The inverse of the incomplete beta integral

Newby, M.J.

Published: 01/01/1991

Document Version
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):
THE INVERSE OF THE INCOMPLETE BETA INTEGRAL

Dr. M.J. Newby

Intern rapport TUE/BDK/ORS/91/7
THE INVERSE OF THE INCOMPLETE BETA INTEGRAL

Martin Newby¹,

Faculty of Industrial Engineering and Management,
Eindhoven University of Technology,
and
Frits Philips Institute for Quality Management, Eindhoven

Keywords: beta integral; F-distribution; beta distribution; inverse beta function

LANGUAGE

FORTRAN 77

DESCRIPTION AND PURPOSE

For a given probability $\alpha \epsilon [0,1]$ and $m>0$, $n>0$ the subprogram returns \( x_\alpha \epsilon [0,1] \), the percentile of the beta distribution satisfying

\[
I_x(m,n) = \frac{1}{B(m,n)} \int_0^{x_\alpha} u^{m-1}(1 - u)^{n-1} du = \alpha
\]

NUMERICAL METHOD

Algorithm AS 64/AS 109 (Majumder and Chattarjee, 1973; Cran, et al., 1977) uses an approximation to determine an initial value for \( x_\alpha \) and thereafter a modified Newton-Raphson method to produce the required accuracy. The modifications are required to ensure that the returned value lies in the appropriate range. When, for example, $m>1$ and $n<1$ the convergence is very slow because the iteration tries to push the \( x \) values outside the interval \([0,1]\). These difficulties are remarkably easily resolved and a clearer and simpler algorithm obtained by remarking that the integral is a monotone increasing function of \( x \) for $x \epsilon [0,1]$. Because the support of the beta function is \([0,1]\) repeated

¹ Address for correspondence: Faculty of Industrial Engineering and Management, Eindhoven University of Technology, Den Dolech 2, PO Box 513, 5600 MB Eindhoven, The Netherlands.
bisection converges rapidly to the root without the need for any special precautions. Thus although the algorithm is not uniformly better than AS 64/AS 109 it is reliable and cannot fail. No initial approximations to the solution are needed and it is expressed entirely in terms of the incomplete beta integral.

**STRUCTURE**

```plaintext
REAL FUNCTION BTAINV(M,N,ALPHA,B,IFAIL)

Formal parameters

M     Real    input: parameter m of the beta integral
N     Real    input: parameter n of the beta integral
ALPHA Real    input: the probability level
B     Real    input/output: the logarithm of the beta function. If B>0 it is used as the value of B(p,q). If B<0 the value of B(p,q) is evaluated within the BETAIN subprogram and is available through B for later use
IFAIL Integer output: error flag, IFAIL=0 indicates success
               IFAIL=1 P<0 or Q<0

AUXILIARY ALGORITHMS

BTAINV uses the function BETAIN(X,P,Q,B,IFAIL) to evaluate the incomplete beta function, BETAIN in turn requires BETA0 and the logarithm of the gamma function.

REFERENCES


```
REAL FUNCTION BTAINV(M,N,ALPHA,B,IFAIL)
IMPLICIT REAL (A-H,O-Z)

C*******************************************************
C
C FUNCTION BTAINV(M,N,ALPHA,B,IFAIL)
C
C*******************************************************

REAL P,ALPHA,X0,X1,X2,B1,M,N,
1 ZERO,ONE,HALF,TOL
INTEGER NOUGHT,UNITY,TWO,IFAIL

PARAMETER ( ZERO = 0.0E0,
1 ONE = 1.0E0,
2 TWO = 2,
3 HALF = 0.5E0,
4 UNITY = 1,
5 NOUGHT = 0,

C*******************************************************
C
C NSTEP IS SUCH THAT 2**(-NSTEP)<=TOL.
C I.E X IS IN [X-.5*TOL,X+.5*TOL]. ABOUT 3.3 STEPS PER DECIMAL PLACE
C ARE NEEDED. 17 STEPS GIVE 5DP, 20 GIVE 6DP AND 23 7DP
C
C*******************************************************

6 NSTEP = 23,
7 TOL = 1.0E-5

P=ALPHA

C*******************************************************
C
C DEAL WITH TRIVIAL CASES AND ARGUMENT CHECKING
C
C*******************************************************

IFAIL = NOUGHT
IF ( P .LT. ZERO .OR. P .GT. ONE ) THEN
    IFAIL = UNITY
    GO TO 1000
ELSEIF ( M .LE. ZERO .OR. N .LE. ZERO ) THEN
    IFAIL = TWO
    GO TO 1000
ELSEIF ( P .EQ. ZERO ) THEN
    BTAINV = ZERO
    GO TO 1000
ELSEIF ( P .EQ. ONE ) THEN

    BTAINV = ONE
    GO TO 1000

ELSEIF ( M .EQ. ONE ) THEN

    BTAINV = ONE - (ONE - P)**(ONE/N)
    GO TO 1000

ELSEIF ( N .EQ. ONE ) THEN

    BTAINV = P**(ONE/M)
    GO TO 1000

ELSE

    C**********************************************************************
    C
    C FIND THE INVERSE ONLY FOR THOSE P THAT ARE NOT ZERO OR ONE
    C USE SIMPLE BISECTION. IF B <= 0 THE VALUE IS RETURNED BY THE
    C FIRST EVALUATION OF BETAIN AND THEN CARRIED FORWARD
    C
    C**********************************************************************

    X0 = ZERO
    X2 = ONE

    DO 10,I=1,NSTEP
        X1 = HALF*(X0 + X2)
        B1 = BETAIN(X1,M,N,B,IFAULT)
        IF ( P .GT. B1) THEN
            X0 = X1
        ELSE
            X2 = X1
        ENDIF
    10 CONTINUE

    BTAINV = X1

    ENDIF

1000 CONTINUE

RETURN
END