Control of a tailless fighter using gain-scheduling

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Published: 01/01/2004

Document Version
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Control of a Tailless Fighter using Gain-Scheduling

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DCT.2004.19

Traineeship report

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Eindhoven, January, 2004
Abstract

In this report the nonlinear model and flight control system (FCS) for the longitudinal dynamics of a tailless fighter aircraft are presented. The nonlinear model and flight control system are developed as an extension of earlier work on a conceptual design project at RMIT Aerospace Engineering, referred to as the AFX TAIPAN. The aircraft is based on a hypothetical specification aimed at replacing the rapidly ageing RAAF fleet of F-111 and F-18 aircraft with a single airframe.

The presented six degree of freedom model consists of six second order differential equations. The lateral and the longitudinal dynamics are highly coupled and nonlinear. Based on this full order model, the longitudinal dynamics of the aircraft are formulated.

A gain scheduled controller is developed for the longitudinal dynamics of the aircraft. The gain scheduled controller is based on 25 operating points. An approach is provided to select the operating points systematically, such that the stability robustness of the overall Gain-Scheduled (GS) control is guaranteed a priori. The longitudinal model is linearized around these operating points and a linear quadratic regulator (LQR) controller is designed for each operating point to stabilize the aircraft. These controllers are interpolated using spline interpolation. Nonlinear simulation of the aircraft and flight control system is executed to analyze the performance of the flight control system.
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Chapter 1

Introduction

1.1 Motivations and Objective

In 1999, a conceptual design project of a new fighter aircraft, referred as the AFX TAIPAN, was started at RMIT Aerospace Engineering, based on a hypothetical specification aimed at replacing the rapidly ageing RAAF fleet of F-111 and F-18 aircraft with a single airframe. This has resulted in a design without a traditional vertical tailplane. The main advantage of a tailless design is the reduction of the radar cross section. In addition, the weight of the aircraft is reduced significantly. However, a major disadvantage of a tailless design is the tendency of autorotation and the spinning due to insufficient damping. The aircraft features two engines with thrust vectoring capability to enhance its manoeuvrability. It was proposed to investigate if the thrust vectoring could be used to artificially augment the stability of the aircraft without a vertical tailplane. Moreover, the vectored thrust property enables additional control authority.

Subsequent to the design of the aircraft in 1999, a new research project was started in 2001 to derive the nonlinear mathematical model of the aircraft to study the stability. From this study it appeared that the AFX-TAIPAN can be stabilized with a LQR state feedback for a certain flight condition. However, the simple LQR technique to design a controller for a certain operating point, is not sufficient to ensure the controllability of the aircraft over its entire flight envelope. Due to the large changes in the aircraft dynamics, caused by the significant changes in lift, altitude and speed of the aircraft, as well as aerodynamic angles or the change in the aircraft mass, a dynamic mode that is stable and adequately damped in one flight condition may become unstable, or at least inadequately damped in the other flight condition.
Chapter 1. Introduction

The objective of this study is to develop a Gain Scheduled controller for the longitudinal dynamics of the AFX-TAIPAN. A gain scheduled controller is a linear feedback controller for which the feedback parameters are variable. The feedback gains are scheduled according to the instantaneous values scheduling variables, which are signals or variables with which an operating condition of the plant is specified.

1.2 Research Approach

To achieve the objective, the following research approach is followed.

- First, the Tailless aircraft is considered and a dynamic model for the six degree of freedom is derived. Moreover, the longitudinal dynamics are formulated.
- Subsequently, an equilibrium point is determined using nonlinear optimization techniques. This equilibrium point is used as the nominal operating point, which is considered to be the middle of the region in which the controller is designed.
- The longitudinal model of the AFX-TAIPAN is linearized around the nominal operating point and a LQR controller is designed to stabilize the aircraft around this operating point.
- A method is developed to automatically generate a grid of operating points around the nominal operating point, such that the stability in the entire control area is guaranteed. This method is based on the stability radius concept. The control area is determined by the grid of operating points.
- For all operating points LQR controllers are designed to stabilize the aircraft within a certain area around the operating points. Using gain scheduling techniques, a global controller is designed, for which the state feedback gains of the individual LQR controllers are scaled according to the operating point in the control area.
- Simulation of the flight control system and the nonlinear model of the longitudinal dynamics of the AFX-TAIPAN is executed to analyze the performance of the flight control system.
Chapter 1. Introduction

1.3 Outline

In Chapter 2 the aircraft which is considered in this report, the AFX-TAIPAN, is discussed. First the description of the tailless aircraft is provided, after which the modelling is treated. The equations of motion and the force equations of the full six degree of freedom model are presented. The subject of Chapter 3 is Linear Quadratic Regulator (LQR) control. LQR control in general is discussed and is applied for the linearized longitudinal dynamics of the AFX-TAIPAN. In Chapter 4 Gain Scheduling is studied. A method to select the operating points is presented, such that the stability of the entire control area is guaranteed. The gain scheduled controller is applied for the longitudinal dynamics of the AFX-TAIPAN in chapter 5. Finally, some conclusions are drawn and recommendations are given in Chapter 6.
Chapter 2

Model of the AFX-TAIPAN

In this chapter the AFX-TAIPAN is introduced. First, a general description the aircraft is provided. Subsequently, the six degree of freedom dynamic model of the aircraft is derived. The assumptions which are needed for this derivation are presented, just as the axes systems used in this report. The equations of motion and the force model are described in body axes.

2.1 AFX-Taipan

The AFX-Taipan was designed in 1999 to be a multirole aircraft. The aircraft is based on a hypothetical specification aimed at replacing the rapidly ageing RAAF fleet of F-111 and F-18 aircraft with a single airframe. The airframe is a fighter-bomber, which is capable to carry a large and diverse payload for the strike role. To attain supermanoeuvrability in air, the thrust-to-weight ratio is larger than one. This resulted in a tailless design. The aircraft is depicted in Figure 2.1. The full specification of the aircraft is given in [GPT99].

The aircraft is equipped with two JSF119-611 Afterburning Turbofans. These engines enable the use of a 3D thrust vectoring system in order to improve the manoeuvrability of the aircraft. The maximum pitch and yaw angle of the nozzles are fixed to 20°. Moreover, the control system of the aircraft contains outboard split ailerons. The application of thrust vectoring allowed the designers to consider a tailless design. The main advantages of a tailless configuration are:

- By removing the tail and shortening of the fuselage, the drag of the aircraft can be reduced by 20% - 35%.
- The structural weight of the aircraft can be reduced with 10%
- The range for a given fuel load can be increase because of the reduced drag.
Chapter 2. Model of the AFX-TAIPAN

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The stealth characteristics can be improved because of the reduced surface area. However, a major disadvantage of a tailless design is the tendency of autorotation and the spinning due to insufficient damping. Moreover, the tail provides a downward force that counteracts the pitching tendency of an aircraft. Therefore a flight control system has to be designed to stabilize the aircraft.

2.2 Aircraft Model

For the design of a flight control system for the AFX-TAIPAN, a mathematical description of the dynamical behaviour is needed. In this section the equations of motion are derived and the force model is provided. The model is an improvement of the model derived [Pan02].

2.2.1 Assumptions

For the modelling of the aircraft dynamics, several assumptions are used. These assumptions follow from [Pan02].

- The aircraft is considered as a rigid body with six degrees of freedom; three translational and three rotational degrees.
- The mass of the aircraft is constant.
• The Earth is flat and therefore the global coordinate system is fixed to it. Consequently, the Earth is considered as an inertia system and Newton’s second law can be applied.

• $OX_b$ and $OZ_b$ are planes of symmetry for the aircraft, therefore: $I_{xy} = \int yz \, dm$ and $I_{xy} = \int xy \, dm$ are equal to zero and $I_{xx} = I_{zz}$ in the inertia tensor matrix.

• The atmosphere is fixed to the Earth.

• The airflow around the aircraft is assumed quasi-steady. This means that the aerodynamics forces and moments dependent only on the velocities of the vehicle relative to the air mass.

• As the aerodynamics is quasi-steady, the forces and moments of the aircraft are considered to act with respect to the aircraft centre of gravity (CG).

• The vector thrust, aerodynamic forces and moments can be resolved into body-axis components at any instant of time.

• The atmosphere is still (no wind).

• The ailerons are symmetrical with respect to the plane that is spanned by the x-axis and z-axis of the body fixed axes system of the aircraft.

• The contribution to the inertial coupling of all portions of the control system other than the aerodynamic surfaces is neglected.

• For the purpose of calculating the inertia coupling, the control surfaces are approximated as laminae lying in the coordinate planes.

• The control systems are frictionless.

• Each control system has one degree of freedom relative to the body axes.

• Each control system consists of a linkage of rigid elements, attached to a rigid airplane.

• The acceleration due to gravity is considered as constant with a value equal to 9.81 $m/s^2$.

• There are no gyroscopic effects acting on the aircraft.

2.2.2 Axes Systems

There are three primary axes systems considered, which are depicted in figure 2.2.
Chapter 2. Model of the AFX-TAIPAN

1. The first axes system is the Earth initial axes system \( (\mathbf{e}^e) \). Because \( \mathbf{e}_1^e \) is pointed towards the North, \( \mathbf{e}_2^e \) towards the East and \( \mathbf{e}_3^e \) Downward, this reference frame is also known as the NED axes system. The inerial frame is required for the application of Newton’s laws.

2. The second axes system is the aircraft-carried inertial axes system \( (\mathbf{e}^i) \). This axis system is obtained if the Earth inertial axes frame is translated to the center of gravity of the aircraft with a vector

\[
\mathbf{r}_{CG} = [x \ y \ z] \mathbf{e}^0
\]  

(2.1)

3. The body axes system \( (\mathbf{e}^b) \) is also an aircraft-carried axes system, with \( \mathbf{e}_1^b \) pointed towards the nose of the aircraft, \( \mathbf{e}_2^b \) towards the right wing and \( \mathbf{e}_3^b \) to the bottom of the aircraft. The axis system is obtained through successive rotations of the aircraft-carried inertial frame with Tait-Bryant angles \( \psi, \theta \) and \( \phi \). The velocity vectors along these axes are \( u, v \) and \( w \) and the angular velocity vectors are respectively: roll rate \( q \), pitch rate \( \dot{q} \) and yaw rate \( r \). These velocity vectors are shown in figure 2.3.

Figure 2.2: Aircraft axes systems
2.2.3 State, input and output variables

The motion of an aircraft considered as a rigid body can be described by a set of six coupled nonlinear second order differential equations. In general the model can be described as

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= g(x, u)
\end{align*}
\]

(2.2)

where \(x \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}^m\) is the input vector and \(y \in \mathbb{R}^d\) is the output vector.

\[
\begin{align*}
x &= [x \ y \ z \ \phi \ \theta \ \psi \ u \ v \ w \ p \ q \ r]^T \\
u &= [\delta A \ T_1 \ p_{n1} \ y_{n1} \ T_2 \ p_{n2} \ y_{n2}]^T \\
y &= [x \ y \ z \ \phi \ \theta \ \psi \ u \ v \ w \ p \ q \ r]^T
\end{align*}
\]

(2.3)

The elements of the state and output vector are described in table 2.1, while the elements of the input vector are shown in table 2.2. The translational velocities, \(\dot{u}, \dot{v}\) and \(\dot{w}\) are selected instead of the total velocity, \(V_t\), the angle of attack, \(\alpha\), and the sideslip angle, \(\beta\), which are chosen in [Pan02]. The relation between the velocities of the aircraft with respect to the inertia axis frame can be expressed in terms of the velocities with respect
Chapter 2. Model of the AFX-TAIPAN

Table 2.1: State definitions

<table>
<thead>
<tr>
<th>States</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>x component of velocity in body axes</td>
<td>m/s</td>
</tr>
<tr>
<td>v</td>
<td>y component of velocity in body axes</td>
<td>m/s</td>
</tr>
<tr>
<td>w</td>
<td>z component of velocity in body axes</td>
<td>m/s</td>
</tr>
<tr>
<td>p</td>
<td>Roll rate in body axes</td>
<td>rad/s</td>
</tr>
<tr>
<td>q</td>
<td>Pitch rate in body axes</td>
<td>rad/s</td>
</tr>
<tr>
<td>r</td>
<td>Yaw rate in body axes</td>
<td>rad/s</td>
</tr>
<tr>
<td>x</td>
<td>x position of CG in inertial frame</td>
<td>m</td>
</tr>
<tr>
<td>y</td>
<td>y position of CG in inertial frame</td>
<td>m</td>
</tr>
<tr>
<td>z</td>
<td>z position of CG in inertial frame</td>
<td>m</td>
</tr>
<tr>
<td>φ</td>
<td>Roll angle</td>
<td>rad</td>
</tr>
<tr>
<td>θ</td>
<td>Pitch angle</td>
<td>rad</td>
</tr>
<tr>
<td>ψ</td>
<td>Yaw angle</td>
<td>rad</td>
</tr>
</tbody>
</table>

Table 2.2: Input Definitions

<table>
<thead>
<tr>
<th>Input</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>δₐ</td>
<td>Aileron angle</td>
<td>rad</td>
</tr>
<tr>
<td>T₁</td>
<td>Thrust force engine 1</td>
<td>N</td>
</tr>
<tr>
<td>p₁₁</td>
<td>Pitch angle of nozzle 1</td>
<td>rad</td>
</tr>
<tr>
<td>y₁₁</td>
<td>Yaw angle of nozzle 1</td>
<td>rad</td>
</tr>
<tr>
<td>T₂</td>
<td>Thrust force engine 2</td>
<td>N</td>
</tr>
<tr>
<td>p₂₂</td>
<td>Pitch angle of nozzle 2</td>
<td>rad</td>
</tr>
<tr>
<td>y₂₂</td>
<td>Yaw angle of nozzle 2</td>
<td>rad</td>
</tr>
</tbody>
</table>
Chapter 2. Model of the AFX-TAIPAN

to the body fixed axes frame using the direction cosine matrix.

\[
\begin{pmatrix}
  \dot{x} \\
  \dot{y} \\
  \dot{z}
\end{pmatrix}
= \mathbf{A}^{cb}
\begin{pmatrix}
  u \\
  v \\
  w
\end{pmatrix}
\]

(2.4)

where:

\[
\mathbf{A}^{cb} =
\begin{pmatrix}
  \cos \psi \cos \theta & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\
  \sin \psi \cos \theta & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\
  -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{pmatrix}
\]

(2.5)

The angular velocity vector gives the relation between the Tait-Bryant angles and their derivatives and the roll (p), pitch (q) and yaw (r) rates of the aircraft.

\[
\mathbf{b}^{bc}\mathbf{\omega}^{b} =
\begin{pmatrix}
  p \\
  q \\
  r
\end{pmatrix}
= \begin{pmatrix}
  -\dot{\psi} \sin \theta + \dot{\phi} \\
  \dot{\theta} \cos \phi + \dot{\psi} \sin \phi \cos \theta \\
  -\dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta
\end{pmatrix}
\]

(2.6)

2.2.4 Equations of Motion in Body Axes

The non-linear model of the tailless aircraft can be derived using Newton-Euler equations. Newton’s second law is applied for the translational dynamics and Euler equations are used to describe the rotational dynamics of the aircraft.

Newton’s second law

Newton’s second law for a rigid body is:

\[
\sum \mathbf{F}^{b} = \frac{d}{dt} \mathbf{m} \mathbf{v}_{CG}^{b}
\]

(2.7)

where \(\sum \mathbf{F}^{b}\) is the resultant of all external forces applied to the body with mass \(m\). \(\mathbf{v}_{CG}^{b}\) is the linear velocity vector of the center of gravity (CG) of the body relative to the Earth inertial frame. Because the mass of the aircraft is considered to be constant, (2.7) can be written as

\[
\sum \mathbf{F}^{b} = m \mathbf{v}_{CG}^{b}
\]

(2.8)

where \(\mathbf{v}_{CG}^{b}\) is the linear acceleration vector of the body relative to the Earth inertial frame. Based on (2.8), the equations of motion derived in the body axes system are:

\[
\sum \mathbf{F}^{b} = m \left( \frac{d}{dt} \mathbf{v}_{CG}^{b} \right) + \mathbf{b}^{bc} \mathbf{\omega}^{b} \times \mathbf{v}_{CG}^{b}
\]

(2.9)
Chapter 2. Model of the AFX-TAIPAN

with

\[
\begin{align*}
\sum F^b &= \left[ \sum F_1^b \sum F_2^b \sum F_3^b \right]^T \\
\dot L_{CG}^b &= \begin{bmatrix} u \\ v \\ w \end{bmatrix}^T \\
\dot L_{CG}^b &= \begin{bmatrix} p \\ q \\ r \end{bmatrix}^T
\end{align*}
\]

(2.10)

where \( \omega^b \) (2.6) is the angular velocity vector of the aircraft with respect to the body fixed frame. This results in the following force equations

\[
\begin{align*}
\sum F_1^b &= m (\dot u + q \omega - r \nu) \\
\sum F_2^b &= m (\dot v + r \omega - p \nu) \\
\sum F_3^b &= m (\dot w + p \omega - q \nu)
\end{align*}
\]

(2.11)

Euler Equations

Euler equations are applied for the rotational dynamics of the aircraft. The rotational dynamics can be described as

\[
\sum \bar M_{CG} = \bar \dot H_{CG}
\]

(2.12)

where \( \sum \bar M_{CG} \) is the resultant of the external moments applied to a body relative to its center of gravity. \( \bar H_{CG} \) is the angular momentum vector relative to the center of gravity of the aircraft, which can be described by

\[
\bar H_{CG} = J_{CG} \omega
\]

(2.13)

where \( J_{CG} \) is the inertia tensor of the rigid body. From (2.12) and (2.13), the rotational equations of motion derived in the body axes system are:

\[
\sum \bar M^b = \frac{d}{dt} \left( J^b_{CG} \omega^b \right) + J^b_{CG} \omega^b \times J^b_{CG} \omega^b
\]

(2.14)

with

\[
J^b_{CG} = \begin{pmatrix}
I_x & 0 & -I_z \\
0 & I_y & 0 \\
-I_z & 0 & I_x
\end{pmatrix}
\]

(2.15)

This results in the following equations.

\[
\begin{align*}
\sum M_1^b &= I_x \dot \phi - I_{xz} \dot \theta + qr (I_z - I_y) - I_{xz} \rho q \\
\sum M_2^b &= I_y \dot \theta + rq (I_x - I_z) + I_{xz} (p^2 - r^2) \\
\sum M_3^b &= -I_{xz} \dot \phi + I_x \dot \theta + pq (I_y - I_z) + I_{xz} qr
\end{align*}
\]

(2.16)
2.2.5 Forces and Moments

The forces and moments at the center of gravity of the aircraft have components due to the gravitational effects, aerodynamics effect and the forces and moments due to the 3d thrust vectoring system. The force model is based on the force model derived in [Pan02].

Gravitational forces

The components of the gravitational forces in the aircraft center of gravity with respect to the body axes are:

\[
\begin{align*}
F^b_{G1} &= -mg \sin(\theta) \\
F^b_{G2} &= mg \sin(\phi) \cos(\theta) \\
F^b_{G3} &= mg \cos(\phi) \cos(\theta)
\end{align*}
\]

(2.17)

Aerodynamic forces and moments

The aerodynamic forces acting on the aircraft are basically the drag, lift, side-force and the aerodynamic rolling, pitching and yawing moment.

\[
\begin{align*}
F_{Drag} &= \bar{q} S C_D \\
F_{Lift} &= \bar{q} S C_L \\
F_{Side} &= \bar{q} S C_y \\
M^{b}_{A1} &= \bar{q} S b C_l \\
M^{b}_{A2} &= \bar{q} S mac C_m \\
M^{b}_{A3} &= \bar{q} S b C_n
\end{align*}
\]

(2.18)

where \( \bar{q} = \frac{1}{2} \rho V_A^2 \) is the dynamic pressure of the free-stream, \( S \) the wing reference area, \( b \) the wing span and \( mac \) the mean aerodynamic chord. The values of these parameters are determined in [GPT99] and given in table A.1 of Appendix A. \( C_D, C_L, C_Y, C_l, C_m \) and \( C_n \) are the dimensionless coefficients of drag, lift, side-force, rolling-moment, pitching-moment and yawing-moment respectively. The aerodynamic coefficients are mainly a function of the angle of attack and the side-slip angle. Although the dependency of the aerodynamic coefficients to these parameters can be very nonlinear in high Mach number region, a linear approximation is used. The aerodynamic coefficients are derived
Chapter 2. Model of the AFX-TAIFAN

in [GPT99] and shown below.

\[ C_D = c_{D0} + c_{D1}(\alpha - \alpha_{eq}) + c_{D2} \left( \frac{u - u_{eq}}{u_n} \right) \]
\[ C_L = c_{L0} + c_{L1}(\alpha - \alpha_{eq}) + c_{L2}q \left( \frac{b}{2V_t} \right) + c_{L3} \left( \frac{u - u_{eq}}{u_n} \right) \]
\[ C_y = c_{y0}(\beta - \beta_{eq}) \]
\[ C_l = c_{l0}(\beta - \beta_{eq}) + c_{l1}p \left( \frac{b}{2V_t} \right) + c_{l2}r \left( \frac{b}{2V_t} \right) + c_{l3}\delta_A \]
\[ C_m = c_{m0} + c_{m1} \left( \frac{u - u_{eq}}{u_n} \right) + c_{m2}(\alpha - \alpha_{eq}) + c_{m3} \left( \frac{mac}{2V_t} \right) \]
\[ C_n = c_{n0}(\beta - \beta_{eq}) + c_{n1}p \left( \frac{b}{2V_t} \right) + c_{n2}r \left( \frac{b}{2V_t} \right) + c_{n3}\delta_A \]

(2.19)

The values for the aerodynamic parameters \( c_i \) are given in table A.2. In (2.19) the parameters \( \beta_{eq}, \alpha_{eq}, u_n, \) and \( u_{eq} \) are obtained from [Pan021] and are given in table A.3.

Since the drag force is defined in negative flight direction and the lift force perpendicular to the flight direction pointed to the top of the aircraft, the components of these aerodynamic forces in the body fixed frame are:

\[ F_{A,1}^b = F_{Lift}\sin(\alpha) - F_{Drag}\cos(\alpha) \]
\[ F_{A,3}^b = -F_{Lift}\cos(\alpha) - F_{Drag}\sin(\alpha) \]

(2.20)

**Thrust forces**

The thrust forces are generated by a 3D thrust vectoring system. The thrust forces in body axes frame are derived using goniometric relations

\[ F_{T,1}^b = T_1 \cos(p_{n1}) \cos(y_{n1}) + T_2 \cos(p_{n2}) \cos(y_{n2}) \]
\[ F_{T,2}^b = T_1 \sin(y_{n1}) + T_2 \sin(y_{n2}) \]
\[ F_{T,3}^b = T_1 \sin(p_{n1}) \cos(y_{n1}) + T_2 \sin(p_{n2}) \cos(y_{n2}) \]

(2.21)

where \( T_1 \) and \( T_2 \) denote the magnitude of the thrust force, \( p_{n1} \) and \( p_{n2} \) the pitch angle of the nozzles and \( y_{n1} \) and \( y_{n2} \) the yaw angle of the nozzles. The moments on the aircraft generated by the thrust forces are

\[ M_{T,1}^b = (F_{T,3,1}^b - F_{T,3,2}^b)d \]
\[ M_{T,2}^b = (F_{T,3,1}^b + F_{T,3,2}^b)X_{cg} \]
\[ M_{T,3}^b = (F_{T,1,1}^b - F_{T,1,2}^b)d - (F_{T,2,1}^b + F_{T,2,2}^b)X_{cg} \]

(2.22)
where $d$ is the distance between the nozzle exit center and the aircraft vertical plane and $X_{cg}$ the distance from the nozzle exit to the aircraft center of gravity.

With this, the dynamical model of the aircraft is derived. The combination of the equations of motion and the force model yields in six coupled nonlinear second order differential equations. The model can be used for the determination of the response of the aircraft.
Chapter 3

LQR Control

In this chapter the design of a Linear Quadratic Regulator (LQR) controller is discussed. First, LQR control in general is studied and then applied for the longitudinal control of the aircraft. A nominal operating point is selected and the input is adjusted such that remaining accelerations in the operating point are minimized. Subsequently, the nonlinear longitudinal model is linearized around this nominal operating point and the stability of the open-loop and closed-loop system is discussed.

3.1 Linear Quadratic Regulator (LQR) State Feedback Design

A most effective and widely used technique of linear control systems design is the optimal Linear Quadratic Regulator (LQR). A brief description of LQR state feedback design is given below. For more details, see [Lew98].

Consider the linear time invariant system

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

(3.1)

with state vector, \(x(t) \in \mathbb{R}^n\), input vector, \(u(t) \in \mathbb{R}^m\) and output vector \(y(t) \in \mathbb{R}^l\). If all the states are measurable, the state feedback

\[
u = -Kx
\]

(3.2)

with state feedback gain matrix, \(K \in \mathbb{R}^{mn}\), can be applied to obtain desirable closed loop dynamics.
Chapter 3. LQR Control

\[
\dot{x} = (A - BK)x = A_d x
\]

(3.3)

For LQR control the following cost function is defined:

\[
J = \frac{1}{2} \int_0^\infty [x(t)^T Q x(t) + u(t)^T R u(t)] \, dt
\]

(3.4)

Substitution of (3.2) into (3.4) yields:

\[
J = \frac{1}{2} \int_0^\infty x(t)^T (Q + K^T R K) x(t) \, dt
\]

(3.5)

The objective of LQR control is to find a state feedback gain matrix, \( K \), such that the cost function (3.5) is minimized. In (3.5), the matrices \( Q \in \mathbb{R}^{n \times n} \) and \( R \in \mathbb{R}^{m \times m} \) are weighting matrices which determine the closed-loop response of the system. The matrix \( Q \) is a weighting matrix for the states and matrix \( R \) is a weighting matrix for the input signals. By the choice of \( Q \) and \( R \) a consideration between response time of the system and control effort can be made. \( Q \) should be selected to be positive semi-definite and \( R \) to be positive definite.

To minimize the cost function, (3.5) should be finite. Since (3.5) is an infinite integral, convergence implies \( x(t) \to 0 \) and \( u(t) \to 0 \) as \( t \to \infty \). This in turn guarantees stability of the closed-loop system (3.3). To find the optimal feedback, \( K \), it is assumed that there exists a constant matrix \( P \) such that:

\[
\frac{d}{dt} (x^T P x) = -x^T (Q + K^T R K)x
\]

(3.6)

Substituting (3.6) into (3.5) results in

\[
J = \frac{1}{2} \int_0^\infty \frac{d}{dt} (x^T P x) \, dt = \frac{1}{2} x^T(0) P x(0)
\]

(3.7)

If the closed-loop system is stable, \( x(t) \to 0 \) for \( t \to \infty \). Now, \( J \) is a constant that depends only on the matrix \( P \) and the initial conditions. At this point, it is possible to find a state feedback, \( K \), such that the assumption of a constant matrix \( P \) holds. Substituting the differentiated form of (3.6) into (3.3) yields:

\[
x^T (A_d^T P + PA_d + Q + K^T R K) x = 0
\]

(3.8)
and therefore
\[ A^T P + PA_d + Q + K^T R K = 0 \] (3.9)
Substitution of (3.3) into (3.9) yields
\[ A^T P + PA + Q + K^T R K - K^T B^T P - PBK = 0 \] (3.10)
Assuming that the following identity is selected
\[ K = R^{-1} B^T P \] (3.11)
the following result can be obtained.
\[ A^T P + PA + Q - PBR^{-1} B^T P = 0 \] (3.12)
This result is the Algebraic Riccati Equation (ARE). It is a matrix quadratic equation, which can be solved for \( P \) given \( A, B, Q \) and \( R \), provided that \((A, B)\) is controllable and \((Q^{-1}, A)\) is observable. In that case (3.12) has two solutions. There is one positive definite and one negative definite solution. The positive definite solution has to be selected.

Summarizing, the procedure to find the LQR state feedback gain matrix \( K \) is:

- Select the weighting matrices \( Q \) and \( R \)
- Solve (3.12) to find \( P \).
- Compute \( K \) using (3.11).

With this, the optimal feedback \( u = -R^{-1} B^T P x \) is obtained.

### 3.2 Longitudinal LQR-Control application

The above LQR-theory is applied for the longitudinal flight control of the tailless aircraft. First the longitudinal dynamics are formulated and a nominal operating point is selected. The nonlinear model is linearized around this operating point and an LQR controller is designed.
3.2.1 Longitudinal dynamics

The full model of the tailless aircraft is described in Chapter 2. In this chapter only the longitudinal dynamics are considered. The longitudinal model of the AFX-TAIPAN in body axes are defined as

\[ \dot{x}_{lon} = f_{lon}(x_{lon}, u_{lon}) \]
\[ y = g_{lon}(x_{lon}, u_{lon}) \]

where \( x_{lon} \in \mathbb{R}^4 = [\theta \ u \ w \ q]^T \) is the state vector, \( u_{lon} \in \mathbb{R}^2 = [T_i \ p_{na}]^T \) is the input vector and \( y \in \mathbb{R}^4 = [\theta \ u \ w \ q]^T \) is the output vector. It should be noted that for the longitudinal model, there are three degree of freedom and two independent inputs. The longitudinal dynamics are considered for a side velocity \( v = 0 \), roll angle \( \phi = 0 \) and a yaw angle \( \psi = 0 \). The angular velocities \( p \) and \( r \) are also equal to zero. The thrust force and pitch angle of both engines are assumed to be equal and the yaw angle for the nozzle is considered to be zero. The dynamic equations, \( f_{lon}(x_{lon}, u_{lon}) \) are defined as follows:

\[
\dot{\theta} = q
\]
\[
\dot{u} = \frac{qS}{m - qw} \left( c_{L0} + c_{L1} \left( \arctan \left( \frac{u}{w} \right) - \alpha_{eq} \right) + \frac{1}{2} c_{L2} q \frac{b}{(u^2 + w^2)^{3/2}} + c_{L3} \frac{w - U_e}{U_n} \right) \frac{w}{u}
\]
\[
\dot{w} = \frac{lfqS}{m + qu} \left( -c_{L0} - c_{L1} \left( \arctan \left( \frac{u}{w} \right) - \alpha_{eq} \right) - \frac{1}{2} c_{L2} q \frac{b}{(u^2 + w^2)^{3/2}} - c_{L3} \frac{w - U_e}{U_n} \right)
\]
\[
\dot{q} = \frac{qS \text{mac}}{I_{yy}} \left( c_{m0} + c_m \frac{u - U_e}{U_n} - c_{m2} \left( \arctan \left( \frac{u}{w} \right) - \alpha_{eq} \right) + \frac{1}{2} c_{m3} q \text{mac} \frac{(u^2 + w^2)^{3/2}}{(u^2 + w^2)^{3/2}} \right)
\]
\[
+ \frac{1}{I_{yy}} (2T_i \sin (p_{na}) \ X_{eq})
\]
Chapter 3. LQR Control

3.2.2 Operating point

In order to design a LQR controller, the nonlinear model (3.15) has to be linearized around a certain equilibrium point. $\mathbf{x}_{\text{eq}}$ is an equilibrium point of (3.13) if there exists $\mathbf{u}_{\text{eq}}$ such that $f_{\text{eq}}(\mathbf{x}_{\text{eq}}, \mathbf{u}_{\text{eq}}) = 0$.

Because it is not possible to find an equilibrium analytically, nonlinear optimization techniques are used to find values for the elements of the inputs $T_i$ and $p_{ni}$ such that the sum of the squared remaining acceleration is minimized for a given operating point.

The objective function $O$ is:

$$O(\mathbf{x}_{\text{opt}}) = \dot{u}^2 + \dot{w}^2 + \dot{q}^2$$

The upper and lower bound for the optimization parameters are:

$$0 \, N \leq T_i \leq 134460 \, N$$

$$-\frac{20}{180} \pi \, \text{rad} \leq p_{ni} \leq \frac{20}{180} \pi \, \text{rad}$$

The next thing to do is to determine an initial operating point. For the initial operating point it is desired that this is an equilibrium point. From (3.13) it can only be concluded that the pitch velocity, $q$, has to be equal to zero. Further, the inputs $T_i$ and $p_{ni}$ are kept at a fixed value of respectively 50.000 N and 0 rad. The remaining states $u, w$ and $\theta$ are used as optimization parameters to minimize the sum of the squared accelerations on the aircraft (3.16). The results are listed in table 3.1. This operating point is considered as the nominal operating point. It can be observed that the maximal remaining acceleration is $-3.3358 \cdot 10^{-6} \, \text{rad/s}^2$. This means that no equilibrium point is found.
Chapter 3. LQR Control

Table 3.2: Eigenvalues

<table>
<thead>
<tr>
<th>Open-loop eigenvalues</th>
<th>Closed-loop eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5607</td>
<td>-9.5580</td>
</tr>
<tr>
<td>0.0334 + 0.0700i</td>
<td>-0.1192 + 0.1435i</td>
</tr>
<tr>
<td>0.0334 - 0.0700i</td>
<td>-0.1192 - 0.1435i</td>
</tr>
<tr>
<td>-0.1030</td>
<td>-0.8962</td>
</tr>
</tbody>
</table>

3.2.3 Stability

Now the nominal operating point is determined, the longitudinal model (3.15) is linearized around this point. Subsequently an LQR controller is designed using the function 'lqr' in the MatLab control toolbox. The weighting matrices $Q$ and $R$ that are used are:

\[
Q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},
R = 0.1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}^2
\]  

(3.18)

It should be noted that the values for the weighting matrices are chosen arbitrary. In this report performance requirements for the aircraft are not specified. The only requirement is to stabilize the aircraft. However, if performance requirements are taken into account, they can be satisfied by adjusting $Q$ and $R$. The selected weighting matrices result in the following feedback gain for the nominal operating point.

\[
K = \begin{bmatrix} -0.1430 & 0.0044 & 0.0009 & -0.1692 \\ 226.8362 & -2.4890 & 1.6478 & 311.0628 \end{bmatrix}
\]

The eigenvalues of the open-loop and the closed-loop systems are given in table 3.2. From this it can be concluded that the unstable modes are now fully stabilized using a LQR controller. In figure 3.1 the closed loop response of the four states on a step in the pitch angle of the nozzle is depicted. From figure 3.1 it can be observed that a steady state value is reached after 50 seconds. Also from this figure it can be concluded that the state feedback matrix $K$, can stabilize the the system.

Summarizing, it can be observed that the longitudinal dynamics of the AFX-Taipan are derived in this chapter and linearized around an operating point. The nominal operating point is found using optimization techniques, but it is not an equilibrium point. In addition it can be concluded that a LQR controller can be used to stabilize the system around the nominal operating point.
Figure 3.1: Response to a step change in command pitch angle of the nozzle
Chapter 4

Gain Scheduling

The subject of this chapter is gain-scheduling. First, gain-scheduling is discussed in general. An approach is developed to construct automatically a regular grid of operating points. This approach is based on [Akm01] and uses the concept of the stability radius.

4.1 Introduction

Gain-scheduling is the main technique that is used in flight control systems. It is a control method that can be applied to linear time-varying and nonlinear systems. The difference between classical control techniques and gain-scheduled control, is that a gain-scheduled controller can provide control over an entire flight envelope, while a classical controller is only valid in the neighbourhood of a single operating point. Many different controller strategies, for example precompensation of a nonlinear gain with the inverse gain function, can be viewed as gain scheduling. However, in this report gain scheduling will be considered as the continuously variation of the controller coefficients according to the current value of the scheduling signals.

According to [RS00] the design of a gain scheduled controller for a nonlinear plant can be described with a four step procedure.

- The first step involves the computation of a linear parameter varying model for the plant. The most common approach is to linearize the nonlinear plant around a selection of equilibrium points. This results in a family of operating points.
- The second step is to design a family of controllers for the linearized models in each operating point. Because of the linearization, simple linear controller design methods can be used to stabilize the system around the operating point.
- The third step is the actual gain scheduling. Gain scheduling involves the implementation of the family of linear controllers such that the controller coefficients are scheduled according to the current value of the scheduling variables.
The last step is the performance assessment. This can be done analytically or using extensive simulation.

4.2 Stability of the Gain Scheduled Controller

Consider the nonlinear system
\[ \dot{x} = f(x, u) \] (4.1)
Let \( \Gamma \subset \mathbb{R}^q \) denote the set of GS-parameters, such that for every \( \gamma \in \Gamma \) there is an equilibrium \( (x_\gamma, u_\gamma) \) of 4.1. Consider the control law for system (4.1)
\[ u = u_\gamma - K_\gamma (x - x_\gamma) \] (4.2)
where \( K_\gamma \in \mathbb{R}^{m \times n} \) is a feedback gain that strictly stabilizes the linearized plant at equilibrium \((x_\gamma, u_\gamma)\).

The stability and the robustness of the nonlinear plant (4.1) with controller (4.2) as the system moves from one equilibrium to another along an arbitrary path is studied in [AkmOl]. Theorem 4.2 in [AkmOl] proves that the control law (4.2) is robust with respect to perturbations if the scheduling variables vary slowly.

4.3 Stability Radius

In most gain scheduling applications, the operating points are selected heuristically. In [AkmOl] a systematic way to select the operating points, such that the stability of the entire control area is guaranteed, is explored, based on the idea of the complex stability radius. The automatic construction of a regular grid of operating points for the control area of the AFX-TAIPAN, is based on this technique.

Consider the linear system of the form:
\[ \dot{z}(t) = A_z(t) + B u(t) \] (4.3)
where \((A,B)\) is assumed to be stabilisable. Applying the state feedback stabilizing controller
\[ u(t) = -K_z z(t) \] (4.4)
(4.3) can be written as
\[ \dot{z}(t) = A_d z(t) \] (4.5)
where \( A_d = A - BK \). Since \( A_d \) is stable, the eigenvalues of \( A_d \) are in the left half plane.
Chapter 4. Gain Scheduling

Consider a perturbation of the closed-loop system with a structured perturbation $E \Delta H$, where $E \in \mathbb{R}^{m \times p}, H \in \mathbb{R}^{q \times m}$ are scale matrices that define the structure of the perturbation and $\Delta$ is an unknown linear perturbation matrix. The closed-loop perturbed system then becomes

$$\dot{x}(t) = (A_{cl} + E \Delta H)x(t)$$

(4.6)

According to [HK89], the complex structured stability radius of (4.6) is defined by

$$r_c(A, E, H) = \inf \{||\Delta||; \Delta \in \mathbb{R}^{p \times q}, \sigma(A_{cl} + E \Delta H) \cap \mathbb{C}_+ \neq \emptyset\}$$

(4.7)

where $\mathbb{C}_+$ denotes the closed right half plane and $||\Delta||$ the spectral norm (i.e. largest singular value) of $\Delta$. Further, in [HK89] it is proved that the determination of the complex stability radius (4.7) is equivalent to the computation of the $H^\infty$-norm

$$\|G\|_{H^\infty} = \max_{\omega \in \mathbb{R}} \|G(i\omega)\|$$

(4.8)

of the associated transfer function $G(s) = H(sI - A_{cl})^{-1} E$.

4.4 Selection of the operating points

Using the concept of the complex stability radius, new operating points can be selected in such a way that the stability and performance robustness of the overall design is guaranteed.

Given the complex stability radius, $r_c(\gamma_i)$, of operating point $\gamma_i$, a sphere, $B_{\gamma_i}(\alpha)$, can be constructed in which the state feedback gain matrix $K_{\gamma_i}$ can be used to stabilize the aircraft. This method is described in [Akm01] and is depicted in figure 4.1 for the 2D case with 2 scheduling variables. In this figure the construction of a circle around $\gamma_i$, contained in the stability radius, is shown. Mathematically the construction of the sphere can be formulated as follows:

$$E \Delta_i H = (A_i - B_i K_i) - (A_\gamma - B_\gamma K_i)$$

$$r(\gamma) = \|\Delta_i\|$$

$$\Gamma_i = \{\gamma : r(\gamma) \leq r_c\}$$

$$B_{\gamma_i}(\alpha) = \{\|\gamma - \gamma_i\| \leq \alpha\}$$

$$B_{\gamma_i}(\alpha) \subset \Gamma_i$$

(4.9)

The next thing to do is to find a systematic way to fill the operating space with operating points, such that the stability of the whole control area is guaranteed. The idea is to fill the control area with a grid of adjoining squares of the same size. The size of the squares is determined by the point in the control area for which the radius of the sphere
in which the system can be stabilized is the smallest. The centers of the squares are the new operating points. This approach is depicted in figure 4.2. With this, the procedure to construct a regular grid of operating points is completed.
Figure 4.2: Neighbourhood
Chapter 5

Longitudinal Gain Scheduling Control

In this chapter the gain scheduling technique will be applied for the longitudinal control of the AFX-TAIPAN. The longitudinal model is described in Chapter 3. A regular grid of operating points is constructed according the technique of Chapter 4. For these operating points LQR controllers are designed. The resulting global gain scheduled controller is applied to the longitudinal flight control system of the AFX-TAIPAN. In figure 5.1 a schematic representation of the design approach is depicted.

5.1 Scheduling Variables

The first step in the design approach is the selection of the scheduling variables. In [Akm01] it was proved that the scheduling variables should vary slowly to maintain stability when the aircraft moves from one equilibrium point to another. From [Rug91] and [RS00] it also follows that the scheduling variables should be slowly varying and capture the nonlinearities of the system. However, this is only a qualitative result. In this report the forward velocity, $u$, and the downward velocity, $w$, are chosen as scheduling variables. Intuitively, these variables are slowly time varying and capture the dynamic nonlinearities. However, not all nonlinearities are captured. To capture all nonlinearities, the density of the air and the pitch angle should also be considered. In this report the pitch angle and the density of the air are kept on a fixed value, such that the nonlinearities caused by these parameters not occur.

5.2 Selection of the Operating Points

The next step is the selection of the operating points as described in Section 4.4. The nominal operating point was earlier determined in Section 3.2.2. The nominal values of
Chapter 5. Longitudinal Gain Scheduling Control

Selection of scheduling variables

Selection of operating points

Linearization

LQR Controller Design

Scheduling

Selection Operating Points

Initial operating point

Selection of the control area

Find Control Neighbourhood
  * Input Optimization
  * Linearization
  * LQR Control Design
  * Stability Radius Computation
  * Construction of Stability Ball

Construction grid of operating points

Figure 5.1: Step approach Gain Scheduling
the scheduling variables are

\[ u_0 = 166.1051 \]
\[ w_0 = 12.7580 \]

which corresponds to a total velocity of 166.59 m/s and an 4.4° angle of attack. These values are chosen because the remaining accelerations are minimized for this combination of scheduling variables. The nominal operating point is considered to be in the middle of the area for which the controller is developed. The ranges of the scheduling variables for which the controller is developed are:

\[ 166.1001 \leq u \leq 166.1101 \]
\[ 12.7530 \leq w \leq 12.7630 \]

It should be noted that this is only a very small area of the entire flight envelope of the AFX-Taipan. However, the principle of the selection of the operating points can be applied in the same way for the entire envelope.

For a regular grid of 25 points within this control area, the radius of the sphere is determined, in which the state feedback of this point is able to stabilize the system. For each of these 25 points the input is optimized to minimize the remaining accelerations and the model is linearized around these equilibrium points. Subsequently the stability radius is computed. For the computation of the stability radius the structure of the perturbations matrices \( E \) and \( H \) has to be defined. Because there is no uncertainty in the first equation of (3.15), \( \theta = \dot{q} \), and unstructured disturbance is assumed, the perturbation matrices are:

\[
E = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

\[
H = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

The singular values of \( G(s) = H (sI - A_d)^{-1} E \) are computed between \( 10^{-2} \text{ rad/s} \) and \( 10^2 \text{ rad/s} \) for all the points. The stability radius is the inverse of the largest singular value. Now the stability radius is known, the radius of the sphere \( B_\eta(\alpha) \) can be constructed as was shown in figure 4.1. In eight directions the value for \( \alpha \) is determined such that \( B_\eta(\alpha) \subseteq \Gamma_i \). This is illustrated for the nominal operating point in figure 5.2.

The maximal values for \( \alpha \) for each of the 25 points are depicted in figure 5.3. From this figure it appears that \( \alpha_{min} = 0.002 \) for all the points in the entire control area. This
Chapter 5. Longitudinal Gain Scheduling Control

Figure 5.2: Determination of $\alpha$

Figure 5.3: Stability Radius for the control area
Chapter 5. Longitudinal Gain Scheduling Control

5.3 Scheduling

Now the operating points are selected, input optimization is applied to minimize the remaining accelerations and LQR controllers are designed for each of the operating points as explained in Chapter 3. Subsequently, the scheduling of the gains of these 25 controllers has to be specified.

The simplest interpolation scheme is to use the pre-computed controllers as a lookup table. This nearest neighbour interpolation scheme is very simple in implementation, but if the number of reference setpoints is too small significant discrete jumps will be seen as the controller moves though the setpoint space.

An improvement to this scheme can be achieved with spline interpolation. This ensures continuous variation of the controller coefficients. The controller gain of a point in the scheduling space will be composed of the controller gains of the LQR controllers in conformance with the distance from this point to the operating points. The distance
Chapter 5. Longitudinal Gain Scheduling Control

from the current point $\gamma$ to operating point $i$ is calculated as follows:

$$d_i = \sqrt{(u_\gamma - u_i)^2 + (w_\gamma - w_i)^2}$$  \hspace{1cm} (5.3)

The equilibrium input matrix and feedback gain matrix are computed using a percentage of the U and K matrices according to the distance.

$$K_\gamma = \sum_{i=1}^{\eta} \xi_i K_i$$

$$U_\gamma = \sum_{i=1}^{\eta} \xi_i U_i$$  \hspace{1cm} (5.4)

where

$$\xi_i = \frac{1}{\eta - 1} \left( \frac{\sum_{i=1}^{\eta} d_i - d_i}{\sum_{i=1}^{\eta} d_i} \right)$$  \hspace{1cm} (5.5)

with $\eta$ the number of operating points. Using (5.4) the resulting controller becomes:

$$u_{\text{lon}} = U_\gamma - K_\gamma (x_{\text{lon}} - x_{\text{des}})$$  \hspace{1cm} (5.6)

5.4 Simulation

5.4.1 Flight path

In order to analyse the tracking capacities of the gain scheduled controller, a flight path is defined. The flight path has to lie in the control area of figure 5.4. The initial operating point is:

$$x_{\text{lon},0} = [ \begin{array}{cccc} 0.3278 & 166.1026 & 12.7605 & 0 \end{array} ]^T$$

The first 100 seconds, it is desired to stay in this initial operating point. During the next 100 seconds a step of $5 \cdot 10^{-5}$ m/s$^2$ acceleration is applied in $x_b$ direction and a step of $-5 \cdot 10^{-5}$ m/s$^2$ in $y_b$ direction. After that the desired accelerations in $x_b$ and $y_b$ direction are put to zero again. This means that after 200 seconds the desired values for $u$ and $w$ are constant again. It is desired to stay in that operating point for 100 seconds. This makes it possible to analyse the steady state response.

5.4.2 Simulation Results

Using the nonlinear longitudinal model of the AFX-TAIPAN and the gain scheduled flight control system, simulations are executed. In figure 5.5 the desired and the actual
Figure 5.5: Desired response and actual response
Figure 5.6: Error as function of time
response are depicted during a 300 seconds simulation. From this figure it appears that the desired trajectory and the actual trajectory do not agree. The error, which is defined as the difference between the actual value of the states and the desired value of the states, is depicted in figure 5.6. It can be observed that a steady state error remains.

The reason for the steady state error is that the operating points which are used are not equilibrium points. From table 3.1 it is shown that for the initial operating points the acceleration of the aircraft is not equal to zero. Remaining accelerations for the operating points are equivalent to a constant disturbance which acts on equilibrium points. That is why a steady state error can be observed in figure 5.6. The error is small enough such that the aircraft stays in the control area. This means that the gain scheduled controller is able to stabilize the aircraft. However, if the desired trajectory is chosen at the boundary of the control area, it could be possible that the remaining accelerations take the aircraft out of the area for which the controller is developed. This can lead to instability.

Summarizing, it can be concluded that the gain scheduled controller has been successfully applied to the stabilization of the aircraft. However, a steady state error remains.
Chapter 6

Conclusions and Recommendations

6.1 Conclusions

- In this report the six degree of freedom dynamical model of the AFX TAIPAN is derived. Based on this six degree of freedom model, the longitudinal dynamics are formulated.

- In order to find an equilibrium point for the longitudinal dynamics of the aircraft, optimization techniques are used to minimize the remaining accelerations. However, it appeared that the remaining accelerations are non-zero. This means that the operating point found is not a real equilibrium point.

- LQR Control can be used to stabilize the AFX TAIPAN around an operating point. The concept of the stability radius is used to determine the neighbourhood of an operating point in which the LQR controller is able to stabilize the nonlinear system.

- An approach to construct a regular grid of operating points for the gain scheduled controller is provided to assure global stability of the entire control area. This approach is based on the smallest control neighbourhood in a predetermined control area.

- The gain scheduled controller, based on 25 LQR controllers, is applied for the longitudinal dynamics of the AFX-TAIPAN. The gain scheduled controller is able to stabilize the aircraft. However, because the control area does not consist of a link up of equilibrium points, a steady state error can be observed.
6.2 Recommendations

- In this report gain scheduling is applied for the longitudinal dynamics of the AFX-TAIPAN. The scheduling variables used are the forward and the downward velocity of the aircraft. However, to capture all nonlinearities the pitch angle, $\theta$, and the density of the air, $\rho$, should also be considered. Moreover, gain scheduling can be applied for the full order model.

- The idea of gain scheduling is to move through the operating space along a path of equilibrium points. However, the operating points which are found are not equilibrium points. Additional control surfaces are needed to ensure that the operating points become equilibrium points and that the gain scheduling theory can be applied.

- The stability radius of the closed-loop system is very small. Since the stability radius determines the number of operating points in the control area, the relation between controller design and stability radius can be explored. After all, the selection of the weighting matrices $Q$ and $R$ determines the state feedback gain matrix, $K$, and the placement of the poles. By exploring the relation between the controller design and the stability radius, the controller can be designed in such a way that the stability radius is maximized.

- The way to construct a regular grid as shown in Chapter 4, is based on the smallest stability radius in the control area. If the stability radius changes a lot within the control area this results in a large overlap of the stabilizing areas of the operating point. More research can be done on minimize the overlap area.

- In this report the performance requirements for the aircraft are not taken into account. However, in general military standard requirements are specified. The only requirement was to stabilize the system. Further research is required to find a control design and an approach to select the operating points, such that the performance requirements are satisfied.
Appendix A

Parameters of the AFX-Taipan

The physical parameters of the AFX-Taipan as derived in are listed in Table A.1. A full description of the aircraft is given in [GPT99]. In table A.2 the aerodynamic parameters are listed as derived in [GPT99]. The equilibrium parameters for the aerodynamic coefficients are depicted in A.3. These parameters follow from [Pan02].
## Appendix A. Parameters of the AFX-Taipan

Table A.1: Physical parameters of the AFX-TAIPAN

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Wing span</td>
<td>m</td>
<td>15.9593</td>
</tr>
<tr>
<td>d</td>
<td>Distance between nozzle exit center and aircraft</td>
<td>d</td>
<td>1.2000</td>
</tr>
<tr>
<td>( I_{xx} )</td>
<td>Moment of inertia about x-axis</td>
<td>( km^2 )</td>
<td>2.0454 \times 10^7</td>
</tr>
<tr>
<td>( I_{yy} )</td>
<td>Moment of inertia about y-axis</td>
<td>( km^2 )</td>
<td>5.1393 \times 10^7</td>
</tr>
<tr>
<td>( I_{zz} )</td>
<td>Moment of inertia about z-axis</td>
<td>( km^2 )</td>
<td>5.7754 \times 10^7</td>
</tr>
<tr>
<td>( I_{xz} )</td>
<td>Moment of inertia about xz-axis</td>
<td>( km^2 )</td>
<td>6.0502 \times 10^7</td>
</tr>
<tr>
<td>m</td>
<td>Mass of the Aircraft</td>
<td>kg</td>
<td>33000</td>
</tr>
<tr>
<td>S</td>
<td>Wing surface Area</td>
<td>( m^2 )</td>
<td>113.1559</td>
</tr>
<tr>
<td>( X_{cg} )</td>
<td>Distance from nozzle exit to aircraft center of gravity</td>
<td>m</td>
<td>8.4300</td>
</tr>
</tbody>
</table>

Table A.2: Aerodynamic parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{L0} )</td>
<td>Lift Coefficient 0</td>
<td>0.166773</td>
</tr>
<tr>
<td>( c_{L1} )</td>
<td>Lift Coefficient 1</td>
<td>5</td>
</tr>
<tr>
<td>( c_{L2} )</td>
<td>Lift Coefficient 2</td>
<td>4.037</td>
</tr>
<tr>
<td>( c_{L3} )</td>
<td>Lift Coefficient 3</td>
<td>0.41103</td>
</tr>
<tr>
<td>( c_{D0} )</td>
<td>Drag Coefficient 0</td>
<td>0.0163</td>
</tr>
<tr>
<td>( c_{D1} )</td>
<td>Drag Coefficient 1</td>
<td>0.24208</td>
</tr>
<tr>
<td>( c_{D2} )</td>
<td>Drag Coefficient 2</td>
<td>0.027</td>
</tr>
<tr>
<td>( c_{y0} )</td>
<td>Side force Coefficient 0</td>
<td>-0.06233</td>
</tr>
<tr>
<td>( c_{\theta 0} )</td>
<td>Roll Moment Coefficient 0</td>
<td>-0.0022</td>
</tr>
<tr>
<td>( c_{\theta 1} )</td>
<td>Roll Moment Coefficient 1</td>
<td>-0.01624</td>
</tr>
<tr>
<td>( c_{\theta 2} )</td>
<td>Roll Moment Coefficient 2</td>
<td>0.24757</td>
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<td>( c_{\theta 3} )</td>
<td>Roll Moment Coefficient 3</td>
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<td>( c_{m0} )</td>
<td>Pitch Moment Coefficient 0</td>
<td>0.02304</td>
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<tr>
<td>( c_{m1} )</td>
<td>Pitch Moment Coefficient 1</td>
<td>0.082</td>
</tr>
<tr>
<td>( c_{m2} )</td>
<td>Pitch Moment Coefficient 2</td>
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<tr>
<td>( c_{m3} )</td>
<td>Pitch Moment Coefficient 3</td>
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<td>( c_{\eta 0} )</td>
<td>Yaw Moment Coefficient 0</td>
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</tr>
<tr>
<td>( c_{\eta 1} )</td>
<td>Yaw Moment Coefficient 1</td>
<td>0.05774</td>
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<tr>
<td>( c_{\eta 2} )</td>
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<tr>
<td>( c_{\eta 3} )</td>
<td>Yaw Moment Coefficient 3</td>
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### Table A.3: Equilibrium parameters

<table>
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<th>Symbol</th>
<th>Name</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{eq}$</td>
<td>Equilibrium angle of attack</td>
<td>0</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta_{eq}$</td>
<td>Equilibrium side slip angle</td>
<td>0</td>
<td>rad</td>
</tr>
<tr>
<td>$u_{eq}$</td>
<td>Nominal forward velocity</td>
<td>205</td>
<td>m/s</td>
</tr>
<tr>
<td>$u_{n}$</td>
<td>Equilibrium forward velocity</td>
<td>205</td>
<td>m/s</td>
</tr>
</tbody>
</table>
Appendix B

Specification of the operating points

The operating points which are selected for the gain scheduled controller are listed in table B.1. For these operating points the thrust force and the pitch angle of the nozzle are optimized to minimize the largest remaining acceleration. The acceleration for each operating point are given in table B.2.
Appendix B. Specification of the operating points

Table B.1: State values of the operating points

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>$\theta$ [rad]</th>
<th>$u$ [m/s]</th>
<th>$w$ [m/s]</th>
<th>$q$ [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP 1</td>
<td>0.3278</td>
<td>166.1001</td>
<td>12.7530</td>
<td>0</td>
</tr>
<tr>
<td>OP 2</td>
<td>0.3278</td>
<td>166.1001</td>
<td>12.7555</td>
<td>0</td>
</tr>
<tr>
<td>OP 3</td>
<td>0.3278</td>
<td>166.1001</td>
<td>12.7580</td>
<td>0</td>
</tr>
<tr>
<td>OP 4</td>
<td>0.3278</td>
<td>166.1001</td>
<td>12.7605</td>
<td>0</td>
</tr>
<tr>
<td>OP 5</td>
<td>0.3278</td>
<td>166.1001</td>
<td>12.7630</td>
<td>0</td>
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<tr>
<td>OP 6</td>
<td>0.3278</td>
<td>166.1026</td>
<td>12.7530</td>
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<tr>
<td>OP 7</td>
<td>0.3278</td>
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<tr>
<td>OP 8</td>
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<td>166.1026</td>
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<tr>
<td>OP 9</td>
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<td>12.7605</td>
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</tr>
<tr>
<td>OP 10</td>
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<td>166.1026</td>
<td>12.7630</td>
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<tr>
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<td>166.1051</td>
<td>12.7530</td>
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<tr>
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<tr>
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<td>OP 14</td>
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<td>OP 16</td>
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<td>166.1076</td>
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<tr>
<td>OP 17</td>
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<td>166.1076</td>
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<tr>
<td>OP 19</td>
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<td>166.1076</td>
<td>12.7605</td>
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<tr>
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<td>166.1101</td>
<td>12.7530</td>
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<tr>
<td>OP 22</td>
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<td>OP 23</td>
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<td>166.1101</td>
<td>12.7580</td>
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</tr>
<tr>
<td>OP 24</td>
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<td>166.1101</td>
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<td>OP 25</td>
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<td>12.7630</td>
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</tbody>
</table>
### Appendix B. Specification of the operating points

Table B.2: Derivatives of the states in the operating points

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>θ [rad/s]</th>
<th>(\dot{\theta} \cdot 10^{-3} \text{m/s}^2)</th>
<th>(\dot{\omega} \cdot 10^{-3} \text{m/s}^2)</th>
<th>(\dot{\varphi} \cdot 10^{-3} \text{rad/s}^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP 1</td>
<td>0</td>
<td>-0.3438</td>
<td>0.0005</td>
<td>-0.0223</td>
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<tr>
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<td>-0.0144</td>
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<tr>
<td>OP 3</td>
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<td>0.0198</td>
<td>0.0001</td>
<td>-0.0065</td>
</tr>
<tr>
<td>OP 4</td>
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<td>0.2006</td>
<td>0.0001</td>
<td>0.0014</td>
</tr>
<tr>
<td>OP 5</td>
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<td>0.3806</td>
<td>0.0003</td>
<td>0.0093</td>
</tr>
<tr>
<td>OP 6</td>
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<td>-0.3525</td>
<td>0.0005</td>
<td>-0.0207</td>
</tr>
<tr>
<td>OP 7</td>
<td>0</td>
<td>-0.1705</td>
<td>0.0002</td>
<td>-0.0128</td>
</tr>
<tr>
<td>OP 8</td>
<td>0</td>
<td>0.0109</td>
<td>0.0001</td>
<td>-0.0049</td>
</tr>
<tr>
<td>OP 9</td>
<td>0</td>
<td>0.1915</td>
<td>0.0001</td>
<td>0.0030</td>
</tr>
<tr>
<td>OP 10</td>
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<td>0.3714</td>
<td>0.0003</td>
<td>0.0109</td>
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<td>-0.0005</td>
<td>-0.0192</td>
</tr>
<tr>
<td>OP 12</td>
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<tr>
<td>OP 13</td>
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<td>0.0019</td>
<td>0.0000</td>
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<tr>
<td>OP 14</td>
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<td>0.1824</td>
<td>0.0001</td>
<td>0.0045</td>
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<tr>
<td>OP 15</td>
<td>0</td>
<td>0.3622</td>
<td>0.0003</td>
<td>0.0124</td>
</tr>
<tr>
<td>OP 16</td>
<td>0</td>
<td>-0.3699</td>
<td>0.0004</td>
<td>-0.0176</td>
</tr>
<tr>
<td>OP 17</td>
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<td>-0.1881</td>
<td>0.0001</td>
<td>-0.0097</td>
</tr>
<tr>
<td>OP 18</td>
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<td>0.0000</td>
<td>-0.0018</td>
</tr>
<tr>
<td>OP 19</td>
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<td>0.0001</td>
<td>0.0061</td>
</tr>
<tr>
<td>OP 20</td>
<td>0</td>
<td>0.3530</td>
<td>0.0003</td>
<td>0.0140</td>
</tr>
<tr>
<td>OP 21</td>
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<td>-0.3786</td>
<td>0.0004</td>
<td>-0.0161</td>
</tr>
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<td>OP 23</td>
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<td>0.0000</td>
<td>-0.0003</td>
</tr>
<tr>
<td>OP 24</td>
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<td>0.1642</td>
<td>0.0001</td>
<td>0.0076</td>
</tr>
<tr>
<td>OP 25</td>
<td>0</td>
<td>0.3438</td>
<td>0.0003</td>
<td>0.0155</td>
</tr>
</tbody>
</table>
Appendix C

Matlab Files

This appendix contains a short description of the MATLAB files which are used.

Model Derivation

- **parameters.m**
  The file `parameters.m` contains the parameters of the aircraft as specified in [Pan02].

- **Kinematics.m**
  The program `Kinematics.m` is used to formulate the kinematics of the 6th order model of the AFX-Taipan. The file `dir_cos_matrix` is used for the computation of the direction cosine matrix. The kinematics are saved in the file `Kinematics.mat`.

- **Forces.m**
  In the file `Forces.m` is used to formulate the force model of the AFX-Taipan. The force model is saved in the file `Forces.mat`.

- **nonlinear6dof.m**
  In the file `nonlinear6dof.m`, the force model is substituted in the kinematics. This results in the 6 dof dynamic model of the AFX-Taipan.

- **Aircraft_lon.m**
  The program `Aircraft_lon` is used to compute the derivatives of the longitudinal states as a function of the values of the states and inputs.

- **Linearization_lon.m**
  The file `Linearization_lon.m` is used to compute the linear longitudinal dynamics of the AFX-Taipan.

- **matrix_A_lon.m** and **matrix_B_lon.m**
  The files `matrix_A_lon.m` and `matrix_B_lon.m` are used for the evaluation of the A-matrix and B-matrix as function of the values of the states and inputs.
Appendix C. MATLAB Files

Initial Operating Point

- **InitOP.m**
  The program `InitOP.m` is used for the computation of the initial operating point. Optimization techniques are used to find the values of the states for which the remaining accelerations of the aircraft are minimized, given the values of the input signals. The file is used in combination with `EqOP lon.m`, in which the optimization is performed, `objfOP lon.m` for the calculation of the objective function, `Aircraft lon.m` for the evaluation of the aircraft’s dynamics and `parameters.m`.

Input Optimization

- **InputOptimization lon.m**
  The program `InputOptimization lon.m` is used to find the magnitude of the thrust force and the pitch angle of the nozzle for which the remaining accelerations of the aircraft are minimized, given the states of the aircraft. The file is used in combination with `EqIO lon.m`, in which the optimization is performed, `objfIO lon.m` for the calculation of the objective function, `Aircraft lon.m` for the evaluation of the aircraft’s dynamics and `parameters.m`.

LQR Controller

- **LqrController lon.m**
  In the file `LqrController lon.m` the feedback gain matrix is computed using the MATLAB function `lqr`. The file is used in combination with the files `matrix A lon.m` and `matrix B lon.m` for the computation of the linear model. The stability of the closed-loop system is evaluated using the file `stability lon.m`.

Neighbourhood

- **BallRadius.m**
  The file `BallRadius.m` is used to compute the radius of the sphere around an operating point for which the linear controller is able to stabilize the aircraft, as explained in Chapter 4. The complex stability radius of the system is computed with the program `StabilityRadius.m`.

- **operatingpoints lon.m**
  The file `operatingpoints lon.m` is used to construct a regular grid of operating points as explained in Chapter 4. The distance between the operating points is determined by the program `BallRadius.m`. 
Gain-Scheduling

- **gs_controller.m**
  The file `gs_controller.m` is used to compute the control input for a random point in the control area of the aircraft according to the distance to the individual operating points. The LQR Controllers are saved in the file `gs_con.mat`.

- **gs_sim.mdl**
  This program is a SIMULINK program which is used for the simulation of the longitudinal model of the aircraft and the gain scheduled controller.

- **run_GS_sim.m**
  The program `run_GS_sim.m` is used to set all parameters for the simulation of the longitudinal model of the aircraft and the gain scheduled controller. The file `plotaircraft_lon.m` is used for the visualization of the desired and actual response of the AFT-Taipan.
Bibliography


[Pan02] Panella, I. *Application of modern control theory to the stability and manoeuvrability of a tailless fighter*, RMIT University, Undergraduate Thesis in Bachelor of Engineering (Aerospace), 2002
