The minimum energy principle applied to the cutting process of various workpiece materials and tool rake angles
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THE MINIMUM ENERGY PRINCIPLE APPLIED TO
THE CUTTING PROCESS OF VARIOUS WORKPIECE
MATERIALS AND TOOL RAKE ANGLES.

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SUMMARY.

Recently, a cutting model has been developed based on plasticity theory. Thus, a relation between the normalized frictional force and the shear angle has been obtained. In the model the tool geometry is represented by the tool rake angle while the workpiece material properties are governed by two plasticity material constants: the strain hardening exponent and the specific stress. The theoretical result was tested experimentally on a small scale.

In the present work, this testing is expanded to a great number of workpiece materials with different strain hardening exponents and specific stresses ranging from normal steels to alloyed steels as well as to a non-iron-metal. The rake angle of the tool is varied over the whole technically applied range. For each workpiece material and tool rake angle the cutting conditions are varied to a great extent. Different methods to determine the shear angle are tested and discussed. In addition to this, it is shown that the upper bound theorem from the plasticity theory leads to the same results as the minimum energy principle formerly applied.
1. INTRODUCTION

In [1] we derived a model for the cutting process. In this model we assumed that the deformation takes place in two regions: the primary and secondary deformation zone. The power for these two zones can be formulated as a function of the shear angle. Furthermore, we assumed that the deformation process will take that geometry which needs the least power ('minimum energy principle'). Applying this principle to the total deformation power we get a differential equation. This equation gives the first derivative of the normalized frictional force with respect to the shear angle as a function of the shear angle. This normalized frictional force is defined as the frictional force on the tool divided by the specific stress and the product of the width of cut and the feed. The shear angle is defined by the geometry of the cutting process. In the differential equation the workpiece material is represented by the two plasticity material constants: the specific stress and the strain hardening exponent. The tool material properties are included in the frictional force; the geometry is represented by the rake angle of the tool. This differential equation can be solved numerically if we know a boundary condition. This boundary condition is derived from the upsetting test. This analysis is already extensively described in Ref. [1]. We shall summarize it in Sec. 2.

The validity of the model has already been tested on a moderate scale. In Ref. [2] the validity of the differential equation itself is tested. Fig. 1 represents an example of these tests. In this figure we compare the numerically calculated dependency of the first derivative of the normalized frictional force with respect to the shear angle as a function of the shear angle with the experimental measured values. The agreement between theory and experiment was excellent, in particular for large values of the shear angle.

In Ref. [1] we have tested the validity of the integrated differential equation. The agreement between theory and experiment was good. However, we have to observe that there was some scatter of the experimental results around the theoretically predicted curves.

The experimental results represented in Refs. [1] and [2] are obtained from only two workpiece materials: C45 and X38CrMo5 and two rake angles of the tool: +6° and -6°. The aim of this work is to expand this comparison to more workpiece materials and rake angles of the tool. The guide for the choice of the different workpiece materials was to take materials with different strain hardening exponents and specific stresses ranging from a simple steel to stainless steel as well as to non-iron metals. The rake angle of the tool was varied from -6° to 33.5°. Just as in [1] we limit the comparison of experimental and theoretical results to the integrated differential equation.
Additionally, we pay attention to different methods determining the shear angle being a function of the feed and the chip thickness. The chip thickness can be determined with a micrometer, gravimetric by weighing, or by lightmicroscopical observation of a fine ground section of the chip. Also it is shown that the chip form has to be taken into account. In the discussion we shall prove that the differential equation as derived with the minimum energy principle can be derived with the upper bound theorem of the plasticity theory as well. Finally, we compare the specific stress derived from the cutting and the tensile test for the different workpiece materials.

2. CUTTING MODEL

For the description of our cutting model we have to derive three relations:
- differential equation (Sec. 2.1),
- boundary condition (Sec. 2.2),
- determination of the specific stress (Sec. 2.3).

2.1. Differential equation.

We assume cutting to be a two dimensional process (i.e. plane strain condition). This process takes place in two regions: the primary and secondary shear zone. Additionally, we assume that
the deformation in the primary shear zone takes place in one shear plane (Fig. 2).

\[ \phi = \text{shear angle} \]
\[ \gamma_O = \text{rake angle of the tool} \]
\[ v = \text{cutting speed} \]
\[ v_c = \text{chip speed} \]
\[ f = \text{feed} \]
\[ h_c = \text{chip thickness} \]
\[ F_f = \text{feed force} \]
\[ F_v = \text{cutting force} \]
\[ F_w = \text{frictional force} \]

Figure 2: Schematic representation of the cutting process in two dimensions (plane strain).

It holds for the process power [1]:

\[ E_p = \frac{C}{n+1} \left( \frac{\cotan \phi + \tan (\phi - \gamma_O)}{\gamma} \right)^{n+1} \]

where:
- \( C \) = specific stress
- \( n \) = strain hardening exponent
- \( \phi \) = shear angle
- \( \gamma_O \) = rake angle of the tool
- \( b \) = width of cut
- \( f \) = feed
- \( v \) = cutting speed

From the geometry of the process (Fig. 2) it holds:

\[ \phi = \text{arc tan} \left( \frac{\cos \gamma_O}{\frac{h_c}{f} \cdot \sin \gamma_O} \right) \]

where \( h_c \) = chip thickness
For the power in the secondary shear zone we calculate:

\[(3) \quad E_s = \frac{F_v \sin \varphi}{\cos (\varphi \gamma_o)}\]

where \(F_v\) = cutting force
\(F_f\) = feed force
\(F\) = frictional force on the tool
\(F_w\) = frictional force on the chip

Taking into account that the process will choose the geometry which needs the minimum power it holds:

\[(5) \quad \frac{d(E_s + E_p)}{d\varphi} = 0\]

Combination of Eqs. (1), (3) and (5) yields:

\[(6) \quad \frac{1}{T_c b_f} \frac{dF}{d\varphi} = -\frac{F \cos \gamma_o}{T_c b_f \cos (\varphi \gamma_o) \sin \varphi} + \frac{\cos (\varphi \gamma_o)}{\sqrt{3} \sin \varphi} *\]

\[\left( \cotan \varphi + \tan (\varphi \gamma_o) \right)^n \left\{ \frac{1}{\sin^2 \varphi \cos^2 (\varphi \gamma_o)} \right\}^{-1}\]

In this relation we assume that the frictional force on the tool is a function of the shear angle.
Eq. (6) describes the dependency of the first derivative of the normalized frictional force with respect to the shear angle as a function of the specific stress, the strain hardening exponent, the rake angle of the tool, the shear angle, the frictional force, the width of cut and the feed. This relation can be solved numerically for a given boundary condition.

2.2. Boundary condition.

The boundary condition can be derived by comparing the start of the cutting process with the upsetting test. (Fig. 3). By pressing the workpiece material against the tool material the plastic deformation starts in a plane 45° inclined to the pressing force. For the frictional force on the chip it holds:

\[(7) \quad F_w = \frac{\sin \gamma_o}{\sin^2 45} \tau_{shi} b_f\]

where \(\tau_{shi}\) = flow stress in the shear plane.
Assuming ideal plastic material it holds with Eq. (7):

\[
F_w = \frac{\sin \gamma}{\sqrt{3} \sin 45}
\]

Eq. (8) gives the normalized frictional force of a given rake angle of the tool for a shear angle of 45°.

2.3. Determination of the specific stress.

In order to control the numerical solution of Eq. (6) in combination with Eqs. (8) and (4) we need the value of the specific stress and the strainhardening exponent; the remaining unknown variables are geometrical cutting conditions. Normally, the two values \(C\) and \(n\) are derived from the tensile test. It holds:

\[
F_T = CA_0 \left( \ln \frac{I}{I_0} \right)^n \frac{1}{10}
\]

where \(F_T\) = tensile force
\(I\) = actual length of the tensile test bar
\(I_0\) = original length of the tensile test bar
\(A_0\) = cross sectional area of the original test bar

Apart from deriving the specific stress \(C\) from the tensile test, \(C\) can be derived directly from the cutting test.
So, its value is corrected for the high deformation rate and enhanced process temperature. Starting from Fig. 2 we deduce for the average shear stress \( \tau_{\text{she}} \) in the primary shear plane:

\[
\tau_{\text{she}} \text{med} = \frac{\sqrt{F_v^2 + F_f^2 \cos(\phi+\beta) \sin \varphi}}{F_b}
\]

(10) where \( \beta = \arctan \frac{F_f}{F_v} \)

For a constant shear velocity in the shear plane and a Nadai stress-strain relation for the workpiece material, it holds with Eq. (10) [1]:

\[
C = (n+1) \frac{\sqrt{F_v^2 + F_f^2 \cos(\phi+\beta) \sin \varphi}}{F_b} \left( \frac{\sqrt{3}}{\cotan \varphi + \tan(\varphi - \gamma_o)} \right)^{-n}
\]

(12) C = (n+1) \frac{\sqrt{F_v^2 + F_f^2 \cos(\phi+\beta) \sin \varphi}}{F_b} \left( \frac{\sqrt{3}}{\cotan \varphi + \tan(\varphi - \gamma_o)} \right)^{-n}

3. RESULTS

- Variation of the rake angle of the tool.
For different rake angles of the tool the model is checked by comparing the theoretical values of the normalized frictional force with experimental results. The theoretical curves are derived from Eqs. (6) and (8) by numerical integration. The choice of the different rake angles ranging from \(-6^\circ\), \(0^\circ\), \(6^\circ\), \(12^\circ\), \(18^\circ\) and \(33.5^\circ\) is made in accordance with practical applications. The workpiece material for this test range was C45 except for the rake angle of \(33.5^\circ\) where AlSi1Mg (Table 1) was used. The tool material was varied from H.S.S. to different types of carbide inserts (Table 1). Moreover, the cutting conditions were varied in a wide range. Fig. 4 shows the comparison between theory and experiment for the normalized frictional force as a function of the shear angle for different feeds and cutting speeds. The tool rake angle was \(-6^\circ\). The curved line is the theoretical result, the multiplicator signs are the experimental data. Every sign has different cutting conditions. The specific stress is determined from the cutting test data in accordance with Eq. (12). The strain hardening exponent is determined by a tensile test (Table 3). The shear angle \( \varphi \) defined by Eq. (2) is computed by measuring the chip thickness with a micrometer. The agreement between theory and experiment is good except for small values of the shear angle. The differences increase with decreasing shear angle. Fig. 4b shows the same results as Fig. 4a but now for a rake angle of \(0^\circ\). The method for measuring the chip thickness is different too. Here the chip thickness is determined with the specific mass method. This method will be discussed later on. In all the figures the results obtained with this method are denoted by the addition sign. In Fig. 4b the agreement between theory and experiment is rather well.
Fig. 4c shows the agreement between theory and experiment for a rake angle of 6°. Again there is an increasing difference between theory and experiment for decreasing shear angle. Fig. 4d represents the agreement between theory and experiment for a rake angle of 12°. The accordance between theory and experiment is well. Fig. 4e gives the comparison between theory and experiment for a rake angle of 18°. The number of experimental data is restricted in comparison with previous figures due to tool breakage for feeds above 0.315 mm/rev. Fig. 4f shows the comparison between theory and experiment for a rake angle of 33.5°. The workpiece material is AlSi1Mg. Also this figure supports the model.

- Variation of the workpiece material.

Another possibility for checking the cutting model is to vary the workpiece material, in particular the specific stress and strain-hardening exponent. The workpiece materials used are cementation steel, heat-treatable steels, tool steel, free cutting steel and aluminium. A survey of these materials together with their chemical composition is shown in Table 2. Table 3 gives the strain-hardening exponent for these materials as derived from the tensile test. For every material three tensile tests are carried out. Also the averages as used in the cutting test model are reported. Fig. 5a shows the comparison between theory and experiment for the normalized frictional force as a function of the shear angle for different feeds and cutting speeds. The workpiece material is X210Cr12. The experimental results are lying above the theoretical curve. This behaviour is clearly different from the previous results. The shear angle is determined by measuring the chip thickness with a micrometer. The same deviation as found in Fig. 5a is also found in Fig. 5b, for the workpiece material 34CrNiMo6. In Fig. 5c and 5d we compare the theoretical and experimental results for X38CrMoV51 and 95Mn28. The agreement is well for both workpiece materials. In Fig. 5e and 5f we find the results for C15 and C22. Here too a good agreement between theory and experiment is shown for large values of the shear angle. An increasing difference is obtained for a decreasing shear angle. In both figures the shear angle is determined by measuring the chip thickness with a micrometer. The comparison between theory and experiment for CH5 and AlSi1Mg can be found in Figs. 4f and 4c.

- Determination of the shear angle.

The determination of the shear angle is made by measuring the chip thickness. In order to get more information about the accuracy and variance of this measurement we determined the chip thickness in three different ways.
- Micrometer. In this method the chip thickness is measured with a micrometer [1].
- Specific mass. Here the weight of the chip is divided by the
Table 1: Survey of the different workpiece materials, tool materials, and rake angles of the applied cutting conditions as well as the number of tests.

<table>
<thead>
<tr>
<th>WORKPIECE-MATERIAL</th>
<th>FEED [mm/rev]</th>
<th>CUTTING SPEED [m/s]</th>
<th>TOOL MATERIAL</th>
<th>RAKE ANGLE OF TOOL</th>
<th>WIDTH OF CUT [mm]</th>
<th>NUMBER OF TESTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>X210Cr12</td>
<td>0.16, 0.315, 0.45</td>
<td>1.15, 2.3</td>
<td>P10, P25, P30</td>
<td>γ=6°</td>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>34CrNiMo6</td>
<td>0.145, 0.315, 0.45</td>
<td>1.15, 2.5</td>
<td>P10, P25, P30</td>
<td>γ=6°</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>X38CrMoV51</td>
<td>0.15, 0.30, 0.45</td>
<td>2.0, 2.5, 3, 3.5, 4</td>
<td>P20, P40, M20, M40, K10</td>
<td>γ=6°</td>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td>95Mn28</td>
<td>0.1, 0.31, 0.5</td>
<td>1.3</td>
<td>P20</td>
<td>γ=6°</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>C15</td>
<td>0.16, 0.315, 0.45</td>
<td>1.15, 2.2, 2.5, 3</td>
<td>P10, P25, P30</td>
<td>γ=6°</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>C22</td>
<td>0.16, 0.315, 0.45</td>
<td>1.15, 2.2, 2.5, 3</td>
<td>P10, P25, P30</td>
<td>γ=6°</td>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>C45</td>
<td>0.10, 0.84</td>
<td>1.0, 5</td>
<td>(P20, P40, M20, M40, K10</td>
<td>γ=6°, 10°, 12°, 18°</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>AISi1Mg</td>
<td>0.1, 0.31, 0.50, 0.71</td>
<td>1.3, 5</td>
<td>H.S.S., P10, P25, P30</td>
<td>γ=33.5</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 4: Comparison of theory and experiment for the normalized frictional force $F_w/Cbf$ as a function of the shear angle $\varphi$ for a number of tool rake angles. For every figure a number of different feeds and cutting speeds were used.
Table 2. The chemical composition of the different workpiece materials.

<table>
<thead>
<tr>
<th>DIN</th>
<th>STANDARD NO.</th>
<th>% C</th>
<th>% Si</th>
<th>% Mn</th>
<th>% P</th>
<th>% S</th>
<th>% Cr</th>
<th>% Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>X210Cr12</td>
<td>1.2080</td>
<td>1.9 -2.2</td>
<td>0.2 -0.4</td>
<td>0.2 -0.4</td>
<td>≤ 0.035</td>
<td>≤ 0.035</td>
<td>11.00-12.00</td>
<td></td>
</tr>
<tr>
<td>34CrNiMo6</td>
<td>1.6582</td>
<td>0.30-0.38</td>
<td>≤ 0.4</td>
<td>0.4 -0.7</td>
<td>≤ 0.035</td>
<td>≤ 0.035</td>
<td>1.40-1.70</td>
<td></td>
</tr>
<tr>
<td>X38CrMoV51</td>
<td>1.2343</td>
<td>0.36-0.42</td>
<td>0.9 -1.2</td>
<td>0.30-0.50</td>
<td>≤ 0.030</td>
<td>≤ 0.030</td>
<td>4.8 - 5.8</td>
<td></td>
</tr>
<tr>
<td>9SnMo28</td>
<td>1.0715</td>
<td>≤ 0.14</td>
<td>≤ 0.05</td>
<td>0.9 -1.3</td>
<td>≤ 0.100</td>
<td>≤ 0.045</td>
<td>0.24-0.32</td>
<td></td>
</tr>
<tr>
<td>C15</td>
<td>1.0401</td>
<td>0.12-0.18</td>
<td>0.15-0.35</td>
<td>0.30-0.50</td>
<td>≤ 0.045</td>
<td>≤ 0.045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C22</td>
<td>1.0402</td>
<td>0.18-0.25</td>
<td>0.15-0.35</td>
<td>0.30-0.60</td>
<td>≤ 0.045</td>
<td>≤ 0.045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C45</td>
<td>1.0503</td>
<td>0.42-0.50</td>
<td>0.15-0.35</td>
<td>0.50-0.80</td>
<td>≤ 0.045</td>
<td>≤ 0.045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AlSiMg</td>
<td>3.2316</td>
<td>-</td>
<td>0.8 -1.2</td>
<td></td>
<td>1</td>
<td>0.6% Mg</td>
<td>&lt; 0.1% Cu</td>
<td>&lt; 0.1% Zn</td>
</tr>
</tbody>
</table>

Figure 5:
Comparison of theory and experiment for the normalized frictional force $F_w/Cbf$ as a function of the shear angle $\Phi$ for a number of workpiece materials. For every figure a number of different feeds and cutting speeds were used.
specific mass of the workpiece material, the length and the width of cut of the chip. The constance of the specific mass for different cutting conditions is shown in Table 4.

- Sectioning of the chip. In this method a chip with a known width of cut is ground very carefully perpendicular to its length dimension. The ground surface is enlarged with a light microscope and measured with a planimeter.

<table>
<thead>
<tr>
<th>WORKPIECE MATERIAL</th>
<th>C[N/mm$^2$] tensile test</th>
<th>C[N/mm$^2$] average</th>
<th>C[N/mm$^2$] cutting test</th>
<th>n tensile test</th>
<th>n average</th>
</tr>
</thead>
<tbody>
<tr>
<td>X210Cr12</td>
<td>1243 1121 1145</td>
<td>1170</td>
<td>1445±69</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>34CrNiMo6</td>
<td>1399 1432 1451</td>
<td>1427</td>
<td>1400±111</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>X38CrMoV51</td>
<td>1023 1023 1049</td>
<td>1032</td>
<td>1268±104</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>9SMn28</td>
<td>799 846 672</td>
<td>772</td>
<td>1010±35</td>
<td>0.27</td>
<td>0.31</td>
</tr>
<tr>
<td>C15</td>
<td>830 810 827</td>
<td>822</td>
<td>951±70</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td>C22</td>
<td>937 965 1037</td>
<td>980</td>
<td>1022±76</td>
<td>0.24</td>
<td>0.26</td>
</tr>
<tr>
<td>C45</td>
<td>1377 1533 1106</td>
<td>1339</td>
<td>1335±100</td>
<td>0.25</td>
<td>0.29</td>
</tr>
<tr>
<td>AlSi1Mg</td>
<td>500 504 451</td>
<td>485</td>
<td>539±19</td>
<td>0.18</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 3: The different properties of the workpiece materials derived from the tensile and cutting test.
C = specific mass,
n = strain hardening.
The specific mass of C45 chips for different cutting conditions.

Table 4

<table>
<thead>
<tr>
<th>cutting speed [m/s]</th>
<th>feed [mm/rev]</th>
<th>specific mass [gr/cm³] (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>7.867 ± 0.022</td>
</tr>
<tr>
<td>1</td>
<td>0.315</td>
<td>7.832 ± 0.013</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>7.821 ± 0.015</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>7.863 ± 0.034</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>7.859 ± 0.026</td>
</tr>
<tr>
<td>5</td>
<td>0.315</td>
<td>7.843 ± 0.015</td>
</tr>
</tbody>
</table>

specific mass of C45

7.86

rake angle of the tool: 6°
width of cut: 3 mm

(1) Average of 8 measurements.

Table 5 gives a comparison of the three methods for cutting steel C45 with a feed of 0.45 mm/rev. and several cutting speeds. From this table it follows that the values of the chip thickness increases notably in the sequence specific mass, micrometer and planimeter method.
4. DISCUSSION

Summarizing the results of Figs. 4 and 5 we can conclude that the agreement between the theoretical curves and the experimental results for the normalized frictional force as a function of the shear angle is fair. The agreement for the first derivative of the normalized frictional force with respect to the shear angle as a function of the shear angle is striking [2]. These results support the idea that the correct process determining parameters are used. However, in the several figures we find some deviations. The deviations can be divided in two groups. In some figures the experimental results are lying above, in other figures below the theoretical curves. This behaviour will now be discussed.

- Experimental results above the theoretical curve:
  We discuss two possibilities:
  • measuring errors. It means that we measure a too low value of the chip thickness or/and a too high value of the cutting forces. Both assumptions are unlikely in relation with the magnitude of the deviations.
  • heterogenuous velocity distribution in the chip: the chip
thickness varies, or the chip is curled. Both will give the same effect. A prove for the inhomogenous velocity distribution or deformation can be derived from Table 5. The more than linear increase of the normalized frictional force with decreasing shear angle gives a chip with a variable chip thickness a higher theoretical normalized frictional force. The chips of both workpiece materials represented in Figs. 5a and 5b were heavily curled. This curled form can explain the difference between the theoretical curves and experimental results.

- Experimental results below the theoretical curve. As already remarked in [2] we find an increasing difference for a decreasing shear angle. Different reasons are possible:
  - Simplicity of the cutting model. In that case expansion of the primary shear zone from a shear plane to a shear volume has to result in a smaller difference.
  - Another possibility may be found in the value of the boundary condition. By using a strain hardening metal without prestrain it means that the first plastic flow occurs at a lower flow stress than used in Eq. (8). This effect lowers the theoretical curve. This lowering effect has to be striking for metals with a high strain hardening exponent; in our case for instance C15 and C22.
  - This deviation is also possible for a chip with a velocity component in the thrust force direction. In this case the frictional force computed with Eq. (4) gives a too low value. It has to be enhanced with a component of the thrust force.

- In nearly all cases we used the micrometer to determine the chip thickness. Another method is possible by means of the specific mass. A comparison of both thickness measurement methods has to show that the micrometer method gives the highest value of $h_C$ or the smallest shear angle. This effect can be seen in the Figs. 5a, 5e and 5f.

- More information on the value of the specific stress from the tensile and cutting test can be found in Table 3. On the measure of agreement we have to make two remarks.
  - The specific stress in the tensile test is determined at room temperature and at low strain rates. However, the specific stress for the cutting test is determined at a temperature fully different from room temperature and at extremely high strain rates.
  - In the tensile test the specific stress for a brittle material is determined by extrapolating the stress from a very low strain to a strain that equals 1. This extrapolation can be the reason for a big variation of C (Table 3; C45). In cutting the strain in many cases is higher than 1. The problem can be solved by replacing the tensile test by the upsetting test or forging test. The same holds also for the strain hardening exponent.
The upper bound theorem of the plasticity theory applied to the cutting model. In the plasticity theory it often is impossible to obtain theoretically exact solutions for technical problems [3]. The reason is that the equations of the process describing quantities are mathematically unsolvable. For these kind of problems one has derived approximative solutions: the lower and upper bound theorem. These theorems start from the principle of virtual work with boundary conditions and make it possible to choose the best solution from a number of approximate solutions [4, 5]. Basically there are two kind of approaches:
- the choice of a velocity-field,
- the choice of a tension-field.

The first method gives higher values (= upper bound) and the second solution method gives lower values (= lower bound) for the real tensions [3]. We discuss only the upper bound theorem.

Fig. 6 is a body with two parts, both having a different velocity field. That difference is $\Delta \dot{u}^*$ and is built up in a plane $A_i$, the boundary plane of the two parts. On a part $S^t$ of the surface $S$ is a tension field $\tau_i$. On the part $S^0 = S - S^t$ the velocity field is $\dot{u}_i$.

\[ J^* = \int \sigma_{ij} \dot{\varepsilon}_{ij}^* \, dV + \sum \int \tau_i / \Delta \dot{u}_i^* / dA_i - \int \tau_i \dot{u}_i^* \, dS \geq \int \tau_i \dot{u}_i \, dS \]

Figure 6: Body under load.
where \( \dot{u}_i^* \) the chosen permitted velocity field that means:
- \( \dot{u}_i^* = \dot{u}_i \) on the surface \( S \)
- \( \dot{u}_i^* \) satisfies the invariance of the volume
- \( \dot{u}_i^* \) can be differentiated step by step

\[
\dot{\varepsilon}_{ij}^* = \frac{1}{2} [\dot{u}_{i,j}^* + \dot{u}_{j,i}^*] \text{; definition of the incremental strain tension}
\]

\( \sigma_{ij}^* \) determined by \( \dot{\varepsilon}_{ij}^* \) and the Levy von Mises relations

\[
\dot{\varepsilon}_{ij}^* = \frac{3}{2} \frac{\dot{\varepsilon}}{\sigma} \left[ \sigma_{ij}^* - \sigma_{m}^* \delta_{ij} \right] \text{ Levy von Mises relations}
\]

\[
\sigma_{m}^* = \frac{1}{3} (\sigma_{11}^* + \sigma_{22}^* + \sigma_{33}^*) \text{ hydrostatic pressure}
\]

\[
\sigma_{ij}^* = \frac{1}{\sqrt{2}} \left( (\sigma_{11}^* - \sigma_{22}^*)^2 + (\sigma_{22}^* - \sigma_{33}^*)^2 + (\sigma_{33}^* - \sigma_{11}^*)^2 + 6(\sigma_{12}^* + \sigma_{13}^* + \sigma_{23}^*)^2 \right)^{1/2}
\]

\( \delta_{ij} \) is the Kronecker delta

\( \tau_v \) is the flow stress of the body L

\( \sigma_{ij} n_j = t_i \) on the surface \( S_t \); \( n_j \) is the normal on \( S_t \)

The first term of Eq. (13) is the deformation power of the body. The second term represents the power loss on the plane \( A_i \). The third and fourth terms represent the power of the outer tension. The choice of the velocity field can be improved by taking a free parameter \( (\dot{\varphi}) \) in Eq. (13). For the best solution it holds:

\[
\frac{\partial J^*}{\partial \dot{\varphi}} = 0
\]

with the supplementary condition:

\[
\frac{\partial^2 J}{\partial \dot{\varphi}^2} > 0
\]

Applying of the upper bound theorem on our cutting model results in:

\[
\int_{A_i} \sigma_{ij} \dot{\varepsilon}_{ij}^* dV = \frac{C}{n+1} \left( \cotan \alpha + \tan(\varphi-\gamma_O) \right)^{n+1} bh_v
\]

\[
\int_{A_i} \tau_v \sqrt{\Delta \dot{u}_i^*} dA_i = F_v \frac{\sin \varphi}{\cos(\varphi-\gamma_O)}
\]

\[
\int_{S^t} t_i \dot{u}_i^* dS = 0
\]
Combination of Eqs. (20), (21), (22) and (18) yields Eq. (6). It means that the used minimum energy principle leads to the same equation as the upper bound theorem of the plasticity theory.

5. CONCLUSIONS

The experimental results for the different rake angles of the tool as well as the different workpiece materials support the proposed model. Needless to say that again it is proven that it also holds for different tool materials, feeds and cutting speeds. The deviations for some workpiece materials are understood. In agreement with former results there is an increasing difference with decreasing shear angle between theoretical and experimental results. A refinement of the proposed model will be necessary. Also different methods for determining the chip thickness are discussed. Finally, we prove that the upper bound theorem of the plasticity theory gives the same results for the cutting model as the applied minimum energy principle.

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