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A MODEL STUDY OF A FEEDDRIVE FOR A NUMERICALLY CONTROLLED LATHE.

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MODEL STUDY OF A FEEDDRIVE FOR A NUMERICALLY CONTROLLED LATHÈ.

SUMMARY.

In this study an experimental analysis has been made of an existing feeddrive consisting of a dc-motor, a tachogenerator, a spindle and a carriage. Based on this analysis a mathematical model has been built in order to predict the time behaviour of such a system in a generalized way i.e. for different parameter values and/or different components.

Starting from the motor-tacho combination - and a gradually extension with the other parts - the dynamic behaviour has been measured by means of a Fourier Analyzer. Afterwards a computer-model has been built and evaluated. The model parameters are according to the component data. As far as the unknown parameters are concerned, the model response fits the dynamic response of the actual construction.

The problems of non-uniform diameter of the spindle-axis and the varying position of the carriage on the spindle have been treated by a Raleigh-Ritz method.
1. INTRODUCTION

In many modern machine tools the feeddrives often consist of a dc - motor with tachogenerator because of the good control properties of this type of motor and the recently obtained possibilities for the electronic control amplifiers. Usually, the dc - motor drives the carriage by means of a coupling and a spindle as shown in Fig. 1.

The behaviour of such a feeddrive is rather simple, but the tendency in machine tools to higher accuracies and speeds and lower tolerances, requires a deeper insight into the dynamic behaviour of the construction as a whole and the influences of individual components in particular on the complete feeddrive. To augment the knowledge of these feeddrives an analysis has been made of an existing feeddrive in such a way that the obtained results and models are generally applicable to feeddrives with other components. In a step by step approach the different components like motor, tachogenerator and coupling have been measured and identified seperately.

Based on these partial analyses, computer models of the complete feeddrive have been built, from which the behaviour in time and/ or frequency domain can be predicted in a generalized way i.e. for different parameter values of the components.

In Sec. 2 an analysis is made of the motor-tacho generator combination of the existing feeddrive. The dynamic behaviour has been measured by a Fourier Analyzer. From this, a computer model has been built, in which the model parameters are mainly set according to the component data. Concerning the unknown parameters, they are chosen in such a way that the model response fits best the dynamic response of the actual construction.
In Sec. 3 a separate analysis is made of the spindle-carriage part of the feeddrive and the coupling to the motor shaft. Here we dealt with the problem of the coupling, the non-uniform diameter (i.e. stiffness) of the spindle and the effect of the varying position of the carriage on the spindle. Therefore, a solution with a simple model is impossible for this part. We constructed for all these components an artificial equivalent spindle with varying diameter along the length and determined the lowest natural frequency by a Raleigh-Ritz method, which is rather close to a solution by the finite elements method. In Sec. 4 an analysis is made of the complete feeddrive. The models of the parts are coupled to each other and time- and frequency responses are shown of these mathematical models as well as responses of the real feeddrive. The conclusion is that with these models the reality can be fairly described and the effect of changing parameters or components can be predicted in this way.

2. D.C. MOTOR-TACHOGENERATOR COMBINATION.

2.1. Measurements.

These two components belong to the velocity feedback loop of the feeddrive as shown in Fig. 2. The dc-motor belongs to the disc-armature type, while the stator consists of permanent magnets. So this motor has a relatively low moment of inertia and it may run at maximum speed within short time.

![Diagram](image)

Figure 2: Position and velocity feedback loop of a feeddrive.

The dynamic behaviour of the motor-tacho combination has been measured by a white noise excitation with a Fast Fourier Analyzer. The white noise is used as an input signal to a power amplifier
which controls the motor voltage $U_m$, while the tachogenerator voltage $U_t$ is considered as the output signal of this system. So, the transfer function $U_t(j\omega)/U_m(j\omega)$ is determined. (See Fig. 3).

**Figure 3:** Transfer function of the motor-tacho combination.
Some remarks:
- The transfer function of the motor-tacho combination has been measured from 0 to 800 Hz. This transfer function is a combination of several measurements in which for the most important parts a total width of 25 Hz is used and for the other parts a width of 100 Hz. The resonance peak has the following characteristics:
  Resonance frequency : \( f_r = 440 \text{ Hz} \).
  Amplitude : \(|H(f_r)| = 0.257\).
  Damping ratio : \( \zeta = 8.5 \times 10^{-3} \).

If the coupling is attached to the motor shaft, the resonance frequency decreases to 436 Hz, due to an increase of the moment of inertia.

The point 0 Hz cannot be measured by the F.F.A. However, this is carried out directly by introducing a dc voltage to the motor and measuring the resulting tachogenerator voltage. This is carried out for a number of voltages. We found the motor to be linear (no saturation effects) over a large range, to more than 50 Volts, with the amplification at 0 Hz: \( |H(0)| = 0.248 \).

- By using the white noise signal, the disturbances are small and around a fixed position of the shaft. The transfer function has also been measured by superimposing on this white noise a dc voltage to the motor - so with a motor running in one direction - but the results are the same.

2.2. The model study.

It is necessary to derive the following mechanical and electrical schemes. (See Fig. 4).

![Schematic representation of the motor-tacho combination](image)

**A)** Mechanical  

**B)** Electrical

Figure 4: Schematic representation of the motor-tacho combination.

In this figure we can observe the following parameters:
- \( J_m \) = moment of inertia of the motor-rotor
- \( J_t \) = moment of inertia of the tacho-rotor
\( J_c \) = moment of inertia of the coupling
\( D_m \) = damping coefficient of the motor bearings
\( D_t \) = damping coefficient of the tacho bearings
\( k \) = torsional stiffness of the connection shaft
\( D_s \) = damping coefficient of the connection shaft
\( l \) = length of the connection shaft
\( d \) = diameter of the connection shaft
\( R \) = resistance of the motor rotor
\( L \) = inductance of the motor rotor
\( c_m \) = proportional constant (E.M.F.) of the motor
\( c_t \) = proportional constant (E.M.F.) of the tacho
\( \theta_m, \theta_t \) = angular position of the motor, tacho
\( \omega_m, \omega_t \) = angular velocity of the motor, tacho.

The specific data of the actual combination is given in Table 1.

| J_m = J_1 = 1.2 \times 10^{-3} \text{ [kgm}^2\text{]} | D_m = D_1 = 7.64 \times 10^{-4} \text{ [Nms/rad]} |
| J_t = J_2 = 1.5 \times 10^{-4} \text{ [kgm}^2\text{]} | D_t = D_2 = \text{unknown} |
| J_{coupling} = 2.5 \times 10^{-4} \text{ [kgm}^2\text{]} | D_3 = 10^{-2} \text{ [Nms/rad]} |
| J_{spindle} = 1.33 \times 10^{-3} \text{ [kgm}^2\text{]} | D_{12} = 8 \times 10^{-3} \text{ [Nms/rad]} |
| J_{carriage} = 6 \times 10^{-5} \text{ [kgm}^2\text{]} | D_{13} = 1.5 \times 10^{-2} \text{ [Nms/rad]} |
| J_3 = 1.43 \times 10^{-3} \text{ [kgm}^2\text{]} | k_1 = 1.02 \times 10^{3} \text{ [Nm]} |
| l = 7.2 \times 10^{-2} \text{ [m]} | k_2 = 7.9 \times 10^{3} \text{ [Nm]} |
| d = 1 \times 10^{-2} \text{ [m]} | k_{spindle} = 1.27 \times 10^{4} \text{ [Nm]} |
| R = 4.6 \times 10^{-1} \text{ [\Omega]} | k_{coupling} = 2.3 \times 10^{4} \text{ [Nm]} |
| L = 1 \times 10^{-4} \text{ [H]} | \text{mass carri}ge = 100 \text{ [kg]} |
| c_m = 2.44 \times 10^{-1} \text{ [Vs/rad=NmA}^{-1}\text{]} | \text{mass spindle} = 8.75 \times 10^{-1} \text{ [m]} |
| c_t = 5.72 \times 10^{-2} \text{ [Vs/rad]} | d_{spindle} = 3.75 \times 10^{-2} \text{ [m]} |
| G = 8 \times 10^{10} \text{ [Nm}^{-2}\text{]} | \text{pitch} = 5 \times 10^{-3} \text{ [m]} |

Table 1: Motor-tacho-spindle-carriage data.

The mechanical equivalent (see Fig. 4 (A)) covers the physical representation under the following conditions:
1. \( J_1 \approx J_m + J_c + J_{shaft} \) (\( J_{shaft} \) is negligible)
2. \( \theta_1 \approx \theta_m \) and \( \theta_2 \approx \theta_t \)
3. The internal torsional damping which arises in the shaft is very small. So, the damping \( D_{12} \) is considered to be the connection damping of the fixation of the two rotors to the
common shaft.

4. The torsional stiffness \( k_1 \) of the common shaft can be determined from the length \( l \) and diameter \( d \) by using the polar moment of inertia \( J_p \) and the shear modulus \( G \).

\[
k_1 = \frac{G J_p}{l} = \frac{G \pi d^4}{32 l}, \text{ it follows } k_1 = 1.09 \times 10^2 \text{ [Nm]}
\]

From the above described assumptions of this motor-tacho generator combination a graph is shown in Fig. 5.

![Figure 5: Graph of the motor-tacho combination.](image)

The following equations may be derived for this model:

(2.1) \( U_m(t) = R i(t) + L \dot{i}(t) + E(t) \)

(2.1a) \( E(t) = c_m \dot{\theta}_1(t) = c_m \omega_1(t) \)

(2.2) \( M_t(t) = J_1 \ddot{\theta}_1(t) + D_1 \dot{\theta}_1(t) + k_1(\theta_1(t) - \theta_2(t)) \\
+ D_{12}(\dot{\theta}_1(t) - \dot{\theta}_2(t)) \)

(2.2a) \( M_t(t) = c_m l(t) \)

(2.3) \( k_1(\theta_1(t) - \theta_2(t)) + D_{12}(\dot{\theta}_1(t) - \dot{\theta}_2(t)) = J_2 \ddot{\theta}_2(t) + D_2 \dot{\theta}_2(t) \)

(2.4) \( U_t(t) = c_t \dot{\theta}_2(t) = c_t \omega_2(t) \)

After Laplace transformation, substitution and rearranging, the following matrix notation arises:

\[
\begin{bmatrix}
R + L & \frac{1}{s c_m} & 0 \\
-c_m & s^2 J_1 + s D_1 & s^2 J_2 + s D_2 \\
0 & -(s D_{12} + k_1) & s^2 J_2 + s(D_2 + D_{12}) + k_1
\end{bmatrix}
\begin{bmatrix}
1(s) \\
\theta_1(s) \\
\theta_2(s)
\end{bmatrix}
= \begin{bmatrix}
U_m(s) \\
0 \\
0
\end{bmatrix}
\]
From Eqs. (2.5) and (2.6) the motor transfer function $H_1(s)$ and the tacho transfer function $H_2(s)$ can easily be determined.

$$H_1(s) = \frac{s \theta_1(s)}{U_m(s)} = \frac{c_m(s^2 J_2 + s(D_2 + D_{12}) + k_1)}{\Delta/s}$$

$$H_2(s) = \frac{U_t(s)}{U_m(s)} = \frac{c_t c_m (s D_{12} + k_1)}{\Delta/s}$$

The determinant matrix (2.5) is denoted by $|\Delta|$.

Some remarks on the tacho transfer function $H_2(s)$:
From Eq. (2.8) it can easily be derived that $H_2(s)$ has one zero and four poles, and

$$H_2(s) = H_2(0) \cdot \frac{(1 + s/s_0)}{(1 + s/s_1)(1 + s/s_2)(1 + 2 \frac{\zeta s}{\omega_r} + \frac{s^2}{\omega_r^2})}$$

In Fig. 6 a schematic diagram of $H_2(s)$ with these breakfrequencies is given.

![Diagram](image)

Figure 6: Transfer function $H_2(s)$, schematic diagram.

With a computer model the characteristics of $H_2(s)$ are calculated.

<table>
<thead>
<tr>
<th>measured</th>
<th>calculated with Table 1</th>
<th>calculated with $k_1=1$, $D_{12}=8 \times 10^{-3}$</th>
<th>$f$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2(o)$</td>
<td>0.248</td>
<td>0.233</td>
<td>0.233 f_o = 1.74 \times 10^4</td>
</tr>
<tr>
<td>$</td>
<td>H_2(fr)</td>
<td>$</td>
<td>0.257</td>
</tr>
<tr>
<td>$f_r$ [Hz]</td>
<td>440</td>
<td>450</td>
<td>440 f_2 = 1.57 \times 10^1</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>$8.5 \times 10^{-3}$</td>
<td>1.47 $10^{-2}$</td>
<td>1.2 $10^{-2}$ f_r = 4.40 $10^2$</td>
</tr>
</tbody>
</table>
As can be observed a slight change in $k_1$ and $D_{12}$ was necessary to fit the measured and calculated responses more accurately. The transfer function $H_2(s)$ of the computer model is also shown in Fig. 3 and corresponds very good to the experimental response.

Approximations for the break frequencies:

\[(2.11) \quad f_0 = \frac{1}{2\pi} \frac{k_1}{D_{12}}\]

\[(2.12) \quad f_1 \approx \frac{1}{2\pi} \frac{R}{L}, \text{ the electrical time constant of the motor.}\]

\[(2.13) \quad f_2 \approx \frac{1}{2\pi} \frac{c_m}{R(J_1 + J_2)^2}, \text{which corresponds to the mechanical time constant of the motor and tacho.}\]

\[(2.14) \quad f_r \approx \frac{1}{2\pi} \sqrt{\frac{k_1(J_1 + J_2)}{J_1}}, \text{which represents the resonance frequency of the motor tacho system.}\]

3. SPINDLE-CARRIAGE SYSTEM

Due to the mechanical structure of these elements of the feeddrive several types of variations may occur - like bending, torsion and longitudinal vibrations - which are not equally important with respect to the performance quality of the system. Furthermore, it depends upon the direct or indirect position measuring device whether these vibrations are detected. In our case the carriage position is derived from the spindle rotation, - an indirect method - so the only important vibration is torsion and that is why the following analysis is based only on this phenomenon.

Concerning the coupling-spindle-carriage part we refer to Fig. 1. Two methods of analysis are considered:

I A mass-spring method with discrete moments of inertia.

II A Raleigh-Ritz method (an equivalent spindle).

Both analyses are subdivided in a system with one clamped end and with two free ends.

I The mass-spring method.

![Figure 7: Mass-spring equivalent.](image-url)
In Fig. 7 A) we determine:

\[ J_a = J_{\text{coupling}} + \frac{1}{2} J_{\text{spindle}} \]  
\[ J_b = \frac{1}{2} J_{\text{spindle}} + J_{\text{carriage (reduced)}} \]

\( J_{\text{carriage (reduced)}} \) is the moment of inertia of the carriage with respect to the pitch of the spindle. The resonance frequency:

\[ f_{\text{res}} = \frac{1}{2\pi} \sqrt{k \left( \frac{1}{J_a} + \frac{1}{J_b} \right)} \]

In Fig. 7 B) we also determine \( J_a \) and \( J_b \) like in Fig. 7 A) and further \( k_a = k_{\text{coupling}} \), \( k_b = k_{\text{spindle}} \).

For the resonance frequency, it follows according to Ref. [6]

\[ 2\omega_r^2 = \frac{k_a+k_b}{J_a} + \frac{k_b}{J_b} + \left[ \left( \frac{k_a+k_b}{J_a} - \frac{k_b}{J_b} \right)^2 + 4k_b^2 \right]^{-1/2} \]

With the data of Table 2, we have found the results of Table 3.

II The Raleigh-Ritz method.

For the carriage on the spindle an equivalent shaft is constructed. Depending upon the moment of inertia (\( J \)) and the torsional stiffness (\( k \)) an equivalent shaft with diameter (\( d \)) and length (\( I \)) is calculated with Eqs. (3.3) and 3.4). For the ballscrew nut the axial stiffness can be taken, which is of the order of \( 10^7 \text{ to } 10^8 \text{Nm} \).

\[ k = \frac{G\pi d^4}{32I} \]  
\[ l = \frac{\sqrt{G\cdot J}}{\rho k} \]

\[ J = \frac{\pi d^4 l}{32} \]  
\[ d = \sqrt{\frac{k\cdot J}{G\rho}} \left( \frac{2l}{\pi} \right)^2 \]

A) B)

Figure 8: Equivalent shaft for the Raleigh-Ritz method.

In this Raleigh-Ritz method we consider the most important natural frequencies and this determines the number of modes which are studied. In order to estimate the lowest natural frequency
the first three modes are sufficient as shown in Fig. 9 for a free vibrating shaft.

\[ \phi_1(x) = \cos \left( \pi \frac{x}{l} \right) \]

\[ \phi_2(x) = \cos \left( 2\pi \frac{x}{l} \right) \]

\[ \phi_3(x) = \cos \left( 3\pi \frac{x}{l} \right) \]

Figure 9: Mode shapes of a free vibrating shaft.

It is assumed that the real deviation \( \theta(x,t) \) may be written as a linear combination of the modes so:

\[ \theta(x,t) = \sum_{i=1}^{n} q_i(t) \phi_i(x) \]

where \( n \) = number of modes.

For the torsional vibrations both the potential energy \( E_p \) and the kinematic energy \( E_k \) may be determined.

\[ E_p = \frac{1}{2} \int_0^l G \cdot J_p(x) \left( \frac{\partial \theta(x,t)}{\partial x} \right)^2 dx \]

\[ E_k = \frac{1}{2} \int_0^l J(x) \left( \frac{\partial \theta(x,t)}{\partial t} \right)^2 dx \]

where \( G \) = shear modulus
\( J_p \) = polar moment of inertia
\( J(x) \) = mass moment of inertia per meter.

The Eqs. (3.6) and (3.7) may be substituted in the equations of Lagrange:

\[ \frac{d}{dt} \left( \frac{\partial E_k}{\partial q_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} = 0 \]

From this, it yields:

\[ -\omega^2 [J][q] + [k][q] = \{0\} \]

where \([J]\) = matrix with the moments of inertia of the parts.
\([k]\) = matrix with torsional stiffness.
\([q]\) = contribution of the vibration mode.
\( \omega = \) angular natural frequency.

The results of a computer program based on this Raleigh-Ritz method with the data of Table 2 are shown in Table 3.

| Carriage: mass | 235 [kg] | \( J_a = 1.23 \times 10^{-3} [\text{kgm}^2] \) |
| Carriage: moment of inertia | 2.1 \times 10^{-4} [kgm^2] | \( J_b = 1.08 \times 10^{-3} [\text{kgm}^2] \) |
| Coupling: \( J_c \) | 2.5 \times 10^{-4} [kgm^2] | \( k = 5.05 \times 10^3 [\text{Nm}] \) |
| Coupling: torsional stiffness | 2.3 \times 10^{4} [Nm] | \( k_a = 2.3 \times 10^4 [\text{Nm}] \) |
| Spindle: diameter | 3.5 \times 10^{-2} [m] | \( k_b = 7.8 \times 10^3 [\text{Nm}] \) |
| Spindle: pitch | 5 \times 10^{-3} [m] | coupling: \( l = 3.3 \times 10^{-1} [\text{m}] \) |
| Spindle: length | 1.5 [m] | \( d = 3.1 \times 10^{-2} [\text{m}] \) |
| Spindle: moment of inertia | 1.7 \times 10^{-3} [kgm^2] | carriage: \( l = 10^{-2} [\text{m}] \) |
| Spindle: torsional stiffness | 7.8 \times 10^3 [Nm] | \( d = 7.2 \times 10^{-2} [\text{m}] \) |

Table 2: Data of the CNC-200 numerical lathe.

<table>
<thead>
<tr>
<th>Vibrating System</th>
<th>I Mass-Spring</th>
<th>II Raleigh-Ritz</th>
</tr>
</thead>
<tbody>
<tr>
<td>complete free</td>
<td>520 Hz</td>
<td>786 Hz</td>
</tr>
<tr>
<td>one end clamped</td>
<td>360 Hz</td>
<td>680 Hz</td>
</tr>
</tbody>
</table>

Table 3: Resonance frequencies of the coupling-spindle-carriage part of the lathe CNC-200.

- The effect of the non uniform thickness of the spindle has been investigated by this Raleigh-Ritz method. The diameter varies between 32 and 35 mm. The effect on the natural frequency is shown in Table 4.

<table>
<thead>
<tr>
<th>diameter of the spindle</th>
<th>31</th>
<th>33</th>
<th>35</th>
<th>37</th>
<th>39</th>
</tr>
</thead>
<tbody>
<tr>
<td>natural frequency Hz</td>
<td>701</td>
<td>747</td>
<td>786</td>
<td>818</td>
<td>845</td>
</tr>
</tbody>
</table>

Table 4: The effect of spindle diameter on the natural frequency.
The effect of the carriage position on the natural frequency has also been investigated by the Raleigh-Ritz method and the result is given in Fig. 10.

![Figure 10: Natural frequency as a function of the carriage position.](image)

### 4. MOTOR-TACHO-SPINDLE-CARRIAGE COMBINATION.

#### 4.1. Measurements.

As carried out in Sec. 2 the white noise signal as the input \( U_m(t) \) and the tacho output \( U_c(t) \) are analysed in the same way. Some remarks with respect to Fig. 11.

<table>
<thead>
<tr>
<th>MEASURED VALUES</th>
<th>( f_{r_1} = 409 \text{ Hz} )</th>
<th>( f_{r_2} = 577 \text{ Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H(0) = 0.23 )</td>
<td>( H(f_{r_1}) = 0.096 )</td>
<td>( H(f_{r_2}) = 0.167 )</td>
</tr>
<tr>
<td>( \zeta = 5.3 \times 10^{-3} )</td>
<td>( \zeta = 8 \times 10^{-3} )</td>
<td></td>
</tr>
</tbody>
</table>

- The amplification at 0 Hz is smaller than in the uncoupled case, due to the fact that more damping is introduced.
- Two peaks occur, the lowest frequency is merely due to the resonance of the motor-tacho system, while the other represents the resonance of the motor-spindle-carriage system, which is most dominant.
- Measurements made without a connected carriage do not show a significant change in the transfer function. The first peak changes from 409 Hz to 412 Hz, while the other remains the same. Referring to Table 1 it follows that \( J_{\text{spindle}}/J_{\text{carriage}} > 20 \), so \( J_{\text{carriage}} \) is negligible in this relation.
- A zero occurs at 400 Hz. This will be treated in detail in the model study.
Figure 11: The transfer function of the complete feeddrive.
One typical example of a step function with the complete feeddrive is - on a small time scale - shown in Fig. 12.

![Graph showing step function response](image)

Figure 12: Step function response of the tacho generator (with the complete feeddrive).

From this, it is clear that the response shows a damped oscillation of 550 Hz which is close to the dominant second peak of Fig. 11 which was 577 Hz.

4.2. The model study.

A model of the complete feeddrive is shown in Fig. 13.

![Diagram of model](image)

Figure 13: Model of the complete feeddrive.

With regard to Fig. 4 the parameters $J_3$, $D_3$, $k_2$ and $D_{13}$ are added. These parameters may not be quite equal to the physical values, so two problems arise:

1. What is the best value of $p$ in the next statement?

$$J_1 = J_{\text{motor}} + p (J_{\text{coupling}} + J_{\text{spindle}} + J_{\text{carriage}}) \quad 0 < p < 1$$
\[ J_2 = J_{tacho} \]
\[ J_3 = (1-p)(J_{coupling} + J_{spindle} + J_{carriage}). \]

2. What is the best value of \( k_2 \) because of the various components like coupling, spindle, carriage?

Ad. 1. By varying \( p \) in a computer model both the resonance frequencies varied according to the next table.

<table>
<thead>
<tr>
<th>( p )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>&quot;Measured&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{r1} ) (Hz)</td>
<td>425</td>
<td>427</td>
<td>430</td>
<td>433</td>
<td>435</td>
<td>437</td>
<td>409</td>
</tr>
<tr>
<td>( f_{r2} ) (Hz)</td>
<td>626</td>
<td>615</td>
<td>613</td>
<td>619</td>
<td>635</td>
<td>663</td>
<td>577</td>
</tr>
</tbody>
</table>

From this, it is concluded that the best value of \( p = 0 \) i.e. a sharing of the moments of inertia according to the physical values.

Ad. 2. An approximation of \( k_2 \) is made by two methods I and II.

I. The spindle is assumed to be of a uniform diameter \( d \) and length \( l \), with:

\[
k_{sp}(l) = \frac{G l d^4}{32 l} \quad \text{we define:} \quad \frac{1}{k_2} = \frac{1}{k_c} + \frac{1}{k_{sp}(l)}
\]

II. The spindle has a real configuration, given in Fig. 14.

![Spindle configuration and its model.](image)

Figure 14: Spindle configuration and its model.

With:

\[
k_i = \frac{G l d_{j}^4}{32 l_1} \quad \text{and} \quad J_i = \frac{\pi d_{j}^4 l_1}{32}
\]

we define:
The computer results of method I and II are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Method I</th>
<th>Method II</th>
<th>Measured and calculated data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_C$ [Nm]</td>
<td>2.3 $10^4$</td>
<td>2.3 $10^4$</td>
<td></td>
</tr>
<tr>
<td>$k_{sp}$ [Nm]</td>
<td>1.8 $10^4$</td>
<td>7.8 $10^3$</td>
<td>1.27 $10^4$</td>
</tr>
<tr>
<td>$k_2$ [Nm]</td>
<td>$10^4$</td>
<td>$5.8 10^3$</td>
<td>$7.9 10^3$</td>
</tr>
<tr>
<td>$f_{r1}$ [Hz]</td>
<td>425</td>
<td>403</td>
<td>409</td>
</tr>
<tr>
<td>$f_{r2}$ [Hz]</td>
<td>626</td>
<td>520</td>
<td>577</td>
</tr>
</tbody>
</table>

The best of the resonance frequencies is obtained by using $k_{sp} \approx 1/2 (k_{sp}(I) + k_{sp}(II))$, with a subsequently value for $k_2$.

With the previous made assumptions the following equations for the model in Fig. 13 are made (with time dependent variables):

\[
(4.5) \quad U_m = R_1 + L \dot{I} + E
\]

\[
(4.5a) \quad E = c_m \dot{\theta}_1 = c_m \omega_1
\]

\[
(4.6a) \quad M_t = c_m l
\]

\[
(4.6) \quad J_1 \ddot{\theta}_1 + D_1 \dot{\theta}_1 + k_1 (\theta_1 - \theta_2) + D_{12} (\dot{\theta}_1 - \dot{\theta}_2) + k_2 (\theta_2 - \theta_3) + D_{13} (\dot{\theta}_1 - \dot{\theta}_3) = M_t
\]

\[
(4.7) \quad k_1 (\theta_1 - \theta_2) + D_{12} (\dot{\theta}_1 - \dot{\theta}_2) = J_2 \ddot{\theta}_2 + D_2 \dot{\theta}_2
\]

\[
(4.8) \quad k_2 (\theta_1 - \theta_3) + D_{13} (\dot{\theta}_1 - \dot{\theta}_3) = J_3 \ddot{\theta}_3 + D_3 \dot{\theta}_3
\]

\[
U_t = c_t \dot{\theta}_2 = c_t \omega_2
\]

After Laplace transformation, the next matrix (4.9) notation arises.
\[
\begin{bmatrix}
R+sL & sJ_1+sD_1 & 0 & i & 0 \\
-cm & s^2J_2+sD_2 & s^2J_3+sD_3 & \theta_1(s) & 0 \\
0 & \sigma & s^2J_2+s(D_2+D_{12})+k_1 & \theta_2(s) & 0 \\
0 & \sigma & s^2J_3+s(D_3+D_{13})+k_2 & \theta_3(s) & 0
\end{bmatrix}
\begin{bmatrix}
1(s) \\
U_m(s)
\end{bmatrix}
= \begin{bmatrix}
1(s) \\
U_m(s)
\end{bmatrix}
\]

and

(4.10) \[ U_t(s) = c_t s \theta_2(s) \]

This leads to the next motor transfer function \( H_1(s) \) and tacho transfer function \( H_2(s) \)

(4.11) \[ H_1(s) = \frac{s \theta_1(s)}{U_m(s)} = \frac{cm(s^2J_2+s(D_2+D_{12})+k_1)(s^2J_3+s(D_3+D_{13})+k_2)}{\Delta/s} \]

(4.12) \[ H_2(s) = \frac{U_2(s)}{U_m(s)} = \frac{cr cm(sD_{12}+k_1)(s^2J_3+s(D_3+D_{13})+k_2)}{\Delta/s} \]

The determinant of the matrix of (4.9) is denoted by \(|\Delta|\).

Some remarks on the tacho transfer function \( H_2(s) \):
From Eq. (4.12) it follows that:

(4.13) \[ H_2(s) = \frac{cr cm}{R(D_1+D_2+D_3)+cm^2} \]

This is lower than in the motor-tacho case (2.9). By comparison of the measurements 0.23 and 0.248 the value of \( D_3 \) may be calculated \( 9.6 \times 10^{-3} \). Furthermore, it follows that:

- \( H_2(s) \) has 3 zero's (one real, one complex set)
- 6 poles (two real, two complex sets)

Approximations for these breakfrequencies:

<table>
<thead>
<tr>
<th>ZERO'S</th>
<th>POLE'S</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.14) ( f_0 = \frac{1}{2\pi} \frac{k_1}{D_{12}} )</td>
<td>4.16) ( f_1 = \frac{1}{2\pi} \frac{R}{L} )</td>
</tr>
<tr>
<td>4.15) ( f_{ro} = \pm \frac{1}{2\pi} \sqrt{\frac{k_2}{J_3}} )</td>
<td>4.17) ( f_2 = \frac{1}{2\pi} \frac{cm^2}{R(J_1+J_2+J_3)} )</td>
</tr>
<tr>
<td>4.18) ( f_{r1} = \pm \frac{1}{2\pi} \sqrt{k_1\left(\frac{1}{J_1} + \frac{1}{J_2}\right)} )</td>
<td>4.19) ( f_{r2} = \pm \frac{1}{2\pi} \sqrt{k_2\left(\frac{1}{J_1} + \frac{1}{J_3}\right)} )</td>
</tr>
</tbody>
</table>

Both Eqs. (4.18) and (4.19) arise from the equation:

(4.20) \[ \omega^4 \omega^2 \left(\frac{k_1+k_2}{J_2} + \frac{k_1}{J_2} + \frac{k_2}{J_3}\right) + k_1k_2\left(\frac{1}{J_1J_2} + \frac{1}{J_1J_3} + \frac{1}{J_2J_3}\right) = 0 \]
• $f_{ro}$: The dynamic stiffness of the motor tacho shaft reaches a minimum.
• $f_1$: The electrical time constant of the motor.
• $f_2$: The mechanical time constant of the complete feeddrive.
• $f_{r1}$: The resonance frequency of the motor-tacho system.
• $f_{r2}$: The resonance frequency of the motor-spindle system.
These frequencies can not always be measured, because either they are hidden or too high.

<table>
<thead>
<tr>
<th>Break freq. Hz</th>
<th>$f_0$</th>
<th>$f_{ro}$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_{r1}$</th>
<th>$f_{r2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>-</td>
<td>400</td>
<td>-</td>
<td>7.4</td>
<td>409</td>
<td>577</td>
</tr>
<tr>
<td>Model</td>
<td>$210^4$</td>
<td>374</td>
<td>732</td>
<td>7.4</td>
<td>410</td>
<td>579</td>
</tr>
</tbody>
</table>

Table 5: Measured and calculated break frequencies.

<table>
<thead>
<tr>
<th>$H_2(0)$</th>
<th>$f_{r1}$</th>
<th>$H_2(f_{r1})$</th>
<th>$\zeta(f_{r1})$</th>
<th>$f_{r2}$</th>
<th>$H_2(f_{r2})$</th>
<th>$\zeta(f_{r2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>0.23</td>
<td>409</td>
<td>$9.6 \times 10^{-2}$</td>
<td>5.3 $10^{-3}$</td>
<td>577</td>
<td>$1.67 \times 10^{-1}$</td>
</tr>
<tr>
<td>Model</td>
<td>0.233</td>
<td>410</td>
<td>$8.34 \times 10^{-2}$</td>
<td>$8.8 \times 10^{-3}$</td>
<td>579</td>
<td>$1.59 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Table 6: Data of remarkable transfer function points.

Based on the analyses of the previous sections several computer models are made. The results are shown in Table 5 and 6.

Figure 15: Transfer function of the complete feeddrive on logarithmic scale with a mathematical model.
Frequency domain:

- On a HP 9025 A (Results in Fig. 11)
- On a Burroughs 7700 (in Fortran) (Results in Fig. 15).

Time domain:

- On a Burroughs 7700 (C.S.M.P.) (Results in Fig. 12)

NOTES

1. The calculation of the transfer function by the frequency domain models are based on the matrix equation (4.9)
   \[ A(s) \Theta(s) = U(s) \] so \( \Theta(s) = A^{-1}(s) U(s) \).
   The inversion of a complex matrix \( A(s) \) has to be made.
   For \( s = j\omega \), we may define:
   \[(4.21) \quad A(j\omega) = B(\omega) + jC(\omega)\]
   \[(4.22) \quad A^{-1}(j\omega) = P(\omega) + jQ(\omega)\]
   Matrix theory learns that in that case
   \[(4.23) \quad P(\omega) = [B(\omega) + C(\omega) B^{-1}(\omega) C(\omega)]^{-1}\]
   \[(4.23) \quad Q(\omega) = -[C(\omega) + B(\omega) C^{-1}(\omega) B(\omega)]^{-1}\]
   The transfer function can easily be derived from \( A^{-1}(s) \) also if the content of the elements of \( A(s) \) changes.

2. In the time domain we are interested in the behaviour at short time. For a reliable response we need the highest frequencies very accurate and because of the fact that the number of points is limited the total function can not be inverted accurately enough. A C.S.M.P-program (Continous System Modelling Program) performs a numerical handling (integration) of a set of differential equations like (4.5) to (4.8) and is based on Fortran.

CONCLUSIONS.

1. The performance of NC-machine tools is limited by the dynamic behaviour of the feeddrives, which may be examined from the transfer functions.
2. Resonances with high peaks at high frequencies in these feeddrives are undesired and inadmissible the more so as feedback loops are applied.
3. It is possible to make a model of such a feeddrive based on a coupled mechanical mass-spring (moment of inertia-torsion) system.
4. The model parameters (which cover the motor-tacho part of the combination) are well in accordance with the physical quantities and may be easily derived from the component data.
5. The model parameters representing the coupling-spindle-carriage part on the contrary are artificial ones and approximations are based on the next considerations:
- Natural frequencies of this combination can not be obtained by simple methods but need advanced methods like finite elements - or Raleigh-Ritz methods.
- The moment of inertia of the carriage is negligible with respect to that of spindle and coupling.
- With respect to the torsional stiffness, the spindle may be considered to have uniform dimensions or consisting of a series of springs and masses. A good approximation of the real torsional stiffness is to be in the middle of these two models.

6. The described model is quite capable to predict responses in time- and frequency domain also with components with slightly different parameter data.

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