency response analysis method but of linear identification in general. The step and PRBS output of the identified models almost perfectly fit the output of the original linear model. Because the method requires a relatively long simulation time and high sample frequency to give good results, it is a quite expensive method.

4.3 Approximate realization method

4.3.1 Introduction

Approximate realization is concerned with finding a state-space description, given some data of an LTI system. The data are typically the impulse response of the system, the step response, I/O measurements, frequency response data, or more general frequency measurements. An overview of minimal state-space realization in linear system theory can be found in [Sch00]. Because the step signal can be approximated more accurately than an impulse signal, a realization theory based on a step signal is chosen. From step response data, the finite block Hankel matrices are computed. Then, the system matrices are computed using singular value decomposition (SVD). For reference, see [HWA80] and [Bos04].

4.3.2 Algorithm

The algorithm used in MATLAB is STEPMODELID. First, the finite block Hankel matrices are computed. Then, the system matrices are computed from these matrices using singular value decomposition (SVD).

```
[SYS_C, SYS_D, S] = STEPMODELID(Y, N, FS) returns an estimation of order N for the dynamical system from step response data Y. SYS_C is the continuous time state space realization for the approximated mode. SYS_D is the original discrete time model. S is a vector containing singular values. FS is the sample frequency.
```

A parameter that can be changed is the order of the estimated model. When there is no prior knowledge of the system, one can choose the model order by looking at the logarithmic plot of the singular values. The number of singular values that are responsible for most of the input/output behavior are called ‘dominant singular values’. When the output behavior does not or hardly change when the amount of singular values is increased, there is little information left in the remaining data. Thus, increasing the model order gives no or little improvement of the resulting model. When the sample frequency is chosen to be 500 Hz and the simulation time 0.1 second, there are 51 data points and 25 singular values, which are shown in Figure 4.4.

Figure 4.4 shows there are 5 dominant singular values, implying that the model order should be chosen as 5. Decreasing the model order would result in a (significant) loss of information from the data and increasing the model order would hardly improve the model. However, when choosing a system order 5, the algorithm automatically increases the system order to 6 to handle real negative poles.
Again, many combinations of sample frequency and simulation time have been tested. When the simulation time is increased, there are more data points and the list of singular values grows without having a lot of effect on the value of the singular values. When the sample frequency is increased, there are not only more data points and singular values, but it also has (little) influence on the value of the singular values. In fact, the resulting model becomes more accurate because we are able to capture more high frequent behavior. When the amount of data points increases, the required data storage increases significantly due to the increasing size of the Hankel matrices that have to be computed. Despite that a sample frequency of 1000 Hz gives better results, a sample frequency of 500 Hz is chosen to have a better comparison with the other methods. A simulation time of 0.03 s is chosen. This is almost the smallest simulation time possible if the model order is chosen to be equal to 5 (7 singular values are computed). Increasing the simulation time does not improve the model significantly.

4.3.4 Results

The output of the algorithm is a discrete and a continuous time state-space model. To compare it with the original linear model, it is transformed into an I/O model. The transfer function of the resulting (continuous time) model is of the form

\[ G(s) = \frac{\sum_{i}^{n} a_i s^i}{\sum_{i}^{n} b_i s^i} \]

where \( a_i \) and \( b_i \) with \( i = 0, 1, \ldots, n \) are constant parameters, see Table 4.3.

The transfer function does neither look like the transfer function of the linear model (see Table 4.2), nor it has common parameters. This does not mean that the estimated model is bad. Note that the value of \( a_6 \) is very small compared to the other parameters in the numerator. This
means that the estimated model is close to a system of relative degree one. This result was also observed in the frequency response method. To improve the model, one could choose to manually set this parameter to zero. In practice however, a frequency response measurement might not be possible and one does not know if the system has relative degree one and setting the parameter to zero could mean important loss of system information.

### 4.3.5 Model validation

To see whether the estimated model is a good approximation of the nonlinear system, the outputs from different inputs are compared.

![Comparison of output data](image)

By looking at the output data, a few things can be remarked. The error becomes high when the pressure drop is very steep. This means that the identification method did not capture the
4.3. APPROXIMATE REALIZATION METHOD

high frequent behavior of the system very well. This resulted in a poor fit of the pulse (high frequent input) and of the parts of the step and PRBS where the signal changes rapidly. The obvious solution for this problem is increasing the sample frequency to get more data points in the high frequent parts of the data. Changing the system order does not make the fit any better. Another remarkable observation is that the error tends to go to zero. This is because the system is identified using a step. Therefore, system information of the shifted operation point is obtained instead of the original operation point. Note that the exact opposite was observed in the frequency response method, where a pulse was used to identify the system. A maximum absolute error of $10^3$ Pa is observed. This is about 10% of the measured pressure difference, which seems pretty large. However, when looking at the PRBS output for example, it can be seen that the predicted output closely matches the measured output. The integrated error for the step data is $1.31 \cdot 10^2$. This is two orders of magnitude smaller than the previous method (frf analysis). This is because the error is defined as the difference between the measurement data and fitted data. The response of the original linear model has a pretty large off-set compared to the measurements. Because a pulse has been used for the frf method, the identified models closely match the original model instead of the measurement data. This static gain off-set causes the relatively large integrated error in the previous method.

Also the frequency response function of the resulting model can be compared with the frequency response of the original linear model, see Figure 4.6.

![Figure 4.6: Comparison of frequency response functions.](image)

When taking a closer look, it can be seen that there is an offset of about $1.14$ dB in magnitude in the low frequency region. This offset becomes somewhat larger when moving towards the high frequency region. The frequency response functions of the identified models are pretty accurate.
up to about 20 Hz.

### 4.3.6 Conclusions

An approximate realization algorithm (from now on called the Hankel-step method in this report) is used to compute the system matrices of step response data. The state-space model has been transformed into a transfer function model, to make comparisons easier. The output of a pulse, step and PRBS input of the estimated model have been compared with the measurements and output of the original linear model. It is shown that the method did not quite capture the high frequent behavior of the system. Increasing the sample frequency would improve the performance a lot. Because the method requires only a small simulation time and thus a small amount of data point, it is a very cheap and effective method which gives good results. Also little a priori knowledge is needed, which is a big advantage.

### 4.4 Prediction error method

#### 4.4.1 Introduction

The third method that is investigated is a parameter estimation method. First, a specific model structure of the system has to be chosen. Then, the parameters within the model structure have to be found. The problem transforms into an optimization problem for finding the optimum set of parameters to minimize the prediction error; hence the name PEM. The prediction error is the difference between the measured output and the output of the estimated model. For reference, see [Lju99].

Several criterion functions can be used in the optimization problem for minimizing the prediction error. Here, the error is minimized in a least-squares (LS) sense (thus a quadratic function). More information about transfer function models can be found in Appendix A. A big advantage of a PEM is that any I/O data can be used. To see whether the type of data has a lot of effect on the
performance, both the step and PRBS are used to identify the compressor. The pulse is only used for validation and not for identification, because the signal-to-noise ratio of the pulse response will be very poor when measurement noise is taken into account. Note that a pulse has been used for frequency response analysis, because a system’s transfer function is defined as being the Laplace transform of an impulse response.

4.4.2 Algorithm properties

For this research the algorithm PEM of the Identification Toolbox is used.

\[
A(q)y(t) = B(q)u(t - nk) + C(q)e(t)
\]

The first objective is to select an appropriate model structure. The question is what number of parameters should be chosen to obtain a good fit with the data, without having unnecessary many parameters. Usually, a priori knowledge about the system, intuition and ingenuity is used to answer this uneasy question. More about model structure selection can be found in Chapter 16 of [Lju99]. The chosen model structure here is an ARMAX model \( (n_d = n_f = 0) \). This means the model structure becomes

\[
A(q)y(t) = B(q)u(t) + C(q)e(t)
\]

It is also assumed there is no time delay of the input \( (nk = 0) \). The size of the polynomials has been tuned to give the best fit to the data. This resulted in the following parameter settings: \( na = 2, nb = 2, nc = 1 \), so \( Mi = [2, 2, 1, 0, 0, 0] \). The property/value pairs are shown in Table 4.4.

<table>
<thead>
<tr>
<th>Property</th>
<th>Options</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus</td>
<td>Prediction / Simulation / Filter / Stability</td>
<td>Simulation</td>
</tr>
<tr>
<td>Initial State</td>
<td>Zero / estimate / Fixed / Backcast / Auto</td>
<td>Zero</td>
</tr>
<tr>
<td>Disturbance Model</td>
<td>None / Estimate</td>
<td>None</td>
</tr>
<tr>
<td>nk</td>
<td>Any non-negative integer</td>
<td>0</td>
</tr>
<tr>
<td>Limit Error</td>
<td>A positive number</td>
<td>1.6 (default)</td>
</tr>
<tr>
<td>Max Iter</td>
<td>Any positive integer</td>
<td>20 (default)</td>
</tr>
<tr>
<td>Tolerance</td>
<td>A positive number</td>
<td>0.01 (default)</td>
</tr>
<tr>
<td>Max Size</td>
<td>Any positive integer / Auto</td>
<td>Auto</td>
</tr>
<tr>
<td>Search Direction</td>
<td>Gns / Gn / Lm / Auto</td>
<td>Auto</td>
</tr>
<tr>
<td>Trace</td>
<td>On / Off</td>
<td>Off</td>
</tr>
</tbody>
</table>

For detailed information about these algorithm properties see [Lju], or try the MATLAB commands help pem or idprops('algorithm').
4.4.3 Simulation parameters

Again, many combinations of sample frequency and simulation time have been tested. The conclusion is that increasing the sample frequency results in a better performance, because the estimated models give a better description of the high frequent parts of the data. The performance of the PEM based on PRBS signal was more dependent on the sample frequency and simulation time. The frequency content of the PRBS signal depends on the simulation time, so it cannot be chosen too small. Also, it cannot be chosen too large because of increasing calculation time. Finally, a simulation time of 1 second has been chosen.

4.4.4 Results

The estimated discrete time model is of the form

\[ A(q)y(t) = B(q)u(t) + C(q)e(t) \]

which is converted to a continuous time transfer function model to compare it with the original linear model. This is possible because the initial state of the system is assumed to be zero. The transfer functions are of the form

\[ G(s) = \sum_{i=0}^{n} a_i s^i \sum_{i=0}^{n} b_i s^i \]

where \( a_i \) and \( b_i \) with \( (i = 0, 1, \ldots, n) \) are constant parameters, see Table 4.5 and \( n = 2 \).

Table 4.5: Transfer function parameters.

<table>
<thead>
<tr>
<th></th>
<th>( a_2 (\times 10^4) )</th>
<th>( a_1 (\times 10^6) )</th>
<th>( a_0 )</th>
<th>( b_2 (\times 10^2) )</th>
<th>( b_1 )</th>
<th>( b_0 (\times 10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear model</td>
<td>0</td>
<td>-4.55</td>
<td>-7.48</td>
<td>1</td>
<td>1.78</td>
<td>7.91</td>
</tr>
<tr>
<td>PEM step</td>
<td>-78.1</td>
<td>-5.91</td>
<td>-10.0</td>
<td>1</td>
<td>2.06</td>
<td>9.31</td>
</tr>
<tr>
<td>PEM prbs</td>
<td>-59.9</td>
<td>-6.48</td>
<td>-17.4</td>
<td>1</td>
<td>3.25</td>
<td>16.39</td>
</tr>
</tbody>
</table>

Table 4.5 shows there is a pretty large difference between the obtained transfer functions. When looking at the Bode diagram of these models, it can be seen that the high frequent behavior is significantly different from that of the original linear model. This results in large errors in the data fit. One may conclude that the order of the chosen model structure is too high. However, lowering the model order did not improve the results. Another option is to set the \( a_2 \) parameter to zero, thus removing high frequent information of the estimated model and implying a relative degree of one. This resulted in a much better fit of the data, as well as the frequency response function. Setting \( a_2 \) to zero can be done for this simulation, but it can only be done for real world situations when enough information about the system is available.

4.4.5 Model validation

The output of the estimated models is compared with the actual model and the results are shown in Figure 4.8. Moreover the frequency response data is compared in a Bode diagram, as is shown in Figure 4.9.
Again, the large errors in the output data are present at the fast changing (high frequent) parts of the data. The maximum absolute error is 22 Pa for the identified model based on a step and 37 Pa for the identified model based on a PRBS. Besides that, the step and PRBS data is pretty accurately described. It can be seen that the PEM based on PRBS data performs somewhat worse than the PEM based on step data, despite the larger frequency content of this data. The most remarkable difference is that there is a pretty large static error (15 Pa) compared to the step data of the model identified with PRBS data (middle bottom plot of Figure 4.8). This causes the large difference between integrated error: the integrated error for the identified model based on a step is $6.96$, while it is $2.51 \cdot 10^2$ for the identified model based on a PRBS.

### 4.4.6 Conclusions

Both step and PRBS I/O data have been used to identify the compressor using a prediction error method. The estimated discrete time polynomial models have been converted to continuous time transfer function models and compared with the original linear model. One parameter ($a_2$) of the transfer function has been set to zero to improve the performance. By doing so, a relative degree of 1 is enforced. Comparing the output data and frequency response functions showed that the estimated models were pretty accurate, except for the high frequent behavior of the system. Increasing the sample frequency will improve the performance a lot, but will also increase the costs.
The fourth and last identification method that is investigated is a subspace method called Numerical algorithms for Subspace State Space System Identification, or N4SID. Subspace identification algorithms yield state-space models and consist of two steps. Most subspace methods first estimate the states of the system using a projection of certain subspaces generated from the data. Next, they determine the state-space model by a linear least squares method. For more detailed information about subspace methods and the N4SID algorithm, see [Kat05]. Subspace methods have the advantage that they can handle any I/O data. Both the step and PRBS data have been used to identify the compressor.

4.5 Subspace method N4SID

4.5.1 Introduction

The fourth and last identification method that is investigated is a subspace method called Numerical algorithms for Subspace State Space System Identification, or N4SID. Subspace identification algorithms yield state-space models and consist of two steps. Most subspace methods first estimate the states of the system using a projection of certain subspaces generated from the data. Next, they determine the state-space model by a linear least squares method. For more detailed information about subspace methods and the N4SID algorithm, see [Kat05]. Subspace methods have the advantage that they can handle any I/O data. Both the step and PRBS data have been used to identify the compressor.

4.5.2 Algorithm properties

For this research the algorithm N4SID of the Identification Toolbox is used.

\[
\text{MODEL} = \text{N4SID}(\text{DATA},N,\text{Property/Value pairs}) \text{ returns a state-space model of dimension } N.\
\]

First, the dimension (order) of the state-space model has to be chosen. In the algorithm, the singular values are computed using SVD.
The plot in Figure 4.10 is constructed fully automatically by the algorithm. It shows that an order 2 would be a logical choice for the step signal. For the PRBS signal order 5 is indicated to be a logical choice. Of course, any other order can be specified. Different model orders have been tested and indeed, an order 2 gave the best results for the step signal. For the PRBS signal there was not much difference between different orders, so order 5 was chosen. The property/value pairs are shown in Table 4.6.

For detailed information about these algorithm properties see [Lju], or try the MATLAB commands `help n4sid` or `idprops('algorithm')`.

### 4.5.3 Simulation parameters

Again, many combinations of sample frequency and simulation time have been tested. It can be concluded that increasing the sample frequency has a positive influence on the results. The simulation time has very little effect on the results. Finally, a simulation time of 1 second has been chosen to keep calculation time low while the output of the PRBS still has a reasonable frequency content.
4.5.4 Results

The estimated model is of the form

\[
\begin{align*}
    x(t + Ts) &= Ax(t) + Bu(t) + Ke(t) \\
    y(t) &= Cx(t) + Du(t) + e(t)
\end{align*}
\]

which is converted to a transfer function model of the form

\[
G(s) = \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{i=0}^{n} b_i s^i}
\]

with \( n = 2 \) and \( a_i, b_i \) given in Table 4.7

<table>
<thead>
<tr>
<th></th>
<th>( a_2 ) ((\times 10^4))</th>
<th>( a_1 ) ((\times 10^6))</th>
<th>( a_0 ) ((\times 10^2))</th>
<th>( b_2 )</th>
<th>( b_1 ) ((\times 10^2))</th>
<th>( b_0 ) ((\times 10^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear model</td>
<td>0</td>
<td>-4.55</td>
<td>-7.48</td>
<td>1</td>
<td>1.78</td>
<td>7.91</td>
</tr>
<tr>
<td>N4SID step</td>
<td>-6.877</td>
<td>-3.97</td>
<td>-8.14</td>
<td>1</td>
<td>1.78</td>
<td>7.55</td>
</tr>
<tr>
<td>N4SID prbs</td>
<td>-5.18</td>
<td>-6.16</td>
<td>-9.42</td>
<td>1</td>
<td>2.00</td>
<td>8.87</td>
</tr>
</tbody>
</table>

Note that the \( a_2 \) parameter is not really close to zero, especially for the estimated model based on PRBS output. This results in a different high frequent behavior than the original system. Therefore, the \( a_2 \) parameter was set to zero and the performance of both estimated models improved a lot. Note that this can be done for a real world system if enough information about the system is available.

4.5.5 Model validation

The output of the estimated models is compared with the data in Figure 4.11. Also the frequency response data is compared in a bode diagram in Figure 4.12. The maximum absolute error for the identified model based on a step and PRBS are 71 and 28.3 Pa respectively. The integrated error for the identified model based on a step and PRBS are \( 1.29 \cdot 10^2 \) and \( 7.54 \cdot 10^2 \) respectively. Again, the large difference in integrated error is caused by the static gain error.

4.5.6 Conclusions

Again, the largest errors arise at the high frequent parts of the data, which can be improved by increasing the sample frequency. Note that the estimated model based on PRBS data performs somewhat better, except for the static error of the simulated step.
4.5. SUBSPACE METHOD N4SID

Figure 4.11: Comparison of output data.

Figure 4.12: Comparison of frequency response functions.
4.6 Concluding remarks

A group of selected methods have all been evaluated with a similar approach. I/O data without signal noise and valve dynamics was generated by a simulation model. This data was used for identification purpose. Identification is done with several input signals, to investigate the influence of the different input signals. A pulse, step and PRBS signal are used as inputs.

The resulting model is validated by applying input signals to the resulting model. The reproduced output is compared to the original output data. In order to compare the data, two error quantities are defined, the maximum absolute error and the integrated error (see Section 4.1).

The maximum absolute error and integrated error are summarized in Table 4.8. These results are all obtained using a step signal for validation. From these results it is clear that in case of a perfect sensor and perfect actuator, all methods seem to work well. Each identification method is able to produce a parametric model that has approximately the same I/O behavior as the compressor simulation.

At this point it is still hard to say which identification method is better. However, some advantages and disadvantages of each method have become clear. The frequency response analysis is an easy method which requires little a priori knowledge. From the frf measurement, a good guess for the model structure can be obtained. However, an accurate frf measurement is needed, which is not always possible. Also high simulation times and sample frequencies are needed, which requires large calculation times and data storage. The Hankel-step method is also a very easy method which requires even less a priori knowledge. SVD is used to make a good guess for the model order. It does not require large simulation times which also makes the method very cheap. The PEM and N4SID method have the advantage that any type of I/O data can be used for identification, which makes them both very attractive. A signal with a large frequency content can be used so they are able to capture more dynamic behavior. Both methods are already implemented in MATLAB which makes them easy to use. A big advantage of the N4SID method compared to the PEM is that it uses SVD to make a good guess for the model order and it uses state-space models. State-space models require matrix calculation which can be handled very well by MATLAB. PEM requires more a priori knowledge because the model structure and order has to be chosen. It also uses non-linear optimization algorithms which are in general more expensive to solve. The I/O behavior of the identified models is very accurate for both methods.

In the next chapter the influence of valve dynamics and measurement noise is investigated.

<table>
<thead>
<tr>
<th>Method</th>
<th>Signal used for identification</th>
<th>Maximum Absolute Error (Pa)</th>
<th>Integrated Quadratic Error (Pa²s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. resp. analysis (invfreqs)</td>
<td>pulse</td>
<td>145</td>
<td>2.04 \times 10^4</td>
</tr>
<tr>
<td>Freq. resp. analysis (fitfrd)</td>
<td>pulse</td>
<td>145</td>
<td>2.04 \times 10^4</td>
</tr>
<tr>
<td>Hankel-step method</td>
<td>step</td>
<td>103</td>
<td>1.31 \times 10^4</td>
</tr>
<tr>
<td>PEM</td>
<td>step</td>
<td>21.5</td>
<td>6.96</td>
</tr>
<tr>
<td></td>
<td>PRBS</td>
<td>36.6</td>
<td>2.51 \times 10^2</td>
</tr>
<tr>
<td>N4SID</td>
<td>step</td>
<td>71.1</td>
<td>1.29 \times 10^4</td>
</tr>
<tr>
<td></td>
<td>PRBS</td>
<td>28.3</td>
<td>7.54 \times 10^2</td>
</tr>
</tbody>
</table>

Table 4.8: Overview of results: the maximum absolute error and integrated quadratic error for a step as validation signal
Chapter 5

Identification under realistic conditions

5.1 Introduction

In the previous chapter, all identification methods have been tested using a perfect actuator and perfect sensors; thus omitting valve dynamics and measurement noise. In practice, problems will arise and it will be much harder to identify the compressor dynamics. To investigate the influence of these problems, valve dynamics and measurement noise are simulated to obtain more realistic measurement data. Identification of the valve has already been done; a state-space model of the valve dynamics is available. When identifying the compressor dynamics, valve dynamics are not of interest. Therefore, an inverse model of the valve is used to compensate for the valve dynamics. This idea is shown in Figure 5.1. A bode diagram of the valve dynamics is shown in Figure 5.2.

![Diagram](image)

Figure 5.1: Compensation for valve dynamics (subscript \( v \) = valve, \( c \) = compressor).

In the following sections, the effects of the valve dynamics and measurement noise on the performance of the identification methods will be studied. Step by step, valve dynamics, inverse valve dynamics and measurement noise will be introduced.

It is obvious that the valve is not a perfect actuator. When a pulse signal as shown in Figure 4.1 is sent to the valve, almost nothing happens. The valve is simply to slow to react on such a fast input signal. The same holds for the very short pulses that are present in the PRBS signal. When using these signals, the performance of the identification methods is very poor. Therefore, the input data is adjusted such that there are no pulses shorter than 0.1 second, see Figure 5.3.

There are four outputs shown for each input. The first one represents the output data without (inverse) valve dynamics or measurement noise. The second one represents the output data with valve dynamics, but without compensation or measurement noise. The third one represents the output data with valve dynamics and inverse valve dynamics, but without measurement noise. Finally, the fourth one represents the data with (inverse) valve dynamics and measurement noise (variance 0.2 kPa). The changes of the output data after each step are clearly visible. Note that
the measurement noise is pretty large; approximately up to one third of the measured pressure difference. The signal-to-noise ratio is not very good.

5.2 Frequency response analysis

After introducing valve dynamics, the performance of the frequency response analysis method becomes very poor. First of all, the pulse signal that is used is not a good approximation for an impulse. The longer the pulse, the more it looks like a step signal. The Fourier transform of a step signal is equal to zero ($\forall \omega \neq 0$), which results in a poor estimation of the compressor’s frequency response function. As expected, the frf shows resonance peaks at multiples of 10 Hz due to a 0.1 second pulse length. Also, at high frequencies, the gain and phase behavior is different from earlier observations. The phase drop seems to be much greater than 90 degrees. This is probably caused by the introduced valve dynamics, resulting in a higher order system. The output of the identified systems does not make a good fit with the measurement, nor the output of the original linear model. There is also a large difference between the original linear model and the measurements, which is very bad for linear identification. See Appendix B for data plots. The maximum absolute error has slightly increased from 144 to 177 Pa for both algorithms. The integrated error has decreased one order of magnitude from $2.04 \cdot 10^4$ to $1.91 \cdot 10^3$.

After compensating for the valve dynamics, the performance somewhat improves, but is still bad. The frf still looks bad, due to the poor approximation of the impulse. The output of the identified systems fits the measurements a little bit better than the case without compensating
5.2. FREQUENCY RESPONSE ANALYSIS

valve dynamics. The maximum absolute error is 108 Pa for both algorithms. The integrated error has again decreased one order of magnitude to $1.18 \cdot 10^2$. Note that despite these low values, the fit is still much worse than the case with perfect actuator.

Finally, measurement noise is added, which is disastrous for the frf data. The high frequent behavior is completely invisible due to noise. This makes it impossible to make a good fit. When only the frf data up to 1 Hz is used to make a fit, the gain fit is still surprisingly close to the true compressor dynamics. However, in practice the compressor dynamics are unknown and these results cannot be trusted. For the first time, a difference is observed between the identified systems of the two algorithms. The frf of the identified system of the algorithms does not go to a static value when the frequency moves towards zero. This results in an unstable system. More study would be needed to confirm the conclusion that the INVFREQS algorithm is less robust than the FITFRD algorithm. The maximum absolute error for the model computed by FITFRD is 315 Pa, while it is $1.76 \cdot 10^3$ for the model computed by INVFREQS. The integrated error for the model computed by FITFRD and INVFREQS is $4.98 \cdot 10^3$ and $2.34 \cdot 10^4$. Of course, both error quantities will go to infinity for the unstable system for $t \to \infty$.

The results could be improved by shortening the pulse, but then the valve might not have time to fully open. The measured pressure difference and hence the signal-to-noise ratio will decrease. This is an important trade-off.

![Image of I/O data](image-url)
5.3 Hankel-step method

After introducing valve dynamics, the logarithmic singular value plot shows 13 dominant singular values, implying that the order of the system should be 13. Dominant singular values are the singular values which are responsible for most of the input/output behavior. That is, the singular values which have the highest gain. In this case, the gain of the 14th singular value is a few orders of magnitude smaller compared to the 13th singular value. After identification of the compressor using an order 13, the output accurately describes the measurements. However, the measurements are very different from the output of the original linear model. The identified model does not only include the compressor dynamics, but also the valve dynamics. The maximum absolute error has dropped to 69.4 Pa and the integrated error has dropped to $9.86 \cdot 10^1$.

After compensating for the valve dynamics, the system order implied by the singular value plot has dropped to 9. Again, the output of the identified system closely matches the measurements. However, due to the difference between the output of the original linear model and the measurements, the identified system does not accurately describe the original linear model. When taking a closer look at the measurements, a time delay of about 12 ms is observed. To improve the performance, one could try to first shift the data 12 ms back in time to get rid of this time delay. Doing so further reduces the amount of dominant singular values to 7. The output of the identified model fits the measurements almost perfectly. The maximum absolute error is 100 Pa and the integrated error is $1.34 \cdot 10^2$. The results are very similar for the case without (inverse) valve dynamics at all. Again the question arises if it is allowed to remove the time delay on the output manually. Is the time delay part of the compressor dynamics, or isn’t it? In practise this question might not always be easy.

The amount of dominant singular values drops to 1 after identifying with measurement noise. Earlier observations already showed that the compressor behaves as a first order system. The output of the identified model of order 1 makes a reasonable fit with the output of the original linear model. For a step signal, the static error between the output of the original linear model and the identified model is almost zero. The results compared to the measurement data is somewhat worse. The maximum absolute error is 918 Pa and the integrated error is $1.43 \cdot 10^4$, due to the large static gain error.

5.4 Prediction error method

The prediction error method is based on minimizing the error between the identified system and the measurements. Therefore, it will not be very hard to fit the data for this method. There are a lot of variables that can be changed. The model structure hasn’t been changed during this analysis, accept the time delay on the input ($n_k$), which has a lot of influence on the resulting model.

After introducing valve dynamics, setting $n_k$ equal to 18 resulted in a good fit. The output does not change until after 18 time samples ($18/500 = 0.036$ s), so the input is shifted 18 time samples ahead in time. Alternatively, the output could be shifted 18 time samples backward in time. As expected, the output of the identified systems closely matches the measurements. However, due to valve dynamics, there is a large difference between the output of the original linear model and the measurements. After compensating for the valve dynamics, $n_k$ is set to 10 to get the
best possible fit to the data. Measurement noise does not seem to have a dramatic effect on the performance. The type of input data that is used for identification (step or PRBS) seems to have some effect on the performance. The output of the identified system of a PRBS hardly changes after introducing measurement noise. The maximum absolute error and integrated error for both identified models are about 100 Pa and $1.90 \cdot 10^2$ respectively. The method seems to be more sensitive to noise when a step signal is used. This is not a very surprising result, since the system is only excited once when a step is used, while the PRBS signal is constantly exciting the system, making it less sensitive to noise.

5.5 Subspace method N4SID

N4SID also uses the SVD technique. Thus, a choice for the order of the system can be made by looking at the logarithmic singular value plot (shown in Figure 5.4). After introducing valve dynamics, the amount of dominant singular values is 4 for a step input. For a PRBS input, the distinction between singular values is not that clear. A system order 7 is chosen. Similar to the PEM method, a time delay of the input ($n_k$) can be specified. When $n_k$ is set to 14, the best fit with the measurements is obtained. The computed error quantities are approximately the same as without valve dynamics. The method seems to be more robust, when a PRBS signal is used for identification. When using a PRBS signal, the output of the identified model made a reasonable fit with the measurements for various values of $n_k$ and system order. When using a step signal, the method is a lot more sensitive to these changes. Changes in system order or $n_k$ resulted in very poor fits with the measurements. This is in accordance with the logarithmic singular value plots. For step data, large differences are visible between singular values while for PRBS data the singular values are almost equally decreasing.

![Log of Singular values](image)

Select model order in command window.

Red: Default Choice

(a) Step data

(b) PRBS data

Figure 5.4: Logarithmic singular value plot for step and PRBS data with valve dynamics.

After compensating for the valve dynamics, the system order that is implied by the logarithmic singular value plot is clearly 2 for step data. Again, it is not that clear for PRBS data. Any system order gives reasonable results. However, a system order 2 seems to give the best results. The time delay is set to 8 to get the best possible fit with the measurements. The output of the model
identified with PRBS data clearly fits the measurements a lot better. The maximum absolute error for the model identified with a step is 211 Pa compared to 73 Pa for the model identified with a PRBS. The integrated error is about the same for both: about $1.10 \cdot 10^3$.

The implied system order reduces to 1 for step data and 2 for PRBS data when measurement noise is added. This was also observed when testing the Hankel-step method. The method still works fine and a reasonable fit with the measurement is obtained. The maximum absolute error is 50.4 Pa for both identified models. The integrated error is $7.54 \cdot 10^1$ for the identified model based on a step and $1.52 \cdot 10^2$ for the identified model based on a PRBS.

## 5.6 Concluding remarks

Valve dynamics and measurement noise have been included in the compressor simulation to create more realistic measurement data. The same procedure of identification and evaluation of the previous chapter has been used to test the identification methods. The maximum absolute error and integrated error are summarized in Table 5.1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Signal used for identification</th>
<th>Valve dynamics</th>
<th>Inv. valve dyn.</th>
<th>Meas. noise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Max (Pa)</td>
<td>Int (Pa$^2$s)</td>
<td>Max (Pa)</td>
</tr>
<tr>
<td>Freq. resp. analysis</td>
<td>pulse</td>
<td>177</td>
<td>1.91 $10^4$</td>
<td>108</td>
</tr>
<tr>
<td>(invfreq)</td>
<td>pulse</td>
<td>177</td>
<td>1.91 $10^4$</td>
<td>108</td>
</tr>
<tr>
<td>Hankel-step method</td>
<td>step</td>
<td>69.4</td>
<td>9.86 $10^1$</td>
<td>100</td>
</tr>
<tr>
<td>PEM</td>
<td>step</td>
<td>37</td>
<td>4.94 10</td>
<td>56.7</td>
</tr>
<tr>
<td></td>
<td>PRBS</td>
<td>33.5</td>
<td>2.92 $10^2$</td>
<td>56.3.5</td>
</tr>
<tr>
<td>N4SID</td>
<td>step</td>
<td>81.5</td>
<td>1.66 $10^2$</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>PRBS</td>
<td>93.6</td>
<td>3.98 $10^2$</td>
<td>72.7</td>
</tr>
</tbody>
</table>

Table 5.1: Overview of results: the maximum absolute error and integrated quadratic error for a step as validation signal

It has been shown that all methods perform much worse when valve dynamics and measurement noise is included. However, some methods do better that others. The frequency response analysis performs very bad. The signal-to-noise ratio is very bad when measurement noise is added, because a pulse signal is used for identification. The pulse length has to be increased to improve the signal-to-noise ratio, which worsens the frf measurement even more. The INVFRQS algorithm produces an unstable system when measurement noise is included.

The Hankel-step method is able to produce a parametric model which has approximately the same I/O behavior as the compressor simulation (measurements), but much worse than in case of a perfect actuator and sensor. A disadvantage of the method is that there is only one parameter that can be adjusted in the algorithm, so it is hard to deal with time delays for example.

The PEM and N4SID method are both able to produce parametric models with the same I/O behavior as the compressor simulation. The results are much worse than in case of a perfect actuator and sensor, but the methods seem to perform better than the other two methods when measurement noise is added. The PEM method becomes pretty complicated because there are so many parameters that can be adjusted. For example, for the model structure alone there are
six parameters, so it is difficult to make a good guess for the model structure. The SVD which is used in the N4SID method is a good tool to make a guess for the model order and becomes a bigger advantage compared to PEM when valve dynamics and measurement noise is included.
Chapter 6

Conclusions and recommendations

The goal of this research was to find an appropriate identification method to identify a centrifugal compression system. A Greitzer lumped parameter model of the system already exists, but a problem is that the local dynamic behaviour can not always be determined uniquely from the measurement data. Therefore a better model of the centrifugal compression system has to be found.

An inventory of identification methods was made. To reduce the number of methods, they are evaluated, based on certain criteria. These criteria are based on the specific requirements and limitations that apply to the compression system. These criteria include practical and numerical restrictions, accuracy and costs. Based on these criteria the following selection of methods is made: frequency response analysis, the Hankel step method, PEM methods and N4SID.

The methods have been used to identify the system and the resulting models are validated. The quality of the resulting fit is checked qualitatively as well as quantitatively. First, the data are compared visually. Secondly, two error quantities are computed, the maximum absolute error and an integral of the squared error over an amount of time.

Frequency response analysis does not work well on realistic measurements, due to the fact that this method cannot handle measurement noise well. The FITFRD algorithm gives an unstable system for measurements with noise. The INVFRREQ algorithm gives a stable system, but results in a poor model for the compressor dynamics. Another disadvantage of FRF methods is that an impulse input is necessary, which is not possible in practice due to a bad signal-to-noise ratio.

The approximate realization method works quite well. From the singular values one can see what the order of the system is. Using this order of the model gives good results, also when measurement noise is present. The method is very simple, which makes it easy to use. The only disadvantage is that the user can not have a great influence on the model. Time delay for example is not implemented in the method (However, this could quite easily be implemented by the user).

With PEM methods it is always possible to make the model output resemble the measurement output very well, also when measurement noise is present, because there are many parameters the user can influence. The model structure can be chosen freely. Noise has a slightly larger influence when a step is used as input signal for identification than when a PRBS signal is used (probably because of the repetitive excitations on the system with PRBS). A disadvantage of PEM methods is that one needs to have a lot of knowledge about the system.
With N4SID methods the model output resembles the measurement output very well, also when measurement noise is present. Like in PEM methods, noise has a slightly larger influence when a step is used as input signal for identification than when a PRBS signal is used. A great advantage of N4SID methods is that very little system knowledge is required. The method finds an appropriate order of the system by itself by evaluating the singular values. The algorithm has some extra options to improve the results when system knowledge is available, for example an option to fill in the time delay.

After evaluating the methods, one can conclude that N4SID methods work best for this system. The Hankel-step method works quite well too. The PEM methods work well if enough system knowledge is available. The FRF methods do not work well when measurement noise is present. However, it is hard to compare the methods, because there is a large difference between measurements and the output of the original linear model, due to dynamics of the valve and due to measurement noise. One cannot know if time delay is caused by the valve or by the compressor itself. Therefore one cannot say for sure how well the methods approximate the actual compression system.
Bibliography


Appendix A

Model structures

The general model structure of transfer function models is

\[ A(q)y(t) = \frac{B(q)}{F(q)}u(t - nk) + C(q)\frac{D(q)}{D(q)}e(t) = G(q)u(t) + H(q)e(t) \]  (A.1)

The polynomials are defined as

\[ A(q) = 1 + a_1q^{-1} + \ldots + a_nq^{-na} \]  (A.2)
\[ B(q) = b_1q^{-1} + \ldots + b_nq^{-nb} \]  (A.3)
\[ C(q) = 1 + c_1q^{-1} + \ldots + c_nq^{-nc} \]  (A.4)
\[ D(q) = 1 + d_1q^{-1} + \ldots + d_nq^{-nd} \]  (A.5)
\[ F(q) = 1 + f_1q^{-1} + \ldots + f_nq^{-nf} \]  (A.6)

where \( q \) is called the forward shift operator and is defined as

\[ qx(t) = x(t + 1) \]  (A.8)

Analogously, \( q^{-1} \) is called the backward shift operator and is defined as

\[ q^{-1}x(t) = x(t - 1) \]  (A.9)

Obviously, the general model structure is too complicated to use. Most of the time, most parameters are assumed to be zero. A list of model structures that are most used in practice is shown in Table A.1.
Table A.1: Model structures.

<table>
<thead>
<tr>
<th></th>
<th>Polynomials</th>
<th>Model structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIR (Finite impulse response)</td>
<td>$B$</td>
<td>$y = Bu + e$</td>
</tr>
<tr>
<td>ARX</td>
<td>$AB$</td>
<td>$Ay = Bu + e$</td>
</tr>
<tr>
<td>ARMAX</td>
<td>$ABC$</td>
<td>$Ay = Bu + Ce$</td>
</tr>
<tr>
<td>ARMA</td>
<td>$AC$</td>
<td>$Ay = u + Ce$</td>
</tr>
<tr>
<td>ARARX</td>
<td>$ABD$</td>
<td>$Ay = Bu + \frac{1}{D}e$</td>
</tr>
<tr>
<td>ARARMAX</td>
<td>$ABCD$</td>
<td>$Ay = Bu + \frac{1}{D}e$</td>
</tr>
<tr>
<td>OE (Output error)</td>
<td>$BF$</td>
<td>$y = \frac{B}{F}u + e$</td>
</tr>
<tr>
<td>BJ (Box-Jenkins)</td>
<td>$BFCD$</td>
<td>$y = \frac{B}{F}u + \frac{C}{D}e$</td>
</tr>
</tbody>
</table>
Appendix B

Model validation

In this Appendix the results of model validation of identified models, with more realistic measurement, are given. For each figure it is stated in the caption what method is regarded, and which realistic disturbances are present.

Figure B.11
Figure B.1: Comparison frequency response data frequency response analysis method with valve dynamics.

Figure B.2: Comparison output data frequency response analysis method with valve dynamics.
Figure B.3: Comparison frequency response data frequency response analysis method with valve dynamics and inverse valve dynamics.

Figure B.4: Comparison output data frequency response analysis method with valve dynamics and inverse valve dynamics.
Figure B.5: Comparison frequency response data frequency response analysis method with valve dynamics and measurement noise.

Figure B.6: Comparison output data frequency response analysis method with valve dynamics and measurement noise.
Figure B.7: Comparison output data Hankel-step method with valve dynamics.

Figure B.8: Comparison output data Hankel-step method with valve dynamics and inverse valve dynamics.
Figure B.9: Comparison output data Hankel-step method with valve dynamics and inverse valve dynamics and manually removed time delay of 12 ms.

Figure B.10: Comparison output data Hankel-step method with valve dynamics and inverse valve dynamics and measurement noise.
Figure B.11: Comparison output data PEM method with valve dynamics.

Figure B.12: Comparison output data PEM method with valve dynamics and inverse valve dynamics.
Figure B.13: Comparison output data PEM method with valve dynamics, inverse valve dynamics and measurement noise.

Figure B.14: Comparison output data N4SID method with valve dynamics.
Figure B.15: Comparison output data N4SID method with valve dynamics and inverse valve dynamics.

Figure B.16: Comparison output data N4SID method with valve dynamics, inverse valve dynamics and measurement noise.