Practical verification of ductile failure curves


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This paper deals with the ductility of workpiece materials in connection with forming. The concept of the failure curve is used as a phenomenological means to describe the ductility in dependence on the state of stress.

On the basis of analysis and experiment, two different forming operations are studied in order to evaluate the practical relevance of ductile failure curves. These processes are sheet bending and backward can extrusion. As an illustrative material, recycled aluminium of moderate ductility is used.

KEY WORDS: forming, material properties, ductility.

1 Introduction

Ductility commonly is defined in terms like: "An indication or measure of the amount of plastic deformation which a material will undergo without fracture..." [3]. The ductility of a material is influenced by the environmental parameters during deformation, such as the state of stress, the temperature and the strain rate. With reference to industrial practice, the ductility of a workpiece material is of importance since it may determine the process limits in the application of plastic processes. The ductility therefore is one of the relevant aspects in the evaluation of fracture; ...

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2 Determination of ductile failure curves

A ductile failure curve - after Hancock and Mackenzie [5] - is a curve of the equivalent strain $\varepsilon^F$ versus the stress tri-axiality $(\sigma_a/\sigma_f)^F$ at ductile failure. The stress tri-axiality is the non-dimensional representative of the state of stress, and is defined as the ratio of the mean stress $\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$ and the effective flow stress $\sigma_f$. Such a failure curve can be obtained by means of some basic material tests. Essentially, these tests involve different loading situations in order to vary the state of stress. For each of the experiments, the equivalent strain and the accompanying stress tri-axiality are assessed for which fracture occurs; these determine a single point of the failure curve. By combining the results of the different tests, then, the course of the failure curve is estimated.

With regard to this concept, Bolt explored the non-negative tri-axiality range for several ferrous and non-ferrous metals [1]. Two standard loading tests were employed in this investigation: the torsion test and the tension test. A further variation in the stress tri-axiality was obtained by the use of pre-notched tensile specimens, in addition to conventional cylindrical ones. Thus, the tri-axiality range could be extended up to values of 1.5. The state of stress in the neck of these specimens was studied by means of FEM simulations, from which the tri-axiality value could be derived. For the investigated conditions, the failure curves allowed for a linear approximation. An application relates to the process of punching, in which the occurrence of fracture is an essential feature; this application also was FEM assisted.

2.1 Procedure

On the basis of these previous studies, a simplified procedure can be proposed for a practical determination of the failure curve. This involves the implementation of the standard torsion and tension test.

For torsion, solid bar specimens are used of a radius $R$ and a gauge length $L$. During testing, deformation is assumed to be distributed linearly over the cross-section of the specimen from zero strain on the axis to a maximum strain for the outer radius. The strain at fracture therefore is determined by the strain for the outer radius at fracture. The state of stress in torsion implies a tri-axiality value which equals zero throughout testing, up to fracture. Summarising, the relevant quantities can be calculated from:

$$\tilde{\varepsilon}^F = \frac{1}{3} \frac{R}{L} \tilde{\phi}^F \text{ and } \sigma_{mF}^F = \frac{\sigma_f}{3}, \quad (1)$$

in which $\tilde{\phi}^F$ denotes the twisting angle at fracture and is expressed in radians.

Concerning tensile testing, it is possible to use either bar or sheet specimens. The quantities of strain and stress are to be determined as local quantities, that is, for the smallest cross-section (in case of specimen necking). As for the calculation of strains, this entails a measurement of the diameter $d$ for bar specimens or a measurement of the width $b$ and thickness $s$ for sheet specimens. The stress situation in tension implies a tri-axiality value of 1/3 during uniform straining, but this value needs adjustment for the multi-axial state of stress in the necking range. Bridgman's necking correction [2] provides for this. According to this analysis for bar specimens, the tri-axiality level is highest on the axis; this is the location where fracture is actually initiated. The calculation requires the additional measurement of the profile radius $r_p$ in the neck. The analogous correction for sheet specimens accounts for the necking in thickness direction and necessitates a measurement of the profile radius $r_p$ in this direction. At the centre plane the tri-axiality level is highest and therefore normative. The analytical solutions for the
determination of the equivalent strain and stress tri-axiality at fracture are, for bar specimens:

\[
\tilde{\varepsilon}^F = -2\ln(\frac{d_F}{d_0}) \quad \text{and} \quad (\frac{d_F}{d_0})^2 - \ln(1 - \frac{d_F}{d_0}) \quad \text{(2)}
\]

the corresponding formulas for sheet specimens are:

\[
\tilde{\varepsilon}^F = -\ln(\frac{d_F}{d_0}) - \ln(\frac{s_F}{s_0}) \quad \text{and} \quad (\frac{d_F}{d_0})^2 - \ln(1 - \frac{d_F}{d_0}) - \ln(1 - \frac{s_F}{s_0}) \quad \text{(3)}
\]

The subscript \( F \) refers to the original specimen dimensions, whereas the index \( F \) denotes the moment of fracture – as reconstructed from the final geometry.

The failure curve now is obtained by combining the results of torsion and tension. For simplicity, the intermediate course is interpolated linearly. As a matter of reproducibility, multiple testing is to be done to obtain reliable results. A number of three experiments for each test generally will serve; the results then are to be presented as averages.

2.2 Examples

As an illustration of the introduced technique, some experimental results are shown in figure 1.

The failure curves indicated by solid lines represent the behaviour of recycled aluminium; the testing temperature here is used as a parameter [10]. Experiments were performed quasi-statically. Tensile data originate from tests on sheet specimens, taken from the transverse direction in the material; this direction proved to be the critical one with regard to ductility. From these failure curves it can be seen, that the ductility of the material approximately duplicates between 20 and 200 °C; the most significant improvements, however, manifest between 200 and 300 °C. From 300 °C on no further increase in ductility is observed. These results can be used as a guide-line in determining the proper conditions for a forming operation from a viewpoint of the failure behaviour. This means that if the strains and stresses can be assessed for a particular process, it is feasible to estimate the required minimum temperature to obtain a sound product.

Failure curves of two common grades of aluminium, AlCuBiPb (AA number 2011) and AlMgSi1 (AA number 8082), are included in figure 1 as dashed lines. These are obtained from bar specimens; tests were performed at room temperature and in a quasi-static fashion.

The failure curve of AlCuBiPb near coincides with the one of recycled aluminium at 200 °C; the curves of recycled aluminium at 300 and 350 °C show resemblance to the failure curve of AlMgSi1. These two conventional grades of aluminium are widely used as workpiece materials for cold forming processes, such as backward extrusion. Backward extrusion at temperatures above 200 °C therefore is suggested to be a suitable operation for recycled aluminium from a ductility point of view.

2.3 Sheet bending

A first concrete application of the failure curve concept concerns sheet bending. Bending processes are frequently used in industrial practice.

The sheet bending process can be described by means of a relatively simple rigid plastic model. Basic assumptions in this model:

- A plane strain situation exists in the bend.
- The neutral layer, which experiences no deformation, coincides with the centre plane of the sheet.
- The sheet thickness \( s \) does not change.
- Elastic effects are not accounted for.

The proceeding is characterised by the bending radius \( r \), which appoints the current curvature of the neutral layer in the bend.

For this rigid plastic model it can be derived that the equivalent strain is distributed linearly about the neutral layer and depends on the current curvature. Further, the stress tri-axiality carries an opposite sign on either side of the neutral layer; apart from that, its value is constant.

Of particular interest is the free surface at the outer side of the bend, since this is the location of fracture initiation. The relevant quantities for this position are:

\[
\tilde{\varepsilon} = -\frac{2}{\sqrt{3}} \ln(1 + \frac{v}{2}) \quad \text{and} \quad (\frac{d_F}{d_0})^2 - \ln(1 - \frac{d_F}{d_0}) - \frac{1}{\sqrt{3}} \quad \text{(4)}
\]

Actual bending tests were performed on sheets of recycled aluminium. This semi-finished product shows significant differences in ductility between longitudinal and transverse direction [10]; for these directions, therefore, different failure curves apply. Specimens for the sheet bending experiments were taken longitudinally and transversely from the material. The specimens had a nominal thickness \( s = 3.4 \text{ mm} \); their width \( b = 30 \text{ mm} \) was chosen so as to ensure plane straining. The experiments were done at room temperature.

In figure 2, the failure curves for the two distinct directions are reproduced. Both curves include the same torsional results. Sheet specimens were used for tensile testing. Also, the results of the sheet bending tests are shown in this figure; these stem from three separate experiments for each direction. The data, as they are presented here, are derived from the specimens by measuring the bending radii at fracture and substituting these in the model formula (4).

Concerning the transverse direction, the sheet bending results agree well with the prediction, as obtained from linear extension of the failure curve (dotted line). For the longitudinal direction, on the other hand, the sheet bending experiments yielded failure strains somewhat below the failure curve prediction.

These latter results, however, are possibly obscured by the occurrence of localised necking in the bend; as a consequence, the proposed model is not strictly appropriate. This phenomenon may appear for equivalent strains \( \tilde{\varepsilon} > \tilde{\varepsilon}_c = (2/\sqrt{3})\pi n \), in which \( n \) represents the
strain hardening exponent of the material \([6]\). From the tension tests in longitudinal direction, it was derived that \(n = 0.20\) (Hollomon flow function); the critical strain for localised necking then is \(\varepsilon_c = 0.23\).

A visioplastic grid, applied on the sheets before testing, enabled an additional determination of the failure strains directly from the specimens. Averaged for each trio of experiments, these values are \(\varepsilon_f = 0.174\) for the traverse direction and \(\varepsilon_f = 0.369\) for the longitudinal direction. The linear failure curve predictions deviate from these experimental results for 8 and 2 \% respectively.

4 Backward can extrusion

Processes which proceed in a compressive fashion are favourable with respect to ductility. One of these is backward can extrusion; this process is taken as a further application. Local values of strain and stress are gained from an analytical model. The model in full is treated elsewhere \([5]\); here, only the main features will be summarised. This model involves a combined approach, in a way that it consists of two separate analyses: one for the strains, the other for the stresses. Basic model assumptions:

- Thin-walled, rotationally symmetric products are produced.
- No dead-zone appears in the bottom region of the workpiece.
- The flow stress value is constant across the workpiece: \(\sigma_\gamma = \sigma_f\).
- Frictional effects are quantified according to the von Mises model; this introduces the plastic friction factor \(m\) as a variable.

The model is based on a three-zone representation of the process, as is shown in figure 3. Thus, the extrusion piece is divided into zones, from which the zones I and II experience plastic deformation while zone III is rigid. These zones are separated by so-called surfaces of discontinuity \(r_1\) and \(r_2\). Polar coordinates \((r, \theta, z)\) are used. The geometry of the process is represented by the ram radius \(R_R\), the chamber radius \(R_2\) and the current bottom thickness \(T\), as well as the original billet height \(T_0\). The symbol \(u\) denotes the ram velocity.

![Figure 3](image_url)

**Figure 3** Representation of the backward can extrusion model

### Strain analysis

The next kinematically admissible velocity field is used as an approximation for thin-walled cans:

\[
\begin{align*}
(\omega_1) & = \frac{r}{2T} \omega, & (\omega_2) & = 0, & (\omega_3) & = -\frac{z}{T} \omega, \\
(\omega_1) & = \frac{R_R}{2(R_C-R_R)} \frac{R_C-r}{T} \omega, & (\omega_2) & = 0, & (\omega_3) & = -\frac{R_R}{2(R_C-R_R)} \frac{z}{T} \omega,
\end{align*}
\tag{5}
\]

and \((\omega_1) = 0, (\omega_2) = 0, (\omega_3) = -\frac{R_R}{2(R_C-R_R)} \omega\).

By integration, the flow lines are deduced from this field. This means that the flow of each material point can be tracked during the process. Further, the strain quantities are derived. These comprise of the contributions of internal deformation in the respective zones and shearing deformation along the surfaces of discontinuity. Depending on the flow line, a material point is due to a certain combination of these individual strain components.

### Stress analysis

The calculation of stresses is based on a slab analysis. This double compression model originates from Dipper \([4]\); the version as it applies to the current set of assumptions is borrowed from \([8]\).

The analysis departs from the condition of force equilibrium on strategically defined "straps" in the respective zones. Only the principal stress components are considered. Plastic friction between tools and workpiece is taken into account as far as it concerns the zones I and II. Zone III is assumed to be free of stresses. Derived formulae for the stress components can be combined to obtain the next expressions for the stress tri-axiality in the different zones:

\[
\frac{\sigma_m}{\sigma_0} = \begin{cases} 
\frac{1}{3} \frac{2}{\sqrt{3}} \left(1 - \frac{1-m}{4} \frac{T}{R_C-R_R} \right) + \frac{1}{3} \frac{2}{\sqrt{3}} \frac{R_R-\tau}{T}, & \text{Zone I and II} \\
\frac{1}{3} \frac{2}{\sqrt{3}} \frac{1-m}{4} \frac{T-z}{R_C-R_R}, & \text{Zone III}
\end{cases}
\tag{6}
\]

The minus signs indicate that the process proceeds under pressure.

The flow of a material point of initial coordinates \((r_0, z_0)\) is appointed by the flow line equations. By application of the strain formulas, then, the corresponding development of the equivalent strain is assessed. The results from the slab analysis enable the determination of the accompanying stress tri-axiality. For convenience, the formulae are incorporated in a calculation programme.

Some illustrative results are shown in figure 4. Calculated strains and stresses - as they develop during the process - are visualised for three material points, symbolised by A, B and C. The inset shows the initial positions of these points in the billet. These data apply to a set of conditions as is indicated in the figure: \(T_e\) denotes the final bottom thickness. Passages through the surfaces of discontinuity are represented as dashed segments; involved quantities are undetermined for these. Further, the failure curves of recycled aluminium at 200 and 300 °C are included; the dotted branches are the linear extensions into the negative tri-axiality range. In the backward can extrusion of recycled aluminium at these temperatures, obviously, point C is a critical point which is liable to fracture.

An additional indication of the critical spots is obtained from the calculated strain distribution in the extruded product. For the present geometry, this distribution is shown in figure 5. Locations of equal equivalent strain are marked by solid lines and indexed by their corresponding values. Dotted lines separate the areas of different equivalent strains.
straining history. The final positions of the points A, B and C are also plotted in the figure. Spots which have experienced highest strains deserve special attention; these are located at the exterior side of the can in the near vicinity of the corner.

Actual experiments were done at elevated temperatures. The extrusion geometry matched the one for the calculations: \( R_e = 5 \) mm, \( R_C = 6 \) mm, \( T_e = 4 \) mm and \( T_C = 1 \) mm. Tools (both ram and chamber) were provided with heating devices. The temperature range was confined to 200–300 °C; this is the range where the material's ductility is highly dependent (figure 1). Molykote HTF was used as a lubricant, suitable for these temperatures. The experiments revealed that sound products could be obtained, even at 200 °C. No fracture of the workpiece material was visually detectable; these observations were affirmed by microscopic examination. Anyhow, subsequent forming of the cans by means of an ironing process—in order to reduce the wall thickness to 0.75 mm—showed not to be possible: the extruded cans fractured in the corner. Similar cans, not extruded but machined from the semi-finished material, could be worked by this ironing process at these temperatures without fracture.

There still remains an ambiguity, as expressed by the intersection of the analytical data (like point C in figure 4) and the extended failure curves. Of course, one may count with inaccuracies in the model. On the other hand, though, there are certain indications that the failure curve may deviate from a straight line in the compression range. One of the most prominent of these is from a study by Pugh and Green [7]. They performed tension tests at superimposed hydrostatic pressures; for various materials, it was observed that fracture deferred more than proportionally at increasing pressure. For the determination of a failure curve, by implication, the use of supplementary tests may be considered.

Plain compression tests are a simple means but their outcomes can be obscured by the occurrence of disturbing effects, such as bulging. Alternatives—as employed by Bridgman [2]—may be tension or torsion tests under pressure, yet these methods are quite involving and therefore of limited practical significance.

5 Conclusion

The failure curve concept is a useful approach towards the implications of ductility in the field of metal forming. Findings of the present work can be recapitulated as follows:

- A ductile failure curve can be determined efficiently by means of a torsion test and a tension test. Amplification, anyhow, may be required for the compression range.
- A failure curve of a workpiece material can be employed as a criterion to select suitable forming applications.
- Failure curves are apt to describe the ductility of a material in dependence on the temperature. These curves are useful in assigning the proper working temperature to a particular forming process.
- Failure curves are apt also to describe the ductility of anisotropic materials, such as recycled aluminum.
- Applications include sheet bending and backward can extrusion.

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References