Dipole scattering of electromagnetic waves propagating through a rain medium
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Published: 01/01/1979

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Dipole scattering of electromagnetic waves
propagating through a rain medium

by

C. A. van Duin
DIPOL SCATTERING OF ELECTROMAGNETIC WAVE PROPAGATING THROUGH A RAIN MEDIUM

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C.A. van Duin

TH-Report 79-E-93
ISBN 90-6144-093-9

Eindhoven
January 1979
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Summary

The propagation of a plane linearly polarized monochromatic electromagnetic wave through a rain medium is studied. Section 2 is devoted to the so-called dipole model. At not too high frequencies, i.e. if the characteristic dimensions of the raindrops are small enough with respect to the wavelength of the incident field, this model appears to be suitable for the description of the scattering and absorption of a wave propagating through the medium. In the sections 3 and 4 expressions are derived from which relevant quantities as damping and depolarization parameters can be determined. In section 5 the so-called cross-polarization parameters are introduced. In section 6 a more realistic rain model is presented: in the preceding sections we assumed that the raindrops are equioriented and equivolumic and have the same shape. The orientation of each drop can be described by two parameters, the so-called canting angles, fixing the direction of the symmetry axis of this drop with respect to the direction of propagation and that of the incident field. Besides, complex canting angles are introduced, leading to a generalization of the dipole model. The real parts of these angles are interpreted as the orientation parameters. In section 7 another method is applied, leading to the same expressions for the quantities derived earlier. Section 8 gives numerical data for the complex forward-scattering amplitudes of a linearly polarized wave incident on an axisymmetric water drop. Finally, in section 9, some calculated relevant quantities are compared with experimental data. In the case of real canting angles the dipole model proves to be applicable at frequencies up to about 20 GHz. However, the possibility whether the introduction of complex canting angles might give rise to a more general scattering model, should be considered.

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Acknowledgement

The author is indebted to Professor R. Bremmer for his valuable help in the preparation of this report and wishes to express his thanks to ir. J. Dijk for his research suggestion* and his contribution to the numerical part of this publication.

* A. Mawira and J. Dijk (1975), Depolarization by rain - some related thermal emission considerations, T.H. Report 75-E-61, Eindhoven University of Technology, Eindhoven, the Netherlands.
1. Introduction

In this report the propagation of a plane linearly polarized monochromatic electromagnetic wave through a rain medium is studied theoretically. The presence of the rain medium causes damping and depolarization of the incident wave. The depolarization effects are a consequence of the anisotropy of the rain medium: raindrops are not spherical. We shall consider axisymmetric oblate spheroidal raindrops.

For the description of the scattering and absorption of the wave, travelling through the medium, we introduce the so-called dipole model (1). If the rain medium considered is weakly inhomogeneous, i.e. if its relevant parameters vary only slightly over one wavelength, the expressions to be derived for the relevant quantities, such as damping and cross-polarization parameters, can be simplified rigorously. It will be shown that the scattering problem described can be reduced to a first-order differential equation. In an article by Morrison and Cross (4) numerical data are given for the complex forward-scattering amplitudes of a linearly polarized wave incident on an axisymmetric water drop at different frequencies and for various drop sizes and drop orientations. For the computation of the relevant quantities we used their results.
2. The dipole model

A possible model for the description of the scattering and absorption of a monochromatic electromagnetic wave by a raindrop concerns the assumption that an electric dipole is induced in such a drop and that the field, generated by this dipole, represents the actually scattered field\(^{(1)}\). If the characteristic dimensions of the drop are small enough with respect to the wavelength of the incident field, i.e. if the frequency of this field is not too high, the dipole induced can be regarded as infinitesimal.

The relation between the induced dipole moment \(p\) and the incident electric field \(E\) will be assumed as linear

\[
p = \chi E. \tag{2.1}\]

The elements of the tensor \(\chi\) are dependent on the frequency of the incident wave and on the geometry and orientation of the raindrop, see appendix A. We consider axisymmetric raindrops. In that case the tensor will be symmetric.

The vector potential \(A^{(s)}(x)\), \(x\) being the field point, of the scattered field induced by an electric dipole\(^{(2)}\) at a position \(x_o\), can be written as

\[
A^{(s)}(x) = -\frac{i\omega}{4\pi} \int \frac{\chi^{(s)}(x_o) e^{i k_o |x - x_o|}}{|x - x_o|} dx_o. \tag{2.2}\]

All time dependent quantities are assumed to be proportional to the factor \(\exp(-i\omega t)\).

The vacuum wavenumber \(k_o\) is related to the frequency \(\omega\) and the velocity of light in vacuum \(c\): \(k_o = \omega/c\). We assume the relative permeability to be unity.

If the raindrops are labelled, \(x_m\) being the position of drop \(m\), the vector potential of the scattered field can be represented by

\[
A^{(s)}(x) = -\frac{i\omega}{4\pi} \sum_m \frac{\chi^{(s)}(x_m) e^{i k_o |x - x_m|}}{|x - x_m|}. \tag{2.3}\]

In this section we shall consider identical raindrops, which means that all drops are equioriented and equivolumic and have the same shape. In section 6 we shall introduce a more realistic rain model.

If a density \(N\) of the raindrops can be defined, relation (2.3) can be written as
\[ A(s)(x) = -\frac{i\omega\mu_0}{4\pi} \int_S d^3u \, N(u) \, p(u) \, \frac{ik \, |x - u|}{|x - u|}, \quad (2.4) \]

\( S \) being the scattering space, \( d^3u = du \, du \, du \) its volume element.

We now introduce a Cartesian coordinate system \( xyz \). The primary plane-wave field \( E^{(o)}(x) \), i.e. the electric field in the absence of the scattering medium, is given by

\[ E^{(o)}(x) = E^{(o)}(o)e^{ik \, z}, \quad (2.5) \]

and will be assumed as transverse. This wave thus propagates in the direction of positive \( z \).

We shall now derive a first-order approximation for the scattered electric field, i.e. we put

\[ \mathbf{p}(x) = \chi \, E^{(o)}(x) = p_o \, e^{ik \, z} \quad (2.6) \]

Each drop in the rain medium observes the primary field (2.5) as the scattering one. Since the raindrops are identical, the elements of the tensor \( \chi \) and the components of the vector \( p_o \) are independent of the spatial coordinates.

Substitution of (2.6) in (2.4) yields

\[ A(s)(x) = -\frac{i\omega\mu_0}{4\pi} \, p_o \, \int_S d^3u \, e^{ik \, u \, z} \, N(u) \, \frac{ik \, |x - u|}{|x - u|}, \quad (2.7) \]

The vector potential is normalized in such a way that

\[ \mathbf{B}(s) = \nabla \times A(s). \]

The scattered electric field can be derived from the vector potential in accordance with

\[ E(s) = i\omega A(s) + i(\omega\mu_0\epsilon_0)^{-1} \nabla(\nabla \cdot A(s)) \quad (2.8) \]

The \( x \)-component in the second right-hand term of relation (2.8) can be written as

\[ \frac{\partial}{\partial x} \nabla \cdot A(s)(x) = -\frac{i\omega\mu_0}{4\pi} \int_S d^3u \, e^{ik \, u \, z} \, N(u) \, \frac{ik \, |x - u|}{|x - u|} \]
Let us assume that the scattering space \( S \) is limited. If \( \text{Sc} \subset \text{U} \) and \( \text{U} \) is also limited, relation (2.9) can be rewritten as

\[
\frac{\partial}{\partial x} \nabla \cdot A(s)(x) = \frac{i\omega_0}{4\pi} \int_S d^3u \left[ \frac{\partial}{\partial u_x} \frac{i k_0 |x - u|}{u} \right] N(u) \cdot \left( \frac{\partial}{\partial x} \frac{e^{i k_0 (x - u)}}{|x - u|} \right) \frac{\partial}{\partial u_x} N(u).
\]

In the same way we obtain

\[
\frac{\partial}{\partial y} \nabla \cdot A(s)(x) = \frac{i\omega_0}{4\pi} \int_S d^3u \left[ \frac{\partial}{\partial u_y} \frac{i k_0 |x - u|}{u} \right] N(u) \cdot \left( \frac{\partial}{\partial y} \frac{e^{i k_0 (x - u)}}{|x - u|} \right) \frac{\partial}{\partial u_y} N(u).
\]

The primary electric field (2.5) is transverse. We shall suppose, for simplicity, that the (total) electric field \( E = E^{(o)} + E^{(s)} \) is transverse too, i.e. we assume that the scattered field \( E^{(s)} \) has no component in the propagation direction: \( E^{(s)}_z = 0 \).

If \( L \) is a scale in the vector potential \( A^{(s)} \), i.e. if the condition

\[
|\nabla A^{(s)}| = L^{-2}\left(|A^{(s)}|\right)
\]

has been satisfied, the second right-hand term in relation (2.8) can be neglected with respect to the first, provided

\[
L^{-2} \ll k_0^2
\]

If the partial derivatives of the density of the rain medium with respect to
the coordinates $x$ and $y$ are small enough, i.e. if the density changes (very)
slightly in directions perpendicular to the propagation direction, the second
right-hand term in relation (2.8) can be neglected with respect to the first
term $i\omega A(s)$. Then relation (2.8) reduces to

$$
E_x(s)(x) = \frac{\omega^2 \mu_0}{4\pi} \int_S d^3 u \ e^{i k_0 u.x} N(u) \ N_x(x) \ e^{ik_0 |x-u|} ,
$$

(2.14)

$$
E_y(s)(x) = \frac{\omega^2 \mu_0}{4\pi} \int_S d^3 u \ e^{i k_0 u.z} N(u) \ N_y(y) \ e^{ik_0 |x-u|} .
$$
3. The geometric-optical approximation

If the density of the rain medium changes only slightly over one wavelength, the expression (2.14) for the scattered electric field can be simplified yet more.

The derivation of the simpler relation is in accordance with one given earlier by Bremner (3).

Since the primary and scattered electric fields are supposed to be transverse, the tensor $\chi$ can be represented by

$$\chi = \begin{pmatrix} \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & \chi_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.1)$$

We assume that the scattering medium is contained in the half space $z > 0$ and introduce the symbolic relation

$$N(x', y', z) = e^{(u_x - a) \frac{\partial}{\partial a} + (u_y - b) \frac{\partial}{\partial b}} N(a, b, u_z), \quad (3.2)$$

to be applied for $a = x$, $b = y$, and polar coordinates according to

$$u_x = r \cos \phi, \quad u_y = r \sin \phi. \quad (3.3)$$

The integration with respect to $\phi$ reduces (2.14) to

$$E_s(x) = \frac{\omega^2 \mu_0}{2} \chi \int_0^\infty \frac{dr}{r} \frac{k_0}{\sqrt{r^2 + (z - u_z)^2 + ik_0 u_z}}$$

$$\times \left\{ (r \sqrt{\frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2}} N(a, b, u_z))_{a=x, \ b=y} \right\} \quad (3.4)$$

The integration with respect to $r$ yields, applying Sommerfeld's integral for a point source

$$E_s(x) = \frac{i \omega^2 \mu_0}{2} \chi \int_0^\infty \frac{dz}{\sqrt{k_0^2 + \frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2}}}$$

$$\times \left\{ z - u_z \sqrt{k_0^2 + \frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2}} \right\} \quad (3.5)$$
\( x \times N(a, b, u_z) |_{a=x, b=y} \)  

(3.5)

We again assume that the density of the rain medium changes only slightly over one wavelength in directions perpendicular to the propagation direction, so that

\[
\left| \frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2} \right| \ll k_o^2
\]

(3.6)

This approximation is known as the geometric-optical approximation. Relation (3.5) then further reduces to

\[
E(s)(x) = \frac{i \omega \mu_o}{2k_o} \gamma E(o)(x) \int_0^\infty du_z e^{ik_o(|z - u_z| + u_z)} N(x, y, u_z).
\]

(3.7)

It is well-known that the conditions of this approximation involve a negligible backscattering. In that case the scattered field becomes, finally

\[
E(s)(x) = \frac{i \omega \mu_o}{2k_o} e^{ik_o z} \gamma E(o)(x) \int_0^z du_z N(x, y, u_z).
\]

(3.8)

4. Reduction of the scattering problem to a first-order differential equation

The first-order approximation (3.8) is applicable if, among other things, the condition that the penetration depth \( z \) is small enough has been satisfied. As a matter of fact, this approximation neglects multiple-scattering effects which may cause radiations deviating from the \( z \) axis and thus yielding fields that are not perpendicular to this axis. In this section we shall derive expressions for the relevant quantities that are valid for an arbitrary penetration depth.

For that purpose the propagation path \([o, z]\) is divided into \( m \) sections, the length of each of them being \( z_o \). Besides, \( z_o \) is supposed to be much smaller than the wavelength of the incident field. Backscattering is then negligible along a single section of this type (see section 3). It will be shown that in this case the scattering problem can be reduced to a differential equation of the first order. If the density of the propagation medium is not dependent on the coordinate \( z \), i.e. that this quantity does not change in the propagation direction, this equation can be solved in a simple way.
As \( z_o \) is assumed to be much smaller than the wavelength of the incident field we put

\[
E(s)(x,y,z_o) = \frac{i\omega^2 \mu}{2k_o} e^{ik_z o} \chi \int_0^{z_o} N(x,y,u_z) du_z
\]  

We introduce the definition

\[
\Gamma(x,y,(n+1)z_o) = \frac{i\omega^2 \mu}{2k_o} \chi \int_{nz_o}^{(n+1)z_o} N(x,y,u_z) du_z,
\]  

\( n \) being zero or a positive integer.

Note that the elements of the tensor \( \chi \) are independent of the spatial coordinates, see relation (2.6). The total electric field at the position \((x,y,z_o)\) reads

\[
E(x,y,z_o) = E^{(o)}(x,y,z_o) + E^{(s)}(x,y,z_o)
\]

\[
= \{I + \Gamma(x,y,z_o)\} E^{(o)}(o) \ e^{ik_z o},
\]

\( I \) being the unit matrix.

We assume that within the slab \( nz_o < z < (n+1)z_o \) the scattering field is given by

\[
E^{(s)}(x,y,znz_o) = e^{ik(z-nz_o)}
\]

The quantity \( E(x,y,znz_o) \) is the (total) electric wave entering this slab.

The following recurrence relation can be derived easily

\[
E(x,y,(n+1)z_o) = \{I + \Gamma(x,y,(n+1)z_o)\} E(x,y,znz_o) e^{ik_z o},
\]

with the boundary condition

\[
E(x,y,o) = E^{(o)}(o).
\]

Using this boundary condition and relation (4.5), the relevant quantities can be determined. These quantities are more exact with decreasing \( z_o \).

The electric field in the plane \( z = mz_o \) is given by

\[
E(x,y,mz_o) = \prod_{k=1}^m \{I + \Gamma(x,y,kz_o)\} E^{(o)}(o) e^{ik_mz_o}.
\]
If $z_0$ is taken infinitesimal, relation (4.5) changes into the differential equation

$$\frac{\partial}{\partial z} - ik_0(1 + \frac{\omega^2 \mu_0}{2k_0^2} N X) E = 0$$

(4.8)

Appendix B shows the solution of (4.8). It is assumed that the density of the different raindrops is independent of the coordinate $z$.

5. The cross-polarization parameters

The solution of (4.8) can be written as

$$E(x) = \int E(x) E^{(o)}(o),$$

(5.1)

with the boundary condition

$$E(x, y, o) = 1,$$

(5.2)

following from (4.6).

Note that the matrix $E$, the so-called evolution matrix, is symmetric if the density of the rain medium is independent of the coordinate $z$, i.e. independent of the propagation direction of the incident wave. The elements of this matrix are calculated in appendix B.

In the following a linearly polarized primary wave is assumed as before, parallel to the x or y axis of the coordinate system introduced in section 2.

The cross-polarization parameters are defined by

$$XPLX = \left| \frac{F_{xx}}{F} \right|^2$$

(5.3)

for transmitting x polarization, and by

$$XPLY = \left| \frac{F_{yy}}{F} \right|^2$$

(5.4)

for transmitting y polarization.
These parameters indicate the degree of depolarization, i.e. they give an indication of the degree of anisotropy of the scattering medium. At a fixed frequency and a given rain rate and penetration depth the above parameters are dependent yet on the orientations of the raindrops with respect to the incident wave and the propagation direction. If all drops are equi-oriented, the following relations hold

\[
XPLX(\theta, \phi) = XPLX(\theta, \phi + \frac{\pi}{2})
\]  \hspace{1cm} (5.5)

\[
XPLX(\theta, \phi + \frac{\pi}{2}) = XPLX(\theta, -\phi + \frac{\pi}{2})
\]  \hspace{1cm} (5.6)

\[
XPLX(\theta + \frac{\pi}{2}, \phi) = XPLX(-\theta + \frac{\pi}{2}, \phi)
\]  \hspace{1cm} (5.7)

The orientation parameters \(\theta\) and \(\phi\) are defined in appendix A. The parameter \(XPLY\) has similar symmetry properties with respect to \(\theta\) and \(\phi\). The relations (5.5 - 7) can be deduced from expressions (A.5), see appendix A.

The cross-polarization parameters are zero if the symmetry axes of the raindrops are parallel to the z axis, viz the direction of propagation of the incident field, or if they are parallel to the incident field. Besides, they are zero for \(\phi = n\pi/2\), \(n\) being zero or a positive integer (appendix A, expressions (A.5)).
6. The situation in the case of non identical raindrops

In the preceding sections expressions were derived for the elements of the evolution matrix $F$ (section 5, appendix B). All relevant quantities can be expressed in these elements.

However, we assumed that the raindrops are equioriented and equivolumic and have the same shape. We shall now start from a more realistic rain model.

Let us consider oblate spheroidal raindrops with minor and major axes $a$ and $b$. We then introduce the parameters $\bar{a}$ and $d$, defined by

$$\bar{a} = (ab^2)^{\frac{1}{3}}, \quad d = 2\bar{a}$$  \hspace{1cm} (6.1)

$\bar{a}$ being the effective radius, i.e. the radius of an equivolumic spherical drop. The parameter $d$ is called the effective diameter. Let the distribution function for the various drops be of the form

$$N = N(d,I),$$  \hspace{1cm} (6.2)

$I$ being the rain rate, and $N$ the number of drops with "diameter" $d$ per unit volume.

An example of such a distribution is that of Laws and Parsons. We shall suppose that the distribution of the diameter values is discrete, the only values occurring being fixed by the numbers $d = d_m$ ($m$ is a positive integer). For the drops considered we further assume the property

$$\frac{a}{b} = 1 - \bar{a},$$  \hspace{1cm} (6.3)

the effective radius being expressed in centimetres.

Relation (6.3) is similar to that used by Oguchi. The geometry of each drop can now be described by one parameter, e.g. the effective radius. We assume that all drops with a given diameter are equioriented (section 9). The orientation of a drop with diameter $d_m$ is described by the parameters $\theta_m$ and $\phi_m$ (defined in appendix A). Note that the orientation of this drop is fixed by a single parameter, e.g. the integer $m$.

According to (2.7) the vector potential of the scattered field reads

$$A^{(s)}(x) = -\frac{\imath \omega}{4\pi} \sum_m \int d^3u N(d_m,I(u)) p^{(m)}(u) e^{\imath k_o \frac{|x - u|}{|x - u|}},$$  \hspace{1cm} (6.4)
\( \mathbf{p}^{(m)}(u) \) represents the dipole moment induced in a drop with diameter \( d_m \), at the integration point \( u \). The relation between this dipole moment and the incident field is given by

\[
\mathbf{p}^{(m)}(u) = \chi(d_m, \theta_m, \phi_m) E^{(0)}(u) .
\]  

(6.5)

Since the tensor in this relation is fully determined by the integer \( m \), (6.5) is written as

\[
\mathbf{p}^{(m)}(u) = \chi^{(m)}(u) E^{(0)}(u) .
\]  

(6.6)

Substitution of (6.6) in (6.4) yields

\[
\Lambda^{(s)}(x) = - \frac{i \omega u}{4 \pi} \sum_m \int d^3 u N(d_m, I(u)) \chi^{(m)} E^{(0)}(u) \frac{ik_o |x - u|}{|x - u|} .
\]  

(6.7)

In section 2 we derived a first-order approximation for the scattered electric field, see expression (2.14). If the same conditions are satisfied here the scattered field will be given by

\[
E^{(s)}(x) = \frac{\omega^2 u_o}{4 \pi} \sum_m \chi^{(m)} E^{(0)}(o) \int d^3 u e^{ik_o u} N(d_m, I(u)) \frac{ik_o |x - u|}{|x - u|} .
\]  

(6.8)

Applying the method described in section 3 (we assume that the same conditions are satisfied here again) leads to

\[
E^{(s)}(x) = \frac{i \omega^2 u_o}{2k_o} \sum_m \chi^{(m)} E^{(0)}(o) \int d^3 u \frac{ik_o |x - u|}{|x - u|} .
\]  

(6.9)

The scattering problem described here can again be reduced to a first-order differential equation, as in section 4. This equation is given by

\[
\left[ \frac{\partial}{\partial z} - ik_o \left( \frac{\omega^2 u_o}{2k_o^2} \sum_m N(d_m, I(x)) \chi^{(m)} \right) \right] E(x) = 0 ,
\]  

(6.10)

with the boundary condition (4.6).

As a matter of fact, the vector potential of the scattered field reads as
follows if we pass from (6.7) to the corresponding expression in terms of the contributions of individual drops

$$\Delta^{(s)}(x) = -\frac{i\omega \mu_0}{4\pi} \sum_m \sum_n \chi_{m, n}(x, \theta_m, \phi_m) e^{\frac{ik_o|\mathbf{x} - \mathbf{x}_m(1)|}{|\mathbf{x} - \mathbf{x}_m(1)|}} \frac{\mathbf{E}(\mathbf{x})}{|\mathbf{x} - \mathbf{x}_m(1)|}$$  \hspace{1cm} (6.11)

The orientation of drop 1, with diameter \(d_m\), at a position \(x_1^{(1)}\), is described here by the parameters \(\theta_m^{(1)}\) and \(\phi_m^{(1)}\). In this case the orientation of a drop is not necessarily fixed only by its geometry. If there exist "averaged" angles \(\langle \theta_m \rangle\) and \(\langle \phi_m \rangle\) in such a way that (6.11) can be rewritten as

$$\Delta^{(s)}(x) = -\frac{i\omega \mu_0}{4\pi} \sum_m \sum_n \chi_{m, n}(\theta_m^{(1)}, \phi_m^{(1)}) e^{\frac{ik_o|\mathbf{x} - \mathbf{x}_m(1)|}{|\mathbf{x} - \mathbf{x}_m(1)|}} \frac{\mathbf{E}(\mathbf{x})}{|\mathbf{x} - \mathbf{x}_m(1)|},$$  \hspace{1cm} (6.12)

the assumption that all drops with diameter \(d_m\) are equioriented might be applicable according to a statistical hypothesis. However, the possibility that these averaged angles are complex should also be considered.

The introduction of complex angles leads to a generalization of the dipole model described: the relation between the induced dipole moment \(\mathbf{p}(x_1^{(1)})\) and the incident field \(\mathbf{E}(x_1^{(1)})\) is then described by a tensor which depends, among other things, on complex orientation parameters.

Complex canting angles give rise to one new parameter in the expression for the tensor \(\chi(d_m, \langle \theta_m \rangle, \langle \phi_m \rangle)\), which can be chosen arbitrarily. This will be shown in appendix D.

The possibility that the introduction of complex canting angles gives rise to a more general model for the description of the scattering problem, in the sense that the dipole model may be applicable at higher frequencies, should be considered.

7. Some remarks concerning the dipole model

In this section we shall apply another method, leading to the same expressions for the relevant quantities derived earlier.

Again we assume that in each raindrop an electric dipole is induced which,
however, should not necessarily be infinitesimal. The shape and distribution of the drops are described in section 6. The orientation of each drop should be determined by its effective radius. Besides, we suppose again that the relation between the dipole moment induced and the electric field in a drop with "diameter" $d_m$ can be described by a tensor $\chi^{(m)}$, the index $m$ referring to the diameter $(d_m)$ of the drop. The elements of this tensor are independent of the spatial coordinates, see sections 2 and 6.

Then the rain medium can be described by a relative permittivity tensor $\varepsilon_r$, given by

$$\varepsilon_r(x) = \frac{\varepsilon_0}{\varepsilon - 1} \sum_m N^{(m)}(x) \chi^{(m)},$$

(7.1)

$N^{(m)}$ being the density of the drops with diameter $d_m$. Relation (7.1) is derived in appendix C. We assume the relative permeability to be unity.

With the aid of Maxwell's equations we obtain the wave equation

$$\nabla^2 E - \nabla(\nabla \cdot E) + k_0^2 \varepsilon_r E = 0.$$

(7.2)

If the rain medium is absent, i.e. $\varepsilon_r$ equals the unity tensor, the primary wave (2.5) is a solution of equation (7.2). The complete solution of this equation is determined by the integral equation

$$E(x) = E^{(o)}(x) + \frac{k_0^2}{4\pi} \int d^3 u \frac{e^{ik_0|x - u|}}{|x - u|} \left\{ \varepsilon_r(u) - \frac{\varepsilon_0}{\varepsilon - 1} k_0^{-2} \nabla \nabla \right\} E(u).$$

(7.3)

Equation (7.3) can be solved by the Neumann-Liouville expansion

$$E(x) = \sum_{k=0}^{\infty} E^{(k)}(x),$$

(7.4)

where the terms $E^{(k)}(x), k \geq 1$, can be determined with the aid of the recurrence relation

$$E^{(k)}(x) = \frac{k^2}{4\pi} \int d^3 u \frac{e^{ik_0|x - u|}}{|x - u|} \left\{ \varepsilon_r(u) - \frac{\varepsilon_0}{\varepsilon - 1} k_0^{-2} \nabla \nabla \right\} E^{(k-1)}(u).$$

(7.5)

We assume the rain medium to be contained in the half space $z > 0$. The primary wave $E^{(o)}(x)$ propagates along the z axis and enters at $z = 0$ into the scattering space. If the penetration depth $z$ of the primary wave is not too large, so that multiple-scattering effects can be neglected, the first two terms of the Neumann-Liouville expansion yield a fair approximation. The solution of (7.2),
the so-called Born approximation, is then given by

\[ E(x) = E^{(0)}(x) + E^{(1)}(x) \]

\[ = E^{(0)}(x) + \frac{k^2}{4\pi} \int d^3 u \frac{e^{ik_o|x-u|}}{|x-u|} \{ \varepsilon_x(u) - \frac{1}{\varepsilon_o} - k_o^{-2} \nabla \cdot \nabla \} E^{(0)}(u). \]  

(7.6)

Since the primary wave is transverse and independent of the coordinates \( x \) and \( y \), relation (7.6) can be rewritten as

\[ E(x) = E^{(0)}(x) + \frac{k^2}{4\pi} \int d^3 u \frac{e^{ik_o|x-u|}}{|x-u|} \{ \varepsilon_x(u) - \frac{1}{\varepsilon_o} \} E^{(0)}(u) \]

\[ = E^{(0)}(x) + \frac{\omega^2 \mu_o}{4\pi} \sum_m \chi^{(m)} E^{(0)}(x) \int d^3 u \frac{e^{ik_o|x-u|}}{|x-u|} \frac{ik_o|x-u|}{|x-u|}. \]

(7.7)

remembering (7.1).

The field \( E^{(1)}(x) \), i.e. the second right-hand term of (7.7), can be regarded as the scattered field \( E^{(s)}(x) \). If we assume infinitesimal induced dipoles and a transverse field \( E^{(s)}(x) \), and comparing the second right-hand term of (7.7) with (6.8), we find that the method described presently leads to the same result as previously in section 6. However, we should bear in mind the conditions described in section 2. It is evident that expression (7.7) is a more general one than (6.8), since the dipoles regarded here are not necessarily infinitesimal.
8. Some numerical values for the complex forward-scattering amplitudes

Morrison and Cross\(^{(4)}\) considered the problem regarding the scattering and absorption of a linearly polarized plane electromagnetic wave on an oblate spheroidal raindrop. They obtained nonperturbative solutions by expanding the scattered and transmitted fields in terms of spherical vector wave functions, so that Maxwell's equations are satisfied exactly in the regions exterior and interior to the raindrop, and by combining point matching with least-squares fitting to satisfy the boundary conditions on the surface of the raindrop with sufficient accuracy.

In their model the ratio of minor to major semiaxis depends linearly on the effective radius\(^{(5,6)}\). They present numerical results at different frequencies. At each of these frequencies different orientations of the drop and different values of the eccentricity are considered. In a more recent paper by Oguchi\(^{(7)}\) it is shown that the scattering properties of the Pruppacher and Pitter form raindrops\(^{(8)}\), i.e. axisymmetric oblate spheroids with a flattened base when their size exceeds about 1 mm in radius, do not differ much from those obtained earlier for oblate spheroidal raindrops\(^{(4)}\).

The numerical results obtained by Morrison and Cross will be considered as starting-points for testing the dipole model.

The electric field at the position \((o,o,z), z > 0\), generated by an infinitesimal dipole with dipole moment \(p\), induced in a raindrop situated at the origin of a Cartesian coordinate system \(xyz\), is given by

\[
E_x(o,o,z) = \frac{\omega \mu_o}{4\pi} p_x \frac{ik_z}{z} e^0, \tag{8.1a}
\]

\[
E_y(o,o,z) = \frac{\omega \mu_o}{4\pi} p_y \frac{ik_z}{z} e^0. \tag{8.1b}
\]

We assume that this dipole is induced by the incident field \((2.5)\). This linearly polarized field is parallel to the \(x\) or \(y\) axis of the coordinate system. Besides, we suppose that the \(z\) component of the field, generated by the dipole, can be neglected: \(E_z(o,o,z) = 0\). At \(\theta = \frac{\pi}{2}\) and \(\phi = \frac{\pi}{2}\), i.e. if the axis of symmetry of the raindrop is parallel to the \(y\) axis, we obtain

\[
p_x = \chi_1 E_x(o), \quad p_y = \chi_2 E_y(o), \tag{8.2}
\]
see (A.4) and (A.5), appendix A.
Substitution of (8.2) in (8.1) yields
\[
E_x(o,o,z) = \frac{\omega \mu_0}{4\pi} \chi_1 E_x(o) \frac{e^{ikz}}{z},
\]
\[
E_y(o,o,z) = \frac{\omega \mu_0}{4\pi} \chi_2 E_y(o) \frac{e^{ikz}}{z}.
\]
Let us introduce the functions \(Q_1\) and \(Q_2\), the so-called forward-scattering amplitudes, according to the definitions
\[
E_x(o,o,z) = Q_1(\theta,\phi) E_x(o) \frac{i e^{ikz}}{k_o z},
\]
provided the incident field is parallel to the x axis, and
\[
E_y(o,o,z) = Q_2(\theta,\phi) E_y(o) \frac{i e^{ikz}}{k_o z},
\]
provided the incident field is parallel to the y axis.
If we compare (8.4) with (8.3), the raindrop considered being parallel to the y axis, i.e. \(\theta = \frac{\pi}{2}\) and \(\phi = \frac{\pi}{2}\), we obtain
\[
\chi_1 = \frac{4\pi i}{\omega \mu_0 k_o} Q_1(\frac{\pi}{2},\frac{\pi}{2}),
\]
\[
\chi_2 = \frac{4\pi i}{\omega \mu_0 k_o} Q_2(\frac{\pi}{2},\frac{\pi}{2}).
\]
If the incident wave is parallel to the x axis, we obtain
\[
p_x = \{\chi_1 + (\chi_2 - \chi_1) \sin^2 \theta \cos^2 \phi\} E_x(o) (o),
\]
see (A.4) and (A.5), appendix A.
Substitution of (8.6) in (8.1a) yields
\[
E_x(o,o,z) = \frac{\omega \mu_0}{4\pi} \{\chi_1 + (\chi_2 - \chi_1) \sin^2 \theta \cos^2 \phi\} E_x(o) (o) \frac{e^{ikz}}{z}
\]
From expressions (8.4a) and (8.8) one obtains

\[ Q_1(\theta, \phi) = Q_1(\pi, \pi) + (Q_2(\pi, \pi) - Q_1(\pi, \pi)) \sin^2 \theta \cos^2 \phi \]  

(8.9)

In the same way we can derive

\[ Q_2(\theta, \phi) = Q_1(\pi, \pi) + (Q_2(\pi, \pi) - Q_1(\pi, \pi)) \sin^2 \theta \sin^2 \phi \]  

(8.10)

Morrison and Cross obtained their results for \( \phi = \frac{\pi}{2} \). We introduce the notations: \( Q_1(\theta, \frac{\pi}{2}) = H_1(\theta) \), \( Q_2(\theta, \frac{\pi}{2}) = H_2(\theta) \). For \( \phi = \frac{\pi}{2} \) relations (8.9) and (8.10) then reduce to

\[ H_1(\theta) = H_1(\pi) \]  

(8.11)

\[ H_2(\theta) = H_1(\pi) \cos^2 \theta + H_2(\pi) \sin^2 \theta \]  

(8.12)

Starting from the dipole model we have formulated the expressions for the forward-scattering amplitudes in such a way that a direct comparison of this model with that of Morrison and Cross (4) is possible. The parameters describing the geometry of a given raindrop are defined in section 6.

In the tables 1 and 2 some of their numerical results are presented. In table 1, \( H_1(\pi) \) and \( H_2(\pi) \) are calculated at 30 GHz, for different drop sizes. In table 2, \( H_1 \) and \( H_2 \) are calculated for \( \theta = 70^\circ \). It should be noted that these tables are not complete.

In table 3, \( H_1 \) and \( H_2 \) are calculated with the aid of table 1 and relations (8.11) and (8.12).
<table>
<thead>
<tr>
<th>$-a$ (cm)</th>
<th>$H_1^{(\parallel)}$</th>
<th>$H_2^{(\parallel)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>$3.6235 \times 10^{-4}$ - $3.8595 \times 10^{-3} \text{i}$</td>
<td>$3.4513 \times 10^{-4}$ - $3.7481 \times 10^{-3} \text{i}$</td>
</tr>
<tr>
<td>0.100</td>
<td>$1.6165 \times 10^{-1}$ - $2.1613 \times 10^{-1} \text{i}$</td>
<td>$1.3415 \times 10^{-1}$ - $1.8677 \times 10^{-1} \text{i}$</td>
</tr>
<tr>
<td>0.175</td>
<td>$9.6534 \times 10^{-1}$ - $2.7438 \times 10^{-1} \text{i}$</td>
<td>$7.3731 \times 10^{-1}$ - $3.4278 \times 10^{-1} \text{i}$</td>
</tr>
<tr>
<td>0.250</td>
<td>$1.8221$ - $2.627 \times 10^{-1} \text{i}$</td>
<td>$1.3309$ - $4.693 \times 10^{-1} \text{i}$</td>
</tr>
<tr>
<td>0.325</td>
<td>$3.06$ - $2.4 \times 10^{-2} \text{i}$</td>
<td>$2.20$ - $6.70 \times 10^{-1} \text{i}$</td>
</tr>
</tbody>
</table>

Table 1. Forward-scattering amplitudes at 30 GHz for different drop sizes.

<table>
<thead>
<tr>
<th>$-a$ (cm)</th>
<th>$H_1^{(\parallel)}$</th>
<th>$H_2^{(\parallel)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>$3.6200 \times 10^{-4}$ - $3.8594 \times 10^{-3} \text{i}$</td>
<td>$3.4679 \times 10^{-4}$ - $3.7610 \times 10^{-3} \text{i}$</td>
</tr>
<tr>
<td>0.100</td>
<td>$1.6133 \times 10^{-1}$ - $2.1727 \times 10^{-1} \text{i}$</td>
<td>$1.3701 \times 10^{-1}$ - $1.9313 \times 10^{-1} \text{i}$</td>
</tr>
<tr>
<td>0.175</td>
<td>$9.7408 \times 10^{-1}$ - $2.8097 \times 10^{-1} \text{i}$</td>
<td>$7.7133 \times 10^{-1}$ - $3.4313 \times 10^{-1} \text{i}$</td>
</tr>
<tr>
<td>0.250</td>
<td>$1.8412$ - $2.678 \times 10^{-1} \text{i}$</td>
<td>$1.4123$ - $4.621 \times 10^{-1} \text{i}$</td>
</tr>
<tr>
<td>0.325</td>
<td>$3.11$ - $4.8 \times 10^{-2} \text{i}$</td>
<td>$2.38$ - $6.46 \times 10^{-1} \text{i}$</td>
</tr>
</tbody>
</table>

Table 2. Forward-scattering amplitudes at 30 GHz for different drop sizes.

If we compare the tables 2 and 3, it will be clear that the dipole model agrees better with the model of Morrison and Cross, at least for $\theta = 70^\circ$, according as the characteristic dimensions of the drops are smaller with respect to the wave length of the incident field. The same can be concluded for $\theta = 50^\circ$, see tables 4 and 5.
Table 4. Forward-scattering amplitudes at 30 GHz for different drop sizes (4).

<table>
<thead>
<tr>
<th>a (cm)</th>
<th>$H_1(\theta=50^\circ)$</th>
<th>$H_2(\theta=50^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>$3.6111 \times 10^{-4} - 3.8590 \times 10^{-3} \times i$</td>
<td>$3.5101 \times 10^{-4} - 3.7936 \times 10^{-3} \times i$</td>
</tr>
<tr>
<td>0.100</td>
<td>$1.6054 \times 10^{-1} - 2.2016 \times 10^{-1} \times i$</td>
<td>$1.4431 \times 10^{-1} - 2.0284 \times 10^{-1} \times i$</td>
</tr>
<tr>
<td>0.175</td>
<td>$9.9679 \times 10^{-1} - 2.9777 \times 10^{-1} \times i$</td>
<td>$8.5967 \times 10^{-1} - 3.4211 \times 10^{-1} \times i$</td>
</tr>
<tr>
<td>0.250</td>
<td>$1.8902 - 2.752 \times 10^{-1} \times i$</td>
<td>$1.6145 - 4.252 \times 10^{-1} \times i$</td>
</tr>
<tr>
<td>0.325</td>
<td>$3.24 - 1.03 \times 10^{-1} \times i$</td>
<td>$2.80 - 5.45 \times 10^{-1} \times i$</td>
</tr>
</tbody>
</table>

Table 5. Forward-scattering amplitudes at 30 GHz for different drop sizes. Calculated with the aid of table 1 and relations (8.11) and (8.12).
9. Numerical calculations

In this section the damping and depolarization of a linearly polarized electromagnetic wave, propagating through a rain medium, are calculated at different frequencies and for different values of the penetration depth. The results are compared with experimental data\(^{(10)}\).

As we assume the rain rate to be independent of the propagation direction of the incident field, the differential equation (6.9) is a suitable starting-point for the calculation of the quantities in question. The solution of this equation is given in appendix B.

Morrison and Cross\(^{(4)}\) give numerical values for the complex forward-scattering amplitudes of a plane electromagnetic wave incident on an oblate spheroidal raindrop, see section 8. Assuming that the dipole model is an adequate model for the scattering problem described, these amplitudes can be related to the elements of the tensor \(\chi\), see the sections 2 and 8 and appendix A.

In this section real canting angles are understood (in section 6 complex canting angles are introduced). We used the diameter distribution function of Laws and Parsons\(^{(9)}\), according to which the diameter values \(d\) occurring are discrete: \(d = n \times 0.5\) mm, \(n\) being a positive integer. As the drops considered are not spherical, we assume the "diameter" \(d\) of each drop to be equivalent to the effective diameter: \(d = 2a\), \(a\) being the effective radius, see section 6.

A uniform vertical wind velocity field parallel to the \(y\) axis of a Cartesian coordinate system \(xyz\) is understood, thus the other axes are horizontal.

Since the propagation direction is along the \(z\) axis, the damping and depolarization parameters are calculated for a horizontal path. The scattering medium is contained in the half space \(z > 0\). The axes of symmetry of the raindrops are in the plane normal to the propagation direction (\(\theta = \frac{\pi}{2}\)).

We suppose that for very large raindrops the gravitation force is dominant. Then the axes of symmetry of these drops are (almost) parallel to the \(y\) axis, i.e. the direction of the wind velocity field, yielding \(\phi = \frac{\pi}{2}\) for this category. For \(\phi = \frac{\pi}{2}\) the raindrops do not contribute to depolarization, see section 5. Besides, the density of the large drops is relatively small with respect to that of the small ones\(^{(9)}\). Therefore we can conclude that the contribution of these drops to depolarization is almost negligible.

In accordance with the model of Oguchi\(^{(5,6)}\) the very small drops are almost
spherical, see also section 6. Therefore it seems reasonable to suppose that the contribution of these drops to depolarization is (almost) negligible. We shall put $\phi = 0$ for this category (for $\phi = 0$ the raindrops do not contribute to depolarization, see section 5).

Finally, we shall assume that the orientation of each drop, i.e. the parameter $\phi$, depends only on its geometry. The geometry of each drop is fixed by its effective diameter $d$, see section 6.

We introduce the distribution

$$
\phi = \begin{cases} 
\frac{\pi}{2} e^{-\frac{(d - d_0)^2}{\sigma^2}}, & d \leq d_0 \\
\frac{\pi}{2}, & d > d_0,
\end{cases}
$$

(9.1)

see figure 1 (the tables and figures we refer to are at the end of this section).

The raindrops with a diameter larger than $d_0$ do not contribute to depolarization. We take $d_0 = 6$ mm and assume that the parameter $\sigma$ can be chosen such that the calculated relevant quantities agree with experimental data as close as possible. A value of 0.66 mm for this parameter turns out to satisfy.

A linearly polarized primary electric field along the x axis is understood. The cross-polarization discrimination and the damping are defined, respectively by

$$\text{XPDH} = -20 \times 10 \log(|F_{xy}/F_{xx}|),$$

(9.2)

$$D_h = -10 \times 10 \log(F_{xx} F_{xx}^* + F_{yx} F_{yx}^*),$$

(9.3)

and are expressed in decibels (dB). The quantities $F_{xx}$, $F_{xy}$ and $F_{yx}$ are calculated in appendix B.

The cross-polarization discrimination parameter XPDH is highly dependent on the parameter $\sigma$, i.e. on the distribution of the canting angles of the different raindrops. With increasing frequency this quantity dependence becomes more spectacular, see table 6. The damping of the horizontal polarization ($D_h$) is less dependent on the parameter $\sigma$. With increasing frequency this quantity proves to be more dependent on this parameter too (table 7).
In a paper by Hogg and Chu\textsuperscript{(10)} semi-empirical estimates are given for the relation between the cross-polarization discrimination and the damping of the horizontal polarization at different frequencies and for a rain rate of 100 mm/h.

Computation of the quantities $X_{PDH}$ and $D_h$, for $\sigma = 0.66$ mm, leads to a fairly good agreement with these estimates, at least at frequencies up to about 11 GHz, see figure 2. At higher frequencies the agreement is less good (see fig. 3), at least at greater penetration depths.

The calculated quantities are realistic if the characteristic dimensions of the raindrops are small enough with respect to the wavelength of the incident field.

However, since the density of large drops is small with respect to that of the small ones, as shown by the distribution of Laws and Parsons, the dipole model can be used in this case at relatively high frequencies.

Table 8 shows the contribution of the larger raindrops to damping and depolarization.

In table 9 data are given for some relevant quantities for different values of the penetration depth. These quantities are computed at 11 GHz and for $\sigma = 0.66$ mm, the rain rate being 100 mm/h. In table 10 the same quantities are computed at 19.1 GHz, for the same value of $\sigma$ and the same rain rate.

\[\text{diameter } d\]

**Fig. 1.** Canting angle $\phi$ as a function of the diameter $d$, at fixes $d_o$. 
Fig. 2. Cross-polarization discrimination as a function of the damping of the horizontal polarization at 11 GHz; \( \sigma = 0.66 \text{ mm} \); — semi-empirical estimate \(^{(10)}\); —— calculated curve.

Fig. 3. Cross-polarization discrimination as a function of the damping of the horizontal polarization at 18.1 GHz; \( \sigma = 0.66 \text{ mm} \); — semi-empirical estimate \(^{(10)}\); —— calculated curve.
Table 6. The cross-polarization discrimination at different values of the parameter $\sigma$; penetration depth 500 m, rain-intensity 100 mm/h.

<table>
<thead>
<tr>
<th>$\sigma$ (mm)</th>
<th>XPFH (dB)</th>
<th>$\sigma$ (mm)</th>
<th>XPFH (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>48.52</td>
<td>0.50</td>
<td>47.00</td>
</tr>
<tr>
<td>0.75</td>
<td>40.89</td>
<td>0.75</td>
<td>37.40</td>
</tr>
<tr>
<td>1.00</td>
<td>35.26</td>
<td>1.00</td>
<td>30.58</td>
</tr>
<tr>
<td>1.25</td>
<td>31.35</td>
<td>1.25</td>
<td>26.19</td>
</tr>
</tbody>
</table>

Table 7. Damping of the horizontal polarization at different values of the parameter $\sigma$; penetration depth 500 m, rain-intensity 100 mm/h.

<table>
<thead>
<tr>
<th>$\sigma$ (mm)</th>
<th>$D_h$ (dB)</th>
<th>$\sigma$ (mm)</th>
<th>$D_h$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>1.817</td>
<td>0.50</td>
<td>3.872</td>
</tr>
<tr>
<td>0.75</td>
<td>1.838</td>
<td>0.75</td>
<td>3.921</td>
</tr>
<tr>
<td>1.00</td>
<td>1.864</td>
<td>1.00</td>
<td>4.007</td>
</tr>
<tr>
<td>1.25</td>
<td>1.896</td>
<td>1.25</td>
<td>4.126</td>
</tr>
</tbody>
</table>
The quantities $D_h$ and $XPDH$ at different values of the parameter $\zeta$; $\zeta$ indicates that the drops with diameter greater than $\zeta$ are assumed to be not present in the rain medium. The quantities are calculated at $11$ GHz; $z = 100$ m, $\sigma = 0.66$ mm, $I = 100$ mm/h.

<table>
<thead>
<tr>
<th>$\zeta$ (mm)</th>
<th>$D_h$ (dB)</th>
<th>$XPDH$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.25</td>
<td>0.369</td>
<td>57.95</td>
</tr>
<tr>
<td>5</td>
<td>0.343</td>
<td>61.51</td>
</tr>
<tr>
<td>4</td>
<td>0.308</td>
<td>77.58</td>
</tr>
<tr>
<td>3</td>
<td>0.183</td>
<td>103.92</td>
</tr>
</tbody>
</table>

Table 8.
<table>
<thead>
<tr>
<th>$\text{Re}(E_x)$</th>
<th>$\text{Im}(E_x)$</th>
<th>$\text{Re}(E_y)$</th>
<th>$\text{Im}(E_y)$</th>
<th>$D_h$ (dB)</th>
<th>XPDH (dB)</th>
<th>$z$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.214</td>
<td>0.0618</td>
<td>0.00813</td>
<td>0.00296</td>
<td>3.69</td>
<td>37.57</td>
<td>1000</td>
</tr>
<tr>
<td>-0.336</td>
<td>0.264</td>
<td>-0.00119</td>
<td>0.0108</td>
<td>7.37</td>
<td>31.89</td>
<td>2000</td>
</tr>
<tr>
<td>-0.235</td>
<td>-0.151</td>
<td>-0.0101</td>
<td>0.00135</td>
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<td>28.74</td>
<td>3000</td>
</tr>
<tr>
<td>0.0430</td>
<td>-0.178</td>
<td>-0.00314</td>
<td>-0.00793</td>
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<td>26.63</td>
<td>4000</td>
</tr>
<tr>
<td>0.119</td>
<td>-0.0116</td>
<td>0.00540</td>
<td>-0.00388</td>
<td>18.43</td>
<td>25.10</td>
<td>5000</td>
</tr>
<tr>
<td>0.0327</td>
<td>0.0711</td>
<td>0.00380</td>
<td>0.00319</td>
<td>22.11</td>
<td>23.96</td>
<td>6000</td>
</tr>
<tr>
<td>-0.0370</td>
<td>0.0354</td>
<td>-0.00155</td>
<td>0.00324</td>
<td>25.79</td>
<td>23.09</td>
<td>7000</td>
</tr>
<tr>
<td>-0.0298</td>
<td>-0.0152</td>
<td>-0.00249</td>
<td>-0.000482</td>
<td>29.47</td>
<td>22.42</td>
<td>8000</td>
</tr>
<tr>
<td>0.00302</td>
<td>-0.0217</td>
<td>-0.000117</td>
<td>-0.00176</td>
<td>33.16</td>
<td>21.91</td>
<td>9000</td>
</tr>
<tr>
<td>0.0141</td>
<td>-0.00279</td>
<td>0.00114</td>
<td>-0.000386</td>
<td>36.84</td>
<td>21.53</td>
<td>10000</td>
</tr>
</tbody>
</table>

Table 9.

The electric field, $D_h$ and XPDH at different values of the penetration depth $z$, calculated at 11 GHz for a rain rate of 100 mm/h and for $\sigma = 0.66$ mm. The quantities $E_x$ and $E_y$ are normalized to the incident field.
The electric field, $D_h$ and XPDH at different values of the penetration depth $z$, calculated at 18.1 GHz for a rain rate of 100 mm/h and for $\sigma = 0.66$ mm. The quantities $E_x$ and $E_y$ are normalized to the incident field.

<table>
<thead>
<tr>
<th>$Re(E_x)$</th>
<th>$Im(E_x)$</th>
<th>$Re(E_y)$</th>
<th>$Im(E_y)$</th>
<th>$D_h$ (dB)</th>
<th>XPDH (dB)</th>
<th>$z$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0514</td>
<td>0.402</td>
<td>-0.000935</td>
<td>0.00707</td>
<td>7.84</td>
<td>35.10</td>
<td>1000</td>
</tr>
<tr>
<td>-0.159</td>
<td>-0.0413</td>
<td>-0.00473</td>
<td>-0.00194</td>
<td>15.68</td>
<td>30.14</td>
<td>2000</td>
</tr>
<tr>
<td>0.0248</td>
<td>-0.0619</td>
<td>0.00163</td>
<td>-0.00221</td>
<td>23.51</td>
<td>27.71</td>
<td>3000</td>
</tr>
<tr>
<td>0.0236</td>
<td>0.0132</td>
<td>0.000844</td>
<td>0.000999</td>
<td>31.35</td>
<td>26.32</td>
<td>4000</td>
</tr>
<tr>
<td>-0.00652</td>
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<td>-0.000517</td>
<td>0.000269</td>
<td>39.18</td>
<td>25.49</td>
<td>5000</td>
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<tr>
<td>-0.00322</td>
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<td>-0.000241</td>
<td>47.01</td>
<td>25.03</td>
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</tr>
<tr>
<td>0.00141</td>
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<td>0.000104</td>
<td>-0.0000768</td>
<td>54.85</td>
<td>24.80</td>
<td>7000</td>
</tr>
</tbody>
</table>

Table 10.
10. Conclusions

If we restrict ourselves to a weakly inhomogeneous rain medium (sections 2 and 3), the dipole model, introduced in section 2, proves to be suitable for the description of the scattering and absorption of a plane linearly polarized monochromatic electromagnetic wave propagating through this medium, at frequencies up to about 20 GHz. In this case only real canting angles are considered.

Since the density of the large drops is small with respect to that of the small ones, as shown by the distribution of Laws and Parsons, the dipole model can be used at these (relatively) high frequencies.

It is possible that the introduction of complex canting angles (section 6, appendix D) gives rise to a more general scattering model.
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Appendix A

The tensor $\chi$

The relation between the dipole moment induced in a raindrop and the incident field should be a linear one

$$P = \chi \cdot E$$  \hspace{1cm} (A.1)

We assume axisymmetric raindrops with their axes of symmetry parallel to the $y'$ axis of some Cartesian $x'y'z'$ system. The elements of the tensor $\chi$ are dependent on the frequency and on the geometry and orientation of the drop considered. In figure 1 the geometry of a special raindrop is shown.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The geometry of an oblate spheroidal raindrop. The $y'$ axis is the rotation axis of symmetry.}
\end{figure}

With respect to the above $x'y'z'$ system the relation between the induced dipole moment $p = p' e'_x + p' e'_y + p' e'_z$ and the incident field $E = E'_e e'_x + E'_e e'_y + E'_e e'_z$ reads

$$
\begin{pmatrix}
    p_x' \\
    p_y' \\
    p_z'
\end{pmatrix} =
\begin{pmatrix}
    x_1 & 0 & 0 \\
    0 & x_2 & 0 \\
    0 & 0 & x_1
\end{pmatrix}
\begin{pmatrix}
    E_x' \\
    E_y' \\
    E_z'
\end{pmatrix}
$$  \hspace{1cm} (A.2)
The orientation of the symmetry axis of this raindrop can be described by the angles $\theta$ and $\phi$ in another orthogonal system, the $xyz$ system, say, the $z$ axis of which lies in the $y'z'$ plane, see figure 2.

![Diagram of raindrop orientation](image)

**Fig. 2.** Description of the orientation of a raindrop in a Cartesian $xyz$ system by the parameters $\theta$ and $\phi$; $e'_x = e'_y \times e'_z$.

The relation between the coordinates in the different systems is given by

$$
\begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} =
\begin{pmatrix}
  -\sin\phi & \cos\phi & 0 \\
  \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\
  \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix}
$$

(A.3)

We introduce the relation (see (3.1))

$$
\begin{pmatrix}
  p_x \\
  p_y
\end{pmatrix} =
\begin{pmatrix}
  \chi_{xx} & \chi_{xy} \\
  \chi_{yx} & \chi_{yy}
\end{pmatrix}
\begin{pmatrix}
  E_x \\
  E_y
\end{pmatrix}
$$

(A.4)

Then the following expressions can be derived

$$
\begin{align*}
\chi_{xx} &= \chi_1 + (\chi_2 - \chi_1)\sin^2\theta\cos^2\phi \\
\chi_{xy} &= \chi_{yx} = (\chi_2 - \chi_1)\sin^2\theta\sin\phi\cos\phi \\
\chi_{yy} &= \chi_1 + (\chi_2 - \chi_1)\sin^2\theta\sin^2\phi
\end{align*}
$$

(A.5)
Appendix B

Solution of equation (4.8)

With the aid of the transformation

\[ E(x) = H(x)e^{\frac{ikz}{\varepsilon}} \]  \hspace{1cm} (B.1)

and the tensor

\[ \mathbf{\Sigma} = -\frac{i\omega\mu}{2\varepsilon_0} N \mathbf{\Sigma} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \]  \hspace{1cm} (B.2)

equation (4.8) changes into

\[ \frac{\partial}{\partial z} + N \mathbf{H} = 0. \]  \hspace{1cm} (B.3)

We assume that the density of the rain medium is independent of the coordinate \( z \). In that case the evolution matrix \( \mathbf{F} \) is symmetric, see sections 4 and 5.

The eigenvalues of \( \mathbf{\Sigma} \) are given by

\[ \lambda_1 = \frac{1}{2}(m_{11} + m_{22} + \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}), \]
\[ \lambda_2 = \frac{1}{2}(m_{11} + m_{22} - \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}). \]  \hspace{1cm} (B.4)

The corresponding eigenvectors read

\[ \mathbf{v}_1 = -m_{12}(m_{11} - \lambda_1)^{-1} e_x + e_y, \]  \hspace{1cm} (B.5)
\[ \mathbf{v}_2 = -m_{12}(m_{11} - \lambda_2)^{-1} e_x + e_y. \]

If \( m_{12} = 0 \), equations (4.8) decouple. However, this situation is not interesting from our point of view.

If we put \( \mathbf{H} = H_1\mathbf{v}_1 + H_2\mathbf{v}_2 \), the solution of (4.8) becomes

\[ E = (H_1(\mathbf{v}_1 + \mathbf{v}_2) e^{\frac{-\lambda_1z}{\varepsilon}} + H_2(\mathbf{v}_1 + \mathbf{v}_2) e^{\frac{-\lambda_2z}{\varepsilon}}) e^{\frac{ikz}{\varepsilon}}. \]  \hspace{1cm} (B.6)

Besides, the boundary condition (4.6) has to be satisfied.
Then the elements of the evolution matrix $\mathbf{F}$ read

$$F_{xx} = \frac{1}{\lambda_1 - \lambda_2} \left[ (m_{11} - \lambda_2) e^{-\lambda_1 z} + (\lambda_1 - m_{11}) e^{-\lambda_2 z} \right] i k_o z \quad (B.7)$$

$$F_{xy} = F_{yx} = \frac{m_{12}}{\lambda_1 - \lambda_2} \left( e^{-\lambda_1 z} - e^{-\lambda_2 z} \right) e^{ik_o z} \quad (B.8)$$

$$F_{yy} = \frac{1}{\lambda_1 - \lambda_2} \left[ (\lambda_1 - m_{11}) e^{-\lambda_1 z} + (m_{11} - \lambda_2) e^{-\lambda_2 z} \right] i k_o z \quad (B.9)$$

If we consider the situation with different raindrops, the tensor (B.2) should be rewritten as follows

$$M = -\frac{i \omega}{2 k_o} \sum_m N(d_m, I) \chi^{(m)} \quad (B.10)$$

see section 6.
Appendix C

Derivation of relation (7.1)

The relation between the dipole moment induced in a drop with diameter \( d_k \), at position \( x_k^{(m)} \), and the incident electric field is given by (see section 6)

\[
P(x_k^{(m)}) = \chi^{(k)}(x_k^{(m)}) E(x_k^{(m)})
\]  

(C.1)

Then the polarization \( P \) reads

\[
P(x) = \sum_k N^{(k)}(x) \chi^{(k)} E(x)
\]  

(C.2)

Combination of relation (C.2) with

\[
\frac{D}{\epsilon} = \epsilon_0 E + P
\]

(C.3)

\[
= \epsilon_0 \frac{E}{\epsilon}
\]

enables a derivation of (7.1).
Appendix D

The introduction of complex canting angles

In appendix A the elements of the tensor $\chi$, describing the (linear) relation between the incident field $E$ and the induced dipole moment $\mu$, have been expressed in terms of the real orientation parameters (canting angles) $\theta$ and $\phi$. In section 6 we considered the possibility of complex characteristic angles (i.e. complex canting angles) $<\theta_m>$ and $<\phi_m>$. We shall now simply denote these quantities by $\theta_m$ and $\phi_m$. The elements of the tensor $\chi^{(m)}$ then read

\[ \chi_{xx}^{(m)} = \chi_1^{(m)} + (\chi_2^{(m)} - \chi_1^{(m)}) \sin^2 \theta_m \cos^2 \phi_m, \]
\[ \chi_{xy}^{(m)} = \chi_{yx}^{(m)} = \frac{1}{2}(\chi_2^{(m)} - \chi_1^{(m)}) \sin 2\theta_m \sin (2\phi_m), \]
\[ \chi_{yy}^{(m)} = \chi_1^{(m)} + (\chi_2^{(m)} - \chi_1^{(m)}) \sin^2 \theta_m \sin 2\phi_m, \]

with

\[ \theta_m = \theta_m^{(1)} + i\theta_m^{(2)}, \]
\[ \phi_m = \phi_m^{(1)} + i\phi_m^{(2)} \]

We shall interpret the real parts of $\theta_m$ and $\phi_m$ ($\theta_m^{(1)}$ and $\phi_m^{(1)}$ respectively) as the orientation parameters of the raindrop considered (figure 3). The axes of symmetry of the raindrops are assumed in the plane normal to the propagation direction (see section 9)

\[ \theta_m^{(1)} = \frac{\pi}{2}, \]

Then the condition that $\chi_{xy}^{(m)} = \chi_{yx}^{(m)} = 0$ for $\phi_m^{(1)} = n\pi/2$, $n$ being an integer, has to be satisfied. This implies the condition

\[ \phi_m^{(2)} = 0 \]

Substitution of (D.3) and (D.4) in (D.1) yields

* See section 5
Fig. 3. Description of the orientation of an axisymmetric raindrop in an orthogonal coordinate system xyz by the parameters $\theta_m^{(1)}$ and $\phi_m^{(1)}$. The z, y' and z' axes are in one plane; the z' axis is perpendicular to the y' axis; $e'_x = e'_y \times e'_z$. The index m refers to the effective radius ($a_m$) of the raindrop considered. The y' axis is the rotation axis of symmetry of this drop.

The introduction of complex canting angles gives rise to one new parameter $(\theta_m^{(2)})$ in the expression for the tensor $\chi_{1}^{(m)}$, which can be chosen arbitrarily.
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