Survey on the modelling of columns with internal ribbing and apertures
Hijink, J.A.W.; van der Wolf, A.C.H.

Published: 01/01/1974

Document Version
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the author’s version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.

• The final author version and the galley proof are versions of the publication after peer review.

• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.

• You may not further distribute the material or use it for any profit-making activity or commercial gain

• You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 12. Dec. 2018
SURVEY ON THE MODELLING OF COLUMNS WITH INTERNAL RIBBING AND APERTURES

J.A.W. HIJINK
A.C.H. VAN DER WOLF

DIVISION OF PRODUCTION TECHNOLOGY

REPORT WT-0337

PROFESSOR P.C. VEENSTRA
PROFESSOR A.C.H. VAN DER WOLF

24th General Assembly of CIRP
TC Ma - CAD
KYOTO, 1974
1. Introduction

In order to estimate the static and dynamic behaviour of machine tool structures a number of methods are used. Some methods well known are:

a. making a topological model of beamlike elements, including the use of rigid beams, hinges, springs etc.

b. dividing the structure into a great number of finite elements as beams, plate- and cubic elements.

Especially for machine tools built up out of parts which are rather slender the first method mentioned is often used successfully (1), (2), (3), (4). The number of elements used for the model is small, so the cost of preparation and computations is low.

But many machine tools are built up out of boxtype parts with the height of the same order or smaller as the dimensions of the cross-section. For these machine tools the beam method most times will lead to unsatisfactory results. The main causes for this are local deformations at connection points and the difficulty to obtain the right values for the properties of the beam element. Also the properties of the beam elements for columns with internal ribbing, apertures and transverse partitions are hard to obtain.

In order to overcome most of these difficulties the finite element method can be used. This method will lead to more accurate results, but the number of elements and certainly the number of degrees of freedom will be much higher and the cost of preparation and computing will rise even more.

An approach in between the two methods is the calculation of some difficult elements with the help of a finite element program after which the computed characteristics are fed into a beam-element program.
2. The beam element

In order to give values for the strength and the stiffness of elastic beams approximation theories are used. The theory of Bernoulli-Navier is well known for bending. For torsion the theory of Bredt is used for beams with closed cross-sections and the theory of De Saint-Venant for beams with open cross-sections.

In many beam-element programs only the displacements and rotations of the centre of gravity are calculated. In fig. 2.1 the local coordinates and forces are shown. The normal force $N$, shear-forces $D_y$ and $D_z$, the torsional moment $M_x$ and bending moments $M_y$ and $M_z$ can act at either end of the element. Fig. 2.2 shows the definition of the local displacements and rotations.

From each element the following characteristics must be known:

- $l =$ length of the element
- $A =$ cross-sectional area
- $I_y =$ moment of inertia about the Y-axis
- $I_z =$ moment of inertia about the Z-axis
- $J =$ torsional constant for the cross-section
- $\kappa_y =$ shear distribution factor in Y-direction
- $\kappa_z =$ shear distribution factor in Z-direction.

and the material constants

- $E =$ Young's modulus of elasticity
- $G =$ shear modulus.

For the calculation of $A$, the centre of gravity, $I_y$ and $I_z$ the formulae are well known for simple cross-sections. In the case of more intricate cross-sections one can use a program as described by Döpper (5).

According to De Saint-Venant the torsional constant for an open thin walled cross-section will be

$$J = \frac{1}{3} \int s t^3 \, ds \quad (2.1)$$

where $t =$ the local wall thickness.

The torsional constant for a closed thin walled cross section is according to the theory of Bredt

$$J = \frac{4 A_t^2}{\oint_{\ell} ds} \quad (2.2)$$

where $A_t =$ the total area closed by the profile line.
In order to tell something about the influence of the shear, its distribution factor $\kappa$ can be found from

$$\kappa = \frac{A}{D} \int_A \tau^2 \, dA \quad (2.3)$$

where $\tau$ = shear tension.

Dreyer (6) gives an approximation for $\kappa$ as follows

$$\kappa_y = \frac{A}{I_z} \int_A \frac{S_z^2(y)}{b^2(y)} \, dA \quad (2.4)$$

where $S_z(y) =$ moment of area about the $z$-axis and $b$ is the total thickness of the walls in $z$-direction.

About the $Z$-axis

$$\kappa_z = \frac{A}{I_y} \int_A \frac{S_y^2(z)}{b^2(z)} \, dA \quad (2.5)$$

In the table below the value for $\kappa$ is given for some cross sections:

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>full square</td>
<td>1.2</td>
</tr>
<tr>
<td>full circular</td>
<td>1.1</td>
</tr>
<tr>
<td>full elliptical</td>
<td>1.15</td>
</tr>
<tr>
<td>circular tube</td>
<td>1.9</td>
</tr>
<tr>
<td>square tube</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Schlemper (7) wrote a program to compute the approximate value of $\kappa$ for closed cross sections.
3. Calculation of the displacements of a beam element.

The following displacements can be calculated for a beam clamped at one end and loaded at the other end:

3.1. Elongation of the beam due to a normal force $N$

$$u = \frac{N}{E \frac{I}{F}} \quad (3.1)$$

3.2. Deflections and rotations due to the moments $M_y$ and $M_z$

$$w = -\frac{M_y l^2}{2E I_y} \quad (3.2)$$

$$\varphi_y = \frac{M_y l}{E I_y} \quad (3.3)$$

$$v = \frac{M_z l^2}{2E I_z} \quad (3.4)$$

$$\varphi_z = \frac{M_z l}{E I_z} \quad (3.5)$$

3.3. Deflections and rotations due to the shear forces $D_y$ and $D_z$. 

The shear force $D_y$ causes a deflection $v_b$ due to bending and an additional deflection $v_s$ due to shear

$$v = v_b + v_s = \frac{D_y l^3}{3EI} + \frac{\kappa D_y l}{GA} \quad (3.6)$$

The rotation $\varphi_z$ will be

$$\varphi_z = \frac{D_y l^2}{2EI} \quad (3.7)$$

Due to the shear force $D_z$

$$w = \frac{D_z l^3}{3EL_2} + \frac{\kappa D_z l}{GA} \quad (3.8)$$

$$\varphi_y = -\frac{D_z l^2}{2EL_2} \quad (3.9)$$
Fig. 3.1 shows for a certain shearfactor the influence of the length of the beam on the ratio between \( w_s \) and \( w \). Shear only causes a displacement but no rotation, therefore the influence of shear will decrease with the length of the coupled element. Fig. 3.2 shows the ratio between \( w_s \) and \( w \) when an element of a certain length is coupled to the loaded element.

3.4. Rotation due to the torsional moment \( M_x \)

A moment \( M_x \) causes:

a) rotation
b) warping and
c) distortion of the cross-section (see fig. 3.3).

ad a: The rotation of the cross-section \( S \) is given by

\[
\phi_x = \frac{M_x l}{G J}
\]

(3.10)

In this formula \( J \) is calculated by the formulæ (2.1) or (2.2) according to the theory of Bredt or De Saint-Venant. In these theories there is no axial normal tension and the cross-section is free to warp. Further more it is assumed that the shape of the cross-section does not change.

ad b: Warping does not seem to influence the rotation or displacement of the centre of gravity of the cross-section. Depending on the shape of the cross-section warping causes a relative axial displacement of one point to another.

ad c: The distortion of the cross section is indicated by the change of the angle \( \psi \) between two adjacent walls. Distortion causes extra displacements of points within the cross-sectional area. Fig. 3.4 shows that for some points these displacements can be much higher in comparison with displacements without distortion.
4. Influence of ribbing and transverse partitions.

In order to research the influence of ribbing and transverse partitions, many practical and theoretical work has been done.

Dreyer (6) has measured these influences on a column as shown in fig. 4.1. The results of the measurements for bending are given in fig. 4.2. The figure shows the relative bending stiffness and the bending stiffness weight ratio. The results are also compared with the relative difference of I and the ratio I/A. From these results it can be seen that for this column there is a fair agreement between measurements and the bending theory of Bernoulli-Navier.

The measurements for the torsional moment do not give values for the rotation but for the displacements of one corner point. In those cases where there is a coverplate on the column, this displacement gives a good indication for the rotation of the cross-section. In fig. 4.3 and fig. 4.4 the results of the measurements are shown. The discrepancies between the theoretical value according to De Saint-Venant and the measurements is clear. This is due to the assumption that the cross-section is free to warp and particularly that there is no distortion.

Based on the Vlasov theory Janssen and Veldpaus (8), (9), (10) analysed the strength and stiffness of rectangular box-ginders with transverse partitions. Veldpaus (11) evaluated this theory for all kinds of open and closed cylindrical thin walled cross-sections. With the help of a special program the characteristics can be calculated. In fig. 4.5 the analysis of the column of a milling machine is shown with and without a topplate. The influence of ribbing and topplate can be clearly seen.

5. Apertures.

In many column elements of machine tool structures apertures in walls and transverse partitions are present. To know the influence of these apertures Dreyer (6) and Bielefeld (12) did a number of experiments. In fig. 5.1 the influence of an aperture in a transverse partition is shown. One can see that for $A'/A > 0.3$ the torsional stiffness decreases rapidly. The influence of apertures in the wall is shown in fig. 5.2. These apertures too have a remarkable influence on the stiffness even after been closed by a cover.
6. Finite elements.

To overcome the problems which occur at points with local deformations, torsion and bending of columns with internal ribbing, partitions and apertures the displacements can be calculated by dividing the column into a number of finite elements.

Typical basic elements include beam elements, thin plate elements of triangular, rectangular or general quadrilateral form and prismatic elements (see fig. 6.1). By connecting such finite elements to another at a definite number of nodal points a construction can be formed.

The deformation of the finite elements is constrained to a prescribed pattern which is expressed in mathematical form by a "displacement function". With these displacement functions the stiffness matrix of an element can be formed. For thin plate elements there are displacement functions describing separately the deformations of the plate under plane stress and the deformations of the plate when subjected to bending forces. These two situations are assumed to be independent.

In fig. 6.2 for a number of frequently used plate elements the displacement functions are given. It is obvious that for all these elements the displacement functions for the in plane deformations differ from the displacement functions for the out of plane displacements. This makes that the elements are not fully compatible when connected to each other under an out of plane angle and to calculate the characteristics of columns properly a fine mesh is necessary.

Hinduja and Cowley (13), (14), did compute the displacements of the column of fig. 4.1, which was used by Dreyer (6). A number of different plate elements were used to see the influence of the displacement functions and the division of the elements. In fig. 6.3 the meshes used to compute the column with rectangular elements are shown. Some of the computed and measured results are shown in fig. 6.4. By refining the mesh the results converge to a certain value.

Hinduja and Cowley (13) also computed the influence of the bending stiffness of the element on the total torsional stiffness of the column. They found that, depending of the height of the column, there was a difference varying from 14 to 24% between the deflections computed with elements having only a membrane (in plane) stiffness and elements having a membrane and flexural (bending) stiffness. So even for a thin walled column as used by Dreyer the bending stiffness of the plates have a remarkable influence.
Some examples of computing column structures are given by Noppen (15), Hoshi (16) and Sato (17) (see figs. 6.5, 6.6 and 6.7). They all use in their programs beam and plate elements together. The number of elements used is large and with it the preparation time to make the computer input. The use of mesh generators seems to be necessary to reduce this preparation time and the possibility of making mistakes. Sata (18) has developed a system in which a construction can be built up with some basic elements (fig. 6.8) combined with modification by rib, window or massive volume (fig. 6.9). The basic elements are automatically divided into a number of finite elements. Fig. 6.10 and fig. 6.11 show the idealization and static deformations of a vertical jig boring machine built up out of these basic elements.

7. Conclusions.

Slender columns can be calculated by using beam elements. When shear has to be taken into account, the calculation of the shear distribution factor is approximatively done in most cases. There is a need for further investigation in this field.

Many experiments have been done in order to get insight into the problems of ribbing, transverse partitions and apertures in columns. The findings of these experiments can be successfully used in applying beam elements for actual columns.

Finally, the finite elements method can be used for the calculation of columns. In order to diminish time and costs to an acceptable level, the use of mesh generators and standard elements is necessary. Both subjects—mesh generators and standard elements—need further investigations.


(11) Veldpaus, F.E.: Some procedures for the analysis of thin-walled beam structures (in Dutch).
   Doctor's thesis, Eindhoven, University of Technology (1973)

(12) Opitz, H. and Bielefeld, J.: Modellversuche an Werkzeugmaschinenelementen.
    Forschungsberichte des Landes Nordrhein-Westfalen Nr. 900


    Industrie-Anzeiger 95(1973), Nr. 66/67

(16) Hoshi, T.: Kyoto University. Personal communication.

    Proc. of the 14th International M.T.D.R.-Conference. (1973)

(18) Sata, T. et.al.: Computerunterstütztes System für die Konstruktion des Werkzeugmaschinenaufbaus.
    Werkstatt und Betrieb 106 (1973) Nr. 9
Fig. 2.1 Local coordinates and forces for a beam element

Fig. 2.2 Definition of the local displacements and rotations

Fig. 3.1 Ratio $w_p/w$ for a single beam

Fig. 3.2 Ratio $w_p/w$ for a coupled beam

Fig. 3.3 Rotation (a), warping (b) and distortion (c) of the cross section
Fig. 3.4  
Ratio of displacement with and without topplate  
Dreyer (6)

Fig. 4.1  
Dimensions of the column used by Dreyer (6)

<table>
<thead>
<tr>
<th>case</th>
<th>mode</th>
<th>theoretical bending stiffness</th>
<th>measured bending stiffness</th>
<th>theoretical bending stiffness/weight ratio</th>
<th>measured bending stiffness/weight ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>110.5</td>
<td>113</td>
<td>82.8</td>
<td>90</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>110.5</td>
<td>117</td>
<td>82.8</td>
<td>94</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>111</td>
<td>114</td>
<td>75.2</td>
<td>76</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>111</td>
<td>114</td>
<td>75.2</td>
<td>76</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>115</td>
<td>119</td>
<td>86.5</td>
<td>90</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>115</td>
<td>121</td>
<td>86.5</td>
<td>90</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>128</td>
<td>132</td>
<td>79.2</td>
<td>83</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>128</td>
<td>132</td>
<td>79.2</td>
<td>81</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>100</td>
<td>91</td>
<td>93.5</td>
<td>85</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>100</td>
<td>85</td>
<td>88</td>
<td>75</td>
</tr>
</tbody>
</table>

Fig. 4.2  
Relative bending stiffness of columns
**Fig. 4.3 Cross-Sectional Deformation of Columns with Different Internal Ribs Loaded in Torsion**

<table>
<thead>
<tr>
<th></th>
<th>Relative torsional stiffness</th>
<th>Torsional stiffness/weight ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>51.5%</td>
<td>51.5%</td>
</tr>
<tr>
<td>B</td>
<td>6.5%</td>
<td>6.5%</td>
</tr>
<tr>
<td>A</td>
<td>51.5%</td>
<td>42%</td>
</tr>
<tr>
<td>B</td>
<td>9%</td>
<td>7.5%</td>
</tr>
<tr>
<td>A</td>
<td>51.5%</td>
<td>36%</td>
</tr>
<tr>
<td>B</td>
<td>15%</td>
<td>10%</td>
</tr>
<tr>
<td>A</td>
<td>79.5%</td>
<td>60.5%</td>
</tr>
<tr>
<td>B</td>
<td>65%</td>
<td>48.5%</td>
</tr>
<tr>
<td>A</td>
<td>126%</td>
<td>79%</td>
</tr>
<tr>
<td>B</td>
<td>115%</td>
<td>70.5%</td>
</tr>
<tr>
<td>A</td>
<td>98%</td>
<td>92%</td>
</tr>
<tr>
<td>A</td>
<td>108%</td>
<td>95%</td>
</tr>
</tbody>
</table>

**Fig. 4.4 Torsional stiffness of columns**
<table>
<thead>
<tr>
<th>( \phi_x ) (rad)</th>
<th>( V ) (mm)</th>
<th>( \phi_z ) (rad)</th>
<th>( W ) (mm)</th>
<th>( \phi_y ) (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>( 0.152 \times 10^{-1} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>( 0.8837 \times 10^{-2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>( 0.8820 \times 10^{-2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( 1000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_z )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -0.5223 \times 10^{-2} )</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -0.5224 \times 10^{-2} )</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -0.5265 \times 10^{-2} )</td>
</tr>
<tr>
<td>( 1000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -0.5919 \times 10^{-2} )</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -0.5408 \times 10^{-2} )</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -0.5461 \times 10^{-2} )</td>
</tr>
<tr>
<td>( 1000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_z )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>( 0.9816 \times 10^{-2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>( 0.7761 \times 10^{-2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>( 0.7744 \times 10^{-2} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( 1000 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ M_x, M_y, M_z, P_x, P_y, P_z, V, W, \phi_x, \phi_y, \phi_z \]

\* A with top-plate
\* B without top-plate
Fig. 5.1 Effect of varying the core hole area on the torsional stiffness of the column (14)

Bending stiffness $k_x$  Bending stiffness $k_y$
Torsional stiffness

Fig. 5.2 Static stiffness of a box beam (12)
**Fig. 6.1 Some basic elements**

![Image of basic elements](image)

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>IN-PLANE DISPLACEMENT FUNCTION</th>
<th>OUT-OF-PLANE DISPLACEMENT FUNCTION</th>
</tr>
</thead>
</table>
| ![Triangle](image) | \[ u = a_1 + a_2 x + a_3 y \]  
\[ v = a_4 + a_5 x + a_6 y \] | \[ w = a_1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 y^2 + a_7 xy + a_8 x y^2 \] |
| ![Square](image) | \[ u = a_1 + a_2 x + a_3 y + a_4 xy + \left( \frac{1}{2} - \frac{1}{y} \right) a_5 + a_6 x^2 \]  
\[ v = a_5 + a_6 x + a_7 y + a_8 xy + \left( \frac{1}{2} - \frac{1}{y} \right) a_9 + a_{10} x^2 \]  
\[ (\text{Cheung}) \] | \[ w = a_1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 y^2 + a_7 xy + a_8 x y^2 + a_9 x^3 + a_{10} y^3 + a_{11} x y^3 + a_{12} x^2 y + a_{13} x^3 + a_{14} y^3 + a_{15} x^2 y + a_{16} x^3 \] |

**Fig. 6.2 The displacement functions of some simple plate elements.**
Fig. 6.3 Meshes used for column with/without end-plate (14)

Fig. 6.4 Convergent curves for unribbed column loaded in torsion (14)
Fig. 6.5 Column model of DC Drill Press with a cut out (16)
- Applied loads: $P_1$ for bending
- Young's modulus: $3.1 \times 10^{10} \text{kg/m}^2$
- Poisson's ratio: 0.3
- Mass density: 7.75
- # of total plate elements: 85
- # of total beam el. 14

Fig. 6.6 Frequency responses for bending and torsional loads (16)
($P_1, P_2 = 1N$) on the column of DC boring machine
Fig. 6.7
Machine tool column and its model (15)

Fig. 6.8
Basic elements (18)

Fig. 6.9
Modified elements (18)

Fig. 6.10 Vertical boring machine (18)

Fig. 6.11 Static loading of the vertical boring machine and the associated deflections (18)