Looking for a relation between sensory and instrumental data

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Final report of the postgraduate programme
MATHEMATICS FOR INDUSTRY

LOOKING FOR A RELATION BETWEEN SENSORY AND INSTRUMENTAL DATA

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Subject headings: linear regression / empirical models.
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Chapter 1

Introduction

1.1 Objective of the report

The objective of this report is to have a valid relation between sensory assessment and instrumental measurements. The answer is desirable for several reasons. If a particular sensory property is of interest, only the relevant instrumental properties have to be measured. Secondly the expected sensory score may be predicted from measured instrumental data. Thirdly, an understanding of the mechanical basis of a sensory property should allow the design of systems which would influence this particular physical basis. Similar research is reported in [3] and [5].

The data used for this research were not gathered specifically for this purpose, but to improve the understanding of the effects of washing product formulations, and wash process variables, on the properties of a selection of 'consumer relevant' fabric types.

1.2 Description of the Data

We have three types of fabric construction, interlocks (I), poplins (P), and terry towels (T), and several different fibres within the interlocks, cotton, acrylic, polyester and nylon. These fabrics are washed and dried a number of times, in water or in a product. Water washed fabrics are line dried or tumble dried, all fabrics washed in product are tumble dried. There are no repeated observations.

It is reasonable to believe that the influence of different fabric constructions can be obviated within the instrumental measurements.

1.2.1 Sensory Data

The sensory evaluation was done by a trained panel. The descriptors used in fabric evaluation are:

- Felting
- Thick
- Stretchiness
- Bounciness
- Greasiness
- Flexibility
- Smoothness
- Warmth
- Softness
- Man-made feel

Felting

Felting appears differently according to the construction of the fabric. We can identify:

on rib knits and interlocks Felting is most obvious on rib knits, particularly wool which may start to felt after only one wash. There are a number of physical changes associated with felting. The fabric shrinks and becomes thicker, goes progressively stiffer and loses suppleness, and develops a noticeable 'fuzz' of matted fibres on the surface which gives the fabric a greyish appearance and obscures the grooves in the rib. The amount of felting on the sample
is judged by assessing how much surface fuzz is present, and ignoring all other associated physical changes.

on terry towelling felting appears as matted 'fuzz' inside the loops, and also as matted tufts 'sitting' on the loops. The more fuzz there is blocking up and sitting on the loops, the higher the level of felting.

**Thick**

Thick is defined as the distance between the upper and lower surfaces. On terry towelling the loops are also taken into consideration so that thick is considered from the loop tip on the upper surface to the loop tip in the under surface.

**Stretchiness**

Stretchiness is defined as the ease of distortion by stretching fabric outwards. The further out the fabric stretches, the more stretchiness it has. For most fabrics, assessment for stretchiness is made by taking firm hold of either side of the fabric and pulling outwards to maximum stretch. This is done first in one direction and then the fabric is turned through 90° and repeated in the other direction. The fabric is not stretched on the diagonal as this gives a very false impression of overall stretch.

**Bounciness**

Bounciness is defined as the rate and degree of success with which the fabric 'bounces back' after having been crumpled in the palm. A very bouncy fabric will spring quickly back to shape and will leave less surface indentations, returning much more successfully to its original shape.

**Greasiness**

Greasiness, on all fabric types, is defined as the degree to which the surface of the fabric feels as though it has a greasy coating which gives a slip-slide greasy feel.

**Flexibility**

Flexibility is defined as 'floppiness', and absence of rigidity or stiffness. The more floppiness the fabric has the more flexibility it has.

**Smoothness**

Smoothness is defined as the lack of roughness experienced when moving the flat of the hand across the fabric; an absence of surface friction. The more smoothly and easily the hand glides across the surface and the less roughness is detected, the more smoothness the fabric has.

**Warmth**

Warmth is defined as 'the degree of apparent warmth given off by the fabric'.

**Softness**

Softness is very simply defined as how soft the fabric feels to the touch. This is often described as a lack of stiffness or hardness.
Man-made feel

Man-made feel is described as a synthetic feel to the fabric. As there is a very wide range of synthetic fabric types, this is a subjective measurement. Another way to describe this is as a 'lack of natural feel for that fabric type'.

1.2.2 Mechanical Data

The following measurements were made on all new fabrics, and then after 1, 10, 25, and 50 wash/dry cycles:

<table>
<thead>
<tr>
<th>AREA_SHRINKAGE</th>
<th>THICKNESS</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>WARP.BEND</td>
<td>WEFT.BEND</td>
<td>WARP.RIGID</td>
</tr>
<tr>
<td>WARP.EMT</td>
<td>WEFT.EMT</td>
<td>WARP.RT</td>
</tr>
<tr>
<td>WARP.WT</td>
<td>WEFT.WT</td>
<td>WARP.G</td>
</tr>
<tr>
<td>WARP.2HG</td>
<td>WEFT.2HG</td>
<td>WARP.2HG5</td>
</tr>
<tr>
<td>COMP.INT</td>
<td>COMPLOOP</td>
<td>CROSS.MOVE</td>
</tr>
</tbody>
</table>

AREA_SHRINKAGE

A 10cm-10cm square was marked on the new fabric and this was measured after 1, 10, 25 and 50 washes.

THICKNESS

This was measured using a Shirley micro gauge.

WEIGHT

A 20cm-20cm piece of fabric was weighted and this was transformed to mg/cm².

WARP.BEND, WEFT.BEND, WARP.RIGID, WEFT.RIGID

The bending length of fabric, \( c \), is the length of fabric that will bend under its own weight to a definite angle. It is a measure of the stiffness that determines draping quality. The flexural rigidity, \( G \), is a measure of stiffness associated with handle, i.e. whether the fabric can be handled easily. \( G \) is calculated from the bending length \( c \) and the weight per unit area of the fabric \( w \) as \( G = w \times c^3 \). The bending length and flexural rigidity are measured in both warp and weft directions, giving WARP.BEND, WEFT.BEND WARP.RIGID and WEFT.RIGID.

WARP.EMT, WEFT.EMT, WARP.RT, WEFT.RT, WARP.WT, WEFT.WT

These tensile properties on all fabrics were obtained from tensile hysteresis curves for both warp and weft directions. A typical tensile hysteresis curve is illustrated in figure 1.1, from which the following tensile parameters are obtainable:

1. WARP.EMT and WEFT.EMT, the percentage extension at a specified load (500 gf/cm for poplins or towels or 50 gf/cm for interlocks).
2. WARP.RT and WEFT.RT, the percentage resilience or recovery of the fabric from extension.
3. WARP.WT and WEFT.WT, the work done in extending fabric to this specified load.
CHAPTER 1. INTRODUCTION

Figure 1.1: A typical stress/strain curve with tensile parameters indicated

EMT = max (strain)

$$RT = \frac{a}{a + b}$$

$$WT = 5 \times (a + b)$$

Figure 1.2: A typical shear hysteresis curve with shear parameters indicated
CHAPTER 1. INTRODUCTION

5
3
2
1
0.5
0.25
0
2
3
Crosshead movement, mm

Figure 1.3: A typical compression hysteresis curve

WARP_G, WEFT_G, WARP_2HG, WEFT_2HG, WARP_2HG5, WEFT_2HG5
These shear properties were obtained from the shear hysteresis curves. A similar procedure to tensile measurements was followed. A typical shear hysteresis loop is illustrated in figure 1.2. Shear variables are

1. WARP_G and WEFT_G, the shear stiffness or "elastic shear rigidity" given by the slope of the hysteresis curve between \( \phi = 0.5^\circ \) and \( \phi = 2.5^\circ \).
2. WARP_2HG and WEFT_2HG, the hysteresis at a deformation of 0.5\(^\circ\), and
3. WARP_2HG5 and WEFT_2HG5, the hysteresis at a deformation of 5\(^\circ\).

COMP_INT, COMP_LOOP, CROSS_MOVE
These compression properties were obtained from compression hysteresis curves for a maximum applied force of 5N. A typical hysteresis curve is shown in figure 1.3. Compression variables are:

1. COMP_INT, work done, to the 5N maximum.
2. COMP_LOOP, hysteresis loss, directly related to the loop area.
3. CROSS_MOVE, compression, the distance of cross-head movement (in mm) between 0.25 and 5 N.
Chapter 2

Warp and weft measurements

In most literature, for example in [3], the mean of the warp and weft measurements is used. The purpose of this chapter is to check whether this is a priori possible in our case.

2.1 Introduction

We have eight properties of the fabrics that are measured in warp and weft directions. These are
- WARP.BEND and WEFT.BEND, the bending length,
- WARP.RIGID and WEFT.RIGID, the flexural rigidity (weight per unit area \( \times \) bending length\(^3\)),
- WARP.EMT and WEFT.EMT, the percentage extension at a specified load (500 gf/cm for woven fabrics or 50 gf/cm for fabrics with a knitted construction),
- WARP.RT and WEFT.RT, the percentage resilience or recovery of fabric form extension,
- WARP.WT and WEFT.WT, the work done in extending fabric to a specified load (500 gf/cm for woven fabrics or 50 gf/cm for fabrics with a knitted construction),
- WARP.G and WEFT.G, shear stiffness or elastic shear rigidity,
- WARP.2HG and WEFT.2HG, hysteresis at \( \phi = 0.5^\circ \),
- WARP.2HG5 and WEFT.2HG5, hysteresis at \( \phi = 5^\circ \).

We have plotted warp, weft and mean value in one graph, and also the mean value agains the difference between warp and weft values. If warp and weft values are equal or a linear relation between the mean and the difference is obvious from the graph, it is useless to differentiate between warp and weft value.

2.2 Results from the Plots

2.2.1 BEND

We have one outlier. (See figure 2.1.) The differences within the constructions are small, but the relationship between mean and difference is different for the different constructions. So if we want a model for all fabrics, we cannot a priori use the mean.
CHAPTER 2. WARP AND WEFT MEASUREMENTS

RELATION BETWEEN WARP AND WEFT VALUES

BENDING

RELATION BETWEEN MEAN AND DIFFERENCES

WARP AND WEFT, BENDING

Figure 2.1: Plots for bend
CHAPTER 2. WARP AND WEFT MEASUREMENTS

RELATION BETWEEN WARP AND WEFT VALUES

RIGID

RELATION BETWEEN MEAN AND DIFFERENCES

WARP AND WEFT, RIGID

Figure 2.2: Plots for rigid
CHAPTER 2. WARP AND WEFT MEASUREMENTS

2.2.2 RIGID, flexural rigidity
Again we find one outlier, which is the same as with bending. We expected this, because of the way the flexural rigidity is calculated. (See figure 2.2.) The group in the lower-right corner are all towels. If we excluded the towels, it seems a fairly good relationship, but since the scale of the differences is similar to that of the means, I am not sure we could use just the mean in that case. If we include the towels, we cannot use the mean.

2.2.3 EMT, tensile
We have three fabrics with warp larger than weft. (See figure 2.3.) We can see no special relations, so we need both warp and weft values, even if we consider just one fabric. There are 7 interlocks higher than all the other interlocks, these are all nylons.

2.2.4 RT, tensile
All the towels are close together, no specific relation; it seems that the poplins are almost on one line. (See figure 2.4.) The interlocks are widely spread and we can identify four groups. The first one is a group with a difference of around 10, which are the nylons. The next group are the cotton interlocks, with means between 28 and 38 and differences between -10 and +5. The polyesters are almost on one line, with means from 42 to 54 and differences from -8 to -2. The acrylics are below this group, with means from 44 to 50 and differences from -20 to -8. Clearly we need to try both warp and weft values, except maybe for the poplins.

2.2.5 WT, tensile
The three groups are clearly interlocks, poplins and towels. (See figure 2.5.) For the interlocks the means would be enough, but for the other two groups we need to try both warp and weft values.

2.2.6 G, shear stiffness
We have one outlier, a towel. (See figure 2.6.) We also see 4 poplins with large MEAN.G. We need to try both warp and weft values.

2.2.7 2HG, shear hysteresis at 0.5°
Again we can identify three groups. (See figure 2.7.) Four poplins are a long way from the rest of the fabrics. These are the same as we found with MEAN.G. We also find the outlying towel again.

2.2.8 2HG5, shear hysteresis at 5°
Again we find the three groups. (See figure 2.8.) One towel is far from the other towels, at the far end of the poplins. This is again the same towel. Just below this are the four extreme poplins again.

2.3 Conclusion
Since we are interested in a general model accros different fabrics and constructions, we have to try both warp and weft measurements, instead of just the mean value.
CHAPTER 2. WARP AND WEFT MEASUREMENTS

RELATION BETWEEN WARP AND WEFT VALUES

RELATION BETWEEN MEAN AND DIFFERENCES

Figure 2.3: Plots for emt
CHAPTER 2. WARP AND WEFT MEASUREMENTS

RELATION BETWEEN WARP AND WEFT VALUES

RELATION BETWEEN MEAN AND DIFFERENCES

Figure 2.4: Plots for rt
CHAPTER 2. WARP AND WEFT MEASUREMENTS

RELATION BETWEEN WARP AND WEFT VALUES

\[ wt \]

RELATION BETWEEN MEAN AND DIFFERENCES WARP AND WEFT, WT

\[ \text{MEAN}_\text{WT} \]

\[ \text{DIFFERENCES}_\text{WT} \]

Figure 2.5: Plots for wt
CHAPTER 2. Warp and Weft Measurements

Relation between Warp and Weft Values

Relation between Mean and Differences

Figure 2.6: Plots for $g$
Figure 2.7: Plots for 2hg
CHAPTER 2. WARP AND WEFT MEASUREMENTS

RELATION BETWEEN WARP AND WEFT VALUES

2HG5

RELATION BETWEEN MEAN AND DIFFERENCES

WARP AND WEFT, 2HG5

Figure 2.8: Plots for 2hg5
Chapter 3

Linear Regression Modelling

3.1 Modelling details

We assume a linear model with $p$ terms

$$y = \beta_0 + \sum_{i=1}^{p} \beta_i x_i + e_i \quad e_i \sim N(0, \sigma^2).$$

The $p$ terms are chosen using a stepwise linear regression procedure. A stepwise linear regression procedure starts with an empty model (i.e. no $x_i$'s), and searches for that variable that gives the biggest improvement in the model. Then the resulting model is tested, on whether we can drop one of the variables. The search is then repeated, until no more variables can be added to the model, either because all variables are in the model, or because the influence of the remaining variables is too low.

We choose the variables from all instrumental measurements, untransformed and not taking cross products. This gives 22 possible predictors, while we have 71 observations.

The coefficient of determination

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

is a measure of the proportion of variation in the variable $y$ explained by the model. Since the $R^2$ increases as the number of predictors in the model increases, we adjust the coefficient of determination to:

$$R^2 = 1 - \left(\frac{n-1}{n-p}\right)(1 - R^2).$$

More information on linear regression is available in [4].

As we have a number of explaining variables, and we do not know whether we need all of them, a stepwise regression is a good first start of the analysis.

In the remainder of this chapter we will discuss the results of a stepwise linear regression for all sensory measurements on all fabrics and on the subgroup of interlocks. We will only consider the model for interlocks if the coefficient of determination for this model is higher, since we are mainly interested in models for all fabrics. Regression for other subgroups (poplins and towels) was not possible because a lack of data in these groups. On all graphs the groups are separated (I=interlocks, P=poplins, T=towels).
3.2 Felting

A stepwise linear regression of the sensory measurement FELTING and all instrumental measurements on all fabrics gives the following model:

\[ \text{FELTING} = -26.95 - 27.44 \times \text{THICKNESS} + 5.265 \times \text{WEIGHT} - 0.0103 \times \text{COMP\_INT} - 4.117 \times \text{WARP\_EMT} \]

This model has an \( R^2 = 0.754 \) and two outliers.

Looking at the plots of FELTING against the predicted value and against the residuals, see figure 3.1, we see that the residuals for the poplins are smaller than those for the interlocks or the towels. Also, FELTING is low for the poplins and high for the towels. This is as we would expect, since felting is not significant for poplin cottons. The interlocks are intermediate.

Another thing we can notice is that there are some very large positive residuals, larger than the corresponding predicted value. Therefore we try a stepwise linear regression on the interlocks alone.

The model we get from this analysis is:

\[ \text{FELTING} = -35.13 - 113.9 \times \text{THICKNESS} + 7.11 \times \text{WEIGHT} \]

This model has an \( R^2 = 0.430 \) and one outlier. The \( R^2 \) of this model is fairly low so we will not consider this model any further.

The significance of all parameters is as follows:

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Coefficient</th>
<th>std. error</th>
<th>t-value</th>
<th>sig. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-26.95</td>
<td>11.62</td>
<td>-2.3188</td>
<td>0.0235</td>
</tr>
<tr>
<td>THICKNESS</td>
<td>-27.44</td>
<td>17.32</td>
<td>-1.5842</td>
<td>0.1179</td>
</tr>
<tr>
<td>WEIGHT</td>
<td>5.265</td>
<td>0.849</td>
<td>6.2022</td>
<td>0.0000</td>
</tr>
<tr>
<td>COMP_INT</td>
<td>-0.0103</td>
<td>0.0023</td>
<td>-4.4080</td>
<td>0.0000</td>
</tr>
<tr>
<td>WARP_EMT</td>
<td>-4.117</td>
<td>1.414</td>
<td>-2.9111</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

From this table we can see that the most significant variables in this model are WEIGHT and COMP\_INT. A higher weight and a lower compression integral are associated with higher felting. The higher WEIGHT is associated with towels, which have a much higher chance of felting. The lower compression integral means that it is easier to compress the fabric, or that the fabric is less washed. The variable WARP\_EMT is also a significant variable. A higher value of WARP\_EMT gives a lower value of felting. THICKNESS is not that important.

3.3 Thick

The second sensory measurement is THICK. Linear regression on all fabrics gives:

\[ \text{THICKNESS} = -34.70 + 1.706 \times \text{WEIGHT} + 0.0167 \times \text{COMP\_LOOP} + 46.31 \times \text{WEFT\_BEND} - 3.045 \times \text{WARP\_WT} - 5.165 \times \text{WEFT\_2IIG5} \]

This model has an \( R^2 = 0.929 \) and no outliers.

If we look at the plots of THICK against predicted value and residual (see figure 3.2), we see a reasonable relation between these values. On the plot of THICK against residual, we can see a relation between THICK and residual within the poplins.

The significance of all parameters is as follows:

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Coefficient</th>
<th>std. error</th>
<th>t-value</th>
<th>sig. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-34.70</td>
<td>12.56</td>
<td>-2.7625</td>
<td>0.0075</td>
</tr>
<tr>
<td>WEIGHT</td>
<td>1.706</td>
<td>0.330</td>
<td>5.1628</td>
<td>0.0000</td>
</tr>
<tr>
<td>COMP_LOOP</td>
<td>0.0167</td>
<td>0.00307</td>
<td>5.4539</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT_BEND</td>
<td>46.31</td>
<td>9.056</td>
<td>5.1143</td>
<td>0.0000</td>
</tr>
<tr>
<td>WARP_WT</td>
<td>-3.045</td>
<td>0.580</td>
<td>-5.2467</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT_2IIG5</td>
<td>-5.165</td>
<td>1.794</td>
<td>-2.8785</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

Note that the instrumental measurement THICKNESS is not part of the model for the sensory measurement THICK, because THICKNESS is highly correlated with WEIGHT and
Figure 3.1: Plots of FELTING against predicted value and against residual
Figure 3.2: Plots of thick against predicted value and against residual
COMP_LOOP. High values of THICK are associated with high values of WEIGHT, COMP_LOOP and WARP.BEND and low values of WARP_WT and WEFT_2HG5.

3.4 Stretchiness

A stepwise linear regression for the sensory measurement STRETCHINESS and all instrumental measurements on all fabrics gives the following model:

\[ \text{STRETCHINESS} = 172.2 - 1.239 \times \text{WEIGHT} - 0.0256 \times \text{COMP}_\text{INT} + 122.8 \times \text{CROSS}\_\text{MOVE} + 5.53 \times \text{WEFT}\_\text{EMT} - 1.651 \times \text{WEFT}_\text{RT} - 7.703 \times \text{WEFT}_\text{WT} \]

This model has an $R^2 = 0.888$ and one outlier.

Looking at the plots of STRETCHINESS against the predicted value and against the residuals, see figure 3.3, we see that the model does not predict stretchiness very well. The high coefficient of determination is explained by the difference between fabric constructions. The model separates the fabrics in woven (poplins and towels) and knitted (interlocks) fabrics. All interlock predicted values of stretchiness are around 120. The plot of residuals showed a clear linear relation between value and residual for the woven fabrics. The dispersion of the residuals for the woven values is also very high compared to the value of stretchiness. The above linear relation does not predict stretchiness, but separates woven and knitted fabrics.

Trying a stepwise linear regression on the interlocks alone we find the following model:

\[ \text{STRETCHINESS} = 105.0 + 15.06 \times \text{WARP}_\text{WT} \]

This model has an $R^2 = 0.045$. This model can hardly be called a model. A possible explanation of this phenomenon is that the interlocks have a very high stretchiness. This result indicates that the procedure for assessing stretchiness is not good enough for interlocks. If we are interested in stretchiness for interlocks, the panel, or another one, should be trained on interlocks for measuring stretchiness. A further analysis of these values does not seem useful.

If we consider the first model for STRETCHINESS again, we can test for significance. We get

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-value</th>
<th>Sig. Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>172.22</td>
<td>21.87</td>
<td>7.8753</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEIGHT</td>
<td>-1.239</td>
<td>0.5385</td>
<td>-2.3010</td>
<td>0.0247</td>
</tr>
<tr>
<td>COMP_INT</td>
<td>-0.0256</td>
<td>0.01010</td>
<td>-2.5356</td>
<td>0.0137</td>
</tr>
<tr>
<td>CROSS_MOVE</td>
<td>122.80</td>
<td>31.818</td>
<td>3.8595</td>
<td>0.0003</td>
</tr>
<tr>
<td>WEFT_EMT</td>
<td>5.529</td>
<td>1.3208</td>
<td>4.0866</td>
<td>0.0001</td>
</tr>
<tr>
<td>WEFT_RT</td>
<td>-1.651</td>
<td>0.3368</td>
<td>-4.909</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT_WT</td>
<td>-7.7030</td>
<td>0.4276</td>
<td>-18.0151</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The three most significant variables are three weft tensile measurements, the percentage extension with a positive parameter, the percentage recovery and the work done in extension with negative parameters. Two compression parameters are also in the model. We have variables in the model that we would expect to be in the model.

3.5 Bounciness

A stepwise linear regression of the sensory measurement BOUNCINESS and all Instrumental measurements on all fabrics gives the following model:

\[ \text{BOUNCINESS} = 70.85 + 0.00427 \times \text{COMP}_\text{INT} - 39.99 \times \text{WARP}_\text{BEND} + 0.0354 \times \text{WARP}_\text{RIGID} + 4.126 \times \text{WARP}_\text{EMT} + 0.7636 \times \text{WEFT}_\text{RT} + 8.397 \times \text{WARP}_\text{2HG} \]

This model has an $R^2 = 0.408$ and two outliers.

Looking at the plots of BOUNCINESS against the predicted value and against the residuals, see figure 3.4, we see a group of 6 poplins with large negative residuals, the towels with approximately the same predicted value, 110-120, and a reasonable distribution of the residuals of the interlocks. The six poplins are the two new poplins and the four poplins washed 1 time in water. Washing the poplins in product increases the bounciness to values found with other fabrics. The product has a definite influence here.
Figure 3.3: Plots of STRETCHINESS against predicted value and against residual
Figure 3.4: Plots of BOUNCINESS against predicted value and against residual
Because the above model seems best for the interlocks, we try a stepwise linear regression on
the interlocks alone. The model we get from this analysis is

\[ \text{BOUNCINESS} = -9.365 - 48.63 \times \text{CROSS._MOVE} + 2.146 \times \text{WEFT._RT} + 12.35 \times \text{WARP._2HG} - 7.433 \times \text{WEFT._2HG5} \]

This model has an \( R^2 = 0.617 \) and two outliers. This model is better than the above model
for all fabrics. If we look at the plots for this analysis, see figure 3.5 we see no linear relation on
the plot for residuals.

As the second model is better than the first one, we test the parameters of the second model.

\[
\begin{array}{cccc}
\text{Independent variable} & \text{coefficient} & \text{std.error} & \text{t-value} & \text{sig.level} \\
\text{INTERCEPT} & -9.364 & 21.444 & -0.4367 & 0.6652 \\
\text{CROSS._MOVE} & -48.629 & 26.115 & -1.8620 & 0.0715 \\
\text{WEFT._RT} & 2.1457 & 0.3939 & 5.4474 & 0.0000 \\
\text{WARP._2HG} & 12.349 & 4.5209 & 2.7316 & 0.0100 \\
\text{WEFT._2HG5} & -7.4239 & 3.4279 & -2.1683 & 0.0374 \\
\end{array}
\]

The most significant variable is the warp tensile percentage recovery measurement. The next
significant variable is the shear hysteresis measurements at 0.5°. The intercept is not significant
at all.

### 3.6 Greasiness

A stepwise linear regression of the sensory measurement \( \text{GREASINESS} \) and all instrumental
measurements on all fabrics gives the following model

\[ \text{GREASINESS} = 104.9 - 0.8033 \times \text{WEFT._RT} - 4.417 \times \text{WARP._2HG5} \]

This model has an \( R^2 = 0.381 \) and no outliers. Looking at the plots of \( \text{GREASINESS} \) against
the predicted value and against the residuals, see figure 3.6, we see two horizontal bands of points.
In the lower band are all the poplins and one towel, the towel washed 50 times in water at 60°C
and line dried, in the higher band are all the interlocks and the other towels.

A stepwise linear regression on the interlocks alone gives the following model.

\[ \text{GREASINESS} = 114.9 - 4.690 \times \text{WEIGHT} + 76.37 \times \text{CROSS._MOVE} + 14.33 \times \text{WARP.WT} + 1.446 \times \text{AREA.SHRINKAGE} \]

This model has an \( R^2 = 0.247 \) and one outlier. The \( R^2 \) of both models is too low so we will
not consider them any further.

One possible explanation for the bad models is that greasiness is a surface property. At this
moment no surface measurements are in the database. These will be available in the future, at
that time the analysis can be repeated.

### 3.7 Flexiness

A stepwise linear regression of the sensory measurement \( \text{FLEXINESS} \) and all instrumental
measurements on all fabrics gives the following model

\[ \text{FLEXINESS} = 269.2 + 23.72 \times \text{WARP.BEND} - 138.7 \times \text{WEFT.BEND} + 0.1055 \times \text{WEFT.RIGID} + 14.78 \times \text{WEFT.G} \]

This model has an \( R^2 = 0.703 \) and one outlier.

Looking at the plots of \( \text{FLEXINESS} \) against the predicted value and against the residuals, see
figure 3.7, we see that this model is a fairly good description of the flexiness, with no apparant
discrepancies. In the plot of the residuals a slight linear relation still exists between the flexiness
and the residual. Therefore we also try a stepwise linear regression on the interlocks alone.

The model we get from this analysis is

\[ \text{FLEXINESS} = 50.85 - 0.8366 \times \text{WEFT.RIGID} + 1.089 \times \text{WARP.RT} + 30.46 \times \text{WARP._2HG5} + 1.192 \times \text{AREA.SHRINKAGE} \]

This model has an \( R^2 = 0.659 \) and one outlier. The \( R^2 \) of this model is lower than that of the
model calculated from all fabrics, so we will not consider this model any further.
Figure 3.5: Plots of BOUNCINESS against predicted value and against residual, for interlocks
Figure 3.6: Plots of GREASINESS against predicted value and against residual
Figure 3.7: Plots of FLEXINESS against predicted value and against residual
The significance of all parameters of the first model are as follows:

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>coefficient</th>
<th>std.error</th>
<th>t-value</th>
<th>sig.level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>206.33</td>
<td>16.14</td>
<td>16.6910</td>
<td>0.0000</td>
</tr>
<tr>
<td>WARP.BEND</td>
<td>23.72</td>
<td>6.946</td>
<td>3.4153</td>
<td>0.0011</td>
</tr>
<tr>
<td>WEFT.BEND</td>
<td>-138.73</td>
<td>14.45</td>
<td>-9.6028</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT.RIGID</td>
<td>0.1055</td>
<td>0.0150</td>
<td>7.0523</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT.G</td>
<td>14.78</td>
<td>5.270</td>
<td>2.8046</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

From this table we can see that the most important variables in this model are WEFT.BEND and WEFT.RIGID. A lower WEFT.BEND, and a higher WEFT.RIGID are associated with higher flexiness. These two values are related via

\[
\text{WEFT.RIGID} = \text{WEIGHT} \times (\text{WEFT.BEND})^2.
\]

Other important values are WARP.BEND and WEFT.G, the shear stiffness or 'elastic shear rigidity'. All these measurements are measurements of flexiness.

### 3.8 Smoothness

A stepwise linear regression of the sensory measurement SMOOTHNESS and all instrumental measurements on all fabrics gives the following model

\[
\text{SMOOTHNESS} = 211.5 - 55.23 \times \text{THICKNESS} - 0.01603 \times \text{COMP\_INT} + 67.5 \times \text{CROSS\_MOVE} - 1.581 \times \text{WARP\_RT} - 5.425 \times \text{WARP\_G}
\]

This model has an \( R^2 = 0.620 \) and one outlier.

This model is a fairly good description of the smoothness of towels, judging from plots of smoothness, see figure 3.8, but does not give any information on the smoothness of poplins and interlocks. The plot of the residuals gives the same view, reasonable for towels, but a clear relation left for other fabrics.

Therefore we try a stepwise linear regression on interlocks. This gives the following result

\[
\text{SMOOTHNESS} = 14.6 + 109.6 \times \text{CROSS\_MOVE} + 36.18 \times \text{WEFT\_BEND}
\]

This model has an \( R^2 = 0.088 \) and two outliers. This model gives no useful information.

The significance of all parameters of the first model is as follows:

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>coefficient</th>
<th>std.error</th>
<th>t-value</th>
<th>sig.level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>211.49</td>
<td>27.30</td>
<td>7.7441</td>
<td>0.0000</td>
</tr>
<tr>
<td>THICKNESS</td>
<td>-55.23</td>
<td>10.14</td>
<td>-5.4447</td>
<td>0.0000</td>
</tr>
<tr>
<td>COMP_INT</td>
<td>-0.01603</td>
<td>0.009453</td>
<td>-1.6957</td>
<td>0.0947</td>
</tr>
<tr>
<td>CROSS_MOVE</td>
<td>67.50</td>
<td>31.89</td>
<td>2.1164</td>
<td>0.0381</td>
</tr>
<tr>
<td>WARP_RT</td>
<td>-1.581</td>
<td>0.5737</td>
<td>-2.7556</td>
<td>0.0076</td>
</tr>
<tr>
<td>WARP_G</td>
<td>-5.425</td>
<td>3.645</td>
<td>-1.4884</td>
<td>0.1415</td>
</tr>
</tbody>
</table>

The most important variable is THICKNESS, with a negative coefficient. This means that a thicker fabric is less smooth. This is a understandable variable, since the thinnest fabrics in the experiment are poplins, these are the smoothest fabrics too. Also the thickest fabrics are towels, which are not very smooth. The next important variable is WARP\_RT. This is the warp value of percentage recovery from tensile extension. Again a lower percentage recovery is associated with a smoother fabric. This is also understandable; if a fabric is very smooth, it does not have much elasticity, and the recovery from a tensile extension is lower. The variable CROSS\_MOVE is significant with a positive parameter. This variable is also a measurement of elasticity of the fabric. Again, as soon as surface measurements are available, this analysis should be repeated.

### 3.9 Warmth

A stepwise linear regression of the sensory measurement WARMTH and all instrumental measurements on all fabrics gives the following model
Figure 3.8: Plots of SMOOTHNESS against predicted value and against residual
WARLTH = 176.4 + 0.02485*COMP_LOOP - 11.73*WARP.BEND - 1.965*WEFT.EMT - 1.383*WARP_RT - 2.558*WEFT.WT

This model has an $R^2 = 0.862$ and one outlier.

Looking at the plots of WARLTH against the predicted value and against the residuals, see figure 3.9, we see that this model is a fairly good description of the warmth, with no apparent discrepancies. We can test the significance of all parameters. We get

<table>
<thead>
<tr>
<th>independent variable</th>
<th>coefficient</th>
<th>std.error</th>
<th>t-value</th>
<th>sig.level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>176.41</td>
<td>22.30</td>
<td>7.9099</td>
<td>0.0000</td>
</tr>
<tr>
<td>COMP_LOOP</td>
<td>0.02485</td>
<td>0.00179</td>
<td>13.8770</td>
<td>0.0000</td>
</tr>
<tr>
<td>WARP.BEND</td>
<td>-11.73</td>
<td>3.868</td>
<td>-3.0327</td>
<td>0.0035</td>
</tr>
<tr>
<td>WEFT.EMT</td>
<td>-1.965</td>
<td>0.7285</td>
<td>-2.6969</td>
<td>0.0089</td>
</tr>
<tr>
<td>WARP_RT</td>
<td>-1.3829</td>
<td>0.3429</td>
<td>-4.0326</td>
<td>0.0001</td>
</tr>
<tr>
<td>WEFT.WT</td>
<td>-2.5594</td>
<td>0.2522</td>
<td>-10.1473</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

From this table we can see that the most important variables in this model are COMP_LOOP and WEFT.WT. A higher compression loop and a lower weft value of work done in tensile extension are associated with higher warmth. Other important values are WARP_RT, WARP.BEND and WEFT.EMT.

### 3.10 Softness

A stepwise linear regression of the sensory measurement SOFTNESS and all Instrumental measurements on all fabrics gives the following model

SOFTNESS = 394.6 - 44.23*WEIGHT + 0.04558*COMP_LOOP - 82.88*WEFT.BEND + 0.1433*WEFT.RIGID - 1.624*WARP_RT - 2.712*WEFT.WT + 0.7219*AREA.SHRINKAGE

This model has an $R^2 = 0.728$ and two outliers.

Looking at the plots of SOFTNESS against the predicted value and against the residuals, see figure 3.10, we see that all the poplins have a predicted value lower than 80, whether SOFTNESS is as low as 35 or as high as 95. The residuals of the towels are also very high for low values of softness. Also a linear relation on the residuals plot exists for the interlocks.

Therefore we try a stepwise linear regression on the interlocks alone. The model we get from this analysis is

SOFTNESS = 281.3 - 135.5*WEFT.BEND - 0.04052*WARP.RIGID - 24.07*WEFT.WT + 53.63*WARP.RIG5

This model has an $R^2 = 0.611$ and one outlier. The $R^2$ of this model is lower than that of the above model, so we will not consider this model any further.

The significance of all parameters of the first model are as follows:

<table>
<thead>
<tr>
<th>independent variable</th>
<th>coefficient</th>
<th>std.error</th>
<th>t-value</th>
<th>sig.level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>394.6</td>
<td>49.89</td>
<td>7.9100</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEIGHT</td>
<td>-5.5503</td>
<td>1.097</td>
<td>-5.0586</td>
<td>0.0000</td>
</tr>
<tr>
<td>COMP_LOOP</td>
<td>0.0456</td>
<td>0.00625</td>
<td>7.2960</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT.BEND</td>
<td>-82.88</td>
<td>19.27</td>
<td>-4.3008</td>
<td>0.0001</td>
</tr>
<tr>
<td>WEFT.RIGID</td>
<td>0.1433</td>
<td>0.04991</td>
<td>2.8715</td>
<td>0.0056</td>
</tr>
<tr>
<td>WARP.RT</td>
<td>-1.624</td>
<td>0.5845</td>
<td>-2.7780</td>
<td>0.0072</td>
</tr>
<tr>
<td>WEFT.WT</td>
<td>-2.7116</td>
<td>0.8106</td>
<td>-3.3454</td>
<td>0.0014</td>
</tr>
<tr>
<td>AREA.SHRINKAGE</td>
<td>0.7219</td>
<td>0.4261</td>
<td>1.6942</td>
<td>0.0952</td>
</tr>
</tbody>
</table>

From this table we can see that the most important variables in this model are WEIGHT and COMP_LOOP. A lower weight and a higher compression loop are associated with higher softness. The higher weight is associated with towels, which have a much lower chance of softness, unless the are new. The higher compression loop means that the fabric does not recover immediately when the force decreases, but more slowly. Other important variables are the weft bending length.
Figure 3.9: Plots of WARMTH against predicted value and against residual
Figure 3.10: Plots of SOFTNESS against predicted value and against residual
and flexural rigidity, the first with a negative coefficient and the second with a positive coefficient, and two tensile measurements, \text{WARP.RT} and \text{WEFT.WT}, both with negative coefficients.

We will try to find a transformed linear model for \text{SOFTNESS} in the next chapter.

### 3.11 Man-made feel

A stepwise linear regression of the sensory measurement \text{MM.FEEL} and all instrumental measurements on all fabrics gives the following model

\[
\text{MM.FEEL} = -32.57 - 32.96 \times \text{WEFT.BEND} + 3.262 \times \text{WARP.EMT} + 3.927 \times \text{WEFT.EMT} + 2.536 \times \text{WARP.RT} - 1.1671 \times \text{WEFT.RT} - 4.8954 \times \text{WEFT.WT} + 3.14 \times \text{WARP.2HG} + 0.5531 \times \text{AREA.SHRINKAGE}
\]

This model has an \( R^2 = 0.800 \) and one outlier.

Looking at the plots of \( \text{MM.FEEL} \) against the predicted value and against the residuals, see figure 3.11, we see a group of seven interlocks on a very high constant predicted value, 150. These are the nylon interlocks, a synthetic fabric with a high man-made feel. The model separates these from all other values. The other fabrics also have their predicted value of man-made feel close together, for example the group with predicted values between 82 and 102 are all cotton interlocks. We can see the same on the plot of residuals as parallel vertical lines.

Therefore we try a stepwise linear regression on the interlocks alone. The model we get from this analysis is

\[
\text{MM.FEEL} = 108.8 - 0.05094 \times \text{COMP.INT} + 8.498 \times \text{WARP.EMT}
\]

This model has an \( R^2 = 0.779 \), \( \hat{R}^2 = 0.776 \) and one outlier, the cotton interlock 50 wash, product 40°C. The \( R^2 \) of this model is lower than that of the above model, so we will not consider this model any further.

If we look again at the model for \( \text{MM.FEEL} \) we got from the first analysis, we can test the significance of all parameters. We get

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>coefficient</th>
<th>std.error</th>
<th>t-value</th>
<th>sig.level</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-33.57</td>
<td>30.47</td>
<td>-1.1061</td>
<td>0.2749</td>
</tr>
<tr>
<td>WEFT.BEND</td>
<td>32.95</td>
<td>12.43</td>
<td>2.6516</td>
<td>0.0102</td>
</tr>
<tr>
<td>WARP.EMT</td>
<td>3.262</td>
<td>1.401</td>
<td>2.3285</td>
<td>0.0232</td>
</tr>
<tr>
<td>WEFT.EMT</td>
<td>3.927</td>
<td>1.015</td>
<td>3.8682</td>
<td>0.0003</td>
</tr>
<tr>
<td>WARP.RT</td>
<td>2.536</td>
<td>0.4169</td>
<td>6.0821</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT.RT</td>
<td>-1.671</td>
<td>0.3251</td>
<td>-5.1390</td>
<td>0.0000</td>
</tr>
<tr>
<td>WEFT.WT</td>
<td>-4.895</td>
<td>0.5044</td>
<td>-9.7037</td>
<td>0.0000</td>
</tr>
<tr>
<td>WARP.2HG</td>
<td>3.140</td>
<td>1.686</td>
<td>1.8620</td>
<td>0.0673</td>
</tr>
<tr>
<td>AREA.SHRINKAGE</td>
<td>-0.5531</td>
<td>0.2418</td>
<td>-2.2869</td>
<td>0.0256</td>
</tr>
</tbody>
</table>

From this table we can see that the most important variables in this model are the tensile measurements, \text{WARP.RT}, \text{WEFT.RT} and \text{WEFT.WT}, the first with a positive coefficient and the last two with negative coefficients. The tensile measurement \text{WARP.EMT} is the next significant, with a positive coefficient. This shows that tensile measurements are very important in this model. Whether this is because they are related to man-made feel or because they separate different fabrics is not clear. It is very well possible that surface characteristics are important in man-made feel. Therefore this analysis should be repeated when surface measurements are available.

### 3.12 Conclusion

Most sensory variables can not be predicted from instrumental data using these linear models. The exceptions are \text{THICKNESS}, see section 3.3 and \text{WARMTH}, see section 3.9. One would expect better models using separate intercepts for the three fabrics. Analysis showed us that this is not the case.
Figure 3.11: Plots of MM.FEEL against predicted value and against residual
Chapter 4

Transformed Linear Modelling

As we saw in chapter 3, a simple linear model is not very satisfying. Therefore we shall look at optimal transformations of our data. We will concentrate our efforts on SOFTNESS. After we have found optimal transformations, we will test our model with some other measurements.

Warning: The transformations found using an optimal transformations technique should never be used for hypothesis testing with the same data. If hypothesis testing is required, separate data sets should be used for finding optimal transformations and hypothesis testing. Otherwise a great risk is finding a significant relation where non exists.

4.1 Looking for optimal transformations

In order to find optimal transformations, we used the SAS procedure TRANSREG, an alternating least-squares algorithm. This procedure extends the ordinary general linear model by providing optimal variable transforms that are iteratively derived. The ordinary regression model assumes that the variables are all measured on an equal interval scale and, therefore, can be represented as vectors in an n (the number of observations) dimensional space. Nominal variables, as for example in analysis of variance, cannot be treated as single vectors. These are expanded to design matrices, each column of which can be treated as a vector.

A ordinary general linear model analysis can be described as taking a set of interval and nominal variables, expanding the nominal variables to a set of variables that can be treated as vectors, then fitting a regression or other model to the expanded set of vectors. The alternating least-squares algorithm adds one additional capability to the general linear model; it allows variables whose full representation is a matrix consisting of more than one vector to be represented by a single vector, which is an optimal linear combination of the columns of the matrix. For any type of linear model, an alternating least-squares program can solve for an optimal vector representation of any number of variables simultaneously. PROC TRANSREG iterates until convergence, alternating these two steps: finding the least-squares estimates of the parameters of the model (given the current scoring of the data, that is, the current set of vectors), and finding least-squares estimates of the scoring parameters, that is, the estimates of the optimal vectors (given the current set of model parameters). (A description of this algorithm can be found in [2].)

For a monotonic continuous transformation we can use the monotonic spline option. In the procedure the data are handled by first creating a B-spline basis, of the specified kind, and then regressing the variable onto the basis. A B-spline basis is a way of expressing such a continuous function, which is easy to use in computing. For more information on splines, see [1]. The plot 4.1 gives an example of the kind of output we get from transreg.

After we found transformations, we have to estimate the functions. Because of discussions with experts on fabric handle, we choose to allow for three possibilities, a logarithmic, a linear and an exponential transformation. We choose the following transformations:
Figure 4.1:
CHAPTER 4. TRANSFORMED LINEAR MODELLING

<table>
<thead>
<tr>
<th>variable</th>
<th>transform</th>
<th>variable</th>
<th>transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFTNESS</td>
<td>EXPONENTIAL</td>
<td>WARP.RT</td>
<td>LOGARITHMIC</td>
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<tr>
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<td>WEFT.RT</td>
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<tr>
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<tr>
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<tr>
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<td>LOGARITHMIC</td>
</tr>
<tr>
<td>CROSS.MOVE</td>
<td>EXPONENTIAL</td>
<td>COMP_LOOP</td>
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</tr>
<tr>
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<td>LOGARITHMIC</td>
<td>WARP_BEND</td>
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<td>WEFT.RIGID</td>
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<td>LOGARITHMIC</td>
<td>WEFT.EMT</td>
<td>LOGARITHMIC</td>
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<tr>
<td>WEFT.EMT</td>
<td>LOGARITHMIC</td>
<td>AREA_SHRINKAGE</td>
<td>LINEAR</td>
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</tbody>
</table>

Two possible ways are open now. The first one is taking these transformations and making one large formula for softness. The problem with this is that we get 43 parameters, while we have only 71 observations. Furthermore, this is a very complicated formula, and it is very difficult to make correct initial estimations for the parameters to get the program to converge. After a few trails on this method I abandoned it in favour of the easier, but theoretically less sound procedure. First we estimate all transformations in the above form from the original and transformed values. Then we do a regression on the calculated transformations. This gives a linear model in the calculated transformed values. Then we use the transformations found to get the final formula.

4.2 Estimating the optimal transformations

All transformations were estimated using the nonlinear least squares estimator of SAS, with the Gauss-Newton method. We get the transformations:

T SOFTNESS = 29.850 \times \exp(0.01129 \times \text{SOFTNESS})

T THICKNESS = \text{THICKNESS}

T WEIGHT = 10.477 \times \exp(0.030807 \times \text{WEIGHT})

T COMP.INT = 667.56 \times \exp(0.000319 \times \text{COMP.INT})

T COMP.Loop = \text{COMP.Loop}

T CROSS.MOVE = 0.2055 \times \exp(1.0777 \times \text{CROSS.MOVE})

T WARP.BEND = 0.9225 \times \exp(0.3925 \times \text{WARP.BEND})

T WEFT.BEND = 2.311 \times \ln(\text{WEFT.BEND} + 0.4111)

T WARP.RIGID = 218.12 \times \exp(0.601321 \times \text{WARP.RIGID})

T WEFT.RIGID = 119.49 \times \exp(0.002138 \times \text{WEFT.RIGID})

T WARP.EMT = 3.3945 \times \ln(\text{WARP.EMT} - 0.2950)

T WEFT.EMT = 4.857 \times \ln(\text{WEFT.EMT} - 1.932)

T WARP.RT = 14.123 \times \ln(\text{WARP.RT} - 22.034)

T WEFT.RT = 14.346 \times \ln(\text{WEFT.RT} - 21.314)

T WARP.WT = 1.4866 \times \exp(0.17136 \times \text{WARP.WT})

T WEFT.WT = 5.9696 \times \ln(\text{WEFT.WT} - 0.000134)

T WARP.G = 2.2821 \times \ln(\text{WARP.G} + 0.5216)

T WEFT.G = 2.2623 \times \ln(\text{WEFT.G} + 0.5271)

T WARP.2HG = 1.8265 \times \exp(0.1896 \times \text{WARP.2HG})

T WEFT.2HG = \text{WEFT.2HG}

T WARP.2HG5 = 4.0673 \times \ln(\text{WARP.2HG5} - 0.7975)

T WEFT.2HG5 = 2.2119 \times \exp(0.15886 \times \text{WEFT.2HG5})

T AREA_SHRINKAGE = \text{AREA_SHRINKAGE}. 

CHAPTER 4. TRANSFORMED LINEAR MODELLING

4.3 Estimating the nonlinear model

The next step is taking the transformations we found and doing a linear regression on transformed value of SOFTNESS on the transformed values of the instrumental variables. This is done with a linear regression procedure. The optimal model is

\[
29.849 \times \exp(0.0113 \times \text{SOFTNESS}) = \\
360.0 - 66.45 \times \text{THICKNESS} - 38.573 \times \exp(0.031 \times \text{WEIGHT}) \\
- 17.56 \times \exp(0.00735 \times \text{COMP. INT}) + 0.109 \times \text{COMP. LOOP} \\
- 91.84 \times \ln(\text{WEFT. BEND} + 0.411) + 11.0 \times \exp(0.00214 \times \text{WEFT. RIGID}) \\
- 16.81 \times \ln(\text{WARP. RT} - 22.03) - \ln(\text{WEFT. WT} - 0.000134) \\
- 117.4 \times \ln(\text{WARP. G} + 0.521) + 123.9 \times \ln(\text{WEFT. G} + 0.5271) \\
+ 39.9 \times \exp(0.1896 \times \text{WARP. 2HG}) - 21.79 \times \text{WEFT. 2HG} \\
+ 112.1 \times \ln(\text{WARP. 2HG5} - 0.797) - 62.45 \times \exp(0.159 \times \text{WEFT. 2HG5}) \\
+ 0.927 \times \text{AREA. SHRINKAGE}
\]

This is a very complicated model and it is not clear what it means in a physical sense. To find out whether it is a useful model we will test it with some other data, partly from part 3 of the experiments, see section 1.2, partly from other experiments on textiles. In figure 4.2 are the plots of the predicted value and the residuals against the transformed value of softness. The legend is:

I Interlocks J Jeans' P Poplins R Ribknits
S Shirts T Towels U Underwear

We can see that the model fits interlocks, poplins and towels, as expected, because these data were used to calculate the model, is reasonable for most of the jumpers, except for two woolen ones, and for most of the underwear, but does not fit the ribknits at all.

The conclusion is that this model is not useful for predicting.

4.4 A second model

As we saw in the last section, the model derived from all of the data did not predict the softness of other fabrics. In order to be able to test the model with data from the experiment, the data set was split in approximately 1/3 and 2/3 at random. The largest part will be used to derive the model, then we can use the rest of the data to test the model.

4.5 Looking for optimal transformations again

Running the procedure TRANSREG again with the smaller dataset we find the following transformations. The figures are very similar to the plots before, therefore we will not give the plots again. The optimal transformations are:

<table>
<thead>
<tr>
<th>variable</th>
<th>transform</th>
<th>variable</th>
<th>transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOFTNESS</td>
<td>EXPONENTIAL</td>
<td>WARP. RT</td>
<td>LOGARITHMIC</td>
</tr>
<tr>
<td>THICKNESS</td>
<td>LINEAR</td>
<td>WARP. RT</td>
<td>LOGARITHMIC</td>
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<td>EXPONENTIAL</td>
<td>WARP. WT</td>
<td>LOGARITHMIC</td>
</tr>
<tr>
<td>COMP. INT</td>
<td>LINEAR</td>
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<td>LINEAR</td>
</tr>
<tr>
<td>COMP. LOOP</td>
<td>LINEAR</td>
<td>WARP.G</td>
<td>EXPONENTIAL</td>
</tr>
<tr>
<td>CROSS.MOVE</td>
<td>EXPONENTIAL</td>
<td>WEFT.G</td>
<td>EXPONENTIAL</td>
</tr>
<tr>
<td>WARP. BEND</td>
<td>LINEAR</td>
<td>WARP. 2HG</td>
<td>LINEAR</td>
</tr>
<tr>
<td>WEFT. BEND</td>
<td>LINEAR</td>
<td>WEFT. 2HG</td>
<td>LINEAR</td>
</tr>
<tr>
<td>WARP. RIGID</td>
<td>LINEAR</td>
<td>WARP. 2HG5</td>
<td>LOGARITHMIC</td>
</tr>
<tr>
<td>WEFT. RIGID</td>
<td>LOGARITHMIC</td>
<td>WEFT. 2HG5</td>
<td>EXPONENTIAL</td>
</tr>
<tr>
<td>WARP. EMT</td>
<td>LOGARITHMIC</td>
<td>AREA. SHRINKAGE</td>
<td>EXPONENTIAL</td>
</tr>
<tr>
<td>WEFT. EMT</td>
<td>LOGARITHMIC</td>
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</tr>
</tbody>
</table>

Notice that in this case more variables are not transformed.
Figure 4.2: Predicted value and residuals, using all the data
4.6 Estimating the optimal transformations again

Again we estimate the transformations

\[ \text{TSOFTNESS} = 31.039 \times \exp(0.010899 \times \text{SOFTNESS}) \]
\[ \text{THICKNESS} = \text{THICKNESS} \]
\[ \text{TWEIGHT} = 10.314 \times \exp(0.03126 \times \text{WEIGHT}) \]
\[ \text{TCOMP.INT} = \text{COMP.INT} \]
\[ \text{TCOMP.LOOP} = \text{COMP.LOOP} \]
\[ \text{TCROSS.MOVE} = 0.2140 \times \exp(1.0523 \times \text{CROSS.MOVE}) \]
\[ \text{TWARP.BEND} = \text{WARP.BEND} \]
\[ \text{TWEFT.BEND} = \text{WEFT.BEND} \]
\[ \text{TWARP.RIGID} = \text{WARP.RIGID} \]
\[ \text{TWEFT.RIGID} = 66.036 \times \ln(\text{WEFT.RIGID} - 40.0876) \]
\[ \text{TWARP.EMT} = 3.3625 \times \ln(\text{WARP.EMT} - 0.2279) \]
\[ \text{TWEFT.EMT} = 4.9160 \times \ln(\text{WEFT.EMT} - 1.8731) \]
\[ \text{TWARP.RT} = 14.338 \times \ln(\text{WARP.RT} - 22.637) \]
\[ \text{TWEFT.RT} = 14.348 \times \ln(\text{WEFT.RT} - 21.153) \]
\[ \text{TWARP.WT} = 4.379 \times \ln(\text{WARP.WT} + 0.4742) \]
\[ \text{TWEFT.WT} = \text{WEFT.WT} \]
\[ \text{TWARP.G} = 0.6634 \times \exp(0.5070 \times \text{WARP.G}) \]
\[ \text{TWEFT.G} = 0.6523 \times \exp(0.5187 \times \text{WEFT.G}) \]
\[ \text{TWARP.2HG} = \text{WARP.2HG} \]
\[ \text{TWEFT.2HG} = \text{WEFT.2HG} \]
\[ \text{TWARP.2HG5} = 4.0895 \times \ln(\text{WARP.2HG5} - 0.7930) \]
\[ \text{TWEFT.2HG5} = 2.1757 \times \exp(0.16086 \times \text{WEFT.2HG5}) \]
\[ \text{TAREA.SHRINKAGE} = 2.8028 \times \exp(0.07802 \times \text{AREA.SHRINKAGE}) \]

4.7 Estimating the nonlinear model again

We will do a linear regression procedure again to estimate the new model, and we get

\[ 31.0 \times \exp(0.0109 \times \text{SOFTNESS}) = \\
795.3 - 75.2 \times \exp(0.031 \times \text{WEIGHT}) + 0.028 \times \text{COMP.LOOP} - 1.567 \times \text{WARP.BEND} \\
+ 0.20 \times \text{WARP.RIGID} + 100.7 \times \ln(\text{WARP.EMT} - 0.228) - 67.5 \times \ln(\text{WEFT.EMT} - 1.87) \\
- 72.8 \times \ln(\text{WARP.RT}) - 148.5 \times \ln(\text{WARP.WT} + 0.474) + 10.6 \times \text{WEFT.WT} \\
+ 34.4 \times \text{WARP.2HG} - 29.0 \times \text{WEFT.2HG} + 121.6 \times \ln(\text{WARP.2HG5}) \\
- 55.5 \times \exp(0.161 \times \text{WEFT.2HG5}) \]

A physical interpretation of this model is again difficult to give. However, if we look at the plots of predicted value and residual against the transformed value of softness, see figure 4.3, we see that this model is better than the last. In this plot are only that part of the data set that was not used to estimate the model. Most values fit reasonably well, but there are some outliers, for example the terry towel washed 50 times in product at 15°C and tumble dried. This model is better that the previous one, and could be used for a preliminary test of formulations for detergents. It must not be used for ribknits, the last model showed that these are very different.

4.8 Conclusions

We have found a model for SOFTNESS which is not very good, but usable. Recommendations are:

1. Try to do more experiments, and repeat some of the experiments to get an idea of how much noise is in the data.
2. Repeat the analysis according to the second way using surface measurements too.
Figure 4.3: Predicted value and residual using 2/3rd of the data
Acknowledgements

I thank Unilever Research Laboratory Port Sunlight for giving me the opportunity to work in the laboratory for seven months, and giving me access to the data used in this report.

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Bibliography


