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Microwave Noise Measurements on Double Barrier Resonant Tunneling Diodes

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SUMMARY

Double barrier resonant tunneling (DBRT) devices are investigated in the Electronic Devices Group at the Eindhoven University of Technology. These DBRT diodes have a nonlinear current voltage characteristic with negative differential resistance (NDR) regions. Biased in one of these NDR regions the DBRT diode can be used for microwave amplification purposes so knowledge of the diode’s noise behaviour is important, also from a physics point of view.

Two noise parameter measurement methods have been developed in which the DBRT diode is used in a reflection amplifier configuration with circulator to transform the active one-port device into an active two-port with separate input and output ports.

The noise figure (NF) of the DBRT diode must be de-embedded from the NF of the reflection amplifier. An equation for the NF of the DBRT diode is derived. Two different measurement methods are used. A (complicated) more exact method uses the measured S-parameters of the actual circulator and accounts for reflections at the noise source, NF meter and DBRT diode. A mathematically simple method (three versions) uses only scalar data collected by the NF meter. The results from these two methods are compared and they coincide well.

The noise parameters of the DBRT diode show a distinct dependence on the bias voltage and an almost frequency independence in the range of 1.0 - 1.5 GHz. In the NDR region of a 20 μm mesa diameter DBRT diode the lowest noise measure observed was 5.5 (7.4 dB).

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two intersecting lines (not-connected)

two connected lines

noise generator

\( a_x \)
lattice constant of substance \( x \)

\( E_n \)
total energy of a particle in a quantum state

\( w \)
width of the quantum well

\( n \)
quantum state specifier

\( h \)
Planck constant

\( m_e \)
electron mass

\( F, NF \)
noise figure

\( G_a \)
available gain

\( L \)
attenuation

\( \Gamma_i \)
reflection coefficient of port \( i \)

\( \phi \)
determinant (Mason's gain rule)

\( S_{ij} \)
S-matrix element

\( L_i \)
loop \( i \) (Mason's gain rule)

\( \Delta \)
determinant (2x2 S-matrix)

\( S_{ij}^m \)
measured S-parameter (network analyser)

\( S_{ij}^\Delta \)
element of 3x3 S-matrix (arbitrary loads)

\( P_G \)
available power of generator

\( b_G \)
generator wave amplitude
$Z_{oi}$ characteristic impedance (line i)

$P_G^-$ available power of the equivalent generator

$b_G^-$ equivalent generator wave amplitude

$a_i$ incident wave at port i

$b_i$ emergent wave from port i

$P_{nG}^-$ available input noise power equal to the noise power generated by a resistor at a temperature of 290 K

$P_{nG}^+$ available output noise power

the subscript $n$ states that the variable is a noise variable

the subscript $G$ states that the variable is a generator variable in general

the superscript $-$ states that the variable is an equivalent generator variable

the superscript $*$ states that the variable is the complex conjugated

$N_D$ noise power at the detector

$N_{i1}$ input noise power generated by a resistor at a temperature of 290 K

$N_{si}$ the noise power added by element i

$F_{45}$ NF of the circulator forward circuit part

$F_{89}$ NF of the circulator feedback circuit part

$M$ noise measure

$B$ bandwidth

$T_0$ reference temperature (K)

$T_i$ temperature of circuit element i (K)

$T_s$ noise source temperature (K)

$k$ Boltzmann constant
1 INTRODUCTION

Electronic devices employing a single or multiple quantum well structure are increasing in number. The double barrier resonant tunneling (DBRT) diode is one of them. Research on GaAs-AlGaAs DBRT devices is part of the activities of the Electronic Devices Group (EEA) of the faculty of Electrical Engineering at the Eindhoven University of Technology.

The current-voltage relationship of a DBRT diode is nonlinear with negative differential resistance (NDR) regions. These NDR regions can be observed up to very high frequencies so the DBRT diode can be used in microwave amplifiers and oscillators. Knowing the noise behaviour of the active device is important for such applications but also from a physics point of view.

This report describes the development of two noise figure measurement techniques and the measurements required to characterize the DBRT diode's noise behaviour. Both methods use the microwave reflection amplifier with circulator as the basic circuit in which the DBRT diode is embedded. In this way the one-port DBRT diode is transformed into a two-port device with separate input and output ports, since most noise figure meters only can perform measurements on two-ports. By applying the techniques described in this report the actual noise parameters of the DBRT diode are de-embedded from the overall noise behaviour of the reflection amplifier circuit.

Chapter 2 deals with important features of the reflection amplifier and the strategy of the measurement methods. Chapter 3 describes the noise figure theory of a two-port. Some adaptations are introduced on the general two-port, in order to model the reflection amplifier. Chapter 4 deals with the noise figure of the reflection amplifier. Chapter 5 describes the complicated but more exact noise figure measurement procedure that applies both the NF meter and an Automatic microwave Network Analyzer (ANA&NFM-method). Chapter 6 discusses a mathematically simple measurement method (three versions) that uses only the noise figure meter (NFM-method). Chapter 7 describes the measurements performed on DBRT diodes with both the ANA&NFM and NFM method and reports the measured and calculated results. Most of the formulas are derived in appendices.

Our research on the noise behaviour of DBRT diodes started with the development of the more exact ANA&NFM measurement method, from which the NFM-procedure spun off. This report is based on the M.Sc.-thesis work of J.I.M. Demarteau and has been revised and supplemented by J.J.M. Kwaspen and H.C. Heyker.

Throughout the report the term noise figure is used. In the equations this quantity is a number (noise factor), while in most tables and plots the noise figure is expressed in dB.
The object of the theoretical and experimental research described in this report is to determine the microwave noise behaviour of Double Barrier Resonant Tunneling (DBRT) diodes versus bias voltage and frequency. A brief introduction to the DBRT diode is given in Appendix A, while several research papers deal with the fundamentals of the device in more detail (Ref. 24,25,26).

The negative differential resistance (NDR) regions in the DBRT diode's I-V characteristic can be used for microwave signal amplification.

The general noise figure (NF) measurement concept, where an active two-port is inserted between a noise source and a noise figure meter, can not immediately be applied to the DBRT diode since this device is a one-port in transmission line terms. From transmission line theory it is well known that a device with a NDR or with a negative real part of the device's impedance mounted to terminate a transmission line shows a reflection coefficient larger than 1. The emergent wave coming from the active device is larger in amplitude than the incident wave amplitude so signal amplification is achieved. Both waves however travel in the same line. To split these two waves into an input and an output wave, the DBRT diode is connected to one port of a three-port circulator with the source and receiver connected to the other ports. The combination of an active one-port and a microwave circulator is called a reflection amplifier. Fig. 2.1 shows the configuration of such a circuit (Ref. 23,27,28,29).

![Reflection Amplifier Circuit](image)

Fig. 2.1 Basic reflection amplifier circuit

### 2.1 The reflection amplifier circuit

The reflection amplifier in our research consisted of a 1-2 GHz coaxial microwave circulator (Addington Labs., model 100100004), a self-built bias T to apply DC bias voltage and current to the DBRT diode and the diode itself (Nottingham University, U.K., wafer NU104).
Fig. 2.2 shows the reflection amplifier with bias circuit. The coaxial Y-junction circulator is a microwave component whose three ports are positioned at 120° intervals around the circumference of the structure.

![Diagram of reflection amplifier with bias circuit]

In its interior a ferrite disc is installed on a stripline pattern. The circularly polarized (noise) waves entering the circulator's port 1 (input) are divided at the ferrite disc and part of the signal flows in a clockwise direction, while another part travels in the counterclockwise direction around the disc. Due to the ferrite disc that is biased with a constant magnetic field the velocity and loss characteristics of the waves that travel through the device are different for both directions. A correctly designed circulator causes the phases of the two counter rotating signals to add when they reach port 2 (Fig. 2.1) and to subtract when they reach port 3. The arrow in Fig. 2.1 indicates the signal flow direction to the next output port of the circulator.

Applied to the reflection amplifier, the wave entering input port 1 leaves the circulator at port 2, is amplified at the DBRT diode, is reflected back to port 2 and leaves the amplifier at the output port 3. The insertion loss between two adjacent ports is dependent on the direction of the signal flow (following the arrow: FORWARD direction or against it: REVERSE direction), and typical values are 0.25 dB (forward) and 25 dB (reverse).

When noise flows into the reflection amplifier, the output noise consists of amplified input noise from the noise source and added noise generated by the circulator (due to losses) and by the DBRT diode.

The aim of the investigations described in this report is to determine the DBRT diode noise contribution, de-embedded from the overall noise of the circuit. To be able to do so, the characteristics of the amplifier circuit have to be analysed. We therefore treat
the circulator+bias T as a three-port unit (ports 1,2,3 in Fig. 2.2) that consists of the next three (labelled) transmission segments:

1 - from the input port 1 to the DBRT diode port 2 (including bias T)
3 - from the DBRT diode port 2 to the output port 3 (including bias T)
4 - from the output port 3 to the input port 1

The first two segments are in the FORWARD path while segment 4 is the FEEDBACK path, so the circuit of Fig. 2.2 between ports 1 and 3 (without bias source) can be redrawn as Fig. 2.3 shows. All effects of the actual circulator+bias T are embedded in segments 1,3 and 4.

**Fig. 2.3** Block diagram of segmented reflection amplifier

The definition of noise figure (NF) holds, strictly speaking, only for two-ports and cascaded two-ports but not for one-ports.

We now define the NF of a DBRT diode as the NF a two-port reflection amplifier equipped with an ideal lossless circulator + the same DBRT diode would have (segment 2 in Fig. 2.3). The ideal circulator has no influence on the DBRT diode noise.

At signal level, the microwave behaviour of the actual circulator+bias T three-port fully can be described by its 3x3 scattering parameter matrix (S-parameter matrix; see Fig. 2.4), but on noise level the internal noise production has to be taken into account. This makes the analysis more complicated. To tackle the problem a dual approach was made.

**Fig. 2.4** 3x3 S-parameter matrix of actual circulator + bias T
At **signal level**, the full S-parameter set of the actual circulator+bias T (Fig. 2.4) was measured at the test frequency with an Automatic Network Analyser (ANA). Then, from the measured transfer functions $S_{31}$ with the DBRT diode at several bias voltages, the reflection coefficients $\Gamma_D$ and available gains $G_\alpha$ of the diode were calculated (Appendices C,D and E). Then the gains between each couple of ports are also known. It is important to measure $\Gamma_D$ when the DBRT is mounted in the actual circuit, since the circuit impedance this device sees determines whether the DBRT diode is oscillating or not. Noise measurements are only meaningful on stable devices. Tuning elements that eventually are necessary can be incorporated into the three-port. The DBRT diode is a very broadband device (DC to above microwave frequencies) and since only circulators are available with limited bandwidth the realisation of a stable reflection amplifier with a DBRT diode is quite a job. However this report does not deal with the effort to stabilize the device, but takes it as a prerequisite.

At **noise level**, the circuit of Fig. 2.3 plays a major role. Since full noise analysis is complicated, the microwave behaviour of the actual circulator+bias T, as described by its S-parameter matrix, added with the internal noise generation, is approximated by a simplified circuit consisting of two parallel connected noisy two-ports. One two-port (element 4; feedback) describes the signal transfer between ports 1 and 3 of the three-port ($S_{13}$ and $S_{31}$), when port 2 would be loaded with a cold, matched termination. The other two-port simulates the forward signal transfer from port 1 to 2 and to port 3 of the three-port. This two-port is a cascade of three two-ports as discussed earlier in this paragraph. The noise figure of a two-port is treated in Appendix F. A further simplification is the introduction of an "adapted two-port" (Chapter 3) that contains only one reflection and one transmission S-parameter. The available gain of such an adapted two-port is dealt with in Appendix C and the noise figure formula is derived in Appendix G. On the FORWARD two-port Friis' noise equation for cascaded two-ports is applied to calculate the NF of the DBRT diode from measurements with the NF meter. The noise analysis of the simplified reflection amplifier circuit as given in Fig. 2.3 (but with adapted two-ports) is carried out in Appendix H and Chapter 4. Appendix I derives equations for determining the noise measure of a two-port with the so called two-attenuator method that can be applied when an automatic noise figure meter is not available. Appendix J gives measured data in tabular form.
2.2 Components

The function of the bias T (bias Tee junction) in the reflection amplifier is to apply DC bias to the DBRT diode, to allow the AC signals to pass through the main line and to stop AC signals from flowing into the bias source. The unit applied in our experiments was self-made and designed for the lower microwave region.

In the experiments DBRT diode chips were used mounted in an F5 ceramic package (LEW TECHNIQUES, Taunton, U.K.), and diode chips mounted on a chip-carrier without package. The packaged device was placed against the end of a 7 mm 50Ω air-line (with an APC-7 precision connector attached) and the end of the transmission line was the DBRT diode port (Fig. 2.5).

![Fig. 2.5 DBRT diode in F5 package](image)

In the experiments on unpackaged DBRT diode chips, the die is mounted on top of a modified M1.4 screw, which gently is screwed into the test fixture shown in Fig. 2.6, until the mesa's top metalisation contacts the protruding tap of the SMA stripline launcher (Ref. 20; no plane capacitor and series resistor).

![Fig. 2.6 SMA stripline launcher DBRT diode test fixture](image)
3 AVAILABLE GAIN AND NOISE FIGURE OF A TWO-PORT

The circulator of the reflection amplifier will be modelled by two-ports so this chapter deals with the available gain and noise figure of two-ports. The noise figure (NF) only is defined for two-ports.
In this chapter as well as in later chapters the following type of block-diagram will be used.

\[ \begin{align*}
&\begin{array}{c}
a_1 \\
b_1 \\
\end{array} \\
&\{S\} \\
&\begin{array}{c}
a_2 \\
b_2 \\
\end{array}
\end{align*} \]

Fig. 3.1 Graphical representation of a two-port

Figure 3.1 shows a circuit element connected between two transmission lines. In each of these lines there are two waves traveling in opposite directions. The waves that travel towards the two-port are labeled \( a_i \), those traveling away from the element are labeled \( b_i \). \((S)\) denotes the scattering matrix of the two-port.

3.1 Available gain of a general two-port

In noise calculations only available powers and available gains are used. Therefore it is useful to express the available gain in terms of scattering parameters. In appendix C the equation of the available gain \( G_a \) is derived.

\[
G_a = \frac{|S_{21}|^2 \left( 1 - |\Gamma_G|^2 \right)}{|1 - S_{11}|^2 - |S_{22} - \Gamma_G \Delta|^2} \quad 3.1
\]

where \( G_a \) is the available gain, \( S_{ij} \) is the scattering parameter from port \( j \) to port \( i \), \( \Delta \) is the determinant of the 2x2 \( S \)-matrix of the two-port and \( \Gamma_G \) is the reflection coefficient of the generator. The available gain \( G_D \) of the DBRT diode can be calculated by

\[
G_D = |\Gamma_D|^2 \quad 3.2
\]

where \( \Gamma_D \) is the reflection coefficient of the DBRT diode (see appendix E).
3.2 Equivalent generator principle

In [1] a generalisation of Thévenin's theorem is used which states:

If a two-port contains internal sources, then the equivalent circuit must be modified.

By a generalization of Thévenin's theorem, the two-port may be separated into a source-free network and two generators.

In [2,3,4] Peterson's equivalent noise generator theorem is used which states:

The performance of a noisy, linear two-port network may be described completely by the addition of two noise generators to a noiseless network which is equivalent otherwise.

See also [1, 5 - 14].

Because the $a_i$ and $b_i$ waves have a direct relation with voltages and currents (eq. 3.3), the generators also may be $a_{n_i}$ and $b_{n_i}$ noise generators.

\[
\begin{align*}
  a_i &= \frac{E_i + Z_i J_i}{2 \sqrt{\text{Re}(Z_i)}} & (3.3a) \\
  b_i &= \frac{E_i + Z_i^* J_i}{2 \sqrt{\text{Re}(Z_i)}} & (3.3b)
\end{align*}
\]

where $E_i$ is the voltage at the $i$-th port, $J_i$ the current flowing into the $i$-th port and $Z_i$ is the characteristic impedance of the transmission line connected to port $i$. There are several possible noise generator configurations. They are listed in Appendix B together with the proper equations [9].

The configuration given in figure 3.2 and equation 3.4 is used throughout this report to model the internal equivalent noise sources of the two-port.

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix} +
\begin{bmatrix}
  b_{n1} \\
  b_{n2}
\end{bmatrix}
\]

\[
\text{Fig. 3.2 Two-port with equivalent noise generators}
\]
3.3 Noise figure of a general two-port

Figure 3.3 shows a noise-free two-port with two external noise sources, where all variables necessary for the calculation of the noise figure are defined (see also Appendix C).

By using the noise figure definition the noise behaviour of a two-port can be described with only one variable $F$. Eq. 3.5 gives the definition of the noise figure $F$:

$$F = \frac{P_G}{P_{nG}}$$  \hspace{1cm} (3.5)

where $P_G$ is the available signal power at the input port, $P_{nG}$ the available input noise power equal to the available noise power generated by a resistor at a temperature of 290 K, $P^-_G$ is the available signal power at the output port and $P^-_{nG}$ is the available output noise power.

The formula for the noise figure of a two-port is (see appendix F)
\[ F = 1 + |\Gamma_G|^2 \frac{|b_{n_1}|^2}{|b_{nG}|^2} + \frac{|1 - \Gamma_G S_{11}|^2}{|S_{21}|^2} \frac{|b_{n_2}|^2}{|b_{nG}|^2} + \frac{\Gamma_G (1 - \Gamma_G S_{11})^*}{S_{21}^*} \frac{b_{n_1} b_{n_2}^*}{|b_{nG}|^2} + \frac{\Gamma_G^* (1 - \Gamma_G S_{11})}{S_{21}} \frac{b_{n_1}^* b_{n_2}}{|b_{nG}|^2} \]

3.4 Comparison with literature

In order to compare eq. 3.6 with noise figure formulas in literature, this equation can be written as follows:

\[ F = 1 + \frac{|b_{n_1}|^2}{|b_{nG}|^2} + \frac{|b_{n_2}|^2}{|b_{nG}|^2} + \frac{b_{n_1} b_{n_2}^*}{|b_{nG}|^2} + \frac{b_{n_1}^* b_{n_2}}{|b_{nG}|^2} \]

In [7] the general form is

\[ F = 1 + A + B E E^* + C J J^* + D (E J^* + E^* J) + F (E J^* + E^* J) \]

In [5] the general form is

\[ F = 1 + A i_n^2 + B e_n^2 + C e_n i_n \]

In [1] the general form is

\[ F = 1 + A u^2 + B e^2 \]

(In each reference the Italic capitals have different values)

First eq. 3.8 will be compared with eq. 3.7:
- Eq. 3.8 has a parameter \( A \) that depicts a kind of admittance mismatch. In the calculation of eq. 3.7 it was assumed that the two line impedances were identical.
- The third and fourth term of eq. 3.8 correspond with the second and third term of eq. 3.7.
- The last two terms of eq. 3.8 correspond with the last two terms of eq. 3.7.

Next eq. 3.9 will be compared with eq. 3.7:
- The second and third term of eq. 3.9 correspond with the second and third term of eq. 3.7.
- The last term of eq. 3.9 corresponds with the last two term of eq. 3.7.

Now eq. 3.10 will be compared with eq. 3.7:
- The second and third term of eq. 3.10 correspond with the second and third term of eq. 3.7.
- As stated in [1] \( i_u \) and \( e \) are uncorrelated. This means that there is no corresponding term for the last two terms of eq. 3.7.

### 3.5 The adapted two-port

Figure 3.4 shows the adapted "two-port" which will be introduced to simplify the calculations on the reflection amplifier circuit. We assume the adapted two-port can be described by two S-parameters \( S_{11} \) and \( S_{21} \). The reverse transmission is omitted since it is much smaller than the forward transmission. The output port of the adapted two-port always is connected to the input of another two-port in the reflection amplifier model so the output reflection coefficient of the adapted two-port is the input reflection coefficient of the other element. In this way the use of the un-physical adapted two-port is allowed in the overall model of the three-port. Work is in progress to model the 3x3 matrix in an exact way.

![Diagram of the adapted two-port](image)

**Fig 3.4 The signal circuit diagram of the adapted "two-port".**

The noise figure of the adapted two-port is derived in appendix G and is given by

\[
F = 1 + |\Gamma_G|^2 \frac{|b_{n2}|^2}{|b_{nG}|^2} + \frac{|1 - \Gamma_G S_{11}|^2}{|S_{21}|^2} \frac{|b_{n2}|^2}{|b_{nG}|^2} + \frac{\Gamma_G (1 - \Gamma_G S_{11})^*}{S_{21}} b_{n1}^* b_{n2} + \frac{\Gamma_G^* (1 - \Gamma_G S_{11})}{S_{21}} b_{n1} b_{n2}^* \]  

3.11

Comparing equations 3.6 and 3.11 concludes that the noise figures of the general and adapted two-port are identical.
In order to be able to calculate the noise figure (NF) of the DBRT diode, the NF of the reflection amplifier must be determined first. Then the NF of the DBRT diode can be deembedded from the NF of the reflection amplifier.

The dominating circuit elements of the reflection amplifier are the circulator, bias T and active element (DBRT diode).

As announced in chapter 2 the first attempt to model the reflection amplifier is to describe the device with two parallel connected two-ports as shown in Fig. 4.1. The forward transmission path including the biased DBRT diode is indicated by two-port B labeled FORWARD. The waves that are reflected at the output port are transported by the circulator to the input port, thus creating feedback (two-port A). The feedback two-port consists only of the circulator part from the output port to the input port.

**Fig. 4.1 The reflection amplifier circuit**

The forward two-port B consists of three sections:
1. The circuit part from the input to the DBRT port (part of the circulator + bias T).
2. The DBRT diode (packaged or die)
3. The circuit elements between DBRT port and output port of the reflection amplifier (part of the circulator + bias T).

**4.1 The circuit diagram of the reflection amplifier**

Fig. 4.2 gives the signal and noise circuit of the reflection amplifier. The forward two-port is indicated by the S-parameters \( S_{44}, S_{45}, S_{54}, \text{ and } S_{55} \), while the feedback two-port is indicated by the S-parameters \( S_{88}, S_{89}, S_{98}, \text{ and } S_{99} \).
The incident wave $a_2$ splits up into $a_3$ and $a_9$ which is done by the circulator. The incident wave $a_9$ travels in the main rotation direction and $a_3$ travels in the reverse rotation direction. The reverse transmission is more than 20 dB smaller than the forward transmission. The incident wave $a_4$ splits up into $a_4$ and $a_8$.

The noise behaviour of the passive components of the reflection amplifier circuit are modelled by the equivalent noise sources $b_{n1}$, $b_{n2}$, $b_{n3}$ and $b_{n4}$ (paragraph 3.2 and Appendix B).

In figure 4.3 the branches labeled $a_5$ and $a_8$ are left out, since they are omitted in the simplified circuit diagram of the reflection amplifier built up from adapted two-ports. This simplified circuit diagram is the base for further calculations of the reflection amplifier's NF.
4.2 Noise figure of the reflection amplifier

The derivation of the NF of the amplifier is carried out in Appendix H and the equation (H.26) reads

\[ F = 1 + |\Gamma_G| \frac{|b_{n1}|^2}{|b_{ng}|^2} + |\Gamma_G|^2 \frac{|b_{n3}|^2}{|b_{ng}|^2} + \frac{|1 - S_{44} \Gamma_G|^2 |b_{n4}|^2}{|S_{54}|^2 |b_{ng}|^2} + \]

\[ \frac{|1 - S_{44} \Gamma_G|^2 |b_{n2}|^2}{|S_{54}|^2 |b_{ng}|^2} + \frac{(1 - S_{44} \Gamma_G) \Gamma_G}{S_{54}^*} \frac{b_{n1} b_{n2}^*}{|b_{ng}|^2} + \frac{(1 - S_{44} \Gamma_G) \Gamma_G^*}{S_{54}^*} \frac{b_{n3} b_{n4}^*}{|b_{ng}|^2} \]

where \( F \) is the NF of the reflection amplifier between input and output ports.

The block diagram of the reflection amplifier is shown in Fig. 4.4. The forward adapted two-port consists of the three sections discussed earlier in this chapter where network 1 and 2 are general two-ports, while section 3 is an adapted two-port.
Each adapted two-port has a noise figure according to appendix G. The noise figure $F_{45}$ of the adapted two-port in the forward path that includes the DBRT diode is given by eq. H.27 (appendix G, eq. G.21).

$$F_{45} = 1 + \left| \Gamma_G \right|^2 \frac{|b_{n3}|^2}{|b_{nG}|^2} + \frac{1 - \Gamma_G S_{44}|^2}{|S_{54}|^2} \frac{|b_{n4}|^2}{|b_{nG}|^2} +$$

$$\frac{\left(1 - \Gamma_G S_{44}\right) \Gamma_G b_{n3} b_{n4}^*}{S_{54}^*} + \frac{\left(1 - \Gamma_G S_{44}\right) \Gamma_G^* b_{n3}^* b_{n4}}{|b_{nG}|^2} \tag{4.2}$$

The external noise source wave amplitude $b_{nG}$ is identical in eqns. 4.1 and 4.2, so corresponding terms of 4.1 can be replaced by those of 4.2 which yields

$$F_t = F_{45} + \left| \Gamma_G \right|^2 \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{1 - \Gamma_G S_{44}|^2}{|S_{54}|^2} \frac{|b_{n2}|^2}{|b_{nG}|^2} +$$

$$\frac{(1 - S_{44}) \Gamma_G}{S_{54}^*} \frac{b_{n1} b_{n1}^*}{|b_{nG}|^2} + \frac{(1 - S_{44}) \Gamma_G^*}{S_{54}} \frac{b_{n2}^* b_{n2}}{|b_{nG}|^2} \tag{4.3}$$

Eq. 4.3 is the formula for the total noise figure of the reflection amplifier in which $F_{45}$ is the noise figure of the forward transmission path from input port to output port.
Note: During the preparation of the final manuscript of this report, another reflection amplifier model was worked out, in which general two-ports were incorporated instead of adapted two-ports (Fig. 4.4). Appendix K was supplemented in which the derivation was carried out. The equation of the total noise figure of the reflection amplifier (eq. K.15) is identical with eq. 4.3 if the same approximations and indices are used.
5 DBRT DIODE NF MEASUREMENT PROCEDURE: ANA&NFM-METHOD

In this chapter the ANA&NFM (Automatic Network Analyser and Noise Figure Meter) noise measurement method to determine the noise measure of DBRT diodes will be described after an introduction to the measurement principles of the applied noise figure (NF) meter. The method is based on the theory stated in the chapters 3 and 4 and uses the NF meter and an Automatic microwave Network Analyzer (ANA). First the NF meter is introduced.

5.1 Noise figure measurements with the HP 8970B noise figure meter

In the first part of this paragraph a brief description of the Hewlett-Packard 346A noise source and the HP 8970B noise figure meter is given. Then the measurement method the NF meter uses in order to obtain the insertion gain and the noise figure will be dealt with. In the third part the calibration routine used by the NF meter in order to obtain the correct noise figure is described.

5.1.1 Description of the noise source and the noise figure meter

In this sub paragraph only some topics of the instruments are given. For more detailed information see [18]. The noise source (HP 346A) is a hot/cold noise source (0.01 - 18 GHz) which means that this source can generate two different noise power levels, one as a cold noise source and one as a hot noise source. The on/off switching is done by the noise figure meter (HP 8970B) which has the following specifications:
- Frequency band: 10 - 1600 MHz
- Instrumentation uncertainty: \( \pm 0.15 \) dB (gain); \( \pm 0.10 \) dB (noise figure)
- Measurement range: 0 to 30 dB (noise figure); - 20 to \( > + 40 \) dB (gain)

5.1.2 Gain and noise figure measurement method used in the HP8970B

Measurement of the insertion gain and NF of a DUT is performed with the configuration shown in figure 5.1. The device under test is directly connected between the noise source

![Fig. 5.1 Gain and NF measurement configuration](image)
and the noise figure meter so the gain and NF of the DUT are defined at the noise source/DUT and DUT/NF meter interfaces.

Before measuring the insertion gain and the noise figure of the DUT, a calibration measurement must be conducted in which the noise source directly is connected to the NF meter (noise source/DUT and DUT/NF meter interfaces defined above coincide). The instrument configuration used in this calibration step is sketched in figure 5.2.

![Fig. 5.2 The calibration configuration](image)

In the calibration step the output noise power of the source is measured with the source at $T'_c$ (cold) and $T'_h$ (hot). Plotting the output noise power versus the noise source temperature results in figure 5.3.

![Fig. 5.3 The output noise power v.s. the noise source temperature (calibration step).](image)

The slope of the curve in figure 5.3 is equal to

$$ k \cdot G_c \cdot B = \frac{N'_2 - N'_1}{T'_h - T'_c} \tag{5.1} $$

where $k$ is Boltzmann's constant, $B$ the bandwidth and $G$ the gain.

Now the noise power measurement is done again with the DUT connected between the noise
source and NF meter (measurement step, see Fig. 5.1). The output noise power is plotted against the noise source temperature in figure 5.4.

\[ N - N \]
\[ k G_G B = \frac{G_{\text{OUT}}}{T - T} \]

The noise figure meter does not measure \( F \) directly. Instead the instrument measures the output noise power at two temperatures and calculates from that \( Y \), following eq. 5.4

\[ Y = \frac{N_2}{N_1} \]

where \( N_2 \) is the output noise power when the noise source is at \( T_h \) and \( N_1 \) is the output noise power when the noise source is at \( T_c \). \( F \) can be calculated when \( Y, T_h, T_c \) and \( T_0 \) are known (\( T_0 = 290 \) K), see eq. 5.5.
5.1.3 **The standard NF measurement method of the HP 8970B NF meter**

The standard measurement procedure is designed for the situation shown in Figure 5.5. Only between the DUT and the noise figure meter the insertion of two-ports (cables, adapters) is allowed. The (calibrated) noise source must directly be connected to the DUT to prevent errors in the NF measurement.

First the noise figure of the configuration shown in figure 5.5a is measured and stored (calibration step). This is the noise figure of everything (cables, adapters, NF meter) that will be placed in the circuit behind the DUT. This noise figure will be referred to as \( F_c \).

The next step is to measure the noise figure of the configuration sketched in figure 5.5b including DUT (measurement step). This is the overall noise figure which will be referred to as \( F_f \). The noise figure of the DUT will be referred to as \( F_{\text{DUT}} \). For \( F_f \) the Friis's formula for cascaded two-ports holds \[19\]

\[
F_f = F_{\text{dis}} + \frac{(F_s - 1)}{G_{\text{DUT}}}
\]

where \( F_{\text{dis}} \) is the displayed NF on the NF meter, \( G_{\text{DUT}} \) stands for the gain of the DUT which is determined from data collected in the calibration and measurement steps (see Eq. 5.3). \( G_{\text{DUT}} \) is also displayed on the NF meter.

Note: The Friis' formula requires the use of the available gain of the DUT, while the NF
meter measures the insertion gain. If the difference is large, a correction must be carried out according to Ref. 12.

Eq. 5.6 can be rewritten as

\[
F_{\text{dis}} = G_{\text{DUT}} = F_t \cdot \frac{(F_t - 1)}{G_{\text{DUT}}} \quad \text{5.7}
\]

After the calibration and measurement steps have been carried out the NF meter calculates and displays \( F_{\text{dis}} \) which for this standard measurement procedure is equal to \( F_{\text{DUT}} \)

5.2 DBRT diode noise figure measurement using an Automatic Network Analyzer and the NF meter (ANA&NFM-method).

5.2.1 Theory of ANA&NFM-method

The measurement method that uses the NF meter and an Automatic Network Analyzer is based on eq. 4.3 repeated here as eq. 5.8.

\[
F_t = F_{45} + |\Gamma_G|^2 \left| \frac{b_{n1}}{b_{nG}} \right|^2 + \left| 1 - S_{44} \Gamma_G \right|^2 \left| \frac{b_{n2}}{b_{nG}} \right|^2 + \left( 1 - S_{44} \Gamma_G \right)^* \frac{b_{n1}}{b_{nG}} \left( \frac{b_{n2}}{b_{nG}} \right) + \frac{(1 - S_{44} \Gamma_G)^* \Gamma_G}{S_{54}} \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{(1 - S_{44} \Gamma_G) \Gamma_G^*}{S_{54}} \frac{|b_{n1}|^2}{|b_{nG}|^2} \quad \text{5.8}
\]

In eq. 5.8 \( F_t \) is the noise figure of the total reflection amplifier and \( F_{45} \) indicates the NF of the forward path including DBRT diode. To simplify the equations two measurements are carried out:

- The total NF of the amplifier is measured with the biased DBRT diode in the circuit. This NF result is named \( F_{tD} \).
- A second NF result, named \( F_{tS} \), is obtained from a measurement on the total amplifier circuit in which the DBRT diode is replaced by a short-circuit at the DBRT diode port.

The S-parameters \( S_{44}, S_{54} \) and the NF \( F_{45} \) are dependent on the load impedance at the DBRT diode port, so in the measurement with the short these parameters are named \( S_{44}^*, S_{54}^* \) and \( F_{45}^* \).

Subtraction of the NF results of both measurements leads to:
\[ F_{d} - F_{1S} = F_{45} + |\Gamma_{G}|^2 \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{|1 - S_{44} \Gamma_{G}|^2}{|S_{54}|^2} \frac{|b_{n2}|^2}{|b_{nG}|^2} + \]
\[ \frac{(1 - S_{44} \Gamma_{G})^* \Gamma_{G}}{S_{54}'} \frac{b_{n1} b_{n2}^*}{|b_{nG}|^2} + \frac{(1 - S_{44} \Gamma_{G})^* \Gamma_{G}}{S_{54}'} \frac{b_{n1} b_{n2}}{|b_{nG}|^2} + \]
\[ - \frac{F_{45} - |\Gamma_{G}|^2}{|b_{nG}|^2} \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{|1 - S_{44} \Gamma_{G}|^2}{|S_{54}'|} \frac{|b_{n2}|^2}{|b_{nG}|^2} + \]
\[ \frac{(1 - S_{44} \Gamma_{G})^* \Gamma_{G}}{S_{54}'} \frac{b_{n1} b_{n2}^*}{|b_{nG}|^2} - \frac{(1 - S_{44} \Gamma_{G})^* \Gamma_{G}}{S_{54}'} \frac{b_{n1} b_{n2}}{|b_{nG}|^2} \]

Applying Friis' formula for the three cascaded two-ports in the forward transmission path, the following equations are obtained

\[ F_{45} = F_{1} + \frac{F_{d} - 1}{G_{1}} + \frac{F_{3} - 1}{G_{1} G_{d}} \]
\[ 5.10 \]

\[ F_{45}^* = F_{1} + \frac{F_{3} - 1}{G_{1}} \]
\[ 5.11 \]

where \( F_{1}, F_{d}, \) and \( F_{3} \) are the noise figures of the three consecutive two-ports in the forward path and \( G_{1} \) and \( G_{d} \) are the gains of the first two two-ports. In the measurement with a short-circuited DBRT diode port \( F_{d} = 1 \) and \( G_{d} = 1 \) (eq. 5.11).

Substituting eqns. 5.10 and 5.11 into eq. 5.9 leads to

\[ F_{d} - F_{1S} = \frac{F_{d} - 1}{G_{1}} + \frac{F_{3} - 1}{G_{1}} \left( \frac{1}{G_{d} - 1} \right) + \]
\[ \frac{|b_{n2}|^2}{|b_{nG}|^2} \left( \frac{|1 - S_{44} \Gamma_{G}|^2}{|S_{54}'|^2} - \frac{|1 - S_{44} \Gamma_{G}|^2}{|S_{54}'|^2} \right) \]
\[ \text{........ (next page)} \]
The variables that have to be measured at each frequency are:
- \( F_{\text{dB}} \) and \( F_{\text{Is}} \) (total NFs of amplifier with biased DBRT diode and with short)
- \( \Gamma_G \) and \( \Gamma_L \) (reflection coefficient of noise source and NF meter)
- \( S_{31}^a \) (transfer function of reflection amplifier with DBRT diode for each bias voltage)
- all \( S \)-parameters of the 3x3 matrix (circulator + bias T)

The first quantities are measured with the NF meter after calibrating the instrument according to the standard procedure described earlier in this chapter and the next three items are measured with the Network Analyzer at each frequency of interest. The circulator is a passive three-port and for passive elements the noise figure can be derived from \( F = L \) or \( F = 1/G \), where \( L \) is the attenuation of the element and \( G \) the gain, which quantity can be calculated from the measured \( S \)-parameters.

To estimate the value of the 3rd, 4th and fifth term of equation 5.12 we look at the noise figure of the single feedback path as given by eq. H.27 where the indices should be replaced by corresponding items. The main contribution to \( F_{98} \) is given by the first and third term, so neglecting the other terms \( F_{98} \) is given by

\[
F_{98} \approx 1 + \frac{|b_{n1}|^2}{G_{89} |b_{nG}|^2}
\]

Equation 5.13 can be rewritten where \( F_{98} = 1/G_{89} \) and \( G_{89} \approx |S_{89}|^2 \) (\( = 0.937 \) @ 1.5 GHz)

\[
\frac{|b_{n1}|^2}{|b_{nG}|^2} \approx (F_{98} - 1) |S_{89}|^2 = 0.063
\]

If \( |b_{n1}|^2 = |b_{n2}|^2 \) and because of the fact that \( \Gamma_G \) is very small (see table J.3) the last two terms of eq. 5.12 are neglectable.
The term of eq. 5.12 is estimated by first calculating the coefficient:

\[
\frac{|b_{n2}|^2}{|b_{nG}|^2} \left( \frac{|1 - S_{44} \Gamma_G|^2}{|S_{54}|^2} - \frac{|1 - S_{44}^\dagger \Gamma_G|^2}{|S_{54}^\dagger|^2} \right) \approx \left( \frac{1}{|S_{54}|^2} - \frac{1}{|S_{54}^\dagger|^2} \right) \leq 0.67
\]

This means that an estimate worst case value for the 3-rd term is 0.04 taken into account all measured $S_{54}$ values ($= S_{31}^m$) of the amplifier with biased DBRT diode (Appendix J). Comparing the values of the sum of the first two terms of eq. 5.12 with the value of the third term over the full bias range of our measurements results in a maximum error of 4.6% when the third term is neglected. This maximum error occurs at low bias levels. The 4-th and 5-th terms are two orders of magnitude smaller.

The resulting equation for $F_d$ (neglecting the 3-rd, 4-th and 5-th term) is

\[
F_d = 1 + G_1 \left( F_{1D} - F_{1S} + \frac{F_3}{G_1} \left( 1 - \frac{1}{G_d} \right) \right)
\]

Equation 5.15 is the basis for the calculation of the noise figure of the DBRT diode carried out by a computer program.

### 5.2.2 Measurement configurations

In this paragraph several measurement configurations briefly are described, that are needed to gather the measured quantities for the DBRT diode NF calculations. Measurement of the output reflection coefficient of the noise source (at $T_e$) and the input reflection coefficient of the noise figure meter, is a straightforward test with the calibrated network analyzer, where only one port of the test set is used. Fig. 5.6 shows the setup in this case.

![Network Analyzer Diagram](image)

*Fig. 5.6 Test setup for one-port reflection coefficient measurement of $\Gamma_G$ and $\Gamma_L$*
The S-parameters of the three-port consisting of circulator+bias T were measured using the setup of Fig. 5.7, where two ports are connected to the (calibrated) network analyzer ports, while the unconnected third port is terminated with a matched 50 Ω load. By alternately changing the DUT ports connected to the network analyzer, all 3x3 S-parameters are measured.

![Fig. 5.7 Setup for measurement of 3x3 S-matrix of circulator+bias T](image)

The DBRT diode mounted in its fixture is connected to the appropriate port of the circulator+bias T three-port which is connected with the other ports to the calibrated network analyzer (Fig. 5.8). The diode is biased and the transfer parameter $S_{31}^m$ is measured with the ANA at each bias voltage and frequency of interest. The same bias settings and frequency are used in the NF measurements on the reflection amplifier. From $S_{31}^m$ and the known S-parameters of the passive reflection amplifier $\Gamma_D$ and $G_D$ can be calculated (Appendix D,E).

![Fig. 5.8 Measurement of $S_{31}^m$ transfer function of reflection amplifier with biased DBRT diode and with short](image)

The noise measurements are conducted with the noise source on the input port 1 of the reflection amplifier and the NF meter on output port 3, after calibrating the NF meter according to the standard method (source directly connected to meter). With the DBRT diode
biased, the total NF ($F_{iD}$) of the reflection amplifier is measured at each bias point ($S^m_{31}$ set). Then the DBRT diode is replaced by a short at that port, and $F_{iD}$ is measured (Fig. 5.9).

![Diagram](image)

**Fig. 5.9 Setup for $F_{iD}$ and $F_{iS}$ NF-measurement with calibration**

Observing formula 5.15 the quantities $G_1$ and $F_3$ have to be determined one time only at each frequency. This was done from calculations with the known 3x3 S-matrix ($F_3 = 1/G_3$) and from measurements with the NF setup on the appropriate sections of the reflection amplifier three-port. $F_1 = 0.85$ dB, $G_1 = -0.66$ dB, $f = 1.5$ GHz.
6 DBRT DIODE NF MEASUREMENT PROCEDURE: NFM METHOD

6.1 General

The standard NF measurement method of the Hewlett-Packard NF meter can not be used to measure the NF of a DUT directly if an additional two-port is present between the noise source and the DUT (which is the case when a DBRT diode is installed in a reflection amplifier equipped with a circulator). This inserted two-port changes the noise power delivered to the DUT with respect to the (calibrated) output noise level of the source. Friis’s formula now has to be applied to a cascade of more two-ports, one of which is between the noise source and the DUT (two-port 1) and another (two-port 3) is between the DUT and the NF meter (Fig. 6.2). The DUT is represented by network 2 and the NF meter and its peripherals by network NFM.

![Fig. 6.1 The calibration configuration](image1)

To perform the calibration step in this method the DUT is removed and two-ports 1 and 3 are connected (Fig. 6.1). The noise figure measured on this calibration configuration will be referred to as $F_c$. It can be written as

$$F_c = F_1 + \frac{F_3 - 1}{G_1} + \frac{F_{NFM} - 1}{G_1 G_3}$$

Now the thru connection of network 1 and 3 is replaced by the DUT. The measurement step is depicted in Figure 6.2.

![Fig. 6.2 The measurement configuration](image2)
The total noise figure measured on this configuration will be referred to as $F_i$. It can be written as

$$F_i = F_1 + \frac{F_{DUT} - 1}{G_1} + \frac{F_3 - 1}{G_1 G_{DUT}} + \frac{F_{NFM} - 1}{G_1 G_{DUT} G_3} \quad 6.2$$

where the quantities $F$ and $G$ respectively are the noise figure and the available gain of the networks given by the indices.

Now eqns. 6.1 and 6.2 must be substituted into eq. 5.7 which yields

$$F_{OUT} = F_i + \frac{F_{DUT} - 1}{G_1} - \frac{F_1 - 1}{G_{DUT}} \quad 6.3$$

The problem is that $F_{dis}$ can be read from the display, but $F_{DUT}$ is the quantity that is wanted. This means that the noise figure on the display of the noise figure meter ($F_{dis}$) is not the noise figure of the DUT ($F_{DUT}$).

When eq. 6.3 is rewritten the result is

$$F_{DUT} = F_1 \left( F_{dis} - F_1 + \frac{F_1 - 1}{G_{DUT}} \right) + 1 \quad 6.4$$

This formula is used in the noise figure measurement method using the NF meter (NFM measurement method, version A).

NOTE: When the DUT and network 1 are resistive attenuators than eq. 6.5 is valid

$$F = 1/G \quad 6.5$$

which means for eq. 6.3

$$F_{dis} = F_1 + F_1(F_{DUT} - 1) - F_{DUT}(F_1 - 1)$$

$$F_{dis} = F_{DUT} \quad 6.6$$

This leads to the conclusion that if all networks are resistive attenuators the value on the display of the NF meter is the value of the noise figure of the DUT.
6.2 DBRT diode noise measurement method using only the NF meter:
(NFM method, version A)

Now the measurement method developed in paragraph 6.1 is applied to the determination of the noise behaviour of a DBRT diode.

The input and output ports of the circulator are connected to the noise source and the NF meter respectively (Fig. 6.3a). This means that between the noise source and the DBRT diode a two-port is inserted representing the behaviour of this transmission path, which changes the noise power delivered to the diode with respect to the calibrated output level of the noise source. Friis's formula now has to be applied to a cascade of more two-ports as is done in 6.1. The actual circulator + bias T are represented by an equivalent circuit consisting of networks 1 and 3 and an ideal circulator. Port 4 of this ideal circulator is thought to be connected to the DBRT diode terminals. The noise behaviour of the DBRT diode now is defined at the ports 1' and 3' of the ideal circulator. In the rest of this paragraph the feedback effect of the reflection amplifier will not be taken into account.

Two-port 1 in figure 6.3b represents the transmission path from the output of the noise source to the DBRT diode (port 1'), whereas two-port 3 represents the transmission path between the DBRT diode (port 3') and the input of the noise figure meter. The ideal circulator + DBRT diode is represented by network 2 and the NF meter by network NFM (paragraph 6.1).

Fig. 6.3a,b The reflection amplifier with equivalent circuit
The following procedure has been developed based on the use of eqns. 6.1, 6.2 and 6.4 derived in paragraph 6.1

A) A calibration measurement at a particular frequency \( f \) is carried out with the DBRT diode replaced by a short so actually ports 1' and 3' (Fig. 6.3b) coincide and network 2 is a thru connection.

Calibrating the NF meter on the calibration configuration of Fig. 6.4, the noise figure \( F_c \) (see eq. 6.1) is measured and stored in the instrument.

![Fig. 6.4 The calibration configuration](image)

To perform the calibration step in this method the DBRT diode port (port 4 in Fig. 6.4) is short-circuited as close as possible to the DBRT diode terminals. When a packaged DBRT diode has to be characterised a separate ceramic package can be equipped with a bonding wire internally short-circuiting the package's cap and post. Characterising a DBRT diode die in the special test fixture (Fig. 6.7) the short can be applied actually at the diode terminals by using a metal dummy die.

B) In the measurement step at the frequency \( f \) the short at port 4 is removed. A DBRT diode is installed at this port and a bias voltage is applied to the device (Fig. 6.5). The NF meter now measures \( F_t \) and corrects it for \( F_c \) (step A) resulting in the value \( F_{dis} \) which is displayed on the instrument.

\( G_{DUT} \) is calculated and also displayed after the calibration and measurement steps have been carried out.
Now the actual measurements on a DBRT diode at the frequency $f$ can be performed versus bias voltage. For the calculation of $F_{\text{DUT}}$ (the noise figure of the DBRT diode + ideal circulator), formula 6.7 is used where $G_{\text{DUT}}$ is the available gain of the DBRT diode (in a package or of the die).

$$F_{\text{DUT}} = 1 + G_{\text{L}} \left( F_{\text{dis}} - F_{\text{L}} + \frac{F_{\text{L}} - 1}{G_{\text{DUT}}} \right)$$  \hspace{1cm} 6.7$$

$F_{\text{dis}}$ is the noise figure on the display of the noise figure meter. As can be seen $F_{\text{dis}}$ is not equal to $F_{\text{DUT}}$ but the noise figure of the DBRT diode can be calculated as soon as one additional measurement to determine $F_{\text{L}}$ and $G_{\text{L}}$ has been carried out.

C) Additional measurement: $F_{\text{L}}$ and $G_{\text{L}}$ (the noise figure and available gain of the circuit path from noise source port 1 to DBRT diode port 4) are measured by connecting the noise figure meter to the DBRT port (Fig. 6.6). The influence of the circuit path from port 4 to the actual DBRT diode terminals (package or die) is not included. This circuit path should be as short as possible. $F_{\text{L}}$ and $G_{\text{L}}$ can be measured after the NF meter is calibrated according to the standard calibration method of the instrument as described in paragraph 5.1.3.

This additional measurement must be done only one time for each measurement frequency. The measured data for our circuit at $f = 1.5$ GHz were: $F_{\text{L}} = 0.85$ dB and $G_{\text{L}} = -0.66$ dB.
Now all quantities can be substituted into eq. 6.7 to calculate the NF of the DBRT diode $F_{D_{UT}}$. This DBRT diode NF measurement method and the results obtained with it will be published [Ref. 41].

Note: The HP8970B NF meter measures the insertion gain of two-ports. The gain factors in the Friis' formula are available gains so eventually a correction must be made.

![Fig. 6.7 Special SMA mount for DBRT diode die](image-url)
6.3 DBRT diode noise figure measurement using only the NF meter:
(NFM-method, version B).

Another version of the DBRT diode NF measurement procedure using only the NF meter (NFM-method, version B) follows the calibration and measurement steps of the NF meter as described in paragraph 5.1.3. The measurement setup is similar to Figures 6.3, 6.5. The feedback $S_{13}$ from output to input port and the reverse transmission parameters of the circulator are assumed to be negligible small so network 4 is omitted.

A The standard NF meter calibration is applied (direct connection of noise source to NF meter) and the NF of the noise figure meter $F_{NFM}$ is measured and stored.

B The overall noise figure $F_{123}$ of the cascaded networks 1, 2 (ideal circulator + biased DBRT diode) and 3 is measured and corrected for $F_{NFM}$, following

\[
F_{123} = F_{123NFM} \cdot \frac{F_{NFM} - 1}{G_1 G_2 G_3}
\]

where $F_{123NFM}$ is the overall NF including the NF meter. $G_2 = G_{DUT}$.

C The overall noise figure $F_{13}$ of the cascaded networks 1 and 3 is measured and corrected for $F_{NFM}$. The DBRT diode port of the circulator is short circuited (= network 1 connected to network 3).

\[
F_{13} = F_{13NFM} \cdot \frac{F_{NFM} - 1}{G_1 G_3}
\]

where $F_{13NFM}$ is the overall NF including the NF meter. The gain $G_2$ can be calculated from the measured and displayed data from steps B and C.

According to Friis the overall noise factor $F_{123NFM}$ of the cascade of two-ports 1,2,3 and the NF meter is

\[
F_{123NFM} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_{NFM} - 1}{G_1 G_2 G_3}
\]

and for the cascade of two-ports 1,3 and the NF meter the overall noise factor reads
Substituting 6.10 in 6.8 and 6.11 in 6.9 yields

\[
F_{13\text{NFM}} = F_1 + \frac{F_3 - 1}{G_1} + \frac{F_{\text{NFM}} - 1}{G_1 G_3}
\] 6.11

where \( F_{13} \) and \( F_{13} \) are measurement results (step B and C). Subtracting 6.13 from 6.12 results in

\[
F_{123} - F_{13} = \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} - \frac{F_3 - 1}{G_1}
\] 6.14

Rearranging 6.14 to yield \( F_2 = F_{\text{DUT}} \) gives

\[
F_2 = 1 + G_1 \left( F_{123} - F_{13} + \frac{F_3 - 1}{G_1} \left( 1 - \frac{1}{G_2} \right) \right)
\] 6.15

which equation resembles the result of the ANA&NFM method.

D Two parameters, \( F_3 \) (the NF of network 3) and \( G_1 \) (the available gain of network 1) have to be measured additionally.

The method requires more labour than the one described in paragraph 6.1.
6.4 DBRT diode noise figure measurement using only the NF meter:
  (NFM-method, version C).

The same steps A, B and C have to be carried out as with version B of the NFM-method, only
some mathematical manipulations must be carried out and the additional measurement is
different.

If eq. 6.13 on both sides is divided by $G_2 (= G_{DUT})$ the result is

$$\frac{F_{13}}{G_2} = \frac{F_1}{G_2} + \frac{F_3 - 1}{G_2 G_1 G_2}$$ 6.16

Subtracting 6.16 from 6.12 yields

$$\frac{F_{123}}{G_2} = \frac{F_1}{G_2} - \frac{F_{13}}{G_1} + \frac{F_3 - 1}{G_1}$$ 6.17

which rearranged to express $F_2 = F_{DUT}$ gives

$$F_2 = 1 + G_1 \left( \frac{F_{13}}{G_2} - F_{13} \left( 1 - \frac{1}{G_2} \right) \right)$$ 6.18

The additional measurement requires the determination of the available gain $G_1$ and noise
figure $F_1$ of network 1 to be able to calculate the NF of the DUT.
This version also is more laborious than version A but less than version B.
7 MEASUREMENT RESULTS

This chapter will give the results of noise measurements carried out on DBRT diodes. Both measurement methods as described in chapters 5 and 6 were used to characterize a DBRT diode with a triangular mesa surface (Δ-DBRT diode). The layer structure is given in Appendix A. This device, fabricated at Nottingham University, U.K., had an effective mesa area of 380 µm² and the chip was mounted in a ceramic package (type F5, L.E.W. Techniques, Taunton, U.K., Fig. 2.5). The package terminates a 7 mm 50 Ω airline. Extensive information on the fixture applied can be found in [20]. The measured results using both noise measurement methods will be compared. In a second experiment the noise figure and the noise measure of a DBRT diode have been investigated over the full bias voltage range with the frequency as a parameter. This was done using an unpackaged DBRT diode die with 20 µm mesa diameter mounted in a special fixture [20], see Fig. 2.6.

7.1 Noise figure measurement using the HP NF meter (NFM-method)

The measurement configuration applied is given in paragraph 6.2 and the data measured on the Δ-DBRT diode is printed in tabular form in Table J.1.

![Graph of I-V curve](image)

Fig. 7.1 The I-V curve of the Δ-DBRT diode plus package; T = 290 K.

A measured DC I-V plot, the NF and attenuation versus bias voltage curves of this Δ-DBRT
diode at f=1.5 GHz are shown in Figures 7.1, 7.2 and 7.3. In the bias voltage range between 641-756 mV the device suffered from bias circuit oscillations, so noise measurements are not performed in that region.

Fig. 7.2 The $F_d$-V characteristic of the Δ-DBRT diode plus package using the HP NF meter; $T = 290$ K; $f = 1.5$ GHz.

Fig. 7.3 The $L_d$-V characteristic of the Δ-DBRT diode plus package using the HP NF meter; $T = 290$ K; $f = 1.5$ GHz.
Noise figure measurements can only be carried out if the DBRT diode is stable (does not oscillate at any frequency). A non-oscillating condition might be very difficult to achieve due to the large bandwidth of the DBRT diode in the NDR region so the stability criteria must be met over a frequency range from DC to several tens of GHz. This means that the impedance of the reflection amplifier circuit seen by the active device must be studied far below and beyond the passband of the circulator used in the circuit and frequently adjustments must be made.

7.2 Noise figure measurement using the network analyzer and the NF meter

The measurement configuration applied is shown in paragraph 5.2. The same Δ-DBRT diode was used as in paragraph 7.1. Table J.2 (Appendix J) gives the measured DC I-V curve (shown in Figure 7.4), data on the total noise figure of the reflection amplifier ($F_t$) and the NF of the DBRT diode ($F_d$) as well as its gain $G_d$. To be able to calculate $F_d$ with this measurement method the S-parameters of the reflection amplifier circuit must be measured (see Tables J3, J4 and J5; $f = 0.8-1.6 \text{GHz}$). Also the reflection coefficients of the noise

![Fig. 7.4](image_url)

Fig. 7.4 The I-V curve of the Δ-DBRT plus package using the network analyzer and the NF meter; $T = 290 \text{K}$.

source and NF meter have to be determined (Tables J.6, J.7; $f = 0.1-1.6 \text{GHz}$).

Finally the S-parameters of the reflection amplifier including the biased DBRT diode have to be measured at each bias level one is interested in (Tables J.8-J.19).
Fig. 7.5 The $F_d$-V characteristic of the Δ-DBRT diode plus package using the network analyzer and the NF meter; $T = 290$ K; $f = 1.5$ GHz.

Fig. 7.6 The $L_d$-V characteristic of the Δ-DBRT diode plus package using the network analyzer and the NF meter; $T = 290$ K; $f = 1.5$ GHz.

Also in this measurement method there must be a stable DBRT diode in the circuit in order to be able to perform the S-parameter and noise measurements.
7.3 Comparison between both methods

Results of both measurement methods will be compared with each other starting with the attenuation curves (see figure 7.7).

In the first positive slope of the curve the attenuation values measured and calculated according to the network analyzer method are lower than those given by the noise figure meter method and also the top attenuation value of the ANA&NFM method is lower than the corresponding value of the NFM method. In the negative part the values correspond very well. Small changes in the measurement conditions may give considerable changes in the results.

In the second positive part the difference between the results of both methods are also very small.

Comparing the noise figures of both Δ-DBRT diode measurements (Fig. 7.8) show that the differences between the $F_d - V_{bias}$ curves are small.

The conclusion is that both methods give results with only a small difference although they are based on totally different calculations.
7.4 The noise measure of the DBRT diode

The measurement routine applied (NFM method) is described in paragraph 6.2. A second measurement was carried out on the Δ-DBRT diode. Measured data and the results of the calculations are given in table J.20. The last column specifies the noise measure $M$ [21,22].

The definition for the noise measure is

$$ M = \frac{\overline{V_n^2}}{4 \ |\text{Re} \ Z| \ k \ T_0 \ B} $$

where $T_0$ is the standard temperature of 290 K, $\overline{V_n^2}$ and $\text{Re} \ Z$ are given in figure 7.9 [23].
The noise source with source impedance

![Diagram](image)

The noise measure can also be written as

\[
M = \frac{F - 1}{|1 - 1/G|}
\]

Fig. 7.10 shows the measured DC I-V curve of the Δ-DBRT diode and Fig. 7.11 depicts the DBRT diode noise figure versus bias voltage diagram.

![Graph](image)

Fig. 7.10  The DC I-V curve of the Δ-DBRT diode plus package.

\[ T = 290 \text{ K}. \]
The attenuation of the device is plotted in Fig. 7.12 while Fig. 7.13 gives the noise measure of the diode as a function of $V_{bias}$. The test frequency is 1.5 GHz.

**Fig. 7.11** The $F_d$-$V$ characteristic of the $\Delta$-DBRT diode plus package using the NF meter; $T = 290 \, K$; $f = 1.5 \, GHz$. 

**Fig. 7.12** The $L_d$-$V$ characteristic of the $\Delta$-DBRT diode plus package using the NF meter; $T = 290 \, K$; $f = 1.5 \, GHz$. 

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Fig. 7.13 The M-V characteristic of the Δ-DBRT diode plus package using the NF meter; \( T = 290 \, K; f = 1.5 \, \text{GHz} \).

For clarity the noise measure of the passive regions are plotted in downward direction (Fig. 7.13). In this curve there are two areas (Area 1: \( V \in [0, 300 \, \text{mV}] \), Area 2: \( V \in [1100 \, \text{mV}, 1200 \, \text{mV}] \)) where \( M \) is about constant.

Now the following assumption is made:

Suppose all the noise power is generated by an equivalent noise resistance \( R_n \) and \( R_n \geq 0 \).

This means for eq. 7.1, that \( V_n^2 \) obeys eq. 7.3

\[
\overline{V_n^2} = 4 k T_0 R_n B \tag{7.3}
\]

where \( |\text{Re} \, Z| \) (eq. 7.1) is the differential resistance of the DBRT diode.

The combination of eqns. 7.1 and 7.3 results in

\[
M = \frac{\frac{4 k T_0 R_n B}{4 k T_0 R_d B}}{R_n} = \frac{R_n}{R_d}
\]

\[
R_n = |M R_d| \tag{7.4}
\]

This gives for both areas the following \( R_n \) :
Table 7.1 The equivalent noise resistance

<table>
<thead>
<tr>
<th>Area 1</th>
<th>Area 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(mV)</td>
<td>Rₙ (kΩ)</td>
</tr>
<tr>
<td>51</td>
<td>7.42</td>
</tr>
<tr>
<td>102</td>
<td>4.04</td>
</tr>
<tr>
<td>147</td>
<td>2.92</td>
</tr>
<tr>
<td>200</td>
<td>1.76</td>
</tr>
<tr>
<td>251</td>
<td>1.32</td>
</tr>
<tr>
<td>300</td>
<td>1.16</td>
</tr>
</tbody>
</table>

These results are depicted in figure 7.14.

If all the noise is shot noise then eq. 7.5 is valid

\[
\frac{4 k T_0}{R_n} = 2 q I_{dern}
\]  

Fig. 7.14 The equivalent noise resistance v.s. V_{bias} of the DBRT diode.

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The results are

Table 7.2 The noise powers

<table>
<thead>
<tr>
<th>V(mV)</th>
<th>( \frac{4kT}{R_n} )</th>
<th>( 2qI_{DBRT} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>2.16 ( 10^{24} )</td>
<td>5.44 ( 10^{24} )</td>
</tr>
<tr>
<td>102</td>
<td>3.96 ( 10^{24} )</td>
<td>1.28 ( 10^{23} )</td>
</tr>
<tr>
<td>147</td>
<td>5.48 ( 10^{24} )</td>
<td>2.27 ( 10^{23} )</td>
</tr>
<tr>
<td>200</td>
<td>9.09 ( 10^{24} )</td>
<td>4.03 ( 10^{23} )</td>
</tr>
<tr>
<td>251</td>
<td>1.21 ( 10^{23} )</td>
<td>6.53 ( 10^{23} )</td>
</tr>
<tr>
<td>300</td>
<td>1.38 ( 10^{23} )</td>
<td>9.76 ( 10^{23} )</td>
</tr>
<tr>
<td>1.10 ( 10^{3} )</td>
<td>1.93 ( 10^{23} )</td>
<td>5.28 ( 10^{22} )</td>
</tr>
<tr>
<td>1.15 ( 10^{3} )</td>
<td>2.77 ( 10^{23} )</td>
<td>6.53 ( 10^{22} )</td>
</tr>
<tr>
<td>1.20 ( 10^{3} )</td>
<td>3.36 ( 10^{23} )</td>
<td>7.90 ( 10^{22} )</td>
</tr>
</tbody>
</table>

The following conclusion can be taken:

The shot noise power of \( I_{DBRT} \) is larger than the total noise power. This means that only a part of \( I_{DBRT} \) contributes to the shot noise power.

7.5 Noise figure of a 20 μm mesa diameter DBRT diode at two frequencies and measured over the full bias voltage range (NFM method)

The measurement configuration and procedure is the same as in paragraph 6.2. (NFM-method). Only the connector at the DBRT diode port is changed to an SMA type since a different test fixture was used (as described in [20] that allowed the mounting of unpackaged DBRT diode chips. However, the built-in bias filter of that test fixture was removed since an external bias T was used. In this fixture a DBRT diode with a mesa diameter of 20 μm is installed (see for diode data Appendix A). Here it is possible to measure almost directly at the DBRT diode terminals without a package in between. With this combination of DBRT diode, special fixture and the reflection amplifier circuit, stable conditions throughout the entire NDR region without relaxation oscillations and microwave oscillations could be achieved. This is a requirement to conduct noise parameter measurements. The measurements were conducted at 1.0 and 1.5 GHz over the full bias voltage range. The frequency range was limited by the circulator used (1-2 GHz) and the NF meter (0.01-1.6 GHz). Tables J.21 and J.22 give the measured and calculated data at both frequencies, while this data is shown in graphical form in Figs. 7.15, 7.16, 7.17 and 7.18.
**Fig. 7.15**  The DC I-V curve of the 20 μm DBRT diode. \( T = 290 \) K.

Figure 7.16 is the noise figure characteristic of the DBRT device at 1 and 1.5 GHz.

**Fig. 7.16**  The \( F_d-V \) characteristic of the unpackaged 20 μm DBRT diode using the NF meter method; \( T = 290 \) K; \( f = 1 \) and 1.5 GHz.

Before the noise figure reaches its local maximum, both curves differ only slightly.
the part between the local maximum and the local minimum both curves are almost identical. This is an important feature because it means that the noise figure in the amplification range is only slightly dependent on the frequency (in the $1.0 - 1.5$ GHz range). After the local minimum both curves start to differ.

![Attenuation vs. Vbias](image)

**Fig. 7.17** The $L_d - V$ characteristic of the unpackaged 20 μm DBRT diode using the NF meter method. $T = 290$ K.

The attenuation curves are almost equal except at high bias voltages. In the amplification area the frequency dependence is very small (in the $1.0 - 1.5$ GHz range).

For clarity the noise measure of the passive regions are plotted in downward direction (Fig. 7.18) while the M of the active region is upwards.

In the first negative part of the $M - V_{bias}$ diagram there is a difference in $M$ measured at both frequencies. A feature that can be found here as well as in figure 7.13 is a small increase of the noise measure before the fast decrease. The explanation could lie in the numerical features of eq. 7.2. In the positive part of the plot both curves are almost identical. This is to be expected because the noise figure as well as the attenuation curves here were also identical. In the second negative part there is only a slight difference between both curves. The difference does not increase with bias voltage as in the noise figure and the attenuation curves.
Fig. 7.18 The $M-V$ curve of the unpackaged 20 μm DBRT diode using the NF meter method. $T = 290 \, K$. 
CONCLUSIONS

* Two methods have been developed to perform noise measurements on stable DBRT diodes over the full bias voltage range. To our knowledge experimental data on the DBRT diode noise behaviour has not been reported yet.

* The ANA&NFM measurement method using the network analyzer and the NF meter, accounts for the feedback behaviour of the circulator as well as the internal reflections and the noise contribution of the circulator.

* The NFM measurement method using only the NF meter accounts for the noise contribution of the circulator. Three versions have been worked out.

* The comparison between the ANA&NFM-method and the NFM-method shows that both methods give results that lay close together. The NFM-method can be conducted in less time and is easier than the ANA&NFM-method.

* The comparison of the resulting equation for the noise figure of a two-port is similar with equations in literature based on current-voltage noise sources.

* The frequency dependence of the NF, the noise measure and the attenuation of the DBRT diode biased in the NDR region is small in the 1.0 - 1.5 GHz range.

* In the NDR region, the noise figure peaks while the noise measure has a minimum. Using the noise measure concept, the influence of the gain is decreased, resulting in easier interpretation and comparison of pictures. When \( G = 1 \), the noise measure shows a singularity (eq. 7.2) so \( M \) is not reliable in that region.
ACKNOWLEDGEMENTS

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Appendix A

THE DBRT DIODE

A.1 Introduction

The Double Barrier Resonant Tunneling (DBRT) diode is a so called semiconductor heterostructure. The heterostructure consists of GaAs and AlGaAs layers. The lattice constant (a) of AlGaAs obeys formula A.1

\[
\begin{align*}
    a_{\text{GaAs}} &< a_{\text{AlGaAs}} < a_{\text{AlAs}} \\
    5.65\,\text{ Å} &< a_{\text{AlGaAs}} < 5.66\,\text{ Å}
\end{align*}
\]

This means that the interface between GaAs and AlGaAs is almost perfect and there will be very few defects, so it is possible to make an almost abrupt interface with a low defect density.

In figure A.1 and in table A.1 a detailed description of the layer structure of a DBRT diode is given.

Fig. A.1 The layer structure of a mesa-shaped DBRT diode (numbers correspond with Table A.1).
Table A.1 The layer structure of a mesa-shaped DBRT diode [20]

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>Doping (at/cm³)</th>
<th>Thickness (nm)</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GaAs</td>
<td>$2 \times 10^{18}$</td>
<td>500</td>
<td>facilitate contact</td>
</tr>
<tr>
<td>2</td>
<td>GaAs</td>
<td>$2 \times 10^{16}$</td>
<td>50</td>
<td>buffer</td>
</tr>
<tr>
<td>3</td>
<td>GaAs</td>
<td>undoped</td>
<td>2.5</td>
<td>spacer</td>
</tr>
<tr>
<td>4</td>
<td>AlGaAs</td>
<td>undoped</td>
<td>5.6</td>
<td>barrier</td>
</tr>
<tr>
<td>5</td>
<td>GaAs</td>
<td>undoped</td>
<td>5.0</td>
<td>well</td>
</tr>
<tr>
<td>6</td>
<td>AlGaAs</td>
<td>undoped</td>
<td>5.6</td>
<td>barrier</td>
</tr>
<tr>
<td>7</td>
<td>GaAs</td>
<td>undoped</td>
<td>2.5</td>
<td>spacer</td>
</tr>
<tr>
<td>8</td>
<td>GaAs</td>
<td>$2 \times 10^{16}$</td>
<td>50</td>
<td>buffer</td>
</tr>
<tr>
<td>9</td>
<td>GaAs</td>
<td>$2 \times 10^{18}$</td>
<td>500</td>
<td>grown on substrate</td>
</tr>
<tr>
<td>10</td>
<td>GaAs</td>
<td>$2 \times 10^{18}$</td>
<td>$&gt;10^5$</td>
<td>substrate</td>
</tr>
</tbody>
</table>

* the real formula for this layer is $\text{Al}_{0.4}\text{Ga}_{0.6}\text{As}$

A.2 The quantum well

The energy gap of GaAs is smaller than the energy gap of AlGaAs. In figure A.2 the energy-band diagram of a DBRT diode is shown.

![Energy-band diagram](image)

\[ E = \frac{n^2 \pi^2}{2m_e w^2} \quad n = 1,2,3,... \tag{A.2} \]

where $E_n$ is the total energy of the particle in a quantum state, $n$ specifies the quantum state, $w$ is the width of the quantum well, $h$ is the Planck constant and $m_e$ is the electron mass. In figure A.3 an energy-band diagram is given with one allowed quantum state in the quantum well.
The energy-band diagram of a DBRT diode with one allowed state in the well.

A.3 The I-V curve of a DBRT diode

Figure A.4a shows a typical I-V curve of a DBRT diode.

Fig. A.4 a,b I-V curves of a DBRT diode

a + indicates a positive polarity on the mesa top

b the four segments of the I-V curve

In figure A.4b the I-V curve is divided into four parts. Each of these parts describes a quantum physical resonant tunneling phenomenon. In figure A.5 the conduction band is sketched with these tunnel phenomena.
Fig. A.5 a,b,c,d  The conduction band diagram of a DBRT diode with several applied voltages

a  $qV_{\text{diode}} = 0$

b  $0 < qV_{\text{diode}} < E_1$

c  $qV_{\text{diode}} = E_1$

d  $E_1 < qV_{\text{diode}} < E_{\text{barrier}}$
Part I of figure A.4b corresponds with figure A.5b. The electrons have to tunnel through the entire barrier-well-barrier structure in order to travel from the cathode to the anode. If the device's bias voltage is increased the electrons which tunnel through the first barrier, enter the well with an energy that is approximately equal to the energy of the allowed state in the well. This leads to a more than proportional increase in the current with increasing bias voltage. This situation corresponds with part II of figure A.4b and figure A.5c.

If the bias voltage is further increased the electrons have to tunnel through the whole barrier-well-barrier structure again. This means that the current decreases with increasing bias voltage. This situation corresponds with part III of figure A.4b and figure A.5d.

When the bias voltage is increased even further a new phenomenon occurs, namely thermionic emission currents over the barrier-well-barrier structure. This leads again to an increase in current with increasing bias voltage. This corresponds with part IV of figure A.4b. For further information see [24, 25, 26].
Appendix B

EQUIVALENT TWO-PORT NOISE GENERATOR CONFIGURATIONS

There are several possible equivalent noise generator configurations that can be used with the two-port. They are listed below with the proper equations [9].

\[
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix}
= \begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
+ \begin{pmatrix}
  S_{11} & 1 \\
  S_{21} & 0
\end{pmatrix}
\begin{pmatrix}
  a_{n1} \\
  b_{n1}
\end{pmatrix}
\]

\[ B.1 \]

\[
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix}
= \begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
+ \begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a_{n1} \\
  a_{n2}
\end{pmatrix}
\]

\[ B.2 \]

\[
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix}
= \begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
+ \begin{pmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
  a_{n1} \\
  b_{n2}
\end{pmatrix}
\]

Fig. B.1 First equivalent noise generator configuration

Fig. B.2 Second equivalent noise generator configuration

Fig. B.3 Third equivalent generator configuration.
\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix} + \begin{bmatrix}
  S_{11} & 0 \\
  S_{21} & 1
\end{bmatrix} \begin{bmatrix}
  a_{n1} \\
  a_{n2}
\end{bmatrix}
\]
\[\text{B.3}\]

**Fig. B.4 Fourth equivalent generator configuration.**

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix} + \begin{bmatrix}
  1 & S_{12} \\
  0 & S_{22}
\end{bmatrix} \begin{bmatrix}
  b_{n1} \\
  b_{n2}
\end{bmatrix}
\]
\[\text{B.4}\]

**Fig. B.5 Fifth equivalent generator configuration.**

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix} + \begin{bmatrix}
  b_{n1} \\
  b_{n2}
\end{bmatrix}
\]
\[\text{B.5}\]

**Fig. B.6 Sixth equivalent generator configuration.**

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If eq. B.1 through eq. B.6 are compared with each other, it's clear that eq. B.5 is the simplest. All other equations consist of an addition of two matrix-vector multiplications. Only eq. B.5 consists of only one matrix-vector multiplication and one vector addition. In this report, figure B.5 and eq. B.5 are used to represent the equivalent noise generators of the two-port since the internal noise sources react on the waves that emerge from the two-port.

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix} + \begin{bmatrix}
  0 & S_{12} \\
  1 & S_{22}
\end{bmatrix} \begin{bmatrix}
  b_{n2} \\
  a_{n2}
\end{bmatrix}
\]

\[\text{B.6}\]
Appendix C

THE AVAILABLE GAIN OF A TWO-PORT

The available gain $G_a$ of a two-port can be calculated from its scattering matrix and the reflection coefficients at the terminals. The theory is described in this appendix and the final equation is applied in the computer program that calculates the NF of a DBRT diode.

C.1 Principle and calculation of $G_a$

\[
\begin{align*}
\text{Fig. C.1} \ a,b,c \ & \text{The equivalent generator principle} \\
\text{Figure C.1a shows a generator connected to a load and the available power this generator delivers is } ([30, 31, 32]): \\
P_G = \frac{|b_G|^2}{1 - |\Gamma_G|^2} \tag{C.1}
\end{align*}
\]

where $P_G$ is the available power, $b_G$ the wave amplitude and $\Gamma_G$ the reflection coefficient of the generator. Figure C.1b shows the same generator and load as in figure C.1a but with an inserted two-port element.

Figure C.1c shows an equivalent generator connected to the same load as before. This generator consists of the old generator and the two-port, so the generator plane has shifted from the input side of the two-port to the output side of the two-port.
The available power delivered by this equivalent generator is:

\[ P_G^\sim = \frac{|b_G^\sim|^2}{1 - |\Gamma_G^\sim|^2} \tag{C.2} \]

where \( P_G^\sim \) is the available power, \( b_G^\sim \) the wave amplitude and \( \Gamma_G^\sim \) the reflection coefficient of the equivalent generator.

The assumption made here is that \( Z_{\text{in}} = Z_{\text{out}} \) holds for the characteristic impedances of input and output lines.

The available gain \( G_a \) can be calculated from eq. C.3

\[ G_a = \frac{P_G^\sim}{P_G} \tag{C.3} \]

In order to remove all generator wave amplitude terms, \( b_G^\sim \) has to be expressed as a complex function of \( b_G \) according to

\[ b_G^\sim = C \cdot b_G \tag{C.4} \]

If eqns. C.1 and C.2 are substituted into eq. C.3, the result is

\[ G_a = \frac{|b_G^\sim|^2}{1 - |\Gamma_2|^2} = \frac{|b_G^\sim|^2 \left( 1 - |\Gamma_G^\sim|^2 \right)}{|b_G|^2 \left( 1 - |\Gamma_2|^2 \right)} \tag{C.5} \]

where \( \Gamma_2 \) is identical to \( \Gamma_G^\sim \). The incident wave \( a_i \) can be derived by

\[ a_i = b_G + \Gamma_G b_i \tag{C.6} \]

This means that the incident wave \( a_i \) is an addition of the generator wave \( b_G \) and the reflected part of the emergent wave \( b_i \). The emergent wave \( b_i \) is the reflected part of \( a_i \).

\[ b_i = \Gamma_i a_i \tag{C.7} \]

If eq. C.7 is substituted into eq. C.6, the result is

\[ a_i = \frac{b_G}{1 - \Gamma_G \Gamma_i} \tag{C.8} \]
The emergent wave $b_2$ can be derived by

$$b_2 = b_0^\sim + \Gamma_2 a_2$$  \hspace{1cm} (C.9)

This means that the emergent wave $b_2$ is an addition of the equivalent generator wave $b_0^\sim$ and the reflected part of the incident wave $a_2$. The incident wave $a_2$ is the reflected part of $b_2$.

$$a_2 = \Gamma_L b_2$$  \hspace{1cm} (C.10)

Eq. C.10 substituted into eq. C.9 results in

$$b_0^\sim = b_2 \left( 1 - \Gamma_2 \Gamma_L \right)$$  \hspace{1cm} (C.11)

The S-matrix equations are given in eqns. C.12a,b

$$b_1 = S_{11} a_1 + S_{12} a_2$$  \hspace{1cm} (C.12a)

$$b_2 = S_{21} a_1 + S_{22} a_2$$  \hspace{1cm} (C.12b)

When eqns. C.8 and C.10 are substituted into eq. C.12b, eq. C.13 results

$$b_2 = \frac{S_{21} b_G}{\left[ 1 - \Gamma_G \Gamma_1 \right] \left[ 1 - S_{22} \Gamma_L \right]}$$  \hspace{1cm} (C.13)

Now eq. C.13 is substituted into eq. C.11

$$b_0^\sim = \frac{S_{21} \left[ 1 - \Gamma_2 \Gamma_L \right] b_G}{\left[ 1 - \Gamma_G \Gamma_1 \right] \left[ 1 - S_{22} \Gamma_L \right]}$$  \hspace{1cm} (C.14)

In eq. C.14 the variables $\Gamma_1$ and $\Gamma_2$ have to be replaced. $\Gamma_i$ is defined according to

$$\Gamma_1 = \frac{b_1}{a_1}$$  \hspace{1cm} (C.15)

Now eq. C.12a must be substituted into eq. C.15

$$\Gamma_1 = S_{11} + S_{12} \frac{a_2}{a_1}$$  \hspace{1cm} (C.16)

Eq. C.12b combined with eq. C.10 gives

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\[ b_2 = \frac{S_{21} a_1}{1 - S_{22} \Gamma_L} \quad \text{(C.17)} \]

The combination of eqns. C.10, C.16 and C.17 gives
\[ \Gamma_1 = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - S_{22} \Gamma_L} = \frac{S_{11} - \Gamma_L \Delta}{1 - S_{22} \Gamma_L} \quad \text{(C.18)} \]

Where \( \Delta = S_{11} S_{22} - S_{12} S_{21} \). Eq. C.19 defines \( \Gamma_2 \)
\[ \Gamma_2 = \frac{b_2}{a_2} \quad \text{(C.19)} \]

Now eq. C.12b must be substituted into eq. C.19
\[ \Gamma_2 = S_{22} + \frac{S_{21}}{a_2} \quad \text{(C.20)} \]

Eq. C.15a combined with
\[ a_1 = \Gamma_G b_1 \quad \text{(C.21)} \]
yields:
\[ b_1 = S_{11} \Gamma_G b_1 + S_{12} a_2 = \frac{S_{12} a_2}{1 - S_{11} \Gamma_G} \quad \text{(C.22)} \]

the combination of the eqns. C.20, C.21 and C.22 yields
\[ \Gamma_2 = S_{22} + \frac{S_{21} \Gamma_G S_{12}}{1 - S_{11} \Gamma_G} = \frac{S_{22} - \Gamma_G \Delta}{1 - S_{11} \Gamma_G} \quad \text{(C.23)} \]

Substituting eqns. C.18 and C.23 into eq. C.14 yields
\[ b_G \sim = \frac{S_{21} \left( 1 - \Gamma_L \left( \frac{S_{22} - \Gamma_G \Delta}{1 - S_{11} \Gamma_G} \right) \right) b_G}{\left[ 1 - \Gamma_G \left( \frac{S_{11} - \Gamma_L \Delta}{1 - S_{22} \Gamma_L} \right) \right] \left( 1 - S_{22} \Gamma_L \right)} \]

\[ 63 \]
which can be reduced to

\[
\frac{b_G^*}{b_G} = \frac{S_{21}}{1 - S_{11} \Gamma_G} \quad \text{C.24}
\]

At this point the situation of eq. C.4 is reached. Substituting C.24 into eq. C.5 yields

\[
G_a = \frac{|S_{21}|^2 \left( 1 - |\Gamma_G|^2 \right)}{|1 - S_{11} \Gamma_G|^2 \left( 1 - |\Gamma_2|^2 \right)} \quad \text{C.25a}
\]

or with eq. C.23 (first part) for \( \Gamma_2 \)

\[
G_a = \frac{|S_{21}|^2 \left( 1 - |\Gamma_G|^2 \right)}{|1 - S_{11} \Gamma_G|^2 - |S_{22} - \Gamma_G \Delta|^2} \quad \text{C.25b}
\]

Eqns. C.25a,b can also be found in [30, 31].

C.2 The calculation of \( G_a \) of an adapted two-port

\[
G_a = \frac{|S_{21}|^2 \left( 1 - |\Gamma_G|^2 \right)}{|1 - S_{11} \Gamma_G|^2} \quad \text{C.26}
\]

Fig. C.2 Adapted two-port block diagram
Appendix D

THE 3x3 S-MATRIX OF THE REFLECTION AMPLIFIER

In this appendix the equations of the S-matrix elements $S_{ij}^\Delta$ of an arbitrary terminated three-port (circulator plus connected bias T) will be derived. The three-port is terminated at the ports with loads having reflection coefficients $\Gamma_G$, $\Gamma_D$ and $\Gamma_L$. In the noise figure measurements these quantities represent the reflection coefficients of respectively noise generator, DBRT diode and load (noise figure meter). The S-parameters $S_{ij}$ of the three-port (terminated with matched loads) and the reflection coefficients $\Gamma_G$, $\Gamma_D$ and $\Gamma_L$ have to be determined by network analyser measurements.

D.1 Three-port flow graph calculations

An S-parameter can be regarded as a transfer function from an input node to an output node. Figure D.1 shows the signal flow graph of the reflection amplifier.

![Signal flow graph of the reflection amplifier](image)

Fig. D.1 Signal flow graph of the reflection amplifier.

In [33,34] some properties of signal flow graphs are derived. In [34,35,36] Mason’s gain rule is explained. The S-matrix of the three-port with arbitrary loads is calculated using this rule and the elements are represented by $S_{ij}^\Delta$.

As an example the calculation of $S_{11}^\Delta$ will be given step by step.
There are eight loops in the signal flow graph shown in figure D.1 ([30, 37, 38]), given by equations D.1 - D.8.

\[
\begin{align*}
L_1 &= \Gamma_G S_{11} & \text{D.1} \\
L_2 &= \Gamma_D S_{22} & \text{D.2} \\
L_3 &= \Gamma_L S_{33} & \text{D.3} \\
L_4 &= \Gamma_G S_{21} \Gamma_D S_{12} & \text{D.4} \\
L_5 &= \Gamma_G S_{31} \Gamma_L S_{13} & \text{D.5} \\
L_6 &= \Gamma_D S_{32} \Gamma_L S_{23} & \text{D.6} \\
L_7 &= \Gamma_G S_{21} \Gamma_L S_{32} \Gamma_D S_{13} & \text{D.7} \\
L_8 &= \Gamma_G S_{31} \Gamma_L S_{23} \Gamma_D S_{12} & \text{D.8}
\end{align*}
\]

The first path is \( S_{11} \)

The nontouching loops of this path are: \( L_2, L_3 \) and \( L_6 \)
The loops \( L_2 \) and \( L_3 \) are also nontouching.

This yields the first numerator term:

\[
S_{11} \left( 1 - L_2 - L_3 + L_2 L_3 - L_6 \right) \quad \text{D.9}
\]

The second path is \( S_{21} \Gamma_D S_{12} \)
The nontouching loop is \( L_3 \)

This gives the second numerator term

\[
S_{21} \Gamma_D S_{12} \left( 1 - L_3 \right) \quad \text{D.10}
\]

The third path is \( S_{31} \Gamma_L S_{13} \)
The nontouching loop is \( L_2 \)

This yields the third numerator term

\[
S_{31} \Gamma_L S_{13} \left( 1 - L_2 \right) \quad \text{D.11}
\]

The fourth path is \( S_{31} \Gamma_L S_{23} \Gamma_D S_{12} \)
This path has only touching loops

This gives the fourth numerator term

\[
S_{31} \Gamma_L S_{23} \Gamma_D S_{12} \quad \text{D.12}
\]
The fifth path is: 

\[ S_{21} \Gamma_D S_{32} \Gamma_L S_{13} \]

This path has only touching loops.

This yields the last numerator term:

\[ S_{21} \Gamma_D S_{32} \Gamma_L S_{13} \quad \text{D.13} \]

Eq. D.9 through eq. D.13 added together give the numerator of the equation for \( S_{11}^\Delta \). The determinant of the signal flow graph is:

\[ \Phi = 1 - L_1 - L_2 - L_3 - L_4 - L_5 - L_6 - L_7 - L_8 + L_1 L_2 L_3 + L_2 L_3 L_4 + L_1 L_2 L_6 + L_1 L_6 L_2 L_3 \]

This yields for \( S_{11}^\Delta \):

\[ S_{11}^\Delta = \frac{S_{11}(1-L_2-L_3+L_2L_3-L_6)+S_{21} \Gamma_D S_{12}(1-L_3)+S_{31} \Gamma_L S_{13}(1-L_2)}{\Phi} + \]

\[ S_{31} \Gamma_L S_{23} \Gamma_D S_{12} + S_{21} \Gamma_D S_{32} \Gamma_L S_{13} \quad \text{D.15} \]

In a similar way the other elements of the three-port matrix can be calculated resulting in:

\[ S_{12}^\Delta = \frac{S_{12}(1-L_3)+S_{13} \Gamma_L S_{32}}{\Phi} \quad \text{D.16} \]

\[ S_{21}^\Delta = \frac{S_{21}(1-L_3)+S_{23} \Gamma_L S_{31}}{\Phi} \quad \text{D.17} \]

\[ S_{22}^\Delta = \frac{S_{22}(1-L_1-L_3+L_1L_3-L_2)+S_{21} \Gamma_G S_{12}(1-L_3)+S_{23} \Gamma_L S_{32}(1-L_1)}{\Phi} \]

\[ S_{21} \Gamma_G S_{13} \Gamma_L S_{32} + S_{23} \Gamma_L S_{31} \Gamma_G S_{12} \quad \text{D.18} \]
\[ S_{23}^\Delta = \frac{S_{23} (1-L_1) + S_{21} \Gamma_G S_{13}}{\Phi} \]  \hspace{1cm} \text{(D.19)}

\[ S_{32}^\Delta = \frac{S_{32} (1-L_1) + S_{31} \Gamma_G S_{12}}{\Phi} \]  \hspace{1cm} \text{(D.20)}

\[ S_{33}^\Delta = \frac{S_{33} (1-L_1 L_2 + L_1 L_2 L_4) + S_{32} \Gamma_D S_{23} (1-L_1) + S_{31} \Gamma_G S_{13} (1-L_2)}{\Phi} \]

\[ + \frac{S_{32} \Gamma_D S_{21} \Gamma_G S_{13} + S_{31} \Gamma_G S_{12} \Gamma_D S_{23}}{\Phi} \]  \hspace{1cm} \text{(D.21)}

\[ S_{31}^\Delta = \frac{S_{31} (1-L_2) + S_{32} \Gamma_D S_{21}}{\Phi} \]  \hspace{1cm} \text{(D.22)}

\[ S_{13}^\Delta = \frac{S_{13} (1-L_2) + S_{12} \Gamma_D S_{23}}{\Phi} \]  \hspace{1cm} \text{(D.23)}

The matrix elements are calculated in a computer program.
Appendix E

THE CALCULATION OF $\Gamma_D$

To calculate the noise parameters of a DBRT diode it is necessary to know the available gain of the biased device which can be determined from the reflection coefficient $\Gamma_D$ of the DBRT diode terminating a transmission line. The DBRT diode noise parameters must be measured on a stable device (non-oscillating) so the circuit impedance presented to the diode is critical. The gain of the device preferably should be measured under the same load impedance conditions, so noise and gain measurements have to be carried out on the DBRT diode installed in the reflection amplifier circuit. This appendix derives an expression to calculate $\Gamma_D$ from measurements on the complete amplifier circuit. The calculation of $\Gamma_D$ is done using eq. D.22 (appendix D) which reads for the transmission coefficient $S^\Delta_{31}$ under arbitrary $\Gamma_G$ and $\Gamma_L$ conditions:

$$S^\Delta_{31} = \frac{S_{31} \left[ 1 - L_2 \right] + S_{32} \Gamma_D S_{21}}{\Phi}$$

D.22

Substituting eq. D.2 and eq. D.14 into eq. D.22 yields

$$S^\Delta_{31} = \frac{S_{31} - S_{31} \Gamma_D S_{22} + S_{32} \Gamma_D S_{21}}{1 - \Gamma_G S_{11} - \Gamma_L S_{33} - \Gamma_D S_{22} - \Gamma_G S_{12} - \Gamma_D S_{21} - \Gamma_G S_{13} - \Gamma_L S_{31}}$$

E.1
Rearranging eq. E.1 the following equation for $\Gamma_D$ can be obtained:

$$\Gamma_D = \frac{S_{31} - S^\Delta_{31}}{S^\Delta_{31}} \left( 1 - \frac{\Gamma L S_{12} + \Gamma G S_{32} - \Gamma G S_{11} - \Gamma L S_{33} - \Gamma S_{13} S_{33} + S_{11} G L S_{33}}{\Gamma S_{12} + \Gamma L S_{22} + \Gamma G S_{32} + \Gamma S_{33} + \Gamma S_{13} S_{33}} \right)$$

\[ E.2 \]

Quantities that have to be measured are the full $S$-parameter set $S_{ij}$ of the three-port (circulator + bias T) with all ports matched and separately the transmission coefficient $S^\Delta_{31}$ from (noise generator) port 1 to port 3 with biased DBRT diode connected to port 2. To measure $S$-parameters of microwave components a two-port Vector Network Analyser is used which is calibrated to enhance the accuracy of the measurement. This calibration procedure effectively approximates a perfect match on both ports of the analyser. Measuring the forward transmission coefficient $\Gamma^\Delta_{31}$ of the reflection amplifier with biased DBRT diode with the calibrated analyser, the assumption $\Gamma_G = \Gamma_L = 0$ is allowed, so equation E.2 simplifies to

$$\Gamma_D = \frac{S^m_{31} - S_{31}}{S_{32} S_{31} + S_{32} \left( S^m_{31} - S_{31} \right)}$$

\[ E.3 \]

where $S^m_{31} = S^\Delta_{31} \mid \Gamma_G = \Gamma_L = 0$
Appendix F

THE NOISE FIGURE OF A GENERAL TWO-PORT

In this appendix the formula for the noise figure of a general two-port will be derived. The derivation will account for the scattering parameters of the two-port and the reflection coefficients of generator and load. The formula for the noise figure is

\[
F = \frac{P_G / P_{nG}}{P^\prime_G / P^\prime_{nG}} = \frac{P_G P_{nG}^\prime}{P^\prime G P_{nG}} = \frac{|b_G|^2 |b_{nG}^\prime|^2}{|b_G^\prime|^2 |b_{nG}|^2}
\]

F.1

where \( P_G \) is the available signal power at the input port, \( P_{nG} \) the available input noise power equal to the available noise power generated by a resistor at a temperature of 290 K, \( P_G^\prime \) the available signal power at the output port and \( P_{nG}^\prime \) the available output noise power. The corresponding signal and noise wave amplitudes are indicated with \( b_G \) and \( b_{nG} \).

F.1.1 The relation between \( b_G \) and \( b_{G}^\prime \)

In figure F.1 the signal circuit diagram of a general two-port is shown.

---

In figure F.1 the signal circuit diagram of a general two-port is shown.

---

Eq. C.24 gives the relationship of \( b_{G}^\prime \) to \( b_G \) for a general two-port, which equation also can be written as

\[
\frac{|b_G|^2}{|b_G^\prime|^2} = \frac{|1 - S_{11} \Gamma_G|^2}{|S_{21}|^2}
\]

F.2

F.2.1 Principle
The calculation of the relation between $b_{nG}$ and $b_n^-$ resembles the calculation of the available gain in appendix C. The main difference is that in appendix C the calculations were done with "normal" signals, here the calculations are with noise signals. Figure F.2 shows the two-port with noise generator and load.

![Two-port with noise generator, load and noise sources](image)

**Fig. F.2** The two-port with noise generator, load and noise sources

First $b_{nG}^-$ will be expressed as a function of $b_{nG}$, $b_{n1}$ and $b_{n2}$ as shown in eq. F.3

$$b_{nG}^- = \mathcal{R} b_{nG} + \mathcal{P} b_{n1} + \mathcal{Q} b_{n2}$$  \hspace{1cm} F.3

Because of the fact that the measurement will be conducted in a small frequency band, the noise signals may be treated as if they were deterministic signals.

**F.2.2 Calculation of the noise figure of a general two-port**

In this paragraph all variables used in the calculation are noise variables. Fig. F.2 shows the noise circuit. For the incident wave $a_{i}$ eq. F.4 holds

$$a_{i} = b_{nG} + \Gamma_{G} b_{i}$$  \hspace{1cm} F.4

The emergent wave $b_{i}$ is an addition of two waves, given by

$$b_{i} = b_{3} + b_{n1}$$  \hspace{1cm} F.5

Now eq. F.5 must be substituted into eq. F.4

$$a_{i} = b_{nG} + \Gamma_{G} (b_{3} + b_{n1})$$  \hspace{1cm} F.6

The emergent wave $b_{3}$ of the two-port is given by the S-matrix relation

$$b_{3} = S_{11} a_{i} + S_{12} a_{2}$$  \hspace{1cm} F.7

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Eq. F.7 substituted into eq. F.6 yields

$$a_1 = b_{nG} + \Gamma_G (b_{n1} + S_{11}a_1 + S_{12}a_2)$$  \hspace{1cm} F.8

which can be evaluated to

$$a_1 = \frac{b_{nG} + \Gamma_G b_{n1} + \Gamma_{12}a_2}{1 - S_{11}\Gamma_G}$$  \hspace{1cm} F.9

Figure F.3 shows the equivalent noise generator.

![Equivalent noise generator](image)

**Fig. F.3 Equivalent noise generator with load.**

For the emergent wave $b_2$ eq. F.10 yields

$$b_2 = b_{nG}^\infty + a_2 \Gamma_2 \quad \text{or} \quad b_{nG}^\infty = b_2 - a_2 \Gamma_2$$  \hspace{1cm} F.10

This means that $b_2$ is an addition of the internal noise wave $b_{nG}^\infty$ plus the reflected part from the incident noise wave $a_2$. The internal noise wave $b_{nG}^\infty$ is the addition of $b_{n2}$ and the non-reflected part of $a_1$. Furthermore $b_2$ is an addition of two waves (see figure F.2)

$$b_2 = b_4 + b_{n2}$$  \hspace{1cm} F.11

The S-matrix of the two-port defines $b_4$

$$b_4 = S_{21}a_1 + S_{22}a_2$$  \hspace{1cm} F.12

When eqns. F.11 and F.12 are substituted into eq. F.10 the result is

$$b_{nG}^\infty = S_{21}a_1 + (S_{22} - \Gamma_2)a_2 + b_{n2}$$  \hspace{1cm} F.13

The substitution of eq. F.9 into eq. F.13 yields

$$b_{nG}^\infty = \frac{S_{21}(b_{nG} + \Gamma_G b_{n1} + \Gamma_{12}a_2)}{1 - S_{11}\Gamma_G} + (S_{22} - \Gamma_2)a_2 + b_{n2}$$  \hspace{1cm} F.14

Further evaluation gives
\[ b_{nG}^- = \frac{S_{21} \left( b_{nG} + \Gamma_G b_{n1} \right)}{1 - S_{11} \Gamma_G} + \frac{S_{21} S_{12} \Gamma_G a_2}{1 - S_{11} \Gamma_G} + (S_{22} - \Gamma_2) a_2 + b_{n2} \quad \text{F.15} \]

The equation for \( \Gamma_2 \) (see eq. C.23) reads

\[ \Gamma_2 = S_{22} + \frac{S_{21} S_{12} \Gamma_G}{1 - S_{11} \Gamma_G} \quad \text{or} \quad \frac{S_{21} S_{12} \Gamma_G}{1 - S_{11} \Gamma_G} = \Gamma_2 - S_{22} \quad \text{F.16} \]

Eq. F.16 substituted into the second term of eq. F.15 yields

\[ b_{nG}^- = \frac{S_{21} \left( b_{nG} + \Gamma_G b_{n1} \right)}{1 - S_{11} \Gamma_G} + (\Gamma_2 - S_{22}) a_2 + (S_{22} - \Gamma_2) a_2 + b_{n2} \]

\[ b_{nG}^- = \frac{S_{21} \left( b_{nG} + \Gamma_G b_{n1} \right)}{1 - S_{11} \Gamma_G} + b_{n2} \quad \text{F.17} \]

The result \(|b_{nG}^-|^2\) is

\[ |b_{nG}^-|^2 = \left| S_{21} \left( b_{nG} + \Gamma_G b_{n1} \right) + b_{n2} \right|^2 \quad \text{F.18} \]

so the relation between \(|b_{nG}^-|^2\) and \(|b_{nG}|^2\) becomes

\[ \frac{|b_{nG}^-|^2}{|b_{nG}|^2} = \frac{\left| S_{21} \left( b_{nG} + \Gamma_G b_{n1} \right) + b_{n2} \right|^2}{|b_{nG}|^2} \quad \text{F.19} \]

Combining eqns. F.2 and F.19 into F.1 gives the noise figure of the two-port

\[ F = \frac{1 - S_{11} \Gamma_G^2}{|S_{21}|^2} \quad \frac{\left| S_{21} \left( b_{nG} + \Gamma_G b_{n1} \right) + b_{n2} \right|^2}{|b_{nG}|^2} \quad \text{F.20} \]

Before the equation is written out, a few assumptions are made:

- The equivalent noise generators \( b_{n1} \) and \( b_{n2} \) are correlated.
- The equivalent noise generator \( b_{n1} \) and the noise generator \( b_{nG} \) are uncorrelated.
- The equivalent noise generator \( b_{n2} \) and the noise generator \( b_{nG} \) are uncorrelated.
In statistical expectation notation:

\[ E \left[ b_{n1}(t) b_{n2}(t+\tau) \right] \neq 0 \] \hspace{1cm} F.21a

\[ E \left[ b_{nG}(t) b_{n1}(t+\tau) \right] = 0 \] \hspace{1cm} F.21b

\[ E \left[ b_{nG}(t) b_{n2}(t+\tau) \right] = 0 \] \hspace{1cm} F.21c

The first assumption is made since both equivalent noise sources have their origin in the same noisy two-port. The other two assumptions are made since the external generator is totally independent from the two-port.

F.3 Noise figure of the general two-port

Equation F.20 can be written as

\[
F = \frac{1 - \Gamma_G S_{11}}{|S_{21}|^2} \left\{ \frac{|S_{21}|^2 |b_{nG}|^2}{1 - \Gamma_G S_{11} |b_{nG}|^2} + \frac{|S_{21}|^2 |\Gamma_G|^2 |b_{n1}|^2}{1 - \Gamma_G S_{11} |b_{nG}|^2} + \frac{|b_{n2}|^2}{|b_{nG}|^2} \frac{S_{21} \Gamma_G b_{n1}^* b_{n2}^*}{1 - S_{11} \Gamma_G} + \frac{S_{21}\Gamma_G}{(1 - S_{11} \Gamma_G)^2} \frac{|b_{n1}|^2 |b_{n2}|^2}{|b_{nG}|^2} \right\} \hspace{1cm} F.22
\]

Eq. F.22 can be simplified to result the NF equation of the general two-port

\[
F = 1 + |\Gamma_G|^2 \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{|1 - \Gamma_G S_{11}|^2}{|S_{21}|^2} \frac{|b_{n2}|^2}{|b_{nG}|^2} + \frac{\Gamma_G (1 - \Gamma_G S_{11}) b_{n1}^* b_{n2}^*}{S_{21}^* |b_{nG}|^2} + \frac{\Gamma_G^* (1 - \Gamma_G S_{11}) b_{n1}^* b_{n2}^*}{S_{21} |b_{nG}|^2} \hspace{1cm} F.23
\]
Appendix G

NOISE FIGURE OF THE ADAPTED TWO-PORT

Figure G.1 shows the adapted two-port.

\[ \Gamma_1 \quad \Gamma_G \quad \Gamma_2 \]

\[ b_{nG} \quad a_1 \quad b_1 \quad S_{11} \quad S_{21} \quad b_2 \quad b_{n2} \]

\[ \text{Fig. G.1 The adapted "two-port"} \]

Eq. F.1 is also valid for this adapted two-port. If eqns. F.2, F.5, F.7 and F.8 are substituted into eq. F.1 the result is

\[ F = \frac{|b_G|^2 - |\Gamma_G|^2}{1 - |\Gamma_G|^2} = \frac{|b_{nG}|^2 - |\Gamma_2|^2}{1 - |\Gamma_2|^2} \]

Next the relation between \( |b_G|^2 \) and \( |b_G^-|^2 \) is calculated. Both \( b_G \) and \( b_G^- \) are signal waves.

\[ |b_G|^2 = |b_G^-|^2 \]

\[ \text{Fig. G.2 The signal circuit diagram of the adapted two-port.} \]

Eq. C.9 is no longer valid, it becomes

\[ b_2 = b_G^- \]

Eq. C.6 repeated here states
Eq. C.12 also changes

\[ b_1 = S_{11} a_1 \]  \hspace{1cm} G.4a

\[ b_2 = S_{21} a_1 \]  \hspace{1cm} G.4b

Now eq. G.4a can be substituted into eq. G.3

\[ a_1 = \frac{b}{1 - \Gamma_G S_{11}} \]  \hspace{1cm} G.5

Eq. G.5 substituted into eq. G.4b leads to

\[ b_2 = \frac{S_{21} b_G}{1 - \Gamma_G S_{11}} \]  \hspace{1cm} G.6

Eq. G.6 substituted into eq. G.2 gives

\[ b^- = \frac{S_{21} b_G}{1 - \Gamma_G S_{11}} \]  \hspace{1cm} G.7

This means for \( |b^-|^2 \)

\[ |b^-|^2 = \frac{|S_{21}|^2 |b_G|^2}{|1 - \Gamma_G S_{11}|^2} \]  \hspace{1cm} G.8

The noise circuit diagram of the adapted two-port is given by figure G.1.

Eq. F.15 is no longer valid, it becomes

\[ b_7 = b^-_{nG} \]  \hspace{1cm} G.9

\( b_7 \) is an addition of two waves

\[ b_7 = b_2 + b_{n2} \]  \hspace{1cm} G.10

Substituting eqns. G.4b and G.9 into eq. G.10 yields

\[ b^-_{nG} = S_{21} a_1 + b_{n2} \]  \hspace{1cm} G.11

The only changes that have to be made in eq. F.10 are the subscripts, it becomes

\[ a_1 = b_{nG} + \Gamma_G b_{10} \]  \hspace{1cm} G.12
The emergent wave $b_{10}$ is an addition of two waves

$$b_{10} = b_{n1} + b_1$$  \hspace{1cm} G.13

Now eqns. G.13 and G.4a are substituted into eq. G.12

$$a_1 = \frac{b_{nG} + \Gamma_G b_{n1}}{1 - \Gamma_G S_{11}}$$  \hspace{1cm} G.14

Now eq. G.14 must be substituted into eq. G.11 the result is

$$b_{nG}^{-} = \frac{S_{21}(b_{nG} + \Gamma_G b_{n1})}{1 - \Gamma_G S_{11}} + b_{n2}$$  \hspace{1cm} G.15

This yields for $|b_{nG}^{-}|^2$

$$|b_{nG}^{-}|^2 = \left| \frac{S_{21}(b_{nG} + \Gamma_G b_{n1})}{1 - \Gamma_G S_{11}} + b_{n2} \right|^2$$  \hspace{1cm} G.16

The relation between $|b_{nG}^{-}|^2$ and $|b_{nG}|^2$ becomes

$$\frac{|b_{nG}^{-}|^2}{|b_{nG}|^2} = \left| \frac{S_{21}(b_{nG} + \Gamma_G b_{n1})}{1 - \Gamma_G S_{11}} + b_{n2} \right|^2$$  \hspace{1cm} G.17

Substituting eqns. G.8 and G.17 into eq. G.1 yields

$$F = \frac{\left| 1 - \Gamma_G S_{11} \right|^2}{\left| S_{21} \right|^2} \left| \frac{S_{21}(b_{nG} + \Gamma_G b_{n1})}{1 - \Gamma_G S_{11}} + b_{n2} \right|^2$$  \hspace{1cm} G.18

The assumptions made in appendix F are also valid in this configuration. The assumptions were

$$\mathbb{E} \left[ b_{nG}(t) b_{n1}(t+\tau) \right] = 0$$  \hspace{1cm} G.19a

$$\mathbb{E} \left[ b_{nG}(t) b_{n2}(t+\tau) \right] = 0$$  \hspace{1cm} G.19b

$$\mathbb{E} \left[ b_{n1}(t) b_{n2}(t+\tau) \right] \neq 0$$  \hspace{1cm} G.19c
Now eq. G.18 can be written as

\[
F = \frac{|1 - \Gamma_G S_{11}|^2}{|S_{21}|^2} \left( \frac{|S_{21}|^2 |b_{nG}|^2}{|1 - \Gamma_G S_{11}|^2 |b_{nG}|^2} + \frac{|S_{21}|^2 |\Gamma_G|^2 |b_{n1}|^2}{|1 - \Gamma_G S_{11}|^2 |b_{nG}|^2} + \frac{|b_{n1}|^2}{|b_{nG}|^2} \right)
\]

Eq. G.20 can be simplified

\[
F = 1 + |\Gamma_G|^2 \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{|1 - \Gamma_G S_{11}|^2}{|S_{21}|^2} \frac{|b_{n2}|^2}{|b_{nG}|^2} + \frac{\Gamma_G (1 - \Gamma_G S_{11})^*}{S^*_{21}} \frac{b_{n1} b_{n2}^*}{|b_{nG}|^2} \frac{\Gamma_G^* (1 - \Gamma_G S_{11})}{S_{21}} \frac{b_{n1} b_{n2}}{|b_{nG}|^2}
\]

G.21
Appendix H

NOISE FIGURE OF THE REFLECTION AMPLIFIER

The noise figure of a two-port (Appendix G, F) is

\[ F = \frac{|b_G|^2 |b_{nG}|^2}{|b_G^-|^2 |b_{nG}|^2} \]  \hspace{1cm} H.1

The reflection amplifier is represented by two adapted two-ports (Fig. H.1), one in the forward transmission path from input port to output port that includes the DBRT diode and the other adapted two-port represents the forward transmission path from output port to input port (feedback path). As already indicated in chapter 4 the subscript numbers of the S-parameters of the two adapted two-ports are different from those used in previous appendices and the subscript numbers follow the incident and emergent branch numbers.

H.1 The relation between \( |b_G|^2 \) and \( |b_G^-|^2 \)

The signal circuit diagram is given in figure H.1

![Signal circuit diagram of the reflection amplifier](image)

Fig. H.1 The signal circuit diagram of the reflection amplifier

The relation between \( |b_G|^2 \) and \( |b_G^-|^2 \) can be derived following the procedure used in Appendix C. If the parameters of eq. C.24 are adapted to match Fig. H.1, the wanted relationship reads

\[ \frac{|b_G|^2}{|b_G^-|^2} = \frac{|1 - S_{44}\Gamma_G|^2}{|S_{54}|^2} \]  \hspace{1cm} H.2
H.2 The relation between $|b_{nG}|^2$ and $|b_{nG}^-|^2$

The noise circuit diagram is given in figure H.2

Fig. H.2 The noise circuit diagram of the reflection amplifier.

The incident wave $a_1$ is given by

$$a_1 = b_{n0} + \Gamma_{G} b_1$$

The emergent wave $b_1$ is an addition of two waves each of which is also an addition of two waves.

$$b_1 = b_3 + b_7$$

$$b_1 = b_4 + b_{n3} + b_8 + b_{n1}$$

The emergent wave $b_4$ is

$$b_4 = S_{44} a_4 = S_{44} a_1 = S_{44} (\Gamma_{G} b_1 + b_{nG})$$

The substitution of eq. H.5 into eq. H.4 yields

$$b_1 = S_{44} \Gamma_{G} b_1 + b_{n1} + b_{n3} + b_8 + S_{44} b_{nG}$$

$$b_1 = \frac{b_8 + b_{n1} + b_{n3} + S_{44} b_{nG}}{1 - S_{44} \Gamma_{G}}$$

The emergent wave $b_2$ is an addition of two waves each of which is also an addition of two waves.

$$b_2 = b_6 + b_{10} = b_5 + b_{n4} + b_9 + b_{n2}$$
The emergent wave $b_9$ is

$$b_9 = S_{gg} a_9 = S_{gg} a_2 = S_{gg} \Gamma_L b_2$$ \hspace{1cm} \text{H.8}

Substitution of eq. H.8 into eq. H.7 gives

$$b_2 = b_3 + b_{n4} + b_{n2} + S_{gg} \Gamma_L b_2$$

$$b_2 = \frac{b_3 + b_{n4} + b_{n2}}{1 - S_{gg} \Gamma_L}$$ \hspace{1cm} \text{H.9}

For $b_3$ eq. H.10 holds

$$b_3 = S_{g_4} a_1$$ \hspace{1cm} \text{H.10}

The substitution of eq. H.10 into eq. H.9 yields

$$b_2 = \frac{S_{g_4} a_1 + b_{n4} + b_{n2}}{1 - S_{gg} \Gamma_L}$$ \hspace{1cm} \text{H.11}

The emergent wave $b_8$ is

$$b_8 = S_{gg} a_2$$ \hspace{1cm} \text{H.12}

The combination of eq. H.12 and H.8 gives

$$b_8 = S_{gg} \Gamma_L b_2$$ \hspace{1cm} \text{H.13}

The substitution of eq. H.11 in eq. H.13 and the result into eq. H.6 yields

$$b_1 = \frac{S_{gg} \Gamma_L (S_{g_4} a_1 + b_{n4} + b_{n2}) + (b_{n1} + b_{n3} + b_{nG} S_{44}) (1 - S_{gg} \Gamma_L)}{(1 - S_{gg} \Gamma_L)(1 - S_{44} \Gamma_G)}$$ \hspace{1cm} \text{H.14}

Now eq. H.14 must be substituted into eq. H.3

$$a_1 = \frac{\Gamma_G \Gamma_S S_{gg} (S_{g_4} a_1 + b_{n4} + b_{n2}) + \Gamma_G (1 - S_{gg} \Gamma_L) (b_{n1} + b_{n3} + b_{nG} S_{44})}{(1 - S_{gg} \Gamma_L)(1 - S_{44} \Gamma_G)}$$

$$a_1 = \frac{b_{nG} (1 - S_{gg} \Gamma_L) (1 - S_{44} \Gamma_G) + \Gamma_L \Gamma_G S_{gg} (b_{n4} + b_{n2}) + \Gamma_G (1 - S_{gg} \Gamma_L) (b_{n1} + b_{n3} + b_{nG} S_{44})}{(1 - S_{gg} \Gamma_L)(1 - S_{44} \Gamma_G) - \Gamma_G \Gamma_L S_{gg} S_{44}}$$ \hspace{1cm} \text{H.15}

Now eq. H.15 must be substituted into eq. H.11
\[
S_{54}\left\{ \frac{b_{nG} (1-S_{99} \Gamma_L)(1-S_{44} \Gamma_G)+\Gamma_L \Gamma_G S_{89} (b_{n4}+b_{n2})+\Gamma_G (1-S_{99} \Gamma_L)(b_{n1}+b_{n3}+b_{nG} S_{44})}{(1-S_{99} \Gamma_L)(1-S_{44} \Gamma_G)-\Gamma_L \Gamma_G S_{89} S_{54}} \right\}
\]

\[
b_2 = \frac{b_{n4} + b_{n2}}{1 - S_{99} \Gamma_L} + \frac{b_{n4}}{1 - S_{99} \Gamma_L}
\]

\[H.16\]

For \(b_2\) eq. H.17 holds

\[
b_2 = b_n^r + a_2 \Gamma_b
\]

\[H.17\]

For \(a_2\) eq. H.18 is valid

\[
a_2 = \Gamma_L b_2
\]

\[H.18\]

The combination of eq. H.17 and H.18 gives

\[
b_2 = b_n^r + \Gamma_L \Gamma_b b_2
\]

\[H.19\]

For \(\Gamma_b\) can be derived (see also eq. C.23)

\[
\Gamma_b = S_{99} + \frac{S_{89} \Gamma_G S_{54}}{1 - S_{44} \Gamma_G}
\]

\[H.20\]

which substituted into eq. H.19 yields

\[
b_n^r = b_2 (1 - \Gamma_L\left[ S_{99} + \frac{S_{54} \Gamma_G S_{89}}{1 - S_{44} \Gamma_G} \right])
\]

\[H.21\]

The next step is the substitution of eq. H.16 into eq. H.21
\[ b_{nG}^- = S_{54} \left( \frac{b_{nG} (1-S_{99} \Gamma_L) (1-S_{44} \Gamma_G) + \Gamma_G \Gamma_L S_{89} (b_{n4} + b_{n2}) + \Gamma_G (1-S_{99} \Gamma_L) (b_{n1} + b_{n3} + b_{nG} S_{44})}{(1-S_{99} \Gamma_L) (1-S_{44} \Gamma_G)} \right) \]

\[ + \frac{(b_{n4} + b_{n2}) ((1-S_{99} \Gamma_L) (1-S_{44} \Gamma_G) - \Gamma_G \Gamma_L S_{89} S_{54})}{(1-S_{99} \Gamma_L) (1-S_{44} \Gamma_G)} \]

\[ b_{nG}^- = S_{54} \frac{b_{nG} + \Gamma_G (b_{n1} + b_{n3})}{1 - S_{44} \Gamma_G} + (b_{n4} + b_{n2}) \]

H.22

The relation between \(|b_{nG}^-|^2\) and \(|b_{nG}^-|^2\) now is given by

\[ \frac{|b_{nG}^-|^2}{|b_{nG}^-|^2} = \left( \frac{S_{54} \frac{b_{nG} + \Gamma_G (b_{n1} + b_{n3})}{1 - S_{44} \Gamma_G} + (b_{n4} + b_{n2})}{|b_{nG}^-|^2} \right)^2 \]

H.23

H.3 Total noise figure of the reflection amplifier

The substitution of eq. H.2 and eq. H.23 into eq. H.1 yields

\[ F = \left| \frac{1-S_{44} \Gamma_G}{S_{54}} \right|^2 \left( \frac{S_{54} \frac{b_{nG} + \Gamma_G (b_{n1} + b_{n3})}{1 - S_{44} \Gamma_G} + (b_{n4} + b_{n2})}{|b_{nG}^-|^2} \right)^2 \]

H.24

Before proceeding with eq. H.24 the following assumptions can be made in accordance with appendices F and G:

All internal noise sources are uncorrelated to the external noise source; the noise sources of forward and feedback path are uncorrelated; the noise sources in the forward path are correlated and so are the noise sources in the feedback path.

\[ E \left[ b_{nG}(t) b_{n1}(t+\tau) \right] = 0 \] \hspace{1cm} H.25a
\[ E \left[ b_{nG}(t) b_{n2}(t+\tau) \right] = 0 \] \hspace{1cm} H.25b
\[ E \left[ b_{nG}(t) b_{n3}(t+\tau) \right] = 0 \] \hspace{1cm} H.25c
\[ E \left[ b_{nG}(t) b_{n4}(t+\tau) \right] = 0 \] \hspace{1cm} H.25d
\[ E \left[ b_{nG}(t) b_{nG}(t+\tau) \right] \neq 0 \] \hspace{1cm} H.25e

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\[
E \left[ b_{n1}(t) b_{n3}^*(t+\tau) \right] = 0 \quad \text{H.25f}
\]
\[
E \left[ b_{n1}(t) b_{n4}^*(t+\tau) \right] = 0 \quad \text{H.25g}
\]
\[
E \left[ b_{n2}(t) b_{n3}^*(t+\tau) \right] = 0 \quad \text{H.25h}
\]
\[
E \left[ b_{n2}(t) b_{n4}^*(t+\tau) \right] = 0 \quad \text{H.25i}
\]
\[
E \left[ b_{n3}(t) b_{n4}^*(t+\tau) \right] \neq 0 \quad \text{H.25j}
\]

To find the total noise figure of the reflection amplifier eq. H.24 can be simplified with eqns. H.25 taken into account

\[
F_t = 1 + \left| \Gamma_G \right|^2 \frac{|b_{n1}|^2}{|b_{nG}|^2} + \left| \Gamma_G \right|^2 \frac{|b_{n3}|^2}{|b_{nG}|^2} + \left| 1 - S_{44} \Gamma_G \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2} + \left| S_{54} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2} + \left| S_{54} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2} + \left| S_{54} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2}
\]

The reflection amplifier circuit schematically can be drawn as shown in figure H.1 or H.2. Each adapted two-port has a noise figure according to appendix G. The noise figure \( F_{45} \) of the adapted two-port in the forward path that includes the DBRT diode is given by eq. H.27 (appendix G, eq. G.21).

\[
F_{45} = 1 + \left| \Gamma_G \right|^2 \frac{|b_{n3}|^2}{|b_{nG}|^2} + \left| 1 - \Gamma_G S_{44} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2} + \left| S_{54} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2} + \left| S_{54} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2} + \left| S_{54} \right|^2 \frac{|b_{n4}|^2}{|b_{nG}|^2}
\]

The external noise source wave amplitude \( b_{nG} \) is identical in eqns. H.26 and H.27, so corresponding terms of H.26 can be replaced by those of H.27 which yields
\[ F_{\text{t}} = F_{45} + |\Gamma_G|^2 \left| \frac{b_{n1}}{b_{nG}} \right|^2 + \frac{|1 - S_{44} \Gamma_G|^2 |b_{n2}|^2}{|S_{54}|^2 |b_{nG}|^2} + \]

\[
\frac{(1 - S_{44} \Gamma_G) \Gamma_G}{S_{54}} \frac{b_{n1} b_{n2}^*}{|b_{nG}|^2} + \frac{|1 - S_{44} \Gamma_G|^* |b_{n1} b_{n2}^*|}{S_{54} |b_{nG}|^2}
\]

Eq. H.28 is the formula for the total noise figure of the reflection amplifier in which \( F_{45} \) is the noise figure of the forward transmission path from input port to output port.
Appendix I

MEASURING THE NOISE MEASURE WITH THE TWO ATTENUATOR METHOD

Basically, all techniques for measuring noise rely on measuring the output noise power from the circuit under two conditions of input noise power. Input noise power changes can be accomplished in basically two ways.

- The first involves a change in the operating conditions of the source.
- The second technique employs changes inside the circuit.

The second technique will be used in this paragraph, in the so called two attenuator method. The basic configuration is sketched in figure I.1. The DUT can be a DBRT diode in a reflection amplifier circuit.

![Diagram of two attenuator configuration](image)

**Fig. I.1** The two attenuator configuration (the elements 1 and 3 are variable attenuators)

The noise power at the detector follows eq. I.1

\[ N_D = G_1 G_2 N_i + G_2 G_3 N_{s1} + G_3 N_{s2} + N_{s3} \]  

where \( N_D \) is the noise power at the detector, \( N_i \) is the input noise power generated by a resistor at a temperature of 290 K, \( N_{s1} \) is the noise power added by attenuator 1, \( N_{s2} \) is the noise power added by attenuator 2, \( N_{s3} \) is the noise power added by attenuator 3, \( G_1 \), \( G_2 \) and \( G_3 \) are the available gains of the circuit elements.

Eq. I.2 is valid for a general two-port network

\[ N_o = G N_i + M (G - 1) k T_o B \]  

where \( N_o \) is the output noise power, \( k \) is the Boltzmann constant, \( T_o \) is the reference temperature (mostly room temperature), \( B \) is the bandwidth, \( M \) is the noise measure. If the network is purely resistive eq. I.2 can be simplified

\[ N_o = G N_i + (1 - G) k T_1 B \]  

where \( T_1 \) is the temperature of the network.
The circuit elements I and 3 (figure 1.1) are purely resistive. The substitution of eqns. I.2 and I.3 in eq. I.1 yields

\[ N_D = G_1 G_2 G_3 N_i + G_2 G_3 (1-G_1)kT_B + G_3 M(G_2-1)kT_0 B + (1-G_3)kT_3 B \]  

I.4

\( N_i \) can be written as

\[ N_i = k T_s B \]  

I.5

where \( T_s \) is the noise source temperature. Substitution of eq. I.5 into eq. I.4 yields

\[ N_D = G_1 G_2 G_3 kT_s B + G_2 G_3 (1-G_1)kT_1 B + G_3 M(G_2-1)kT_0 B + (1-G_3)kT_3 B \]  

I.6

Now the measurement method will be discussed.

- First the attenuators I and 3 are set to a certain attenuation value \((G_1, G_3)\)
- Next \( N_D \) is measured with the detector (reference level).
- The setting of attenuator 1 is changed \((\hat{G}_1)\).
- Now the value of attenuator 3 is changed \((\hat{G}_3)\) so that the value of \( N_D \) is the same as the reference value.

The equation for the first setting is

\[ N_D = G_1 G_2 G_3 kT_s B + G_2 G_3 (1-G_1)kT_1 B + G_3 M(G_2-1)kT_0 B + (1-G_3)kT_3 B \]  

I.7

The equation for the second setting is

\[ \hat{N}_D = G_1 G_2 G_3 kT_s B + G_2 G_3 (1-G_1)kT_1 B + G_3 M(G_2-1)kT_0 B + (1-G_3)kT_3 B \]  

I.8

Eqns. I.7 and I.8 are equal (equal detector levels \( N_D \)), this means

\[ G_1 G_2 G_3 kT_s B + G_2 G_3 (1-G_1)kT_1 B + G_3 M(G_2-1)kT_0 B + (1-G_3)kT_3 B = \]

\[ \hat{G}_1 G_2 \hat{G}_3 kT_s B + \hat{G}_2 \hat{G}_3 (1-\hat{G}_1)kT_1 B + \hat{G}_3 M(G_2-1)kT_0 B + (1-\hat{G}_3)kT_3 B \]  

I.9

Now all terms with \( M \) must be brought to the left side of the equality sign

\[ M \left( (G_2-1)G_3 T_0 (G_2-1)\hat{G}_3 T_0 \right) = -G_2 G_3 G_1 \hat{G}_3 \hat{G}_1 (T_1 - T_1) + T_1 G_2 (\hat{G}_2 - G_2) + T_3 (G_3 - \hat{G}_3) \]  

I.10

The result is eq. I.11

\[ M = \frac{G_2}{G_2-1} \left[ \frac{T_3}{T_0 G_2} - \frac{T_1}{G_2} - G_3 G_1 - \frac{\hat{G}_3 \hat{G}_1}{G_3 - \hat{G}_3} \frac{T_5}{T_0} \right] \]  

I.11

This can also be found in [23,39], see also [40].
Appendix J

RESULTS IN TABULAR FORM

Table J.1 NF data obtained using only the noise figure meter. Triangular DBRT diode. $f=1.5$ GHz.

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<th>$F_d$ (dB)</th>
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<td>-6.55</td>
</tr>
<tr>
<td>550</td>
<td>1.54 $10^3$</td>
<td>14.4</td>
<td>14.2</td>
<td>-11.4</td>
</tr>
<tr>
<td>600</td>
<td>2.15 $10^3$</td>
<td>16.4</td>
<td>16.1</td>
<td>-12.5</td>
</tr>
<tr>
<td>620</td>
<td>2.36 $10^3$</td>
<td>14.7</td>
<td>14.3</td>
<td>-9.52</td>
</tr>
<tr>
<td>630</td>
<td>2.44 $10^3$</td>
<td>13.1</td>
<td>12.6</td>
<td>-5.45</td>
</tr>
<tr>
<td>635</td>
<td>2.47 $10^3$</td>
<td>11.6</td>
<td>11.1</td>
<td>-3.27</td>
</tr>
<tr>
<td>638</td>
<td>2.48 $10^3$</td>
<td>11.0</td>
<td>10.4</td>
<td>-1.42</td>
</tr>
<tr>
<td>641</td>
<td>2.48 $10^3$</td>
<td>10.5</td>
<td>9.92</td>
<td>0.13</td>
</tr>
<tr>
<td>756</td>
<td>0.98 $10^3$</td>
<td>7.26</td>
<td>6.66</td>
<td>2.44</td>
</tr>
<tr>
<td>766</td>
<td>0.95 $10^3$</td>
<td>7.30</td>
<td>6.72</td>
<td>1.68</td>
</tr>
<tr>
<td>850</td>
<td>0.90 $10^3$</td>
<td>7.74</td>
<td>7.23</td>
<td>-0.84</td>
</tr>
<tr>
<td>900</td>
<td>0.95 $10^3$</td>
<td>8.13</td>
<td>7.64</td>
<td>-1.48</td>
</tr>
<tr>
<td>950</td>
<td>1.03 $10^3$</td>
<td>8.74</td>
<td>8.25</td>
<td>-2.19</td>
</tr>
<tr>
<td>1000</td>
<td>1.17 $10^3$</td>
<td>9.63</td>
<td>9.15</td>
<td>-3.14</td>
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<tr>
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<td>10.9</td>
<td>10.4</td>
<td>-4.34</td>
</tr>
<tr>
<td>1100</td>
<td>1.63 $10^3$</td>
<td>12.3</td>
<td>11.8</td>
<td>-5.72</td>
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</table>

Table J.2 NF Data obtained using the network analyzer and the NF meter. Triangular DBRT diode. $f=1.5$ GHz.

<table>
<thead>
<tr>
<th>V (mV)</th>
<th>I (µA)</th>
<th>$F_i$ (dB)</th>
<th>$F_d$ (dB)</th>
<th>$G_d$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2.11</td>
<td>0.89</td>
<td>-0.34</td>
</tr>
<tr>
<td>100</td>
<td>39</td>
<td>2.41</td>
<td>1.21</td>
<td>-0.48</td>
</tr>
<tr>
<td>250</td>
<td>200</td>
<td>4.25</td>
<td>3.15</td>
<td>-1.41</td>
</tr>
<tr>
<td>400</td>
<td>611</td>
<td>7.35</td>
<td>6.38</td>
<td>-3.04</td>
</tr>
<tr>
<td>550</td>
<td>1.55 $10^3$</td>
<td>14.4</td>
<td>13.4</td>
<td>-9.52</td>
</tr>
<tr>
<td>600</td>
<td>2.16 $10^3$</td>
<td>16.5</td>
<td>15.6</td>
<td>-10.46</td>
</tr>
<tr>
<td>620</td>
<td>2.38 $10^3$</td>
<td>14.7</td>
<td>13.9</td>
<td>-7.07</td>
</tr>
<tr>
<td>630</td>
<td>2.45 $10^3$</td>
<td>13.2</td>
<td>12.4</td>
<td>-3.44</td>
</tr>
<tr>
<td>766</td>
<td>0.96 $10^3$</td>
<td>7.42</td>
<td>6.67</td>
<td>1.74</td>
</tr>
<tr>
<td>850</td>
<td>0.96 $10^3$</td>
<td>7.85</td>
<td>7.04</td>
<td>-0.59</td>
</tr>
<tr>
<td>950</td>
<td>1.06 $10^3$</td>
<td>8.83</td>
<td>8.01</td>
<td>-1.81</td>
</tr>
<tr>
<td>1100</td>
<td>1.68 $10^3$</td>
<td>12.4</td>
<td>11.6</td>
<td>-4.95</td>
</tr>
</tbody>
</table>

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### Table J.3 Two-port S-parameter measurement (between circulator input-port 1 and DBRT diode port of the reflection amplifier).

<table>
<thead>
<tr>
<th>FREQ. (GHz)</th>
<th>S11 MOD. ARG.</th>
<th>S12 MOD. ARG.</th>
<th>S21 MOD. ARG.</th>
<th>S22 MOD. ARG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8000</td>
<td>0.219 327.05</td>
<td>0.281 34.68</td>
<td>0.851 157.47</td>
<td>0.287 273.42</td>
</tr>
<tr>
<td>0.8500</td>
<td>0.212 314.48</td>
<td>0.213 336.40</td>
<td>0.878 112.68</td>
<td>0.269 235.25</td>
</tr>
<tr>
<td>0.9000</td>
<td>0.161 294.19</td>
<td>0.150 278.73</td>
<td>0.906 69.12</td>
<td>0.210 194.73</td>
</tr>
<tr>
<td>0.9500</td>
<td>0.093 280.48</td>
<td>0.093 218.48</td>
<td>0.926 25.39</td>
<td>0.142 164.51</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.050 300.16</td>
<td>0.047 149.09</td>
<td>0.929 343.13</td>
<td>0.103 151.41</td>
</tr>
<tr>
<td>1.0500</td>
<td>0.059 324.15</td>
<td>0.025 46.78</td>
<td>0.926 301.33</td>
<td>0.103 138.53</td>
</tr>
<tr>
<td>1.1000</td>
<td>0.073 318.67</td>
<td>0.035 310.60</td>
<td>0.929 260.28</td>
<td>0.106 119.50</td>
</tr>
<tr>
<td>1.1500</td>
<td>0.077 305.06</td>
<td>0.047 248.39</td>
<td>0.928 219.25</td>
<td>0.115 93.86</td>
</tr>
<tr>
<td>1.2000</td>
<td>0.065 288.96</td>
<td>0.053 196.28</td>
<td>0.927 178.28</td>
<td>0.112 68.90</td>
</tr>
<tr>
<td>1.2500</td>
<td>0.072 267.83</td>
<td>0.054 154.87</td>
<td>0.926 138.07</td>
<td>0.101 26.95</td>
</tr>
<tr>
<td>1.3000</td>
<td>0.060 240.29</td>
<td>0.054 116.16</td>
<td>0.935 97.47</td>
<td>0.074 353.88</td>
</tr>
<tr>
<td>1.3500</td>
<td>0.065 206.18</td>
<td>0.053 30.40</td>
<td>0.932 56.48</td>
<td>0.049 332.11</td>
</tr>
<tr>
<td>1.4000</td>
<td>0.061 166.57</td>
<td>0.054 51.52</td>
<td>0.925 16.30</td>
<td>0.021 331.18</td>
</tr>
<tr>
<td>1.4500</td>
<td>0.070 127.62</td>
<td>0.062 16.81</td>
<td>0.928 355.88</td>
<td>0.038 20.85</td>
</tr>
<tr>
<td>1.5000</td>
<td>0.085 93.56</td>
<td>0.072 341.98</td>
<td>0.923 295.03</td>
<td>0.078 1.31</td>
</tr>
<tr>
<td>1.5500</td>
<td>0.098 63.34</td>
<td>0.081 300.61</td>
<td>0.913 254.38</td>
<td>0.118 328.30</td>
</tr>
<tr>
<td>1.6000</td>
<td>0.106 37.13</td>
<td>0.089 257.90</td>
<td>0.913 214.84</td>
<td>0.133 286.15</td>
</tr>
</tbody>
</table>

### Table J.4 Two-port S-parameter measurement (between DBRT diode port and circulator output-port 3 of the reflection amplifier).

<table>
<thead>
<tr>
<th>FREQ. (GHz)</th>
<th>S22 MOD. ARG.</th>
<th>S23 MOD. ARG.</th>
<th>S32 MOD. ARG.</th>
<th>S33 MOD. ARG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8000</td>
<td>0.282 272.63</td>
<td>0.251 38.14</td>
<td>0.941 160.83</td>
<td>0.258 322.70</td>
</tr>
<tr>
<td>0.8500</td>
<td>0.265 253.92</td>
<td>0.180 329.45</td>
<td>0.870 116.35</td>
<td>0.244 306.69</td>
</tr>
<tr>
<td>0.9000</td>
<td>0.269 192.99</td>
<td>0.115 266.87</td>
<td>0.902 73.10</td>
<td>0.190 284.14</td>
</tr>
<tr>
<td>0.9500</td>
<td>0.143 161.26</td>
<td>0.062 193.63</td>
<td>0.925 30.05</td>
<td>0.122 264.01</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.105 147.96</td>
<td>0.040 89.91</td>
<td>0.932 347.44</td>
<td>0.060 268.68</td>
</tr>
<tr>
<td>1.0500</td>
<td>0.106 134.99</td>
<td>0.053 356.32</td>
<td>0.928 305.77</td>
<td>0.043 281.57</td>
</tr>
<tr>
<td>1.1000</td>
<td>0.111 115.61</td>
<td>0.070 292.18</td>
<td>0.931 264.86</td>
<td>0.047 291.58</td>
</tr>
<tr>
<td>1.1500</td>
<td>0.120 91.18</td>
<td>0.080 237.37</td>
<td>0.935 223.79</td>
<td>0.045 285.50</td>
</tr>
<tr>
<td>1.2000</td>
<td>0.120 61.01</td>
<td>0.081 188.39</td>
<td>0.929 183.17</td>
<td>0.050 278.28</td>
</tr>
<tr>
<td>1.2500</td>
<td>0.184 27.12</td>
<td>0.076 143.50</td>
<td>0.929 143.05</td>
<td>0.051 268.28</td>
</tr>
<tr>
<td>1.3000</td>
<td>0.076 375.56</td>
<td>0.068 103.09</td>
<td>0.936 102.66</td>
<td>0.057 232.73</td>
</tr>
<tr>
<td>1.3500</td>
<td>0.046 334.03</td>
<td>0.061 68.79</td>
<td>0.930 61.86</td>
<td>0.057 196.67</td>
</tr>
<tr>
<td>1.4000</td>
<td>0.021 342.24</td>
<td>0.057 30.87</td>
<td>0.925 22.09</td>
<td>0.064 162.74</td>
</tr>
<tr>
<td>1.4500</td>
<td>0.038 25.78</td>
<td>0.068 10.76</td>
<td>0.930 341.84</td>
<td>0.069 129.39</td>
</tr>
<tr>
<td>1.5000</td>
<td>0.078 2.43</td>
<td>0.068 336.68</td>
<td>0.924 301.17</td>
<td>0.084 101.05</td>
</tr>
<tr>
<td>1.5500</td>
<td>0.112 321.71</td>
<td>0.079 300.50</td>
<td>0.917 261.29</td>
<td>0.086 71.94</td>
</tr>
<tr>
<td>1.6000</td>
<td>0.127 294.09</td>
<td>0.089 259.21</td>
<td>0.915 221.45</td>
<td>0.107 46.15</td>
</tr>
</tbody>
</table>
Table J.5  Two-port S-parameter measurement (between circulator output port and circulator input-port of the reflection amplifier).

REF1 = OUTPUT CIRC. ; REF2 = INPUT  
CORRECTED S-PARAMETERS  

<table>
<thead>
<tr>
<th>Freq. (GHz)</th>
<th>$S_{33}$ Mod.</th>
<th>$S_{33}$ Arg.</th>
<th>$S_{31}$ Mod.</th>
<th>$S_{31}$ Arg.</th>
<th>$S_{13}$ Mod.</th>
<th>$S_{13}$ Arg.</th>
<th>$S_{11}$ Mod.</th>
<th>$S_{11}$ Arg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8000 0.235</td>
<td>322.30</td>
<td>0.321</td>
<td>235.74</td>
<td>0.886</td>
<td>3.02</td>
<td>0.223</td>
<td>326.91</td>
<td></td>
</tr>
<tr>
<td>0.8500 0.245</td>
<td>307.00</td>
<td>0.248</td>
<td>184.81</td>
<td>0.914</td>
<td>330.92</td>
<td>0.212</td>
<td>314.37</td>
<td></td>
</tr>
<tr>
<td>0.9000 0.193</td>
<td>293.92</td>
<td>0.173</td>
<td>133.44</td>
<td>0.941</td>
<td>299.99</td>
<td>0.166</td>
<td>293.96</td>
<td></td>
</tr>
<tr>
<td>0.9500 0.122</td>
<td>263.79</td>
<td>0.113</td>
<td>72.19</td>
<td>0.960</td>
<td>269.80</td>
<td>0.095</td>
<td>260.33</td>
<td></td>
</tr>
<tr>
<td>1.0000 0.061</td>
<td>258.63</td>
<td>0.081</td>
<td>352.64</td>
<td>0.968</td>
<td>239.91</td>
<td>0.054</td>
<td>295.41</td>
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</tr>
<tr>
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<td>280.46</td>
<td>0.079</td>
<td>267.16</td>
<td>0.970</td>
<td>210.99</td>
<td>0.061</td>
<td>323.17</td>
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</tr>
<tr>
<td>1.1000 0.044</td>
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<td>209.29</td>
<td>0.969</td>
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<tr>
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<td>285.95</td>
<td>0.109</td>
<td>157.05</td>
<td>0.969</td>
<td>154.22</td>
<td>0.080</td>
<td>304.58</td>
<td></td>
</tr>
<tr>
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<td>0.970</td>
<td>126.47</td>
<td>0.076</td>
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<td>65.01</td>
<td>0.969</td>
<td>98.53</td>
<td>0.076</td>
<td>267.54</td>
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</tr>
<tr>
<td>1.3000 0.054</td>
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<td>0.066</td>
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<td>195.86</td>
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<td>321.95</td>
<td>0.971</td>
<td>43.19</td>
<td>0.063</td>
<td>208.46</td>
<td></td>
</tr>
<tr>
<td>1.4000 0.060</td>
<td>161.05</td>
<td>0.017</td>
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<td>15.71</td>
<td>0.058</td>
<td>170.90</td>
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</tr>
<tr>
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<td>346.08</td>
<td>0.079</td>
<td>129.47</td>
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</tr>
<tr>
<td>1.5000 0.082</td>
<td>99.71</td>
<td>0.080</td>
<td>43.62</td>
<td>0.968</td>
<td>320.38</td>
<td>0.083</td>
<td>95.88</td>
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<tr>
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<td>292.11</td>
<td>0.097</td>
<td>62.26</td>
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</tr>
<tr>
<td>1.6000 0.106</td>
<td>45.09</td>
<td>0.125</td>
<td>314.74</td>
<td>0.962</td>
<td>265.45</td>
<td>0.101</td>
<td>37.92</td>
<td></td>
</tr>
</tbody>
</table>
Table J.6  Reflection-coefficient of the noise source HP346A.

<table>
<thead>
<tr>
<th>FREQ.</th>
<th>MOD.</th>
<th>ARG.</th>
<th>VSWR</th>
<th>RE(C')</th>
<th>IM(C')</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.008</td>
<td>155.27</td>
<td>1.015E+00</td>
<td>9.864E-01</td>
<td>9.231E-03</td>
</tr>
<tr>
<td>1.350</td>
<td>0.001</td>
<td>291.17</td>
<td>1.026E+00</td>
<td>1.009E+00</td>
<td>2.655E-02</td>
</tr>
<tr>
<td>1.500</td>
<td>0.015</td>
<td>262.18</td>
<td>1.036E+00</td>
<td>9.935E-01</td>
<td>2.929E-02</td>
</tr>
</tbody>
</table>
**Table 1.7 Reflection-coefficient of the noise figure meter HP8970B.**

<table>
<thead>
<tr>
<th>FREQ.</th>
<th>MOD.</th>
<th>ARG.</th>
<th>VSWR</th>
<th>RE(C')</th>
<th>IM(C')</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.157</td>
<td>139.12</td>
<td>1.37E+00</td>
<td>7.73E-01</td>
<td>1.628E-01</td>
</tr>
<tr>
<td>1.500</td>
<td>0.051</td>
<td>315.81</td>
<td>1.16E+00</td>
<td>1.07E+00</td>
<td>-7.661E-02</td>
</tr>
<tr>
<td>2.000</td>
<td>0.084</td>
<td>66.44</td>
<td>1.19E+00</td>
<td>1.05E+00</td>
<td>1.628E-01</td>
</tr>
<tr>
<td>2.500</td>
<td>0.101</td>
<td>343.93</td>
<td>1.22E+00</td>
<td>1.21E+00</td>
<td>-6.824E-02</td>
</tr>
<tr>
<td>3.000</td>
<td>0.053</td>
<td>228.87</td>
<td>1.11E+00</td>
<td>9.28E-01</td>
<td>-7.399E-02</td>
</tr>
<tr>
<td>3.500</td>
<td>0.060</td>
<td>14.83</td>
<td>1.12E+00</td>
<td>1.12E+00</td>
<td>3.446E-02</td>
</tr>
<tr>
<td>4.000</td>
<td>0.068</td>
<td>267.61</td>
<td>1.19E+00</td>
<td>9.77E-01</td>
<td>-1.730E-01</td>
</tr>
<tr>
<td>4.500</td>
<td>0.062</td>
<td>149.82</td>
<td>1.13E+00</td>
<td>8.95E-01</td>
<td>5.647E-02</td>
</tr>
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<td>-1.551E-01</td>
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<td>1.29E+00</td>
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<td>0.103</td>
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<td>0.087</td>
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<td>0.057</td>
<td>68.89</td>
<td>1.12E+00</td>
<td>1.05E+00</td>
<td>1.044E-01</td>
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<td>75.47</td>
<td>1.03E+00</td>
<td>1.07E+00</td>
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<td>0.070</td>
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<td>1.05E+00</td>
<td>1.333E-01</td>
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<td>0.041</td>
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<td>1.08E+00</td>
<td>9.79E-01</td>
<td>-7.759E-02</td>
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<td>1.32E+00</td>
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</table>
Table J.8  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 0 mV).

<table>
<thead>
<tr>
<th>FREQ. (GHz)</th>
<th>MOD.</th>
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<th>MOD.</th>
<th>ARG.</th>
<th>MOD.</th>
<th>ARG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4000</td>
<td>0.027</td>
<td>108.00</td>
<td>0.814</td>
<td>344.52</td>
<td>0.976</td>
<td>16.11</td>
<td>0.025</td>
<td>119.12</td>
</tr>
<tr>
<td>1.4500</td>
<td>0.019</td>
<td>158.19</td>
<td>0.808</td>
<td>261.66</td>
<td>0.977</td>
<td>348.35</td>
<td>0.014</td>
<td>165.67</td>
</tr>
<tr>
<td>1.5000</td>
<td>0.071</td>
<td>147.20</td>
<td>0.884</td>
<td>171.83</td>
<td>0.975</td>
<td>228.43</td>
<td>0.069</td>
<td>147.72</td>
</tr>
<tr>
<td>1.5500</td>
<td>0.137</td>
<td>98.49</td>
<td>0.766</td>
<td>33.42</td>
<td>0.965</td>
<td>293.08</td>
<td>0.138</td>
<td>91.49</td>
</tr>
<tr>
<td>1.6000</td>
<td>0.172</td>
<td>49.59</td>
<td>0.808</td>
<td>4.30</td>
<td>0.961</td>
<td>265.56</td>
<td>0.170</td>
<td>48.61</td>
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</table>

Table J.9  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 100mV).

<table>
<thead>
<tr>
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<th>MOD.</th>
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<th>MOD.</th>
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<th>MOD.</th>
<th>ARG.</th>
<th>MOD.</th>
<th>ARG.</th>
</tr>
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<tbody>
<tr>
<td>1.4000</td>
<td>0.030</td>
<td>109.44</td>
<td>0.801</td>
<td>347.62</td>
<td>0.977</td>
<td>16.05</td>
<td>0.028</td>
<td>120.59</td>
</tr>
<tr>
<td>1.4500</td>
<td>0.017</td>
<td>149.59</td>
<td>0.794</td>
<td>265.06</td>
<td>0.977</td>
<td>248.31</td>
<td>0.013</td>
<td>156.07</td>
</tr>
<tr>
<td>1.5000</td>
<td>0.068</td>
<td>147.62</td>
<td>0.789</td>
<td>175.32</td>
<td>0.975</td>
<td>228.39</td>
<td>0.066</td>
<td>148.03</td>
</tr>
<tr>
<td>1.5500</td>
<td>0.135</td>
<td>99.45</td>
<td>0.781</td>
<td>96.45</td>
<td>0.966</td>
<td>292.96</td>
<td>0.134</td>
<td>92.53</td>
</tr>
<tr>
<td>1.6000</td>
<td>0.172</td>
<td>50.81</td>
<td>0.795</td>
<td>6.92</td>
<td>0.961</td>
<td>265.56</td>
<td>0.170</td>
<td>41.78</td>
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Table J.10  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 250mV).

<table>
<thead>
<tr>
<th>FREQ. (GHz)</th>
<th>MOD.</th>
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<th>MOD.</th>
<th>ARG.</th>
<th>MOD.</th>
<th>ARG.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4000</td>
<td>0.034</td>
<td>116.27</td>
<td>0.719</td>
<td>352.45</td>
<td>0.976</td>
<td>16.06</td>
<td>0.032</td>
<td>128.37</td>
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<tr>
<td>1.4500</td>
<td>0.021</td>
<td>133.13</td>
<td>0.708</td>
<td>270.27</td>
<td>0.977</td>
<td>348.33</td>
<td>0.018</td>
<td>134.13</td>
</tr>
<tr>
<td>1.5000</td>
<td>0.062</td>
<td>143.22</td>
<td>0.699</td>
<td>180.54</td>
<td>0.975</td>
<td>228.43</td>
<td>0.060</td>
<td>143.49</td>
</tr>
<tr>
<td>1.5500</td>
<td>0.127</td>
<td>99.37</td>
<td>0.706</td>
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<td>0.966</td>
<td>293.08</td>
<td>0.125</td>
<td>92.55</td>
</tr>
<tr>
<td>1.6000</td>
<td>0.185</td>
<td>52.35</td>
<td>0.725</td>
<td>10.74</td>
<td>0.962</td>
<td>265.47</td>
<td>0.163</td>
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Table J.11  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 400 mV).

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<th>$S_{11}$</th>
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<td>MOD. ARG.</td>
<td>MOD. ARG.</td>
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<td>0.592 354.72</td>
<td>0.976 16.03</td>
<td>0.036 139.00</td>
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<tr>
<td>1.5000</td>
<td>0.030 129.42</td>
<td>0.575 272.63</td>
<td>0.976 348.32</td>
<td>0.026 129.30</td>
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<td>1.5500</td>
<td>0.060 134.10</td>
<td>0.563 192.38</td>
<td>0.975 329.44</td>
<td>0.057 139.15</td>
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<td>1.6000</td>
<td>0.113 96.54</td>
<td>0.576 92.02</td>
<td>0.967 293.02</td>
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Table J.12  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 550 mV).

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<th>$S_{11}$</th>
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<tbody>
<tr>
<td>1.4000</td>
<td>0.046 151.33</td>
<td>0.265 348.22</td>
<td>0.975 15.97</td>
<td>0.046 162.31</td>
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<tr>
<td>1.4500</td>
<td>0.052 131.75</td>
<td>0.241 265.60</td>
<td>0.975 348.27</td>
<td>0.048 132.74</td>
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<tr>
<td>1.5000</td>
<td>0.070 112.56</td>
<td>0.237 187.97</td>
<td>0.973 320.49</td>
<td>0.068 110.39</td>
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<td>1.5500</td>
<td>0.106 94.36</td>
<td>0.225 75.14</td>
<td>0.967 293.13</td>
<td>0.104 76.32</td>
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<td>0.129 48.47</td>
<td>0.346 359.18</td>
<td>0.985 265.50</td>
<td>0.127 39.54</td>
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Table J.13  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 600 mV).

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<td>0.975 15.90</td>
<td>0.046 164.30</td>
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<td>0.975 348.25</td>
<td>0.051 138.55</td>
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<tr>
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<td>0.072 112.46</td>
<td>0.211 162.64</td>
<td>0.973 320.44</td>
<td>0.070 108.33</td>
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<td>0.104 74.56</td>
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<td>0.317 355.78</td>
<td>0.965 265.46</td>
<td>0.125 38.85</td>
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Table J.14 Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 620 mV).

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Table J.15 Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 630 mV).

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Table J.16 Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 766 mV).

<table>
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<td>224.13</td>
<td>1.055</td>
<td>267.61</td>
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<tr>
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<td>1.052</td>
<td>179.61</td>
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<td>0.151</td>
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<td>0.999</td>
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<td>0.190</td>
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<td>0.975</td>
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</table>
Table J.17  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 850 mV).

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<th>MOD. ARG.</th>
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<tr>
<td>1.4000</td>
<td>0.034</td>
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<td>353.04</td>
<td>0.975</td>
<td>15.87</td>
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<tr>
<td>1.4500</td>
<td>0.026</td>
<td>140.56</td>
<td>0.787</td>
<td>271.36</td>
<td>0.977</td>
<td>340.12</td>
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<td>0.064</td>
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<td>0.777</td>
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<td>0.976</td>
<td>320.20</td>
</tr>
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<td>1.5500</td>
<td>0.133</td>
<td>101.75</td>
<td>0.766</td>
<td>92.32</td>
<td>0.967</td>
<td>292.34</td>
</tr>
<tr>
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<td>0.172</td>
<td>53.27</td>
<td>0.750</td>
<td>12.46</td>
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Table J.18  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 950 mV).

<table>
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<td>0.976</td>
<td>15.88</td>
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<td>0.023</td>
<td>135.89</td>
<td>0.674</td>
<td>271.95</td>
<td>0.977</td>
<td>340.16</td>
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<tr>
<td>1.5000</td>
<td>0.063</td>
<td>142.04</td>
<td>0.662</td>
<td>182.45</td>
<td>0.975</td>
<td>320.29</td>
</tr>
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<td>1.5500</td>
<td>0.126</td>
<td>99.39</td>
<td>0.664</td>
<td>92.71</td>
<td>0.967</td>
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Table J.19  Two-port S-parameter measurement (between input and output port of the reflection amplifier with the triangular DBRT diode biased at 1100 mV).

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<th>MOD. ARG.</th>
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Table J.20  Second measurement using only the noise figure meter
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Table J.21 NF measurements using only the NF meter (NF meter method).
20 μm DBRT diode; f=1.5 GHz; T=290 K.

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continuation of Table J.21 ($f=1.5$ GHz)

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<th>I(μA)</th>
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<th>$F_d$ (dB)</th>
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Table J.22 NF measurements using only the NF meter (NF meter method).
20μm DBRT diode; $f=1.0$ GHz; $T=290$ K.

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<th>V(mV)</th>
<th>I(μA)</th>
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continuation of Table J.22 (f = 1.0 GHz)

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Appendix K

NOISE FIGURE OF THE REFLECTION AMPLIFIER: a second approach

The noise figure of a two-port is given by (Appendix F):

\[ F = \frac{|b_G|^2 |b_{nG}|^2}{|b^-|^2 |b_{nG}|^2} \]  

A second simplified model of the reflection amplifier is shown in the flow graph of Fig. K.1. The transmission S-parameters of the FORWARD path are indicated by \( S_{54} \) and \( S_{45} \) and those of the FEEDBACK path by \( S_{89} \) and \( S_{98} \), while \( S_{11} \) and \( S_{22} \) are the overall reflection parameters. The internal equivalent noise sources incident on the b-nodes, are \( b_n \) and \( b_{n2} \) (feedback path) and \( b_{n3} \) and \( b_{n4} \) in the forward path.

The relationship between the wave amplitudes \( b_G \) at the input and \( b^- \) at the output is identical to the corresponding relationship of a general two-port (Appendix C) which can be deduced from the calculation of the noise wave relationship \( b_{nG} \) to \( b^- \) by putting the noise wave amplitudes equal to zero.

**K.1 Noise wave relation between \( b^- \) and \( b_{nG} \)**

The calculation of the relation between \( b^- \) and \( b_{nG} \) of the simplified reflection amplifier model uses Mason’s non-touching loop rules and follows the procedure used in Appendix H. The following equations are valid:
To simplify the calculations put \((S_{45} + S_{89}) = S_{12}'\) and \((S_{54} + S_{98}) = S_{21}'\) so eqns. K.4' and K.5' become

\[
\begin{align*}
  b_1 &= S_{11}'a_1 + S_{12}'a_2 + b_{n1} + b_{n3} \\
  b_2 &= S_{21}'a_1 + S_{22}'a_2 + b_{n2} + b_{n4}
\end{align*}
\]

Substitution of eq. K.4 into eq. K.2 gives

\[
a_1 = \frac{b_{nG} + (S_{12}'a_2 + b_{n1} + b_{n3}) \Gamma_G}{1 - S_{11}' \Gamma_G}
\]

Substitute eq. K.6 into eq. K.5, which results in

\[
b_2 = S_{21}'\left[\frac{b_{nG} + (b_{n1} + b_{n3}) \Gamma_G}{1 - S_{11}' \Gamma_G}\right] + \left[\frac{S_{21}' \Gamma_G}{1 - S_{11}' \Gamma_G} + S_{22}\right]a_2 + b_{n2} + b_{n4}
\]

Now equate eq. K.7 to eq. K.3, which yields

\[
b_{\sigma_0}' = S_{21}'\left[\frac{b_{nG} + (b_{n1} + b_{n3}) \Gamma_G}{1 - S_{11}' \Gamma_G}\right] + b_{n2} + b_{n4} + \left[\frac{S_{21}' \Gamma_G}{1 - S_{11}' \Gamma_G} + S_{22}' \Gamma_G\right] a_2
\]

The calculation of \(\Gamma_2\) is carried out in Appendix C (eq. C.23) which, adapted to the situation here states:

\[
\Gamma_2 = S_{22} + \frac{S_{21}' \Gamma_G}{1 - S_{11}' \Gamma_G}
\]

Substitution of eq. K.9 into eq. K.8, makes the second term in brackets equal to zero and gives for \(b_{\sigma_0}'\):

\[
b_{\sigma_0}' = S_{21}'\left[\frac{b_{nG} + (b_{n1} + b_{n3}) \Gamma_G}{1 - S_{11}' \Gamma_G}\right] + b_{n2} + b_{n4}
\]

or

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Setting all internal noise wave amplitudes to zero gives for the relation between $|b_G|^2$ and $|b_G^-|^2$:

$$\frac{|b_G|^2}{|b_G^-|^2} = \frac{|1 - S_{11}^\Gamma G|^2}{|S_{21}^-|^2} \quad K.12$$

For the noise figure of the simplified reflection amplifier we find by substituting eqns. K.11 and K.12 into K.1:

$$F = \frac{|1 - S_{11}^\Gamma G|^2}{|S_{21}^-|^2} \left| S_{21} \left( \frac{b_nG + (b_{n1} + b_{n3})\Gamma_G}{1 - S_{11}^\Gamma G} \right) + b_{n2} + b_{n4} \right|^2 \quad K.13$$

The equation K.13 resembles eq. H.24, and taken into account the assumptions made in eqns. H.25 (correlation of the noise sources) the final equation for the total noise figure of the simplified reflection amplifier model consisting of two two-ports in parallel reads:

$$F = 1 + |\Gamma_G|^2 \frac{|b_{n1}|^2}{|b_nG|^2} + |\Gamma_G|^2 \frac{|b_{n3}|^2}{|b_nG|^2} + \frac{|1 - S_{11}^\Gamma G|^2}{|S_{21}^-|^2} \frac{|b_{n4}|^2}{|b_nG|^2} +$$

$$\frac{|1 - S_{11}^\Gamma G|^2}{|S_{21}^-|^2} \frac{|b_{n2}|^2}{|b_nG|^2} + \frac{(1 - S_{11}^\Gamma G)^* G b_n1 b_n2}{S_{21}^*} \frac{|b_{n2}^*|^2}{|b_nG|^2} + \frac{(1 - S_{11}^\Gamma G)^* G b_n1 b_n2}{S_{21}^*} \frac{|b_{n4}^*|^2}{|b_nG|^2} +$$

$$\frac{(1 - S_{11}^\Gamma G)^* G b_n3 b_n4}{S_{21}^*} \frac{|b_{n3}|^2}{|b_nG|^2} + \frac{(1 - S_{11}^\Gamma G)^* G b_n3 b_n4}{S_{21}^*} \frac{|b_{n4}|^2}{|b_nG|^2} \quad K.14$$

In this simplified model, the transmission $S$-parameters $S_{54}$ and $S_{98}$ from input to output can not be measured separately (only the overall $S_{21}$) and the same is valid for the $S$-parameters $S_{45}$ and $S_{89}$ from output to input (only $S_{12}$). Parameters $S_{54}$ and $S_{89}$ are in the forward rotation direction of the circulator and $S_{45}$ and $S_{98}$ are in the reverse
direction. In a well-designed circulator the reverse transmission between two adjacent ports is at least 25 dB smaller than the forward transmission parameters, so the approximation is made to address the whole transmission from input to output to $S_{44}$ which then can be replaced by $S_{21}$ (overall transmission S-parameter).

The remaining derivation follows Appendix H (eq. H.27-H.28). The noise figure $F_{45}$ of only one general two-port (the forward path that includes the DBRT diode) is given in Appendix F. Corresponding terms in eq. K.14 can be replaced which yields the formula for the total noise figure of the reflection amplifier (second approach, with general two-ports) in which $F_{45}$ is the noise figure of the forward transmission path from input to output.

$$F = F_{45} + \left| \Gamma_G \right|^2 \frac{|b_{n1}|^2}{|b_{nG}|^2} + \frac{|1 - S_{11} \Gamma_G|^2 |b_{n2}|^2}{\left| S_{21} \right|^2 \left| b_{nG} \right|^2} + \frac{(1 - S_{11} \Gamma_G)^* \Gamma_G}{S_{21}^*} \frac{b_{n1} b_{n1}^*}{\left| b_{nG} \right|^2} + \frac{(1 - S_{11} \Gamma_G)^* \Gamma_G}{S_{21}} \frac{b_{n1} b_{n2}}{\left| b_{nG} \right|^2}$$

K.15
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