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HOW TO REPLACE THE FOUR-BAR COUPLER MOTION BY THE COUPLER MOTION OF A SIX-BAR MECHANISM WHICH DOES NOT CONTAIN A PARALLELOGRAM

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Sometimes the motion required to answer a (particular) kinematic problem may be met by the coupler motion of a four-bar linkage. In many cases, however, the dimensions of the four-bar meeting these requirements are not suitable, or the location of the fixed centers is not convenient because of the lack of space.

It is helpful, therefore, if the designer may devise an alternative linkage that produces the same coupler motion. In the paper under consideration, three different types of alternative solutions are suggested. The alternative mechanisms are all six-bar linkages of the Watt-I type. However, they all leave the designer with sufficient design degrees of freedom to meet additional requirements.

The solutions proposed here do not contain linkage parallelograms. They are based either on Burmester's Configuration, or on Kempe's generalized form and focal linkage.

1. INTRODUCTION

In Hartenberg's and Denavit's paper [1] on Cognate Linkages it is shown that a four-bar coupler curve could be reproduced by a proper six-bar linkage. How to obtain such an alternative mechanism has been equally described in their book [2] named the "Kinematic Synthesis of Linkages" (page 179). An extension has been made by Dijksman [3] in his paper presented at the IFTOMM-Bucharest Symposium (June 1973) in which he dealt with "Five-bar curve and coupler cognates having two cranks that rotate with the same speed". In fact, it has been shown in that paper that \( \infty \) six-bar alternative linkages exist that reproduce the entire coupler motion of a four-bar. Thus, these linkages not only reproduce four-bar coupler curves, but more than that: they reproduce also the complete motion of the coupler. In the paper, they were named coupler alternative mechanisms. Since they are to be derived from five-bar linkages, all six-bar alternative mechanisms that reproduce the coupler motion contain a linkage parallelogram in order to enforce the two input cranks to rotate with the same speed.

However, linkage parallelograms are subjected to folding positions, in which the transmission angles are zero.

It is the object of this paper, therefore, to avoid this and to create alternative mechanisms that do not contain parallelograms.

As will be explained in detail here, there are a number of ways to do this. Notwithstanding, each way allows the free choice of a fixed center or a turning-joint of the mechanism. This gives the designer the same freedom as in the case where parallelograms are used. In addition, however, the transmission angles of the six-bar may now be given acceptable values that guarantee a smooth motion.

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In order to derive such mechanisms, we make extensive use of two distinct configurations. They are both overconstrained linkages. One of them is named after I. Burmester [4] and the other one after A. B. Kempe [5].

2. BURMESTER’S CONFIGURATION AND ITS USE TO DERIVE ALTERNATIVE LINKAGES REPRODUCING FOUR-BAR COUPLER MOTIONS

2a. BURMESTER’S CONFIGURATION AND ITS DERIVATION

According to Burmester [4] we may link any point \( D \) with the four sides of a four-bar (see figure 1). For a given four-bar represented by the joints \( A_oABB_o \) for example, this may be done after establishing the location of the four points \( F, D_o, C \) and \( K \) attached to the sides, that we intend to link with the chosen point \( D \). These points are defined by the triangle \( A_oDB \) formed by the diagonal \( A_oB \) of the four-bar and the randomly chosen point \( D \). As shown in figure 1, this is effectuated, namely, if we comply with the similarities such as \( \Delta A_oDB \sim \Delta A_oFA \sim \Delta A_oD_oB_o \sim \Delta B_oCB \sim \Delta AKB \). In the design position of the linkage, the four points so defined may be linked to the chosen point \( D \), which then becomes a common joint to the four links that are adjoined to the four-bar.

The linkage completed this way is an overconstrained one, that is, it is still movable, despite the fact that the linkage now contains more joints than necessary in defining a constrained motion.

![Fig. 1. - Burmester's configuration 1st type.](image)

In order to prove the fact that the real motion is possible for this type of linkage, we will obtain its dimensions in another, more natural way, that shows you that indeed the links may be adjoined without hindering the initial four-bar motion. For this, we retrieve the additional links using the method of stretch rotation. In this case, for example, we may transform or stretch rotate the four-bar \( A_oABB_o \) (about \( B \)) into the similar four-bar \( DKBC \). The transformation carried out here is a contraction and at the same time a rotation about the centre \( B \). Sometimes,
such a transformation is called a *spiral similarity*, since it transforms configurations into similar ones that are changed in size but rotated simultaneously. In the case under consideration, the center of the spiral similarity or stretch rotation coincides with the joint $B$, whereas the *stretch rotation factor* is defined by the vector ratio $\overrightarrow{BD}/\overrightarrow{BA}$, which is made a constant here and thus independent of time. As a consequence, the four-bars $A_0ABB_0$ and $DKBC$ stay similar throughout the complete motion, whereas the corresponding sides rotate at the same speed. Actually, to the four-bar we now merely have adjoined the dyad $\overline{CDK}$. Clearly, this does not obstruct the initial motion at all. But the dyad chosen to adjoin the four-bar is a particular one, since the sides of the dyad have no angular velocity relative to the corresponding sides of the dyad $\overline{BD_0A_0}$. This property allows us to introduce the linkage parallelograms $\overline{CDD_0}$ and $\overline{KAFD}$ by adjoining the links $D_0D$ and $DF$ respectively. Again, this does not affect the initial motion. The adjoining simply sustains the parallel motion of corresponding sides already enforced initially through similarity. This proves the proposition. In addition, it is now easily seen that

$$\square A_0ABB_0 \sim \square DKBC \sim \square A_0FDD_0.$$ 

Also, all triangles attached to the sides are similar to the $\triangle A_0DB$. In case the stretch rotation was carried out with respect to $A$ instead of $B$, we should obtain the configuration as demonstrated in figure 2. Then,

$$\text{Fig. 2. - Burmester's configuration 2nd type.}$$

the triangles which are successively attached to the sides are similar to $\triangle A_0DB$. Since this merely interchanges the points $A$ and $B$, we will disregard this possibility, and confine ourselves to configurations such as shown in figure 1.

Note: * This is effectuated by attaching the joints $C$ and $K$ to the respective sides $B_0B$ and $AB$. 


2b. FIRST METHOD TO REPRODUCE A FOUR-BAR COUPLER MOTION BY THE COUPLER MOTION OF A SIX-BAR LINKAGE

The method proposed here is based on a slightly modified Burmester's configuration.

In figure 3, Burmester's configuration is extended by the linkage parallelogram $DFA_oA'$. Since, in addition, $\Delta A'DK \cong \Delta A_oFA$, triangle $A'DK$ may be made a rigid triangle. Further, $\Delta A'DF \sim \Delta AKB \sim \Delta A_oD_oB_o$.

![Figure 3](image)

Fig. 3. — Slightly modified Burmester's configuration comprising two four-bar curve cognates and two coupler alternative mechanisms.

Therefore, the configuration as it is shown in figure 3 demonstrates a part of Robert's Law, since it comprises two four-bars generating the identical coupler curve. The four-bars mentioned here are $A_oA'DD_o$ and $A_oABB_o$.

They have a common coupler point $K$ that produces the identical coupler curve. The two four-bars shown in the figure are so-called curve cognates deriving from one another. This distinguishes them from coupler cognates or coupler alternative mechanisms producing identical coupler motions.

As mentioned earlier, it is the object of this paper to find coupler alternative mechanisms being six-bars that reproduce a four-bar coupler motion. It is further required that the alternative six-bars do not contain linkage parallelograms.

With the analysis deployed here, the assignments to obtain these six-bars from the source mechanism are now easy to understand:

**Given:** Source mechanism, which is the four-bar $A_oA'DD_o$ (see figure 3).

**Problem:** Find a six-bar not containing a parallelogram, that generates the entire motion of the coupler $A'D$.

**Solution:** According to the assignments:

a) Choose an arbitrary center $B_o$ (This may be done in $\infty^2$ different ways).

b) Determine point $C$ by way of the linkage parallelogram $DDB_oB_oC$. 
c) Determine the points $B$ and $K$ through the similarities:
\[ \Delta B_0CB \sim \Delta A_0D_0B_0 \sim \Delta A'DK \]
d) Find point $A$, using the fact that
\[ \Delta AKB \sim \Delta A_0D_0B_0 \]
e) Connect the points $A$ and $A_0$.
f) Finally, skip the bars $A_0A'$, $DD_0$, $D_0B_0$ and $A_0D_0$.

What remains is a six-bar of the Watt-1 type. It contains the similar four-bars $A_0ABB_0$ and $DKBC$ and, in addition, the similar and rigid triangles $AKB$ and $B_0CB$.

The coupler motion of $\Delta A'DK$ is common to both linkages, to wit the source linkage and the obtained six-bar mechanism. Therefore, both linkages which are distinctly demonstrated in figure 4 are coupler alternatives of each other.

So, generally, any four-bar coupler motion is to be replaced in this way by a six-bar coupler motion. The method represented here has the advantage of the free choice of the fixed center $B_0$, a fact which may be very useful to the designer.

In practice it means that the designer may replace the fixed center $D_0$ at any suitable position at the cost of two additional links. Wherever he chooses point $B_0$, the alternative six-bar linkage will generate the desired coupler motion he has originally been aiming at.
As far as the transmission angles are concerned, we must take care of the fact that the $\angle KDC$ may never obtain the value zero. It is advisable even to make $150^\circ \geq \angle KDC \geq 30^\circ$ in all positions of the linkage. Consequently, $150^\circ \geq \angle AA_{o}B_{o} \geq 30^\circ$. It would be convenient, therefore, to take $B_{o}B$ as the input crank that may rotate the full cycle only. This means a crank-and-rocker mechanism would be the most suitable.

Simultaneously, we have to restrain the values for $\angle A_{o}AB$. They are optimal in the positions where $B$ joins the line $A_{o}B_{o}$. Similarly, optimum values for $\angle KDC$ are obtained in the positions where the coupler $AB$ is aligned with the center $B_{o}$. Naturally, the angles just observed in the four-bar $A_{o}ABR_{o}$ are to be indicated also in its curve cognate, to wit the initial four-bar $A_{o}A'DD_{o}$.

The angles which we have to restrain then are the angles $\angle (A'D, A_{o}D_{o})$ and $\angle (A_{o}A', A'D)$. Thus, the transmission angles which are decisive for a smooth motion are to be observed directly in the source linkage.

2c. SECOND METHOD TO REPRODUCE A FOUR-BAR COUPLER MOTION BY A SIX-BAR ALTERNATIVE MECHANISM

Like before, the alternative mechanisms we are dealing with in this case are closely related through Burmester's Configuration.

Here, the configuration, as it is demonstrated in figure 5, is obtained from figure 3 by adjoining some linkage parallelograms. As we see from the figure, the parallelograms $KDC'D$, $A'DCF$, $EA'A_{o}E_{o}$ and $B_{o}D_{o}A_{o}E_{o}$ are adjoining in the configuration. Since, in addition, $\triangle ECD' = \triangle A'D'K$, triangle $ECD'$ may be made a rigid triangle as it is carried out in the figure. Now, of course, it is easy to understand how to derive the alternative mechanism from the source mechanism. Thus:

*Given:* Source mechanism, which is the four-bar $BoJiJ0Bo$ (see figure 5).

*Problem:* Find a six-bar not containing a parallelogram, that generates the entire motion of the coupler $EC$.

*Solution:*

a) Choose an arbitrary center $A_{o}$ (This may be done in $\infty^2$ different ways);

b) Make both triangles $ECD'$ and $B_{o}CB$ similar to $\triangle B_{o}E_{o}A_{o}$ and so determine the pin-joints $D'$ and $B$ respectively.

c) Draw $D'K$ parallel to $E_{o}A_{o}$ and make $D'K = E_{o}A_{o}$. This gives point $K$.

d) Make $A_{o}A$ parallel and equal to $ED'$ and so determine point $A$.

(Verify that the triangles $AKB$, $B_{o}E_{o}A_{o}$ are similar and that $AK$ is equal and parallel to $E_{o}E$);

e) Finally, erase the bars $E_{o}E$, $A_{o}E_{o}$ and $E_{o}B_{o}$.

The alternative mechanism so obtained resembles a six-bar linkage of the Watt-1 type in which the bar $CD'$ reproduces the coupler (motion) of the source mechanism.

The big advantage of course is the freedom of choice for the fixed center $E_{o}$. For the designer it boils down to the fact that he could replace the fixed center $E_{o}$, he has initially, to any suitable position he wants.

So, if we compare both methods, we see that with the first method one center is replaced and with the second one the other center. With
the first solution, the \( KD \) plane is involved, and with the second solution the \( CD' \) plane. Like with the first method, all the transmission angles involved in this case could be derived from the source mechanism.

Finally, the two linkages, having three common links (to wit \( \Delta ECD' \), \( \Delta B_0CB \) and \( \Delta B_0E_0A_0 \)), are shown separately in figure 6.
Note that the six-bar alternative mechanism resembles one of Hain's linkages [12, 13, 14] that contain a parallel moving bar (in the case under consideration the bar $KD'$ stays parallel to the fixed link $A_6E_6$). Note also that the linkage shown in figure 5 is just part of the configuration demonstrated in figure 5A (see also figure 621 from Burmester's Atlas zum Lehrbuch der Kinematik, Verlag von Arthur Felix, Leipzig 1888). The configuration shown in figure 5A, consists of two identical four-bars and four pairs of identical rigid triangles, all of them similar to one another. The two four-bars comprised in the figure, are rotated about a fixed angle of 180° with respect to one another. Corresponding triangles therefore maintain a relative motion which is only a translating one. No wonder, therefore, that two of Hain's linkages containing a parallel moving bar are to be recognized as a part of this type of configuration.

3. KEMPE'S GENERALIZED FORM [5] AND ITS USE TO DERIVE ALTERNATIVE LINKAGES REPRODUCING FOUR-BAR COUPLER MOTIONS

Quite another way of deriving alternative linkages would be one which is based on Kempe's generalized form. An example of this form is shown in figure 7. It resembles an overconstrained linkage that could be derived from the so-called focal linkage which is demonstrated in figure 8.

A geometric derivation of the focal linkage has been described in the author's book [10] named "Motion Geometry as a Modern Theory of Machines".

In addition, the derivation of Kempe's generalized form has been presented in author's paper [11] about "Kempe's focal linkage generalized, particularly in connection with Hart's (Second) straight-line mechanism" (see the Journal of Mechanism and Machine Theory).
In the last paper, it has been shown in detail that the generalized form could be derived from the focal linkage using stretch rotations in each of the four turning joints of the outer four-bar. This essentially provides the designer with two additional design degrees of freedom, namely, those which are represented by the stretch rotation factor for one of the turning joints (The remaining three factors for the other turning joints are only dependent on the first one).

Clearly, the generalized form could be used to extract alternative six-bars reproducing four-bar coupler motions. For instance, if we contemplate the configuration shown in figure 7, we immediately recognize the sub-chain which is the six-bar $Q'A'R'B_0 - B'T'A'$ of which the side $BT'$ just reproduces the four-bar coupler motion of the coupler $AB$. So, in fact, all we have to do would be to derive the six-bar from the four-bar in order to obtain the alternative mechanism.

The assignments to carry this out are given below:

Given: The source mechanism, which is the four-bar $(A_0ABB_0)$

Problem: Find a six-bar not containing a parallelogram that reproduces the motion of the coupler $AB$.

Solution: (See the figures 8 and 7).

a) Move the four-bar $A_0ABB_0$ until $AB$ runs parallel to $A_0B_0$. (This is done only to simplify the design).

b) Intersect $A_0A$ and $B_0B$ at the instantaneous pole $P$.

c) Draw the pole tangent $p$ of the four-bar by making $\angle pPB = \angle PAB$.

(Here, Bobillier's Theorem is applied).

d) Choose a random point $Q$ on the frame line that joins the points $A_0$ and $B_0$. (This choice provides the designer with an infinite number of possibilities).

e) Connect the pole $P = (A_0A, B_0B)$ with $Q$ and determine the point of intersection $T$ with the coupler $AB$. Thus $T = (AB, FQ)$.

f) Draw the common bisector $N_1N_2$ of the line segments $A_0A$, $QT$ and $B_0B$. The bisector intersects these line segments at the points $N_1$, $M$ and $N_2$ respectively.

* Basically, this factor resembles a complex number.

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Fig. 7. — Kempe's generalized form.
g) Intersect the circles having the respective diameters $A_0A$ and $B_0B$, at the points $C$ and $D$.

h) Intersect the line $PQ$ and the circle joining $C$ and $D$ that has $M$ as its center, at the points $F_1$ and $F_2$ (In case $F_1$ and $F_2$ are unreal points, one is obliged to choose another point $Q$ until they transform into real points of intersection).

\[ \text{Fig. 8. – Design position of Kempe's focal mechanism.} \]

Remark 1: The exact location of the points $F_1$ and $F_2$ are easily calculated using relations $ATQB_0 = TF_1, F_1Q = TF_2, F_2Q = A_0Q \cdot TB$ and $TF_1 + F_1Q = TQ = TF_2 + F_2Q$.

Thus, the distances $TF$ and $FQ$ are the roots of a quadratic equation.

Remark 2: According to Burmester [4] points $F_1$ and $F_2$ are called focal points, since each of them may be considered a focal point of a conic section inscribed in the four-bar.

i) Choose either of the focal points to continue the design with. Say, for the sake of convenience, we have chosen the point $F_1$.

j) Then, intersect the circle joining $F_1$ that touches the line $p$ at $P$, and sides $A_0A$ and $B_0B$ at the points $S$ and $R$ respectively (Verify that $SR$ is parallel to $A_0B_0$).

k) Join the point $F_1$ and the four sides of the four-bar through the links $F_1S, F_1T, F_1R$ and $F_1Q$ respectively (The four-bar adjoined with these links then resembles Kempe’s focal linkage).
1) Next, choose a random point \( Q' \) (This choice provides the designer with two additional degrees of freedom in design. Since, in addition, \( Q' \) is to be taken as a frame center, it is to be observed as a free replacement for the initial center \( A_0 \)).

m) Stretch rotate the four-bar \( QF_RB_0 \) about \( B_0 \) until \( Q \) comes to \( Q' \). We then obtain a new four-bar \( Q'A'R'B_0 \) which stays similar with \( (QF_RB_0) \) as long as \( R' \) is attached to the side \( B_0B \). Note that \( \angle A_0B_0Q' = \angle BB_0R' \) and \( R'B_0/RB_0 = Q'B_0/QB_0 \); etc.

n) Stretch rotate the four-bar \( TF_RB \) about \( B \) until \( R \) comes to \( R' \), giving the new points \( T' \) and \( A_0' \). In this case the four-bar \( T'A'R'B \) is obtained, staying similar to \( TF_RB \) as long as \( T' \) is attached to the side \( AB \). Thus, \( \angle T'B'A = \angle R'B'B_0 \) and \( T'B/TB = R'B/RB \); etc.

o) Turn the \( \Delta A'R'A_0' \) into a rigid triangle. This may be done, since the sides \( A'R' \) and \( A_0'R' \), both coming from the side \( FB \) before stretch rotation was applied, rotate with equal angular velocity and they additionally have a common turning joint \( R' \).

p) The resultant linkage is the six-bar represented by the sub-chains \( Q'A'R'B_0 \) and \( T'A'R'B \), both having the common links \( A'R'A'_{0} \) and \( B_0R'B \). The initial four-bar and the six-bar, so obtained, have the common links \( AT'B, B_0R'B \) and \( A_0B_0Q' \).

In figure 9, the two alternative linkages that demonstrate this, are shown. They both form a part of Kempe's generalized form as shown in figure 7. Although the design of the six-bar using this form is a bit more complicated than the ones based on Burmester's configuration, it

![Fig. 9. — Two coupler alternative mechanisms.](image-url)

has the advantage of having one additional design degree of freedom by comparison. Totally, the design described in this paragraph leaves the designer with three additional design degrees of freedom that could be
used at his convenience. At least it gives the impression that the method displayed in this section, is of a more general value than the ones furnished in the preceding sections.

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