Mean value modelling of a s.i. engine

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Mean value modelling of
a s.i. engine

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Abstract

Despite governmental constraints and legislation, the fuel consumption in traffic and transportation increases persistently. To help solving this severe problem, TUE, Van Doorne's Transmissie (VDT) and TNO started the EcoDrive project to develop a hybrid driveline by using a high or medium speed flywheel, engine, continuously variable transmission and control strategy. This hybrid driveline and the control strategy decreases the fuel consumption (0-20 %) without deteriorating the driveability. To realise a good control strategy, the produced engine output torque \( M_d \) has to be estimated on-line. To estimate this torque the Mean Value Engine Model is applied. This model describes the transfers between input (e.g. the crankshaft angular speed \( \omega_c \)) and throttlevalve angle \( \phi \) and the output (e.g. \( M_d \) and fuel mass flow \( Q_{fc} \)). It is derived from basic physical and thermodynamic principles and seeks to predict the mean values of engine variables (\( \eta_v \) (volumetric efficiency), \( p_e \) (exhaust manifold pressure) or \( M_d \)). The structure of the model is modular and can easily be adapted to other engines. In the past, mean value models were used to generate engine maps, but most of the time the maps are already provided by the manufacturer. The object of this research will be to start from the engine map and go in the opposite direction, trying to estimate the unknown model parameters and engine variables. And if it's possible, to reduce the number of experiments.

For estimating the model parameters and engine variables only the engine output torque \( M_d \), engine speed \( \omega_c \), fuel consumption \( Q_{fc} \), the position \( \phi \) of the throttle valve and intake manifold pressure \( p_m \) have to be known. Under the assumption that the combustion process is stoichiometric and that the intake manifold temperature \( T_m \) is constant, the estimate problem is well defined.
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## Notation

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<th>Meaning</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$p_0$</td>
<td>ambient pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$p_e$</td>
<td>pressure in exhaust manifold</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$p_m$</td>
<td>pressure in intake manifold</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$T_0$</td>
<td>ambient temperature</td>
<td>[K]</td>
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<tr>
<td>$T_m$</td>
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<td>induced mechanical torque</td>
<td>[Nm]</td>
</tr>
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<td>torque due to losses in the engine</td>
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<td>load torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$Q_{atr}$</td>
<td>air mass flow through throttle</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>$Q_{ac}$</td>
<td>air mass flow into cylinders</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>$Q_{fc}$</td>
<td>fuel mass flow into the cylinders</td>
<td>[kg/s]</td>
</tr>
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<td>$\eta_{opt}$</td>
<td>optimal thermal efficiency</td>
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</tr>
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<tr>
<td>$\alpha_{opt}$</td>
<td>optimal spark advance angle</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of specific heat at constant pressure and at constant volume</td>
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<td>quotient of the pressure after and before the throttle plate</td>
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<tr>
<td>$H_0$</td>
<td>fuel heating value</td>
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</tr>
<tr>
<td>$R_a$</td>
<td>specific gas constant of air</td>
<td>[J/kgK]</td>
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Chapter 1

Introduction

The fuel consumption in traffic and transportation increases persistently regardless of governmental constraints and legislations. Moreover, the exhaust emissions resulting from combustion of fossil fuels are a severe threat to our environment. To contribute to the solution of this problem, Van Doorne’s Transmissie (VDT), the Netherlands Organization for Applied Scientific Research (TNO) and the Eindhoven University of Technology (EUT) started the EcoDrive project. The objective of this project is to generate technological knowledge to alleviate these problems for passenger cars without detoriating the driveability.

Shedding new light on the design of drivelines, a perfect match between transmission and engine is persued. This match is not only achieved by the redesign of driveline hardware, but also by the development of an optimising driveline control strategy. Extended possibilities are obtained by applying a high or medium speed flywheel to accumulate kinetic energy. One of the possible lay-outs of a driveline with a flywheel is given in Figure 1.1.

![Figure 1.1: Schematic view of a hybrid driveline with CVT](image-url)
This hybrid drive line uses the engine as the prime mover while the flywheel can be used for short term energy storage. In hybrid mode, the energy stored in the flywheel is used as the main power supply for low vehicle speeds. The energy also can be used for accelerating the vehicle. During braking, kinetic energy of the vehicle is recovered. Because of the use of a Continuously Variable Transmission (CVT) it is possible to operate the engine in almost any desired torque-crankshaft speed working point to meet some predefined criteria like minimal fuel consumption or minimal exhaust emissions. Furthermore, the CVT is used to accelerate or decelerate the flywheel.

To realise a good control strategy the produced engine torque has to be known. Consequently, an engine model is required to predict the torque at the crank shaft of the engine. The engine models presented in literature vary from simple quasi steady state model to very complex models which involve combustion processes. As the engine model has to run on-line for real-time control applications, the Mean Value Engine Model [6] is applied. This model is intermediate between large cycle simulation models and simple phenomenological transfer function models. It has few adjustable parameters and can easily be fitted to a given engine. It can be used for both simulation and control purposes. In earlier research at the EUT, ([8] and [7]), mean value models were presented for a diesel truck engine and a LPG truck engine. Starting points of these models were thermodynamic equations and a lot of measurements to identify the model parameters and to fit the general mean value engine model on the considered engine. With this model it is then possible to produce the engine maps\(^1\) which are required for the EcoDrive project. Often, however, those maps are already generated by the manufacturer. The question then arises whether or not it is possible to go in the opposite direction and to use the manufacturers maps to estimate the model parameters or, at least, reduce the number of experiments that have to be done to get enough information to estimate these parameters.

This is the objective of this report: in what way is it possible to reduce the number of experiments and the number of measurements per experiment without harming the accuracy of the engine model? For the EcoDrive project it is important to obtain a relation between the throttle valve angle and the torque at the crankshaft.

In Chapter 2 the version of the mean value engine model that will be used in this report is defined. In Chapter 3 will be explained how the mean value engine model is fitted to the MB 160 A-class spark ignition (s.i.) engine. Chapter 4 presents some conclusions, which will be the starting point of my future research.

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\(^1\)Tables or graphics with specific fuel consumption as a function of engine angular speed and engine torque
Chapter 2

Mean Value Modelling of the engine

2.1 Introduction

In this chapter the development of a mean value engine (MVE) model for a small engine, typical of those used in passengers cars, is described. The presented model is the result of a literature study into MVE models of spark ignition and diesel engines. An important aspect of mean value modeling is the use of mathematical submodels. As far as possible, the relations in these submodels are derived from basic physical and thermodynamic principles.

Mean value models aim to predict the mean value of the gross external engine variables\textsuperscript{1} and the gross internal engine variables\textsuperscript{2} in steady state and transient situations, with moderate accuracy. It is generally assumed that high accuracy can only be achieved with complicated models. The time scale of the model is just adequate to describe accurately the change of the mean value of the most rapidly changing relevant engine variable. The time scale is of the order of the time for 3 to 5 revolutions, see [5] for more details.

\textsuperscript{1}for example crank shaft speed, engine output torque etc.
\textsuperscript{2}for example thermal and volumetric efficiency.
2.2 The mean value model

The model in this chapter is based on the MVE model in [2] and on the simplified models in [7] and [8]. As said before, the time scale is small compared to the most important 'time constants' of the relevant subsystems. For some quantities equilibrium is established in a few crank shaft revolutions and the relations for these quantities are algebraic equations. Time developing quantities, on the contrary, reach equilibrium in 20 to 2000 crank shaft revolutions and their behaviour is expressed by differential equations.

Figure 2.1: Schematic view of a spark ignition engine and the subsystems

The simplified engine will be divided into three parts [7], subsystems (see Figure 2.1), being the manifold subsystem, the fuel supply subsystem and the crank shaft subsystem. The model has a modular structure and can easily be adapted to other engines.
2.2.1 The intake manifold subsystem

The intake manifold is the part from the airfilter to the cylinder inlet valves. In our model the airfilter isn’t taken into account, so the manifold subsystem consists of the throttle body, plenum and plenumrunners. The function of the plenum is to realise a reasonable constant pressure in stationary situations.

The model of the subsystem relates the pressure between the throttle valve and the cylinder ports to the flow through the throttle and the flow into the cylinders. This pressure $p_m$ and the temperature $T_m$ are supposed to be the same everywhere in the manifold. As in [8], $T_m$ is supposed to be constant. The manifold is modelled as a rigid volume $V_m$ with an input air mass flow $Q_{atr}$ and output air mass flow $Q_{ac}$. The gas in the manifold is assumed to behave as an ideal gas. Hence, the state equation for the intake manifold can be derived with the law of conservation of mass and the ideal gas law [7], yielding
\[
\dot{p}_m = \frac{R_a T_m}{V_m} (Q_{atr} - Q_{ac}).
\] (2.1)

Here $\dot{p}_m$ is the time derivative of $p_m$ and $R_a$ is the specific gas constant of air.

The MB motor has a common rail direct injection system, meaning that the flow through the intake manifold is an air flow without fuel.

**The throttle body flow**

Normally an engine has two mass flows: through the throttle valve and through the by-pass throttle valve. It is assumed that the by-pass is open only in the idle speed mode. However, nearly all available measurements are for other modes than the idle speed mode and the exact working of the by-pass valve is not known at the moment, so we decided to neglect the by-pass air mass flow in the identification process. In fact, the by-pass identification is included in the indentification of the flow through the throttle valve.
The throttle body consist of a cylindrical bore with a throttle plate to control the airflow to
the engine, see Figure 2.2. The throttle body model is based on the theory for one-dimensional,
steady, isentropic, compressible flow of an ideal gas across an orifice (see [4] and [6] for more
details). The equation for the air mass flow through the throttle \( Q_{atr} \) then becomes:

\[
Q_{atr} = \begin{cases} 
    c_d A(\phi) \frac{p_0}{\sqrt{R_0 T_0}} \sqrt{\frac{\kappa}{\kappa+1}} (\frac{2}{\kappa+1})^{\frac{\kappa+1}{\kappa-1}} & \gamma_t \leq 1 \\
    c_d A(\phi) \frac{p_0}{\sqrt{R_0 T_0}} \sqrt{\frac{\kappa}{\kappa+1}} (\frac{2}{\kappa+1})^{\frac{\kappa+1}{\kappa-1}} \cdot h(\gamma_t) & \gamma_t > 1
\end{cases}
\]  

(2.2)

where \( c_d \) is the discharge coefficient and \( \kappa \) the adiabatic exponent of air (\( \kappa \approx 1.4 \)). \( A \) is the opened
area of the throttle, depending on the angle \( \phi \) of the throttle plate [6]:

\[
A(\phi) = \frac{\pi}{4} D^2 \left( 1 - \frac{\cos(\phi + \phi_0)}{\cos(\phi_0)} \right).
\]  

(2.3)

Here, \( D \) is the diameter of the throttle and \( \phi_0 \) the angle of the throttle plate when it is closed.
The function \( h(\gamma_t) \) accounts for the pressure influence in subsonic flow and is given by

\[
h(\gamma_t) = \sqrt{\frac{\gamma_t + 1}{\gamma_t - 1}} \sqrt{\frac{2}{\kappa}} - \left( \frac{\gamma_t}{\kappa + 1} \right)^{\frac{\kappa+1}{\kappa-1}}
\]  

(2.4)

where, apart from a constant factor \( K \), \( \gamma_t \) is the quotient of the pressure after and before the
throttle plate

\[
\gamma_t = \frac{p_m}{p_0} \cdot \left( \frac{\kappa + 1}{2} \right)^{\frac{\kappa-1}{\kappa+1}} = K \cdot \frac{p_m}{p_0}
\]  

(2.5)

For all \( \gamma_t > 0 \), Equation (2.2) can be approximated by (see, for instance, [7]):

\[
Q_{atr} = g_{tr}(\phi, p_m) \frac{p_0}{\sqrt{R_0 T_0}} \cdot K \sqrt{1 - \frac{p_m}{p_0}} \ \forall \ \gamma_t
\]  

(2.6)

where

\[
g_{tr}(\phi, p_m) = (C_1 + C_2 \phi + C_3 \phi^2 + C_4 (\frac{p_m}{p_0}) \phi) \phi
\]  

(2.7)

with parameters \( C_1, C_2, C_3 \) and \( C_4 \) that have to be determined experimentally.
The cylinder flow

The air mass flow $Q_{ac}$ from the intake manifold into the cylinders follows from the so-called speed density formula [7]:

$$Q_{ac} = \frac{1}{2} \eta_v V_c \omega_c \bar{z} \frac{p_m}{R_a T_m}$$

(2.8)

where $V_c$ is the cylinder volume, $z$ is the number of cylinders, $\omega_c$ is the crank shaft angular speed and $\eta_v$ is the volumetric efficiency. The factor $1/2$ shows up, because we are dealing with a four stroke engine. According to [2], experimental results show that it is justified to assume that $\eta_v$ depends on $\omega_c$ and $p_m$ and to use a simple approximation for $\eta_v$:

$$\eta_v(\omega_c, p_m) = C_5 + C_6 \omega_c + C_7 \omega_c^2 + C_8 p_m$$

(2.9)

2.2.2 The crank shaft subsystem

The model for the crank shaft subsystem describes the transformation of chemical energy in the fuel into mechanical energy at the crank shaft. In order to avoid modelling of the cooling and exhaust system losses, the thermal efficiency of the engine is inserted as a multiplier of the fuel mass flow. The equation of motion for this subsystem is

$$J_{tot} \dot{\omega}_c = M_{ind} - M_{loss} - M_d$$

(2.10)

where $J_{tot}$ is the moment of inertia of the engine flywheel, crankshaft, connecting-rod, piston and valve train assembly. It is the sum of a constant and a periodical function of the rotation angle of the crank shaft [9]. The periodical contribution to the moment of inertia is small and is neglected. Furthermore, $M_d$ is the load torque. Under road conditions, this torque is mainly due to rolling resistance, aerodynamic drag of the vehicle and friction in the driveline, etc. Also torques to drive engine accessoires (power-steering, fan and air conditioner) can contribute to $M_d$. Under testing conditions, $M_d$ equals the brake torque applied by the dynamometer (measured). Finally, $M_{loss}$ is the torque due to friction and pumping losses in the engine and generator. An approximation for the loss torque is given by [2]:

$$M_{loss} = \beta_1 + \beta_2 \omega_c + \beta_3 \omega_c^2 + (\beta_4 + \beta_5 \omega_c) p_m$$

(2.11)

where $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, $\beta_5$ are parameters that have to be determined experimentally. The so-called induced torque, $M_{ind}$ in Equation 2.10 can be determined from [2, 3]:

$$M_{ind} = \eta_i H_0 \frac{Q f_c}{\omega_c}$$

(2.12)
Here $H_0$ is the fuel heating value of gasoline ($H_0 = 45 \cdot 10^6 \,[\text{J/kg}]$). The thermal efficiency $\eta_t$ can be approximated by [6]:

$$\eta_t(\mu_c, \alpha) = \eta_{opt} AFI(\mu_c)SI(\alpha)$$

(2.13)

where $\eta_{opt}$ is the optimal thermal efficiency. An approximation of the Air/Fuel ratio Influence (AFI) follows from

$$AFI(\mu_c) = 1 + a_3 \mu_c^2 + a_4 \mu_c^2; \quad \mu_c = \frac{(1 - \lambda)L_{st}}{1 + \lambda L_{st}}$$

(2.14)

where $\lambda$ is the air/fuel equivalence ratio and $L_{st}$ is the stoichiometric air/fuel ratio for gasoline ($L_{st} = 14.7$). The parameters $a_3$ and $a_4$ have to be determined experimentally. An approximation of the Spark advance Influence (SI) is given by

$$SI(\alpha) = 1 + s_3(\alpha - \alpha_{opt})^2$$

(2.15)

where $\alpha$ is the spark advance angle whereas the parameters $s_3$ and $\alpha_{opt}$ have to be determined experimentally. The following constraints have to be satisfied:

$$a_3 \leq 0$$

(2.16)

$$AFI(\mu_c) > 0$$

(2.17)

$$s_3 < 0$$

(2.18)

### 2.2.3 The fuel supply subsystem

The fuel supply subsystem describes the fuel flow from the point of injection to its arrival at the intake valves of the cylinders. The model applied in [7] is a simplified version of the model used in [2, 8], but is sufficient accurate in our situation. The fuel supply model used in [2, 8] accounts for condensation and evaporation of the fuel. However, we are dealing with direct injection above the inlet port with a pressure of 1350 [10^5\,\text{Pa}]. So, condensation and evaporation in the manifold are not likely and we may assume that all injected fuel directly streams into the cylinder.

The injection always begins after the inlet port is opened and is always finished before the intake valve close. The fuel dynamics can be neglected and the fuel supply model becomes:

$$Q_{fc} = \frac{Q_{ac}}{\lambda_{ECU} L_{st}}$$

(2.19)

where $\lambda_{ECU}$ is the air/fuel equivalence determined by the Electronic Control Unit.

In Appendix A a summary of the model equations is listed.
Chapter 3

MB 160 A-class spark ignition engine

3.1 Introduction

In the mean value engine model, as presented in chapter 2, there are two types of relations between engine variables: instantaneous and time developing. The difference between the two types is related to the time scales on which the variables change. For estimating a number of the parameters in the model, we use steady state measurements (provided by TNO). Afterwards we try to estimate the extra parameters that become relevant in transient situations.

3.2 Steady state mean value model

In this section it will be explained how some of the parameters can be estimated using steady state measurements. When these parameters are known, the model can be extended to describe transients. Then also time delays and inertia's have to be estimated, but that will not be done in this report.

In steady state all derivatives with respect to time are equal to zero. We assume that during the tests the engine operates in stoichiometric mode, so \( \lambda = \lambda_{ECU} = 1 \) and that the spark advance angle \( \alpha \) is equal to \( \alpha_{opt} \). The earlier given set of equations then reduces to:
Intake manifold:

\[ \dot{p}_m = 0 \implies Q_{ac} = Q_{atm} \quad (3.1) \]

Throttle body subsystem:

\[ Q_{atm} = g_{tr}(\phi, p_m) \frac{p_0}{\sqrt{R_a T_0}} \cdot K \sqrt{1 - \frac{p_m}{p_0}} \forall \gamma_t \quad K = \left( \frac{\kappa + 1}{2} \right) \frac{\kappa - 1}{\kappa - 1} \quad (3.2) \]

Fuel supply subsystem:

\[ Q_{ac} = (C_0 + C_6 \omega_c + C_7 \omega_c^2 + C_8 p_m) \frac{z V_c \omega_c p_m}{2 \pi R_m T_m} \quad (3.3) \]

\[ Q_{ac} = L_{atm} \lambda_{ECU} Q_{fc} \quad (3.4) \]

Crank shaft subsystem:

\[ M_d = \eta_{opt} H_0 \frac{Q_{fc}}{\omega_c} - (\beta_1 + \beta_2 \omega_c + \beta_3 \omega_c^2 + (\beta_4 + \beta_5 \omega_c) p_m) \quad (3.5) \]

The measured quantities are \( M_d, \omega_c, \phi \) and \( b_e \) [\( \frac{\text{kWh}}{\text{kg}} \)] (specific fuel consumption) where \( b_e \) is related to \( Q_{fc} \) by:

\[ Q_{fc} = \frac{b_e M_d \omega_c}{10^6 \cdot 3600} \quad (3.6) \]

These equations will be the start of the estimation in my research. In what way is it possible to reduce the number of experiments.
Chapter 4

Conclusions

We have to estimate the unknown constant parameters ($\theta_1, \ldots, \theta_{14}$) and unknown variable ($p_m$) in the equations on page 10 for steady state situations. The measured variables are the throttle angle ($\phi$), the air mass flow ($Q_{fc}$), the engine output torque ($T_{brake}$) and the engine angular speed ($\omega_c$). The question remains whether or not it is possible to estimate mentioned constant parameters and variable. Which will be elaborated in my next report.

The other situation is transient. But the provided measurements are for static situations, which contains not enough information to identify e.g. the total inertia or time delays. So, we need extra measurements (transient) or extra assumptions. In the transient situation, the time derivatives are not equal to zero.

In my future report will be elaborated in what way the steady state solution is sufficient enough for the transient solution.
Chapter 5

Bibliography


Appendix A

Model equations

Intake manifold subsystem

\[ \dot{p}_m = \frac{R_a T_m}{V_m} (Q_{atr} - Q_{ac}) \quad \dot{p}_m \rightarrow 0 \quad Q_{atr} = Q_{ac} \]

\[ Q_{atr} = g_{tr}(\phi, p_m) \frac{p_0}{\sqrt{R_a T_0}} \cdot K \sqrt{1 - \frac{p_m}{p_0}} \forall \gamma_t \]

\[ \gamma_t = \frac{p_m}{p_0} \cdot \left( \frac{\kappa + 1}{2} \right)^{\kappa-1} = \frac{K p_m}{p_0} \]

\[ g_{tr} = (C_1 + C_2 \phi + C_3 \phi^2 + C_4 \gamma_t \phi) \phi \]

\[ Q_{ac} = \frac{1}{2} \eta_v V_c \frac{\omega_c}{2\pi} \frac{p_m}{R_a T_m} \]

\[ \eta_v(\omega_c, p_m) = C_5 + C_6 \omega_c + C_7 \omega_c^2 + C_8 p_m \]
Crank shaft subsystem

\[ J_{\text{tot}} \dot{\omega}_c = T_{\text{ind}} - T_{\text{loss}} - Td \]

\[ T_{\text{loss}} = \beta_1 + \beta_2 \omega_c + \beta_3 \omega_c^2 + (\beta_4 + \beta_5 \omega_c) p_m \]

\[ T_{\text{ind}} = \eta_t H_0 \frac{Q_{fc}}{\omega_c} \]

\[ \eta_t(\mu_c, \alpha) = \eta_{\text{opt}} AFI(\mu_c) SI(\alpha) \]

\[ AFI(\mu_c) = 1 + a_3 \mu_c^2 + a_4 \mu_c^3 \]

\[ \mu_c = \frac{(1 - \lambda) L_{st}}{1 + \lambda L_{st}} \]

\[ SI(\alpha) = 1 + s_3 (\alpha - \alpha_{\text{opt}})^2 \]

\[ a_3 \leq 0 \]

\[ AFI(\mu_c) > 0 \]

\[ s_3 < 0 \]

Fuel supply subsystem

\[ Q_{fc} = \frac{Q_{sc}}{\lambda_{\text{ECU}} L_{st}} \text{; stiochiometrisch } \implies \lambda_{\text{ECU}} = \lambda = 1 \]