Radiometric retrieval of tropospheric parameters

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Radiometric retrieval of tropospheric parameters

Proefschrift

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Radiometric retrieval of tropospheric parameters

‘De man die de wolken meet’, Jan Fabre

Fabre: Ik heb ervaren dat je als kunstenaar van de ladder kan vallen. Maar je moet de moed hebben om er elke keer weer op te kruipen. Je mag de droom niet opgeven. Je moet de wolken blijven meten, ook al zijn ze onmeetbaar. Dat is wat ik met dit werk in de stad wilde plaatsen: een monument voor alle kunstenaars en hun streven, dat zo ver staat van alle dagelijkse beslommeringen en toch onmisbaar is voor de gemeenschap.

Dedico questo libro, senza nuvole, a Nicola, il mio sole e amore.
1. Introduction

General
In the last few decades it became evident that clouds play an important role in radiation transfer and as a result for weather forecasting and the study of climate change. Therefore, weather and climate research institutes are more and more interested in measured data on clouds, which are needed to develop a more accurate cloud parameterisation and real time cloud data to feed dynamic models. In order to obtain an optimal cloud-monitoring network, it became also important to determine the instrument combination (referred to as sensor synergy) that is most suitable for this purpose. Research on the above-mentioned topics was done in intense measurement campaigns such as the CLouds And RAdition campaign (CLARA), and the Cloud Lidar And Radar Experiment (CLARE’98) campaign. CLARA was organized by the Royal Dutch Meteorological Institute (KNMI), The National Institute for Public health and Environment (RIVM), the Dutch Energy Foundation (ECN) and the International Research Centre for Telecommunications transmission and Radar (IRCTR, TUDelft). Other participating institutes were TU/e, European Space Research and Technology Centre (ESTEC) and the Rutherford Appleton Laboratory (RAL). CLARE’98 was organized by ESTEC. The TU/e radiometer was used in both measurement campaigns.

The goal of the CLARA campaign was to improve modelling of cloud micro- and macro physical properties and to determine the instruments that are necessary to obtain a reliable cloud monitoring system. In advanced monitoring, lidar, radar and satellites play an important role. Macrophysics includes cloud cover, cloud structure and turbulence and microphysics concerns droplet spectra. Measurement data gathered during this campaign were used for the development and validation of the algorithms that model the micro- and macro physical properties of clouds and to compare the performance of different remote sensing techniques.

In the CLARE’98 campaign, attention was concentrated on optimal sensor synergy with respect to the future Earth Radiation Mission of the European Space Agency (ESA).

Along with the above-mentioned trend in cloud research, radiometers began to play a more and more important role in operational forecasting, climate and environmental monitoring. Therefore, it was proposed in the COST 712 project to study and improve meteorological applications of radiometry. This was done by both improving models of the interaction of microwave radiation with the Earth atmosphere and development and validation of retrieval algorithms.

Atmospheric water project
In this thesis we investigate sensor combinations to optimally measure amount of liquid water and water vapour. These are important parameters for the characterisation of clouds and for the modelling of the creation of clouds, respectively.

The aim of the Atmospheric Water (AWater) project was to design an instrument combination, that gives optimum performance for deriving integrated amounts of water vapour and liquid water in the atmosphere. To reach this goal, the idea was to use sensor synergy retrieval algorithms, where radiometer and radar/lidar measurements are combined. Radar and/or lidar data are used for improving liquid water modelling by giving extra information about cloud boundaries.
Radiometer data were processed at TU/e, whereas radar and/or lidar data were processed by the IRCTR. The infrared radiometer can be used to determine the temperature of the base of the cloud.

**Radiometer technology**
The radiometer measures the amount of radiation emitted by the atmosphere at five frequencies. At 21, 31 and 51/53/54 GHz, most of the atmospheric radiation is caused by water vapour (21 GHz), liquid water (31 GHz) and oxygen (51/53/54) respectively. Measurements of oxygen radiation can be used for temperature profile retrieval, as it is very sensitive to temperature changes. Changes in radiation at 21 and 31 GHz give information on the total amount of water vapour and liquid water.

In contrast to the radiometer, which is a passive instrument because it only receives radiation, radar and lidar are active instruments, because they both send and receive signals. The received signal gives information on the interaction between the emitted signal and the atmosphere and therefore on the characteristics of the atmosphere at that particular moment and place.

Both radiometer, radar and lidar are remote sensing instruments, as they measure a remote object; here clouds and/or the atmosphere at some distance. In contrast to the lidar signal, the radar signal is also able to penetrate inside the cloud, as only a small portion of the total signal is extincted. Lidar, on the other hand, transmits a light pulse instead of a radio wave, which is strongly attenuated, but which makes it possible to determine the bottom of a cloud very accurately.

**Measurement campaigns**
Radiometer data were gathered during several intense measurement campaigns. Publications made on results of these campaigns can be found in the list of all publications, at the end of this chapter.

The first campaign was the CClouds And RAdiation (CLARA) campaign, which was held in the period between April and December 1996. In this campaign, the following co-located instruments took part:

**Airborn**
- Forward Scattering Spectrometer Probe (FSSP)
- Temperature, Pressure and True Air Speed

**Satellite**
- AVHRR
- Meteosat

**Global Positioning System (GPS)**
Integrated Precipitable Water Vapour measurements
Figure 1.1 CLARA measurement configuration of the ‘ground-based’ instruments, situated at the top of the roof of the Electrical Engineering building of the TUDelft.

Figure 1.2 DARR radar of TUDelft.

Data gathered by TU/e were converted into water vapour and liquid water values and retrieval performance of the different algorithms was compared with water vapour values derived from GPS and radiosonde data. It was found that the radiometer could play a significant role in meteorology, as real-time integrated amounts of water vapour and liquid water give important information on the situation of the atmosphere and give continuous information on clouds.
Figure 1.3 The TU/e radiometer in the CLARE’98 campaign.

Figure 1.4 The 3 GHz radar at the Chilbolton site in the CLARE’98 campaign.
Ground based

TU Delft site

- Radiometer
  TU/e and ESTEC: 20/30(/50) GHz radiometer
- Radar:
  TU Delft: FM-CW radar; 3.315 GHz
- Lidar:
  RIVM: 1064 nm; 532 nm
  ESTEC: 906 nm
- Infrared radiometer:
  KNMI: 9.6 - 11.5 µm
- Radiosondes
  At 6, 12 and 18 UTC
- Meteostation
  Temperature, pressure and wind

ECN site

- Cloud chamber

CDN (Cloud Detection Network)

- 10 ground stations in the Netherlands
  - infrared radiometer: 9.6 - 11.5 µm
  - lidar ceilometer: 906 nm
  - pyranometer

The reason for co-location was the ability to compare and validate results of the different instruments. All instruments should measure the same cloud volume as good as possible, in order to determine the temporal and spatial cloud variability. In this way it was, for example, possible to compare radar and lidar cloud boundaries. Fig.1.1 and 1.2 give an impression of the measurement configuration of the ‘ground-based’ instruments, situated at the top of the roof of the Electrical Engineering building of the TUDelft.
Besides, the radiometer measured during several months together with the radar on the roof of the Electrical Engineering building of TUDelft, at Lopik/Cabauw (KNMI) with the Radio Acousting Sounding System (RASS), in Chilbolton during the CLARE’98 campaign (from 9-10-1998 to 23-10-1998) and for the rest of the time at the satellite earth station of the TU/e (tip-curve calibration campaign) or it was in Denmark at the manufacturer Rescom Ltd for reparation. Fig.1.3 and 1.4 show the TU/e radiometer and the Chilbolton radar measuring during the CLARE’98 campaign.

Contents of this thesis

Chapter 2 treats the problem of calculating brightness temperatures from known atmospheric parameters and spectral models (the ‘forward problem’) and compares the performance of the different spectral models. Furthermore, it treats the radiative transfer equation, which can be linearized in the case of small atmospheric changes. In this linearized radiative transfer equation, a new expression for weighting functions in terms of pressure altitudes is derived. Finally, the frequency choice of the radiometer is motivated.

Chapter 3 treats the different retrieval algorithms, used to obtain water vapour and liquid water values from radiometer measurements. Three types of algorithms are discussed: two of these (Linear Algorithm and Matched Atmosphere Algorithm) use 21/31 GHz radiometer data, and one (Linear Perturbation Algorithm) 21/31/51/53/54 GHz data. Of the 21/31 GHz algorithms, the linear algorithm uses statistical information to derive amounts of water vapour and liquid water. The Matched Atmosphere Algorithm (MAA) uses real-time ground data and atmospheric profiles to derive the amount of water vapour \( V \) and liquid water \( L \). The MAA is modified and extended, to produce real-time coefficients for linear retrieval. A new element in the retrieval of atmospheric parameters is the introduction of real-time measured parameters, such as lidar base- and radar top of the cloud. The linear perturbation algorithm solves small changes in atmospheric profiles. In order to avoid singularity, a method is developed for the reduction of the number of parameters to tune profile changes in linear perturbation.

In chapter 4, a functional description of the Rescom radiometer and its conversion of antenna temperatures into brightness temperatures is presented. Subsequently, it describes its calibration procedure. The equipment analysis leads to an improved understanding of the relative importance of various parameters defining the equipment.

Chapter 5 analyses the problem of the accuracy of the retrieval of atmospheric parameters. It describes the tip-curve calibration, which appears to be only useful for calibration of the 21/31/51 GHz channels and proposes a method to calibrate the 53 and 54 GHz channels. This new method uses profile information from the lower frequencies to calibrate the higher frequencies.

Chapter 6 compares the retrieval performance of the algorithms discussed in chapter 3 and concludes with the design of the optimum measurement configuration. The analysis proved that for accurate retrievals of liquid water \( L \), it is necessary to use cloud height and temperature information. This understanding leads to new conclusions regarding the design of the optimum measurement configuration.
List of Publications of the Awater project


[21] Dutch National Research program on Global Air pollution and climate change ‘Clouds and Radiation: intensive observational campaigns in the Netherlands (CLARA)’.

[22] T.Ingold, C.Mätzler and N.Kämpfer, B.Schmid and P.Demoulin, ‘Assessment of water vapour retrievals of Sun photometer data from low and high elevation’ (reference as ‘personal communication’).


2 Theory of microwave radiometry

2.1 Introduction

This chapter discusses the principles of microwave radiometry, which are used to calculate the brightness temperature of the atmosphere. Section 2.2 deals with radiative transfer theory. It describes the propagation of radiation through the atmosphere. The refractive index $n$ (section 2.2.2) is a measure of the interaction of the electromagnetic radiation with the atmosphere. The real part describes the refractivity of the atmosphere and the influence on the velocity of the radio waves. The imaginary part describes the absorption, which affects the amplitude of the radio waves. Besides the absorption coefficient, the parameters optical thickness and attenuation are introduced. For small perturbations to a standard ITU-R profile, it is possible to linearize the radiative transfer equation (section 2.3).

Section 2.4 treats Liebe’s spectral model called ‘Millimeter wave Propagation Model’ (MPM). It calculates the refractive index from knowledge of the resonant and non-resonant spectra of the atmospheric constituents. For the frequency range we use (20-50 GHz), the main contributions come from water vapour, liquid water and oxygen. We chose to use the MPM version of ’92 (MPM’92), as line mixing parameters in MPM’92 are determined by adjustment to laboratory measurements and therefore provides an excellent fit to those measurements [1-7].

Water vapour laboratory data are best represented by a combination of the vapour broadened continuum component from MPM’87 and the self-broadened component of the water continuum from MPM’93. In future, it is necessary to go beyond the MPM formulation with a continuum model that applies both to microwave and infrared spectral ranges. The reason for this is that applications are extended to submillimeter wavelengths and the fact that microwave and infrared continua are a result of the same process (i.e. molecular collisions). At the end of the COST712 project, the water vapour model as described by Rosenkranz was recommended [8-11]. In this thesis for water vapour absorption modelling the MPM’92 version is used as well, for consistency of the model and because of the fact that the radiometer calibration is the main error source in water vapour retrieval (COST 712 [12]). As in the Awater project there was still no application in submillimeter wavelengths, so, it was not necessary to apply Rosenkranz’s model.

The spectra depend on temperature, pressure and relative humidity. The atmospheric profiles and input parameters of MPM are discussed in section 2.5.
Section 2.6 treats the spectral models that are used in linear retrieval. Chapter 2 ends with a motivation for the frequency choice of the radiometer based on the oxygen, vapour and liquid water spectrum.

2.2 Radiative transfer theory

2.2.1 Radiation and brightness temperature

Black body radiation

The atmosphere is an absorbing medium. On the other hand, as stated by Kirchhoff’s law, in thermal equilibrium all energy absorbed is emitted as well. For a black body (an object that absorbs all radiation) the amount of radiated power per unit area, solid angle and bandwidth is given by the specific intensity or spectral brightness

\[
I_{\nu} = \frac{2h_{\nu}f^3}{c^2} \exp\left(\frac{h_{\nu}f}{kT}\right) - 1 \quad [W/(m^2 \text{ sr Hz})] \tag{2.1}
\]

- \(h_{\nu}\): Planck’s constant \(= 6.626 \times 10^{-34} \text{ [Js]}\)
- \(f\): the frequency [Hz]
- \(k\): Boltzmann’s constant \(= 1.38 \times 10^{-23} \text{ [J/K]}\)
- \(T\): the absolute (physical or kinetic) temperature [K]
- \(c\): the speed of light [m/s].

Transfer equation

In practice, a layer of the atmosphere does not absorb all incident radiation, but only part of it. It sometimes is referred to as a grey body. The absorption coefficient \(\alpha\) [Np/m] is a measure of the greyness of the atmosphere. The radiative transfer [13] in the atmosphere is calculated by dividing it into a large number of layers with a certain thickness. Each layer is assumed to be homogeneous. This means that temperature (\(T\)), pressure (\(P\)), relative humidity (\(RH\)) and absorption (\(\alpha\)) may be taken constant inside the layer. In order to calculate brightness temperature \(T_b\) values within the desired measurement accuracy (~1K), the atmosphere should be divided into layers with a maximum thickness of 0.1 km (Fig. 2.1).
The amount of radiation absorbed by the layer is equal to $\alpha(x)dxI(x)$, where $\alpha(x)$ is the absorption coefficient of the layer. It is calculated from the spectra of water vapour, liquid water and oxygen. The amount of radiation emitted by the layer is according to Kirchhoff’s law $e = \alpha(x)dxI_{bl}(x)$, where $I_{bl}$ stands for the radiation intensity of a black body. The difference between the outgoing and incoming intensity $I(x - dx) - I(x)$ is equal to the difference of these two contributions:

$$I(x - dx) - I(x) = -\alpha(x)dxI(x) + \alpha(x)dxI_{bl}(x) \tag{2.2}$$

or

$$\frac{dI(x)}{dx} = -\alpha(x)I(x) + \alpha(x)I_{bl}(x) \tag{2.3}$$
The solution to this equation is:

\[
I(0) = I_{bg} e^{-\tau(0,x_0)} + \int_0^{x_0} I_{bl}(x) e^{-\tau(0,x)} \alpha(x) \, dx
\]  

where

\[
\tau(x', x) \quad \text{the optical thickness or opacity defined by: } \tau(x', x) = \frac{x}{x'} \int \alpha(s) \, ds \quad \text{[Np]}
\]

Defining the black body brightness temperature by

\[
T_{bl}(x) \equiv \frac{c^2}{2kT} I_{bl}(x)
\]  

Eq.2.4 can be written as

\[
T_b(0) = T_{bg} e^{-\tau(0,x_0)} + \int_0^{x_0} T_{bl}(x) \alpha(x) e^{-\tau(0,x)} \, dx
\]  

where

\[
T_{bg} \quad \text{is the brightness temperature [K] of the incident radiation, which is 2.73 K cosmic background noise if we take the whole atmosphere into account (} x_0 > 10 \text{ km).}
\]

The radiometer antenna transforms the incident radiation into a noise signal. An antenna receiving isotropic radiation with specific intensity \( I(0) \) receives in each of two orthogonal polarisations a noise power

\[
P_r = \frac{1}{2} \int I(0) \, dA \, d\Omega \, df' = \frac{1}{2} \frac{2kT^2}{c^2} \frac{\lambda^2}{4\pi} \int T_b(0, f) \, df' = kT_b(0)B
\]  

for a small bandwidth \( B \) [Hz].

In practice, radiation is not isotropic and the antenna pattern of the radiometer should be taken into account.

**Rayleigh-Jeans approximation**

In literature, for frequencies in the millimeterwave range, often the Rayleigh-Jeans approximation is used. In that case, the term \( \exp \left( \frac{h_p f}{kT} \right) \) in Eq.2.1 is approximated by

\[
1 + \left( \frac{h_p f}{kT} \right)
\]

so

\[
I_{bl} = \frac{2h_p f^3}{c^2} \frac{1}{\exp \left( \frac{h_p f}{kT} \right) - 1} \approx \frac{2f^3 kT}{c^2}.
\]
As a result, $T_{bl}(x) = T(x)$ and Eq.2.6 can be rewritten as

$$T_b(0) = T_{bg} e^{-\tau(0,x_0)} + \int_0^{x_0} T(x) \alpha(x) e^{-\tau(0,x)} dx \quad (2.9)$$

To quantify the Rayleigh-Jeans error, the brightness temperature $T_{bl}$ of a black body with physical temperature $T=280$ K is determined. Fig.2.2 shows this approximation. The solid blue line is the exact brightness temperature $T_b$ of a black body at a physical temperature of 280 K and the dotted line its Rayleigh-Jeans approximation. It is clear that for frequencies between 10 and 100 GHz the error due to the Rayleigh-Jeans approximation is in the order of the measurement error (~1 K, see chapter 4). Therefore it is not completely negligible and will not be applied.

![Figure 2.2. Illustration of validity of Rayleigh-Jeans approximation in the frequency range 1-1000 GHz.](image)

**Conversion to attenuation**

In order to convert measured brightness temperatures into attenuation ($A$) /opacity ($\tau$) ($A = 4.343 \tau$) values and vice versa, it is often assumed that the atmosphere is an absorber of constant temperature $T_{eff}$. 

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In that case Eq.2.9 can be written as
\[ T_b(0) = T_{bg} e^{-\tau(0,x_0)} + T_{eff} \int_0^{x_0} \alpha(x) e^{-\tau(0,x)} dx = T_{bg} e^{-\tau(0,x_0)} + T_{eff} (1 - e^{-\tau(0,x_0)}) \] (2.10)
because
\[ \alpha = \frac{\partial \tau}{\partial x} \] (2.11)
and
\[ T_{eff} \int_0^{x_0} \alpha(x) e^{-\tau(0,x)} dx = T_{eff} \int_0^{\tau(0,x_0)} e^{-\tau(0,x)} d\tau = -e^{-\tau(0,x)} \tau(0,x_0) \] (2.12)
and the opacity \( \tau \) and attenuation can be solved directly from Eq.2.10
\[ \tau = \ln \left( \frac{T_{eff} - T_{bg}}{T_{eff} - T_b} \right) \text{ and } A = 10 \log \left( \frac{T_{eff} - T_{bg}}{T_{eff} - T_b} \right) \] (2.13)

\( T_{eff} \) can be calculated from
\[ T_{eff} = \frac{\int_0^{x_0} T(x) \alpha(x) e^{-\tau(0,x)} dx}{(1 - e^{-\tau(0,x_0)})} \] (2.14)

### 2.2.2 Attenuation and refractive index

The propagation of a plane electromagnetic wave in vacuum is described by
\[ E = E_0 e^{i\omega t} e^{-ik_0 r} \] (2.15)
with
- \( E \) = electric field strength [V/m],
- \( E_0 \) = amplitude of \( E \),
- \( k_0 \) = wave number [1/m] \[ \frac{2\pi}{\lambda} \],
- \( \omega \) = angular frequency [rad/s],
- \( r \) = distance [m],
- \( t \) = time [sec].

In reality, the wave propagates in a medium that introduces delay and loss.
The expression for the electromagnetic wave in a medium is

\[ E_m = E_0 e^{i\omega t} e^{-i k_0 n r} \quad [\text{V/m}] \]  \hspace{1cm} (2.16)

where

\[ n = \text{the complex refractive index of the medium}. \]

The refractive index can be divided in a dispersive (frequency dependent) and a non-dispersive (frequency independent) part:

\[ n = n_0 + n'(f) - i n''(f) \]  \hspace{1cm} (2.17)

with

\[ n_0 \quad \text{non dispersive part} \]

\[ n'(f) \quad \text{real dispersive part} \]

\[ n''(f) \quad \text{imaginary dispersive part}. \]

The electric field in a medium \( E_m \) (Eq.2.16) then becomes

\[ E_m = E_0 e^{i\omega t-k_0(n_0+n'(f)-in''(f))r} = E_0 e^{-k_0 n'(f)r} e^{i\omega \left(t-k_0(n_0+n'(f)-1)r\right)} \]  \hspace{1cm} (2.18)

The ratio

\[ \frac{E_m}{E} = e^{-k_0 n''(f)r} e^{i\omega \left(k_0(n_0+n'(f)-1)r\right)} \]  \hspace{1cm} (2.19)

is a measure for the attenuation and excess delay of the wave with respect to vacuum. One of the applications of microwave radiometers is the accurate estimation of excess delay due to water vapour, the so-called 'wet delay'. The non-dispersive excess delay with respect to vacuum is given by

\[ \Delta = \frac{k_0}{\omega} (n_0 - 1)r \quad [\text{s}]. \]  \hspace{1cm} (2.20)

The dispersive excess delay with respect to vacuum is defined by the delay dispersion

\[ \nu = \frac{k_0}{\omega} n'(f) = \frac{1}{c} n'(f) \quad [\text{s/m}]. \]  \hspace{1cm} (2.21)

Because \( n-1 \) is very small, the refractivity \( N(f) \) is introduced

\[ N(f) \equiv (n(f) - 1)10^6, \quad N_0(f) = (n_0(f) - 1)10^6 \]
When expressing the delay dispersion in [ps/km], Eq.2.23 can be written as

\[ \nu \approx 3.336 N'(f) \quad [\text{ps/km}]. \]  

(2.24)

For calculating brightness temperatures (Eq.2.6), only the absorption has to be taken into account. The absorption coefficient \( \alpha \) is calculated from

\[ \alpha = \frac{4\pi f}{c} \frac{N''(f)}{10^6} \quad [\text{Np/m}]. \]  

(2.26)

When \( r \) is expressed in [km] and \( f \) in [GHz]

\[ \alpha = 0.419 f N''(f) \quad [\text{Np/km}]. \]  

(2.27)

The optical thickness of the atmosphere is given by

\[ \tau(0,\infty) = \int_0^\infty \alpha(x)dx \quad [\text{Np}]. \]  

(2.28)

Np values are converted into dB values by multiplying with \( \frac{10}{\ln(10)} \approx 4.343 \).

The total attenuation \( A \) can be written in terms of the optical thickness in dB as

\[ A = 4.343 \tau(0,\infty) \]  

(2.29)

### 2.3 Linearization radiative transfer equation

For small changes in brightness temperature, the radiative transfer equation can be linearized. Weighting functions \( W \) are the brightness temperature response to a small change in one of the standard profiles temperature \( T \) [K], water vapour density \( \nu \) [g/m\(^3\)], liquid water density \( w \) [g/m\(^3\)] and of pressure \( p \) [kPa] (Rec. ITU-R P.835-2, section 2.5) at height \( x \). In terms of weighting functions, the radiative transfer equation can be written as [14]
\[
\delta T_b + \varepsilon = \int_0^x \left( W_T \delta T + W_v \delta v + W_w \delta w + W_p \delta p \right) dx = \int_0^x \bar{W}(x) \bar{\delta \Pi}(x) dx
\] (2.30)

with

\begin{align*}
\varepsilon & \quad \text{measurement error}, \\
\bar{W}(x) &= [W_T(x), W_v(x), W_w(x), W_p(x)], \text{the weighting vector at height } x, \\
\bar{\delta \Pi}(x) &= [\delta T(x), \delta v(x), \delta w(x), \delta p(x)], \text{the profile difference vector at height } x,
\end{align*}

\begin{align*}
W_T & \quad T_b \text{ response to a change in the } T \text{ profile,} \\
W_v & \quad T_b \text{ response to a change in the } v \text{ profile,} \\
W_w & \quad T_b \text{ response to a change in the } w \text{ profile,} \\
W_p & \quad T_b \text{ response to a change in the } p \text{ profile.}
\end{align*}

In case of small perturbations, weighting functions are assumed to be constant, making the transfer equation linear in \( \delta T, \delta \phi, \delta v \text{ and } \delta w \). Then, profile parameters are retrieved from brightness temperature measurements with a linear inversion algorithm. If profile changes are significant, an iteration procedure should be used to adjust the value of the weighting function. The next section describes how weighting functions are calculated \[2\].

### 2.3.1 Calculating weighting functions for small perturbations to reference profile

Weighting functions are calculated by considering the differential form of the radiative transfer equation (Eq.2.3)

\[
\frac{dT_b}{dx} = \alpha(x)(T(x) - T_b(x))
\] (2.31)

Eq.2.31 is a linear differential equation. For small perturbations \( \delta T \) in the (physical) atmospheric temperature it is allowed to write

\[
\frac{d(\delta T_b)}{dx} = \frac{\partial}{\partial T} \left( \frac{dT_b}{dx} \right) \delta T,
\] (2.32)

to approximate \( \alpha, T \) by \( \alpha_r, T_r \) of the reference profile, and \( T_b \) by \( T_{br} \). Then Eq.2.31 reads

\[
\frac{d(\delta T_b)}{dx} = -\alpha_r \delta T_b + \left( \alpha_r + \frac{\partial \alpha}{\partial T} (T_r - T_{br}) \right) \delta T.
\] (2.33)

Since the background radiation is not changed, the solution of Eq.2.33 is given by

\[
\delta T_b = \int_0^x \left( \alpha_r + \frac{\partial \alpha}{\partial T} (T_r - T_{br}) \right) e^{-\alpha(x) dx} \delta T dx,
\] (2.34)
hence

\[ W_r(x) = \left( \alpha_e + \frac{\partial \alpha}{\partial T} (T_r - T_{br}) \right) e^{ - \int_x^0 \alpha_e(x) dx } \]  \hspace{1cm} (2.35)

Note that this differential weighting factor is only equal to the value

\[ W_r = \alpha_e e^{ - \int_x^0 \alpha_e(x) dx } \]  \hspace{1cm} (2.36)

used in literature [1] if \( \frac{\partial \alpha_r}{\partial T} = 0 \). In the same way we find for \( W_p, W_v \) and \( W_w \)

\[ W_\mu(x) = \frac{\partial \alpha_r}{\partial \mu} (T_r - T_{br}) e^{ - \int_x^0 \alpha_e(x) dx } \]  \hspace{1cm} (2.37)

where \( \mu \) is \( p, v \) or \( w \).

### 2.3.2 Calculating weighting functions for pressure altitudes

In order to compare retrievals with actual meteorological observations of atmospheric profiles, weighting functions are often calculated as a function of pressure altitude \( H \) rather than physical altitude. Pressure altitude is the altitude corresponding to a particular pressure, for a given standard reference atmosphere for which the ideal gas law applies. In this way the gas law is imposed for the relation between pressure and temperature and pressure is no longer an independent parameter, just as is the case in radiosonde observations. The ideal gas law in terms of physical altitude is given by

\[ \frac{dP}{dx} = -c_M \frac{p}{T} \]  \hspace{1cm} (2.38)

and in terms of pressure altitude by

\[ \frac{dP}{dH} = -c_M \frac{p}{T_r} . \]  \hspace{1cm} (2.39)

From Eq. 2.38 and 2.39 we learn

\[ dx = \frac{T}{T_r} dH \]  \hspace{1cm} (2.40)

\( W_r \) as a function of pressure height is solved from

\[ \frac{d(\delta T_h)}{dH} = \frac{d(\delta T_h)}{dx} \frac{dx}{dH} = \frac{d(\delta T_h)}{dx} \frac{T}{T_r} . \]  \hspace{1cm} (2.41)
For small perturbations, it is allowed to write
\[
\frac{d(T_b^h)}{dH} = \frac{\partial}{\partial T} \left( \frac{d(T_b^h)}{dH} \right) \delta T
\]  \hspace{1cm} (2.42)

and to approximate \( \alpha, T \) and \( T_b \) by \( \alpha_r, T_r \) and \( T_{br} \). We get
\[
\frac{\partial}{\partial T} \left( \frac{d(T_b^h)}{dH} \right) \delta T = \frac{\partial}{\partial T} \left( \frac{T}{T_r} \alpha_r, (T_r - T_{br}) \right) \delta T .
\]  \hspace{1cm} (2.43)

with the solution
\[
\delta T_r(H) = \int_0^H \left( \alpha_r \left( 1 - \frac{T_{br}}{T_r} \right) + \alpha_r + \frac{\partial \alpha}{\partial T} (T_r - T_{br}) \right) e^{-\int_0^H \alpha_r, (H)dH} \delta T dH
\]  \hspace{1cm} (2.44)

and
\[
W_{T,H} = \left( \alpha_r \left( 2 - \frac{T_{br}}{T_r} \right) + \frac{\partial \alpha}{\partial T} (T_r - T_{br}) \right) e^{-\int_0^H \alpha_r, (H)dH} .
\]  \hspace{1cm} (2.45)

Comparison of Eq.2.36 and 2.44 shows that the expression for \( W_T \), as a function of physical altitudes is different from that in pressure altitudes \( W_{T,H} \). The difference is due to the temperature dependence of the pressure altitudes. The expressions of \( W_r \) and \( W_w \) on the other hand are based on the use of physical altitudes.

### 2.4 The Millimeter wave Propagation Model

#### 2.4.1 General

The Millimeter wave Propagation Model (MPM)[3-7] was developed by H.J. Liebe. It predicts the complex refractive index for frequencies up to 1000 GHz. Only contributions of water vapour, dry air (oxygen), suspended water droplets (hydrosols: haze, fog and clouds) and rain are taken into account. MPM was first published in 1985[3], followed by several new versions. In our research, after comparison with other models [10-13], we used the 1992 version [8], which is considered the best version for radiometric applications at frequencies below 60 GHz.

**Absorption contributions**

In his spectral model, Liebe models the complex refractivity \( N_0(f) \) by

\[
N_0(f) = N_0 + N(f) = N_0 + N'(f) - iN''(f) \quad \text{[ppm]}
\]  \hspace{1cm} (2.46)

For calculating brightness temperatures, only \( N'' \) has to be evaluated. It consists of a resonant (moist air contribution \( N''_r \)) and a non-resonant (dry air spectrum \( N''_d \), suspended waterdroplet refractivity \( N''_w \) and rain approximation \( N''_R \)) spectrum. Besides, an empirical water vapour term \( N''_c \) is introduced to account for the fact that measured absorption rates of moist air are generally higher than expected from the H\(_2\)O resonant spectrum (especially visible in the millimeter wave windows).
Expressions for these terms are summarised below.

### 2.4.2 Resonant contributions

#### Moist air resonant contributions

The resonant contribution consists of the line absorption of water vapour and oxygen. The refractive index term is given by the summation of 44 oxygen resonance and 30 water vapour resonance contributions, which are described by the Van Vleck-Weisskopf [6] shape function

\[ N_L^* = \sum_{i=1}^{44} S_i F_i^*(f) + \sum_{k=1}^{30} S_k F_k^*(f) \]  

(2.48)

where

\[ S = \text{the line strength [kHz]}, \]

\[ F' = \text{complex shape function [GHz}^{-1}]. \]

\[ F_i^*(f) = \frac{f}{f_i} \left( \frac{\gamma_i - \delta_i(f_i - f) + \gamma_i - \delta_i(f_i + f)}{(f_i - f)^2 + \gamma_i^2} \right) \]  

(2.49)

and the line parameters are:

<table>
<thead>
<tr>
<th>Oxygen</th>
<th>Water vapour</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_i ) = ( a_3 10^{-3} (p \theta(0.8-a_4) + 1.1e \theta) )</td>
<td>( \gamma_i = b_3 10^{-3} (p \theta b_5 + b_7 e \theta b_9) )</td>
</tr>
<tr>
<td>( \delta_i ) = ( (a_5 + a_6 \theta)10^{-3} p \theta^{0.8} )</td>
<td>( \delta_i = 0 )</td>
</tr>
<tr>
<td>( S_i = a_1 10^{-6} p \theta^3 \exp(a_2(1-\theta)) )</td>
<td>( S_i = b_1 e \theta^{3.5} \exp(b_2(1-\theta)) )</td>
</tr>
</tbody>
</table>

with

\[ \gamma = \text{pressure-broadened width}, \]

\[ \delta = \text{pressure-induced interference}, \]

\[ \theta = \text{inverse temperature } 300/T [\text{K}^{-1}], \]

\[ p = \text{pressure profile [kPa]}, \]

\[ e = \text{partial water vapour pressure profile [kPa]}, \]

\( a_{1-a6} \) are the spectroscopic parameters,

\( b_{1-b6} \) are the spectroscopic parameters.
$N_L^*$ is calculated from the resonance line centre frequencies $f_i$ and the so-called spectroscopic parameters $a_{1,6}$ and $b_{1,6}$. Table 2.1 shows some of these spectroscopic parameters. The expression for the line parameters shows that $N_L^*$ depends on $\theta$, $p$ and $e$.

**Spectroscopic parameters**

<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$b_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.23</td>
<td>0.109</td>
<td>2.143</td>
<td>28.11</td>
<td>0.69</td>
<td>4.8</td>
<td>1</td>
</tr>
<tr>
<td>67.8</td>
<td>0.0011</td>
<td>8.735</td>
<td>28.58</td>
<td>0.69</td>
<td>4.93</td>
<td>0.82</td>
</tr>
</tbody>
</table>

H$_2$O (V)

<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$A_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.5</td>
<td>6.08</td>
<td>7.744</td>
<td>8.9</td>
<td>0</td>
<td>1.165</td>
<td>5.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.71</td>
<td>('92)</td>
<td></td>
<td>6.73</td>
</tr>
<tr>
<td>54.13</td>
<td>228</td>
<td>3.814</td>
<td>10.2</td>
<td>0</td>
<td>-0.314</td>
<td>5.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.91</td>
<td>('92)</td>
<td></td>
<td>3.75</td>
</tr>
<tr>
<td>54.67</td>
<td>391.8</td>
<td>3.194</td>
<td>10.5</td>
<td>0</td>
<td>-0.706</td>
<td>5.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.25</td>
<td>('92)</td>
<td></td>
<td>2.65</td>
</tr>
</tbody>
</table>

Table 2.1. Spectroscopic parameters of the H$_2$O vapour and O$_2$ resonance lines for the MPM’89 and MPM’92 version. For more details see [4] and [6].

### 2.4.3 Non –resonant contributions

**Dry-air non-resonant spectra**

The non-resonant refractivity terms of dry air give only a small contribution at ground level pressures due to the Debye spectrum of oxygen below 10 GHz and pressure-induced nitrogen absorption above 100 GHz. In our MPM implementation they are neglected.

**Water vapour continuum spectrum**

The continuum absorption $N_c^*$ was determined by series of accurate laboratory measurements. It was found that

$$N_c^*(f) = f(3.57\theta^{7.5}e + 0.113p)10^{-5}e\theta^3. \quad (2.50)$$

**Wet continuum: Suspended water droplet refraction**

The absorption of suspended water droplets in haze, fog and clouds is considerable. If the circumference of the droplet is smaller than one wavelength, it is allowed to apply the Rayleigh approximation for Mie scattering. More precisely: $\frac{a}{\lambda} < 1$ with $a = 2\pi r$ and $r$ is the radius of the droplet. The maximum allowed droplet radius $r_{max}$ in the Liebe model is $50\mu$m, which follows from the fact that frequencies considered in the Liebe model are up to 1000 GHz resulting in
\[ \lambda_{\text{min}} = \frac{c}{f} = \frac{3 \times 10^8}{10^{12}} = 0.3 > 2\pi r_{\text{max}} \text{ mm and hence } r_{\text{max}} \approx 50 \mu\text{m}. \]

In case the Rayleigh approximation is applied, the suspended water droplet refraction can be written as

\[ N^*_w(f) = \frac{4.5w}{\varepsilon''(1 + \eta^2)} \]  
(2.51)

where

\[ w = \text{liquid water density [g/m}^3\text{]}, \]
\[ \varepsilon'' = \text{imaginary part of the permittivity for liquid water}, \]
\[ \eta = \frac{2 + \varepsilon'}{\varepsilon''}, \]  
(2.52)
\[ \varepsilon' = \text{real part of the permittivity for liquid water}. \]

The dielectric permittivity \( \varepsilon \) of water is determined with the double Debye model [4]. Note that Eq.2.51 underestimates \( N^*_w \) at frequencies above 300 GHz.

\[ \varepsilon'(f) = \frac{\varepsilon_0 - \varepsilon_1}{2} + \frac{\varepsilon_1 - \varepsilon_2}{2} + \varepsilon_2 \]
(2.53)

\[ \varepsilon''(f) = \frac{\varepsilon_0 - \varepsilon_1}{f_p} + \frac{\varepsilon_1 - \varepsilon_2}{f_s} \]  
(2.54)

The coefficients are given by Liebe as

\[ \varepsilon_0 = 77.66 + 103.3(\theta - 1), \]
\[ \varepsilon_1 = 5.48, \]
\[ \varepsilon_2 = 3.51, \]
\[ f_p = 20.09 - 142(\theta - 1) + 294(\theta - 1)^2, \]
\[ f_s = 590 - 1500(\theta - 1). \]

**Rain approximation**

The refractivity of rain is mainly caused by absorption and scattering. Scattering becomes substantial if radio wavelength and drop diameters (0.1 to 5 mm) are in the same range. Mie calculations requiring drop shape- and size distributions and the dielectric permittivity of water resulted in the following approximation:

\[ N''_R = c_R R^Z \]  
(2.55)
In our research, rain periods have not been taken into account. They can be recognized in the radiometer signal as spikes (see Fig.2.3). More specific: fast (i.e. within a minute) increases in brightness temperatures of 5 K will be considered to be contaminations by rain.

![Figure 2.3. Recognizing rain periods (1-12-1996) in radiometer signal as spikes.](image)

As we already saw, refractivity depends on the temperature $T$, the partial water vapour pressure $e$, the amount of liquid water $L$ and the pressure $p$. The next paragraph treats the origin of these parameters.

**Implementation of the model**

In the software, designed to simulate the MPM model, we used the exact form of the radiative transfer equation. Eq.2.6 is used in order to get as accurate values as possible. No rain contributions ($N''_R$) and no rain profiling were taken into account, because rain data were not evaluated. Finally $N''_d$ was not calculated, because its contributions were only significant at frequencies below 10 GHz and above 100 GHz, whereas we are evaluating frequencies between 20 and 60 GHz.

**Profile parameters**

As we saw in the above-mentioned absorption contributions, they depend on air conditions and liquid water amount. Air conditions are characterised by barometric pressure $P$ (consisting of the partial pressure for dry air $p$ and for water vapour $e$), temperature $T$ and relative humidity $RH$ [%], whereas liquid water density $w$ [g/m$^3$] is characterised by cloud, aerosol and rain modelling. In order to calculate absorption at each height, atmospheric profiles are necessary. In principle the $P$, $T$ and $RH$ profiles can be retrieved from radiosonde data. However, these are not always available. Alternatively, standard ITU-R profiles, average profiles or profiles as proposed in the ‘profile algorithm’ of Peter and Kämpfer (chapter 3) may be used as well, but this limits the accuracy of the results.
2.5 Spectral models used in linear retrieval

Instead of using MPM line-by-line calculation, the specific attenuation due to dry air, water vapour and liquid water can be estimated using simplified models [15,16]. In linear retrieval, the approximations for modelling the specific water vapour and oxygen specific attenuation as proposed in ITU-R Recommendation P.676-2 and Benoits model for modelling the liquid water specific attenuation will be referred to as the CCIR model. The approximations of ITU-R Recommendation P.676-3 and ITU-R Recommendation P.840-2 will be referred to as the Gamma model [17].

2.5.1 CCIR model

ITU-R Recommendation P. 676-2

In this Recommendation, the oxygen and water vapour specific attenuation are assumed to be \( v \)- and/or \( T \) dependent. They are approximated by

\[
\gamma_v = 0.05 + 0.0021v + \left( \frac{3.6}{(f - 22.2)^2 + 8.5} \right) + \left( \frac{10.6}{(f - 183.3)^2 + 9} \right) + \left( \frac{8.9}{(f - 325.4)^2 + 26.3} \right) f^2 v 10^{-4}
\]

(2.56)

and

\[
\gamma_o \approx \left[ 7.19 \cdot 10^{-3} + \left( \frac{6.09}{f^2 + 0.227} \right) + \left( \frac{4.81}{(f - 57)^2 + 1.5} \right) \right] f^2 10^{-3} \quad [\text{dB/km}]
\]

(2.57)

Benoit

The specific attenuation by liquid water as modelled by the empirical function introduced by Benoit is given by

\[
\gamma_L = f^{1.95} \exp(-6.866(1 + 4.5 \cdot 10^{-3} (T - 273))) w
\]

(2.58)

where \( T \) is the temperature of water droplets [K]

2.5.2 Gamma model

ITU-R Recommendation P. 676-3

This spectral model is based on curve fitting to the line-by-line calculation and holds for altitudes from sea level to 5 km. It is \( T \)-, \( v \)- and \( p \) dependent for \( f \leq 57 \text{ GHz} \).
\begin{equation}
\gamma_v = \left[ 3.27 \times 10^{-2} r_i + 1.67 \times 10^{-3} \frac{w_i}{r_p} + 7.7 \times 10^{-4} f^{0.5} + \frac{3.79}{(f - 22.235)^2 + 9.81 r_p^2 r_i} \right] + 11.73 r_i \frac{4.01 r_i}{(f - 183.31)^2 + 11.85 r_p^2 r_i} + \frac{3.91}{(f - 325.153)^2 + 10.44 r_p^2 r_i} \right] f^2 v_g r_p r_i 10^{-4} \right]
\end{equation}

with \( r_p = \frac{p}{1013} \) and \( r_i = \frac{288}{273 + T} \) and \( \gamma_v \) given in [dB/km]

\begin{equation}
\gamma_o = \left[ \frac{7.27 r_i}{f^2 + 0.351 r_p^2 r_i^2} + \frac{7.5}{(f - 57)^2 + 2.44 r_p^2 r_i} \right] f^2 r_p^2 r_i^2 10^{-3}.
\end{equation}

**ITU-R Recommendation P. 840-2**

In the case of liquid water, the specific attenuation is estimated from

\begin{equation}
\gamma_L = \frac{0.819 f}{\varepsilon''(1 + \eta^2)} w.
\end{equation}

Comparison with the specific attenuation of MPM gives

\begin{equation}
\alpha_L = 0.419 f N''_w = 1.889 f \frac{w}{\varepsilon''(1 + \eta^2)}
\end{equation}

with

\begin{equation}
N''_w = \frac{4.5 w}{\varepsilon''(1 + \eta^2)}.
\end{equation}

### 2.5.3 Comparison MPM with above mentioned spectral models

In order to evaluate the influence of the choice of the spectral model on \( T_b \) calculations, the MPM, CCIR and Gamma spectral models are used as input to the radiative transfer equation and brightness temperatures are calculated for an ITU-R standard atmosphere. In Fig.2.4 and 2.5, results are shown for frequencies between 1 and 35 GHz.
Figure 2.4 Comparison of brightness temperatures at 1-35 GHz, calculated from different spectral models.

Figure 2.5 Comparison of brightness temperature differences of the different spectral models at 1-35 GHz with respect to MPM.
Results show that the Gamma modelling is most accurate and introduces errors in the order of measurement errors. The CCIR approximation on the other hand introduces errors much larger than the measurement error (which is ~1K as will be demonstrated in chapter 4). Furthermore, the sensitivity of the spectral models to the input profiles is evaluated. This is done by calculating $T_b$ of a standard ITU-R Recommendation P. 835-2 atmosphere with a cloud liquid water profile between 1.5 and 1.8 km modelled according to Slobin [18] and comparing it with a homogeneous atmosphere with the same total amount of $V$ and $L$ and $T_{eff}$.

![Figure 2.6 Profile dependence Gamma-, CCIR- and MPM model.](image)

Fig.2.6 shows that both MPM and Gamma model are most sensitive to the choice of the input profiles, whereas the CCIR model neglects the dependence on the profile. This means that the accuracy of $T_b$ calculations using the MPM and Gamma model can be improved by additional information such as radar $H_{top}$, whereas this is not true for the CCIR model. Dependent on the frequency, $T_b$ differences can become larger than the measurement error. This shows the importance of accurate profiling.

**2.6 Input parameters to MPM**

In the software, designed to simulate the MPM model, barometric pressure (consisting of the partial pressure for dry air and water), temperature and relative humidity must be converted into the following internal parameters in order to calculate the $N^\text{th}$ contributions.
\[ p = P - e, \quad (2.64) \]

with

\[ P = \text{barometric pressure} \ [\text{kPa}], \]
\[ e = \text{partial water vapour pressure} \ [\text{kPa}], \]
\[ p = \text{dry air pressure} \ [\text{kPa}], \]
\[ \theta = \frac{300}{T}, \quad (2.65) \]

where

\[ \theta = \text{relative inverse temperature} \ [\text{K}^{-1}], \]
\[ T = \text{temperature} \ [\text{K}], \]
\[ e = \frac{e_s \cdot RH}{100}, \quad (2.66) \]

where

\[ e = \text{partial water vapour pressure} \ [\text{kPa}], \]
\[ e_s = \text{water vapour saturation pressure} \ [\text{kPa}], \]
\[ RH = \text{relative humidity} \ [%] \]

and

\[ w(x) = w_C(x) + w_A(x) + w_R(x), \quad (2.67) \]

where

\[ w = \text{liquid water density} \ [\text{g/m}^3], \]
\[ w_C = \text{cloud liquid water density} \ [\text{g/m}^3], \]
\[ w_A = \text{aerosols liquid water density} \ [\text{g/m}^3], \]
\[ w_R = \text{rain liquid water density} \ [\text{g/m}^3]. \]

The dependence of water vapour saturation pressure \( e_s \) [kPa] on temperature is approximated by

\[ e_s \simeq \frac{1.739 \cdot 10^9}{7.223} \cdot 100 \cdot \theta^4 \exp(-22.64\theta) \quad (2.68) \]

valid over \( \pm 40^\circ\text{C} \). The water vapour density \( \nu \) is then calculated from \( RH \) using

\[ \nu(x) = 7.223 e \theta = 7.223 e_s \frac{RH}{100} = 1.739 \cdot 10^9 RH \theta^5(x) \exp(-22.64 \theta(x)) \quad (2.69) \]
The total water vapour \((V)\) and liquid water content \((L)\) are obtained from

\[
V = \int_0^{\chi_0} v(x)dx \tag{2.70}
\]

and

\[
L = \int_0^{\chi_0} w(x)dx . \tag{2.71}
\]

### 2.7 Choice of frequencies

![Figure 2.7 Oxygen, water vapour and liquid water spectrum according to Liebe. The (–) line represents the oxygen, the (:) the vapour and the (- -) the liquid water absorption.](image)

Fig.2.7 shows the oxygen, water vapour and liquid water spectrum (calculated from Liebes MPM model). The frequencies at which the radiometer is measuring are chosen in such a way that the relative sensibility for \(\text{H}_2\text{O} (V)\), \(\text{H}_2\text{O} (L)\) and temperature changes differs as much as possible\[19\]. Fig.2.7 indicates that the amount of water vapour can be measured best near the vapour peak (20 GHz), liquid water in the window near 30 GHz and temperature at the oxygen peak (60 GHz). The oxygen line is suitable for \(T\) profiling, as it is determined by its density fluctuations, caused by temperature fluctuations (Eq.2.48-2.49).
References

3 Inversion

3.1 Introduction

As already noted in chapter 2, the forward model consists of determining the $T_b$ value from atmospheric parameters. The ingredients necessary to do so are a spectral model, atmospheric parameters and the radiative transfer equation. The inversion problem consists of retrieving the atmospheric parameters (i.e. integrated amount of vapour and liquid water) from brightness temperature measurements.

Among the classes of actual retrieval algorithms, statistical regression, inversion of physical models, neural network and Bayesian techniques[1] can be distinguished. The last two retrieval techniques both rely on the availability of a large statistical set. As this was absent in our situation, we were limited to the evaluation of linear and physical retrievals.

In statistical regression, a fit is calculated between brightness temperature measurements and independent measurements of the variables that have to be retrieved. In this thesis, this class is represented by the linear algorithms.

In the inversion of physical models, the algorithm adjusts the variable(s) to be retrieved until modelled brightness temperatures agree with measured brightness temperatures.

In this thesis, inversion of physical retrieval is represented by the Matched Atmosphere Algorithm (MAA). The MAA was chosen since it is very suitable to evaluate the performance of sensor synergy, consisting of the combination of radiometer-, lidar- and radar measurements. Lidar and radar data are used for measuring geometrical cloud properties.

For profile retrieval, the linear analysis as described in [2] is used. As there are less measurements than the number of variables to be solved, the covariance matrix is ill conditioned. This ill conditioning can be improved by using statistical data, or, as evaluated in this thesis, by constraining the solution and by diminishing the amount of variables to be retrieved.

Section 3.2 discusses the implementations of the forward model in the different algorithms.

Section 3.3 treats solution methods for the retrieval of atmospheric parameters from $T_b$ measurements. Section 3.4 finally describes the different implementations of the algorithms.
3.2 Implementation of the forward model

3.2.1 Linear retrieval

Linear retrieval is frequently used in dual frequency radiometry, because it allows the direct solution of the vapour \( V \) and liquid \( L \) parameters from brightness temperature \( T_b(A) \) measurements, that are dependent on the total attenuation \( A \). In linear retrieval it is assumed that

\[
A_v = \int_0^\infty \gamma_v(x)dx = \int_0^\infty a(x)v(x)dx, \quad A_L = \int_0^\infty \gamma_L(x)dx = \int_0^\infty b(x)w(x)dx, \quad A_\alpha = \int_0^\infty \gamma_\alpha(x)dx \quad (3.1)
\]

where \( a(x) \) and \( b(x) \) are weighting factors calculated from the specific attenuations \( \gamma_v \) and \( \gamma_w \) (sections 2.5 and 2.6). Input for calculation of the weighting factors are atmospheric profiles. \( a(x), b(x) \) and \( c(x) \) are tuned to the site by using real time data instead of standard ground data (reference profiles) as input of the profiles or by introducing an average atmosphere for that site. Note that the accuracy of linear retrieval degrades if profiles differ significantly from the atmosphere used in the calculation of the weighting factors. This is also the case when using simplified models that are sensitive to the real time data.

\( A \) can be calculated directly from \( V \) and \( L \) by introducing the parameters \( a_{\text{eff}}, b_{\text{eff}} \) and \( c_{\text{eff}} \) using

\[
A_v = a_{\text{eff}} \int_0^\infty v(x)dx, \quad A_L = b_{\text{eff}} \int_0^\infty w(x)dx, \quad A_\alpha = c_{\text{eff}} \quad \text{and} \quad (3.2)
\]

\[
A = a_{\text{eff}} V + b_{\text{eff}} L + c_{\text{eff}} \quad (3.3)
\]

where

\[
a_{\text{eff}} = \frac{\int_0^{x_0} a(x)v(x)dx}{\int_0^{x_0} v(x)dx}, \quad b_{\text{eff}} = \frac{\int_0^{x_0} b(x)w(x)dx}{\int_0^{x_0} w(x)dx}, \quad c_{\text{eff}} = \int_0^{x_0} \gamma_\alpha(x)dx. \quad (3.4)
\]

Note that in case water vapour absorption is independent of height, \( a_{\text{eff}} = a \) and \( b_{\text{eff}} = b \).

As the specific attenuation in the gamma and CCIR model is an approximation of that in the MPM model, the MPM model is most sensitive to the choice of the profiles (see section 2.5 and 2.6). A further simplification is often made by assuming that the relation between attenuation \( A \) and brightness temperature \( T_b \) is linear, which is only valid for low attenuation values \( T_b \ll T_{\text{eff}} \).
\[ A = 10 \log \left( \frac{T_{\text{eff}} - T_{bg}}{T_{\text{eff}} - T_h} \right) \approx c \frac{T_h - T_{bg}}{T_{\text{eff}} - T_h} \approx c \frac{T_h - T_{bg}}{T_{\text{eff}}} \]  \hspace{1cm} (3.5) \\

with \( c = \frac{10}{\ln(10)} = 4.343 \)

In that case, Eq.3.3 can be rewritten as [6], [7]

\[ T_h = a'_{\text{eff}} V + b'_{\text{eff}} L + c'_{\text{eff}} \]  \hspace{1cm} (3.6) \\

In our work, Eq.3.6 is not used, because the approximation introduces errors larger than the measurement error (1.5 K at 30 K). (see Fig.3.1 and section 4.4).

![Figure 3.1 Approximated and exact A value.](image)

Note that before, we calculated the constants \( a_{\text{eff}}, b_{\text{eff}}, c_{\text{eff}} \) to determine \( A \) from \( V \) and \( L \) values. In order to retrieve \( V \) and \( L \) values from attenuation measurements, the set of linear equations must be inverted [3-6].

\[
\begin{pmatrix}
A_{31} \\
A_{21}
\end{pmatrix} = 4.343
\begin{pmatrix}
\tau_{31} \\
\tau_{21}
\end{pmatrix} =
\begin{pmatrix}
a_{\text{eff},31} & b_{\text{eff},31} & c_{\text{eff},31} \\
a_{\text{eff},21} & b_{\text{eff},21} & c_{\text{eff},21}
\end{pmatrix}
\begin{pmatrix}
V \\
L \\
1
\end{pmatrix}
\]

or

\begin{equation}
(3.7)
\end{equation}
If a radiosonde set is available, another way to determine the constants is to calculate \( V, L \) and \( T_b \) values at different frequencies from multiple radiosonde data and calculate the constants by inverting the following set of equations:

\[
\begin{pmatrix}
4.343 \tau_{31} - c_{eff,31} \\
4.343 \tau_{21} - c_{eff,21}
\end{pmatrix} = \begin{pmatrix} a_{eff,31} & b_{eff,31} \\ a_{eff,21} & b_{eff,21} \end{pmatrix} \begin{pmatrix} V \\ L \end{pmatrix}.
\]  
\[(3.8)\]

\[
\begin{pmatrix} V \\ L \end{pmatrix} = \text{inv} \begin{pmatrix} a_{eff,31} & b_{eff,31} \\ a_{eff,21} & b_{eff,21} \end{pmatrix} \begin{pmatrix} 4.343 & 0 - c_{eff,31} \\ 0 & 4.343 - c_{eff,21} \end{pmatrix} \begin{pmatrix} \tau_{31} \\ \tau_{21} \end{pmatrix}.
\]  
\[(3.9)\]

3.2.2 Matched Atmosphere Algorithm

The difference between linear inversion and the Matched Atmosphere Algorithm [7,8] is that specific attenuations (or the standard atmospheric models necessary to calculate them) are not only tuned to the site by using ground data, but also by changing the shape of the \( RH \)- and \( w \) input profile. By taking also the shape of the profiles into account, \( V \) and \( L \) values are expected to be more accurate. Specific attenuations were calculated from an atmosphere profile model as proposed by ‘Peter and Kämpfer’ [8] using MPM, where the shape of the \( RH \) profile is determined by the tuning parameter \( RH_{ref} \) and that of the \( w \) profile by the tuning parameter \( H_{base} \). By changing these tuning parameters, a look up table was generated, in which measured brightness temperatures and accompanying \( V \) and \( L \) values were looked up. As the absorption is not very sensitive to \( P \) values, the ‘Peter and Kämpfer’ atmosphere uses the standard pressure \( (P) \) profile

\[
q_{31} = A \backslash R
\]  
\[(3.11)\]

with

\[
A = \begin{pmatrix}
A_1 \\
A_2 \\
\ldots \\
A_n
\end{pmatrix} \quad R = \begin{pmatrix} V_1 & L_1 & 1 \\
V_2 & L_2 & 1 \\
\ldots \\
V_n & L_n & 1
\end{pmatrix} \quad q_{31} = \begin{pmatrix} a_{31} \\
b_{31} \\
c_{31}
\end{pmatrix}
\]  
\[(3.12)\]

and likewise for \( q_{21} \). \( L \) is usually modelled from radiosonde profiles by assuming the presence of a cloud with sub-adiabatic water content if \( RH > 90 \% \) [4].
\[ P(x) = P_g \left( \frac{T_g}{T_g - 6.5x_g} \right)^{\frac{34.163}{-6.5}} \]  

(3.13)

where

\( P_g \) pressure altitude at \( x_g \) [kPa] \( (P_g = 101.325 \text{ kPa under ITU-R standard conditions and } x_g \text{ in [km]}) \) see Fig.3.2.

The temperature (\( T \)) profile on the other hand is modelled as accurate as possible, because especially liquid absorption is very sensitive to it, see Fig.3.3.

**Figure 3.2** ITU-R pressure profile.

**Figure 3.3** Temperature dependence of liquid water absorption, expressed in dB per mm precipitable water.
In the ‘Peter and Kämpfer’ atmosphere, an exponential fit of the initial profile to the surface value with a scale height of 3 km is used. In this way the high variability of the temperature, especially near the surface, is taken into account.

\[
T(x) = T_{\text{init}}(x) + (T_g - T_{\text{init}}(x_g)) \exp\left(-\frac{x}{h} \right)
\]

(3.14)

where

\[
x_g = \text{ground height},
\]

\[
T_{\text{init}} = T_g - 6.5x [\text{K}],
\]

\[
x = h - x_g,
\]

\[
T_g = \text{ITU-R standard temperature at } x_g (288.15 \text{ K}).
\]

The temperature profile is shown in Fig.3.4

![Temperature profile](image)

**Figure 3.4** ITU-R temperature profile.

Temperature profiling can be improved by using a monthly mean of radiosonde measurements at the site or by using 50 GHz data.

Instead of water vapour density \((\nu)\), in the ‘Peter and Kämpfer’ atmospheric model, relative humidity \((RH)\) used (Fig.3.5a and b), because information on ground measurements, is generally expressed in terms of relative humidity. The vapour content of the atmosphere is varied with a tuning parameter \(RH_{\text{ref}}\). This is the value of the relative humidity between 1.5 km above the site and 1.5 km above the top of the cloud. It is a constant value. The relative humidity at 12 km \((RH(12))\) is zero. The profile between ground level and 1.5 km and between 1.5 km above the top of the cloud and 12 km is obtained by linear interpolation between \(RH_{\text{ref}}\) and \(RH(0)\) and \(RH(12)\), respectively.
Figure 3.5a and b Relative humidity profiles.

It should be remarked that the exact shape of the profile of water vapour \( \nu \) is not very critical, because vapour absorption is not very sensitive to \( T \) profiling (and hence also not to the shape of the \( \nu \) profile) (see Fig.3.6).

Figure 3.6 Temperature dependence of water vapour absorption.

When using the Peter and Kämpfer \( RH \) profile, total absorption due to water vapour is still dependent on \( T \). This is a consequence of modelling a temperature-independent relative humidity profile and thereby a temperature-dependent vapour density profile. From a physical point of view it is more likely that \( RH \) instead of \( \nu \) depends on \( T \). Therefore it has been proposed to use a \( T \)-independent vapour density profile (Rec. ITU-R P.835-2).

The liquid contribution of clouds should be modelled from drop size spectra. In case this information is not available, the model of Slobin [9] is used. It states that the atmosphere is full of rising parcels of moist air which will condensate if the mixing ratio becomes larger than the saturation mixing ratio. The condensed water content of a rising parcel is equal to the difference between the water vapour density \( \nu(x) \) inside the cloud and the saturated water vapour density at the base of the cloud \([g/m^3]\) \( \nu_s(H_{base}) \) \([g/m^3]\).
It is called the adiabatic liquid water content. The liquid water density $w_C(x)$ increases with the distance from the base of the cloud due to the decrease of the water vapour saturation density $(v_s)$ with height.

$$w_C(x) = C(v_s(H_{base}) - v_s(H_{top}))(\frac{x - H_{base}}{H_{top} - H_{base}})$$  \hspace{1cm} (3.15)

where

$$H_{top} = \text{height of the top of the cloud},$$

$$H_{base} = \text{height of the base of the cloud}.$$  

$C$ is a parameter for the fine tuning of the total water content in relation to the adiabatic water content of the cloud. According to [9], $C$ may vary within the range of 0.1 to 0.75. It is set to 0.4 in our Matched Atmosphere Algorithm (MAA) implementation.

The top of the cloud is assumed at the height where droplets start to freeze, because ice contributions are not detected by the microwave radiometer. Cloud studies show that this temperature level can range between $-3^\circ C$ and $-10^\circ C$ (occasionally even colder). In the ‘Peter and Kämpfer’ model it is assumed to be at 0°C. In winter time, when ground temperatures drop below 0°C, the cloud is assumed to be at a height of at least 2 km and corresponds in that case to a super cooled cloud. The parameters used to change the $L$ content are $H_{base}$ and for fine tuning $C$.

![Figure 3.7 ‘Slobin’ liquid water profile as used in the MAA.](image-url)
Modelling of hydroscopic aerosols
Solution droplets of hydroscopic aerosols are modelled for RH values of $80% \leq RH \leq 99.9%$. The contribution of these aerosols may lie between 0.01 and 0.5 g/m$^3$, depending on the particular location.

$$w_A = w_0 \frac{20(C_1 + 4) - RH}{C_1(100 - RH)} \text{ (for } 80% \leq RH \leq 99.9%)$$

where

- $w_A =$ liquid water content due to aerosols [g/m$^3$],
- $w_o =$ dry mass concentration of hydroscopic aerosols [g/m$^3$],
- $C_1 =$ parameter that depends on the particular location, i.e.
  - $C_1 = 1.87$ Rural environment,
  - $C_1 = 2.41$ Urban environment,
  - $C_1 = 5.3$ Maritime environment,
  - $C_1 = 5.83$ Maritime and a strong wind.

Modelling rain
The instantaneous suspended liquid water concentration due to rain $w_R(x)$ [g/m$^3$] is equal to

$$w_R(x) = \int_{D_{\text{min}}}^{D_{\text{max}}} \frac{\pi}{6} D^3 N(x, D) dD \text{ for } 0 \leq x \leq x_0$$

where

- $N(x) =$ standard drop size distribution,
- $D =$ diameter of the droplets.

The rain rate $R$ at height $x$ is given by

$$R(x) = \int_{D_{\text{min}}}^{D_{\text{max}}} v(D) \frac{\pi}{6} D^3 N(x, D) dD$$

where

- $v(D) =$ fall speed of droplets with diameter $D$. 

\[39\]
The total liquid water content (due to rain)

\[ L = \int_{0}^{H_{\text{max}}} w_{R}(x)dx \quad [\text{g/m}^{2}]. \quad (3.19) \]

In practice, \( L \) is usually expressed in terms of the height of the equivalent water column [mm].

In the Peter and Kämpfer model, rain is assumed to originate from convective clouds in saturated air. Furthermore, the drop size distribution and thereby \( R \) is assumed to be independent of height and equal to the value measured at the ground.

\[ R = R_{0} = n_{0} \int_{D_{\text{min}}}^{D_{\text{max}}} v(D) \frac{\pi}{6} D^{3} n(D) dD \quad (3.20) \]

where

\[ N(0,D) = n_{0}n(D) \quad (3.21) \]

For a constant droplet distribution

\[ w_{R} = n_{0} \int_{D_{\text{min}}}^{D_{\text{max}}} \frac{\pi}{6} D^{3} n(D) dD \quad (3.22) \]

or

\[ w_{R} = Rm_{R} \text{ for } 0 \leq x \leq H_{\text{top}} \quad (3.23) \]

where

\[ R = R_{0} \quad \text{rain rate [mm/hour]}, \]
\[ m_{R} = 0.1 \quad ([\text{g/m}^{3}]/(\text{mm/hour})] \text{ for } 0 \leq R \leq 2.5 \quad [\text{mm/hour}]. \]

The value of \( m_{R} \) is a measure of the characteristics of the drop size distribution. It should be noticed that this approximation is not generally valid. The drop size contribution (and thereby \( R \)) is not always constant as a function of height, for example when rain evaporates before it reaches the ground. Furthermore, the temperature of the rain is not taken into account in the ‘Peter and Kämpfer’ model. In reality, rain temperatures can differ significantly. In case of heavy showers, rain temperature is approximately equal to temperature at the top of the cloud, whereas for low rain rates, rain temperature is approximately equal to the temperature of the surroundings. Because liquid water absorption is very sensitive to temperature (see Fig.3.3), this approximation may cause considerable errors.

Finally, when raindrops are included, the suspended droplet contribution is only valid in case of small droplets. For large droplets, the Rayleigh-Jeans approximation of Mie scattering, used in modelling the suspended water droplets refractivity, is not valid anymore. In that case rain should be modelled separately as an input parameter of Liebes MPM model. Values of rain refractivity as modelled in MPM are valid for both small and large droplets.
3.2.3 Linear perturbation algorithm

In the linear perturbation algorithm, instead of evaluating $T_b$ values, only small changes $\delta \Pi$ to a reference profile are evaluated. In that case, it is allowed to linearize the radiative transfer equation [10].

$$\delta T_b + \varepsilon_i = A_j(s_j) \delta \Pi(s_j)$$  \hspace{1cm} (3.24)

where

$$\delta \Pi(s_j) = [\delta T(s_j), \delta \rho(s_j), \delta \epsilon(s_j), \delta \Lambda(s_j)]^T,$$  \hspace{1cm} (3.25)

$$A_j(s_j) = w_j W(s_j),$$  \hspace{1cm} (3.26)

$w_j =$ quadrature weight for height $s_j,$
$W(j(s_j) =$ $W$ as calculated from the reference profile at height $s_j$ for frequency $i (i = 1-5),$
$\varepsilon_i =$ measurement error.

The weighting functions give the sensitivity of the $T_b$ value to a change in the profile parameters. If only two profile parameters are introduced, the situation is analogous to that of the Matched Atmosphere Algorithm. The only retrieval difference (see section 3.3.2) is that in linear perturbation, the solution plane is fixed by the values of the weighting functions, whereas in the Matched Atmosphere Algorithm, it is determined by the three points nearest to the measured $T_b$ value (see below). If more than two profile parameters are introduced, the number of measurements is smaller than the number of profile parameters and the inversion matrix is ill conditioned. Section 3.3 will show how the condition of the inversion matrix can be improved.

3.3 Estimating atmospheric parameters from measured brightness temperatures

3.3.1 Linear algorithms

By measuring brightness temperatures at least two frequencies in the linear algorithm, it is possible to give an exact solution of the set of two equations for $V$ and $L.$ In matrix notation the forward problem reads in terms of attenuation

$$A = Q_{lin} r$$  \hspace{1cm} (3.27)

where

$$r = [V \ L \ 1],$$  \hspace{1cm} (3.28)
\[ Q_{lin} = \begin{bmatrix} a_{31} & b_{31} & c_{31} \\ a_{21} & b_{21} & c_{21} \end{bmatrix}. \]  

(3.29)

\( V \) and \( L \) are solved by inversion of the forward problem, which means in terms of attenuation that

\[ r = A \backslash Q_{lin}. \]  

(3.30)

### 3.3.2 Matched Atmosphere Algorithm

As was already discussed in section 3.3.1, in the Matched Atmosphere Algorithm (MAA) \( V \) and \( L \) values are looked up in a table. It is generated by changing the \( RH \) profile with the tuning parameter \( RH_{ref} \) between 0 and 100\% in steps of 10 \% and by changing the \( L \) profile with the tuning parameter \( H_{base} \) (see Fig.3.5) between 0.1 km and \( H_{top} \) in steps of 0.1 km and calculates accompanying \( V, L \) and \( T_b \) values. The set of \( V \) \( (T_{b31}, T_{b21}) \) and \( L \) \( (T_{b31}, T_{b21}) \) values is called convergence area. The location and shape of the convergence area depend on the profiles and/or real time ground data that are used to model the atmosphere. Measured \( T_b \) values then are looked up in the convergence area, spanned by the calculated \( T_{b31} / T_{b21} \) plane. Fig.3.8 shows an example of a convergence area.

![Figure 3.8 Convergence area of April 19th 1996.](image)

\( V \) and \( L \) values are looked up by determining the three points in the surfaces spanned by \( V \) and \( L \) that are closest to the measured \( T_b \) value and by (linearly) interpolating in the plane spanned by these points.
This means that the convergence area is assumed to be locally linear for $T_b$ values generated with the abovementioned $R_{\text{Href}}$ and $H_{\text{base}}$ step size. Note that when approximating the whole convergence area by a plane, the model is reduced to a linear model and the constants can be retrieved from the plane. In case of a bad calibration or inadequate modelling of the atmosphere, measured $T_b$ values will lie outside the convergence area. In that case, $L$ values can become negative. Note that especially outside the convergence area, the retrieval becomes inaccurate, as it is necessary to extrapolate.

### 3.3.3 MAA retrieval using radar/lidar/radiometer derived parameters

The retrieval of atmospheric water content using the MAA algorithm is improved by using radar/lidar and 50 GHz derived parameters because radar/lidar/50GHz derived data contain extra temperature information.

**MAA retrieval using radar/lidar derived parameters**

Radar-derived cloud parameters are $H_{\text{top}}$, $H_{\text{base}}$ and the radar reflectivity profile. When comparing radar $H_{\text{top}}$ with 0°C $H_{\text{top}}$, differences of approximately 1 km have been found (Fig.3.9).

![Figure 3.9 Radar derived and 0 °C estimates of $H_{\text{top}}$.](image)

Because of the strong $T$ dependence of the specific attenuation by liquid water (Fig.3.3), modelling of the cloud at the correct height (i.e. at the right $T$) improves the $L$ retrieval significantly. Fig.3.11 shows the relative differences in retrieval between original MAA (0 °C $H_{\text{top}}$) (Fig.3.10) and MAA using radar $H_{\text{top}}$. 

43
Figure 3.10 Original MAA L retrieval.

Figure 3.11 Difference in $L$ retrieval using Radar Top (RT) and 0 °C $H_{top}$. 
Fig. 3.11 shows that retrieval using radar $H_{\text{top}}$ values gives larger $L$ values than when using $0^\circ C$ $H_{\text{top}}$. This is also expected, as for the 19$^{th}$ of April, radar $H_{\text{top}}$ is below the $0^\circ C$ $H_{\text{top}}$ (Fig. 3.10) so the cloud has a higher temperature. As $L$ absorption decreases with increasing temperature (Fig. 3.3), in order to obtain the same brightness temperature, the $L$ value of the cloud top as ‘seen’ by the radar must be higher. The $L$ differences increase with the difference in $H_{\text{top}}$ value.

**MAA using radar reflectivity profile**

In this version of the MAA algorithm, radar reflectivity instead of a Slobin $L$ profile is used for $L$ profiling. This is valid under the assumption that both the droplet concentration is approximately constant with height (which is shown by measurements in marine stratocumulus clouds (see, e.g., Slingo et al. [1982],[11])) and the width parameter in a lognormal parameterisation of the droplet distribution is constant (Frisch et al. [1995],[12]). In that case the cloud liquid water content is proportional to the square root of the radar reflectivity and consequently can be used for $L$ profiling [13]. Note that for $L$ modelling only reflectivity values between $H_{\text{base}}$ (as determined by lidar) and $H_{\text{top}}$ (as determined by the radar) are used. $L$ values at heights smaller than $H_{\text{base}}$ or larger than $H_{\text{top}}$ are assumed to be zero. The total amount is tuned by multiplying the profile with a tuning parameter $C$. The question remains however if the radar is able to detect clouds. If clouds contain little water, the radar does not detect them and if they contain much water, the radar detects drizzle (individual thick droplets). Fig. 3.12 shows the relative difference between MAA using a radar reflectivity profile and MAA$_{\text{BT}}$ using the Slobin liquid water profile.

![Figure 3.12](image)

**Figure 3.12** Difference between MAA using radar $Z$ profile and MAA$_{\text{BT}}$ using Slobin $L$ profile (5 minute average).
From comparison with Fig.3.10, which is an illustration of the general observation, it can be concluded that the relative difference (in %) in \( L \) retrieval between using a Slobin or a radar \( Z \) profile is small for clouds containing small amounts of liquid water, but is going to be significant for clouds containing much water as in that case, the radar detects drizzle.

### 3.3.4 Perturbation theory

In the perturbation algorithm, the perturbation to the reference profile \( \delta \pi \) is calculated by solving the set of equations

\[
\delta \pi_b = A \delta \pi \tag{3.31}
\]

In our case Eq.3.31 is an underdetermined system of equations i.e. it has an infinite number of solutions. By rewriting Eq.3.31 as

\[
B \delta \pi_b = BA \delta \pi \tag{3.32}
\]

and requiring that

\[
BA = I \tag{3.33}
\]

it is possible to retrieve an unique solution \( \delta \hat{\pi} \)

\[
\delta \hat{\pi} = B \delta \pi_b \tag{3.34}
\]

which is the solution for which \( \delta \hat{\pi}^T \cdot \delta \hat{\pi} \) is minimal, the so-called minimum norm solution. This is the smoothest exact solution of the perturbation algorithm, Eq.3.31. \( B \) is called the pseudo inverse and is determined from

\[
BAA^T = A^T, \tag{3.35}
\]

\[
B = A^T (AA^T)^{-1}. \tag{3.36}
\]

The covariance matrix of the weighting functions \( C = AA^T \) is a square symmetric matrix. In terms of its eigenvalues \( \Lambda_{AA^T} \) and eigenvectors \( U_{AA^T} \) [10] \( C \) can be written as

\[
C = U_{AA^T} \Lambda_{AA^T} U_{AA^T}^T \tag{3.37}
\]
where

\[
\Lambda_{AA^T} = \begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_m
\end{pmatrix}
\quad \text{and} \quad
U_{AA^T} = \begin{pmatrix}
u_1 \\ v_2 \\ \vdots \\ v_m
\end{pmatrix}.
\]  

(3.38)

As it is always possible to decompose \(\delta T_b\) in terms of the orthonormal system of eigenvectors \(U_{AA^T}\) (\(\delta T_b = U_{AA^T} \xi\) with \(\xi = (\xi_1, \xi_2, \ldots, \xi_m)^T\)), Eq.3.34 can be rewritten as

\[
\delta \hat{\pi} = A^T (U_{AA^T} \Lambda_{AA^T} U_{AA^T}^T)^{-1} U_{AA^T} \xi = A^T U_{AA^T} \Lambda_{AA^T}^{-1} \xi ,
\]  

(3.39)

\(\xi\) is calculated from the measurements of \(\delta T_b\) as \(U_{AA^T}^{-1} \delta T_b\). If we define the eigenprofile matrix \(\delta \pi\) as the solution corresponding to the eigenvector matrix \(U_{AA^T}\) with

\[
\delta \pi = A^T U_{AA^T} \Lambda_{AA^T}^{-1} ,
\]  

(3.40)

\(\delta \pi\) can be expanded into the sum of eigenprofiles as

\[
\delta \hat{\pi} = \delta \pi \xi .
\]  

(3.41)

Because of the limited degree of independence in the weighting functions [10], some eigenvalues of the covariance matrix \(AA^T\) are very small and \((AA^T)^{-1}\) is nearly singular. This causes instabilities in the inversion of the radiometry measurements especially when the eigenvalues are in the order of magnitude of the measurement error.

The ill conditioning can be improved by increasing the eigenvalue (regularization technique) or by adding (relevant) statistical information (maximum likelihood estimation). A disadvantage of the use of a-priori statistics is that it does not depend on the radiation measurements themselves.

Therefore other ways to improve the condition of the inversion matrix have been developed. In order to minimize the influence of random errors that make the retrieval instable the choice of the channels may be optimised [14]. Another way to improve the ill conditioning is to reduce the number of parameters that determine the profile.
**Regularization technique**

Especially for eigenvalues in the order of magnitude of the measurement error, the measurement accuracy is very important as it is responsible for the instability of the retrieval. To take errors into account, Eq. 3.31 should be replaced by

\[
\delta T_b = A \delta \pi + \varepsilon
\]

(3.42)

and an unique solution is retrieved by minimizing the quadratic function

\[
q(\delta \pi) = (A \delta \pi - \delta T_b)^T (A \delta \pi - \delta T_b) + \kappa \delta \pi^T \delta \pi.
\]

(3.43)

It can be shown that this means that among the solutions that satisfy

\[
|A \delta \hat{\pi} - \delta T_b| \leq \varepsilon^2
\]

(3.44)

the smoothest solution as stated by the measure

\[
\varphi = \delta \hat{\pi}^T \delta \hat{\pi}
\]

(3.45)

is selected. This solution is given by

\[
\delta \hat{\pi} = A^T (AA^T + \kappa I)^{-1} \delta T_b.
\]

(3.46)

Eq. 3.46 shows that this is equivalent to enlarging the eigenvalues of the covariance matrix with a term \( \kappa \) that is related to the measurement error \( \kappa \leq \varepsilon^2 \). \( \kappa \) is the regularisation term, stabilizing the solution by allowing the retrievals to be a non-perfect solution to the measurements. The choice of \( \kappa \) is determined [2,12] by evaluating the residual \( R \)

\[
R = |A \delta \hat{\pi} - \delta T_b|.
\]

(3.47)

If it is much smaller than the measurement error \( \varepsilon \), \( \kappa \) is too small, \( \frac{1}{\lambda} \) too large and the solution is a wildly varying function i.e. is under constrained. Fig. 3.15 shows an example of an under constrained solution. In this figure the retrieval is compared with the reference profile and the true profile.
Figure 3.15 Example of an under constrained solution of Eq.3.46.

Figure 3.16 Example of an over constrained solution of Eq.3.46.
If the residual $|A\delta\hat{T} - \delta T_b|$ is much larger than the measurement error, the solution is over constrained and $\kappa$ is too large. Fig. 3.16 shows an over constrained solution. If for all reasonable $\kappa$ values $|A\delta\hat{T} - \delta T_b| \geq \epsilon^2$, the choice of the start reference profile was wrong and non-linearity in the transfer equation are causing problems. In that case, a new reference profile and accompanying weighting functions should be determined.

**Linear statistical method: maximum likelihood estimation**

It is also possible to use statistical data [2,10,15] to stabilize the inversion. In the linear statistical method this is done by introducing a statistical set of known profiles $\langle \delta\pi \rangle$ and measurement errors $\epsilon$ to determine a maximum likelihood estimate of the profile change $\delta\pi$.

The likelihood function $L(\delta\pi, \epsilon)$ is the joint probability distribution of $\delta\pi$ and $\epsilon$,

$$L(\delta\pi, \epsilon) = p_{\delta\pi,\epsilon}.$$  \hspace{1cm} (3.48)

When assuming that $\delta\pi$ and $\epsilon$ are uncorrelated and that $p_{\delta\pi}$ and $p_{\epsilon}$ obey Gaussian statistics

$$p_{\delta\pi,\epsilon} = p_{\delta\pi}(\delta\pi)p_{\epsilon}(\epsilon),$$ \hspace{1cm} (3.49)

$$p_{\delta\pi}(\delta\pi) = \frac{1}{(2\pi)^{\frac{m}{2}}|S_{\delta\pi}|} \exp\left(-\frac{1}{2} (\delta\pi - \langle \delta\pi \rangle)^T S_{\delta\pi}^{-1} (\delta\pi - \langle \delta\pi \rangle)\right),$$ \hspace{1cm} (3.50)

$$p_{\epsilon}(\epsilon) = \frac{1}{(2\pi)^{\frac{m}{2}}|S_{\epsilon}|} \exp\left(-\frac{1}{2} \epsilon^T S_{\epsilon}^{-1} \epsilon\right),$$ \hspace{1cm} (3.51)

where $|S_{\delta\pi}|$ and $|S_{\epsilon}|$ are the determinants of the covariance matrices, $m$ is the number of measurements and $n$ is the number of profile samples.

The covariance matrix $S_{\delta\pi}$ of $\delta\pi$, with

$$S_{\delta\pi} = E(\delta\pi - \langle \delta\pi \rangle)(\delta\pi - \langle \delta\pi \rangle)^T,$$ \hspace{1cm} (3.52)

gives the correlation between the profile parameter (i.e. temperature at level $k$ and $j$). In case of no correlation it is a diagonal matrix. The diagonal elements give the variance of the parameter at the $k^{th}$ level. Analogously the covariance matrix $S_{\epsilon}$ of $\epsilon$ is

$$S_{\epsilon} = E(\epsilon\epsilon^T).$$ \hspace{1cm} (3.53)
The maximum likelihood estimate of \( \delta \pi \) is found by solving

\[
\frac{\partial \ln L(\delta \pi)}{\partial (\delta \pi)} = 0 ,
\]

which is equivalent to minimizing the quadratic form

\[
Q(\delta \pi) = (A \delta \pi - \delta T_b)^T S_{de} (A \delta \pi - \delta T_b) + (\delta \pi - \langle \delta \pi \rangle)^T S_e (\delta \pi - \langle \delta \pi \rangle) .
\] (3.54)

The two terms represent the weighted least square solution to the measurements \( \delta T_b \) and the estimate of \( \delta \pi \). The solution \( \delta \hat{\pi} \) is given by

\[
\delta \hat{\pi} = S_{de} A^T (AS_{de} A^T + S_e)^{-1} (\delta T_b - A \langle \delta \pi \rangle) .
\] (3.55)

Note the analogy with the regularization procedure. In fact, the regularization procedure is equivalent to the linear statistical method if the covariance matrices

\[
S_{de} \quad \text{and} \quad S_e \quad \text{are given by} \quad S_{de} = \sigma_{de}^2 I \quad \text{and} \quad S_e = \sigma_e^2 I .
\]

In that case, it is assumed that all profile points and measurement errors are independent and identically distributed parameters. In that case \( \kappa \) may be directly interpreted in terms of a signal to noise ratio, \( \kappa = \frac{\sigma_{de}^2}{\sigma_e^2} \). Further constraints to the profiles can be made by utilising independent ancillary information such as the observation of clouds by various other instruments in the special multi-instrument campaigns like CLARA.

**Using eigenvectors of statistical covariance matrices**

Another way to stabilize the inversion with statistical information is by using the matrix of eigenvectors \( (U_{de} \) and \( U_{st} \)) and eigenvalues \( (\Lambda_s \) and \( \Lambda_{st} \)) of the covariance matrices \( (S_{de} \) and \( S_{st} \)) of the observations of \( \delta \pi \) and \( \delta T_b \) [10]. As the eigenvectors are orthonormal, they give the independent patterns of the variance of \( \delta \pi \) and \( \delta T_b \) and can be written as a linear combination of them. The eigenvectors of \( S_{de} \) and \( S_{st} \) belonging to the highest eigenvalue show the lowest oscillation. Smith and Woolf found that usually the first few (2) eigenvectors account for the largest part of the variance, whereas the remaining ones provide the fitting noise. Therefore \( \delta \pi \) and \( \delta T_b \) can be well represented by a comparatively small number of coefficients. Eq.3.56 gives the expansion of \( \delta \pi \) and \( \delta T_b \) into their eigenvectors \( U_{de} \) and \( U_{st} \).

\[
\delta \pi = U_{de} \xi_{de} \\
\delta T_b = U_{st} \xi_{st} .
\] (3.56)
Note that $\xi_{\delta \ell}$ and $\xi_{\delta \varepsilon}$ can be determined directly using $\xi_{\delta \ell} = U_{a \ell}^{-1} \delta T_b$ and $\xi_{\delta \varepsilon} = U_{a \varepsilon}^{-1} \delta \pi$. Temperature changes can be retrieved from brightness temperatures by introducing the matrix $D$ that converts the coefficients $\xi_{\delta \ell}$ into $\xi_{\delta \varepsilon}$.

$$\xi_{\delta \varepsilon} = D\xi_{\delta \ell},$$  \hfill (3.57)

where $D$ is given by

$$D = \xi_{\delta \ell} \xi_{\delta \ell}^T (\xi_{\delta \ell} \xi_{\delta \ell}^T)^{-1}. \hfill (3.58)$$

This $D$ is determined from a knowledge of the statistics as given by $U_{a \ell}$ and $U_{a \varepsilon}$. Then for individual measurements $\delta T_b$ the retrieval $\delta \hat{\pi}$ can be calculated with

$$\delta \hat{\pi} = (U_{a \ell} D U_{a \ell}^T) \delta T_b \hfill (3.59)$$

The advantage of the eigenvector method is that it only uses the independent pieces that account for the largest part of the variance, which greatly suppresses the influence of errors in the measurements (i.e. improves the condition of the inversion matrix).

**Reduction of the number of parameters**

The condition of the inversion matrix can also be improved by reducing the number of parameters that determine the profile. In the final implementation of the algorithm, the elements of matrix $A$ are the response to changes in the profile parameters:

$$A = \left[ A_T, A_v, A_p, A_L \right]. \hfill (3.60)$$

$A_T$ is the $T_b$ response to a change in the temperature profile at 7 heights. Between these heights the temperature profile is assumed to be linear. $A_v$ is a measure of the $T_b$ response to a change $\delta v_g$ in ground vapour concentration and thereby to a change in the vapour profile $\delta v = \delta v_g \exp \left( - \frac{h}{h_0} \right)$.

As $\delta \phi$ is assumed to be zero, $A_p$ was not calculated. $A_L$ finally is the $T_b$ response to a small change in the concentration of a Slobin cloud. In order to calculate matrix $A$ accurately, the height resolution of the forward model was maintained at 0.1 km. In this way the dimensionality of the inversion is diminished, without compromising the accuracy of the forward model.
Finally we looked at an alternative way to discretise the integral

$$\delta T_b = \int_0^{x_b} W \delta x dx .$$

(3.61)

Instead of using equidistant discretisation, the zeros of Laguerre polynomials were used as the discretisation points, with the corresponding quadrature. This corresponds to the integration of an exponential function. Unfortunately this discretisation did not reduce the number of equations necessary to achieve a particular accuracy. More information on inversion methods can be found in [16-18].
References

4 The radiometer as a measurement instrument

4.1 From total power to noise injection radiometer

This chapter starts with a description of how the noise injection radiometer is developed from the total power radiometer and Dicke-switched radiometer. Section 4.2 gives a functional description of the Rescom radiometer. Due to side lobe pickup, horn spill over, losses in the antenna and wave-guide parts and mismatch to the receiver input, the temperature value measured by the receiver $T_R$ is not equal to the sky noise temperature $T_b$. Section 4.3 describes how $T_b$ is calculated from $T_R$. Values of the parameters necessary for this conversion are both determined in the basic calibration (and referred to with $P_{bc}$) and measured by temperature sensors inside the radiometer ($P_T$).

To ensure a continuous high measurement accuracy, recalibration of the radiometer is necessary. Section 4.4 treats Rescom’s recalibration procedures. They can be divided into two types: the basic calibration and tip-curve calibrations. The basic calibration is performed in-factory. It is used to calibrate the injected noise (cold load calibration) and to determine the values of $P_{bc}$. Tip-curve measurements are performed on-site. They are used to determine a change in one of the $P_{bc}$ values.

4.1.1 Total power radiometer

Like all radiometers, the total power radiometer [1,2,3] consists of an antenna and a receiver (Fig.4.1).

![Antenna and Receiver Diagram](Image)

$T_b$ $T_R$

**Figure 4.1** Total power radiometer.

Fig.4.2 shows the block diagram of the antenna. $T_R$ is not equal to $T_b$, because the antenna introduces side lobe pickup, horn spill over, losses in the antenna and wave-guide parts and mismatch to the receiver.
After the incident wave is focussed by the reflector, the feed horn receives the noise signal. It is connected to the diplexer. The diplexer selects the frequencies and is connected to the receiver. Fig.4.3 shows the block diagram of the receiver.

First the RF signal is amplified in the RF amplifier. After that, the down converter converts the RF signal into a 70 MHz IF signal, which is filtered and further amplified in the IF amplifier. Next the square-law detector detects the power proportional to the input noise temperature. Finally the integrator smoothenes the output fluctuations. The output voltage is equal to

\[ V_{out} = (T_R + T_N)G \]  

(4.1)

- \( G \) : overall gain of the radiometer receiver,
- \( T_R \) : noise temperature entering the receiver,
- \( T_N \) : noise temperature generated by the components in the receiver.

Besides the radiation \( T_R \) received from the antenna, the radiometer also measures the noise \( T_N \) generated by the receiver. The radiometer must be sensitive enough to distinguish the small sky-noise fluctuations from the noise signal generated by the receiver itself. To achieve this, first its sensitivity, determined by the minimal detectable signal change in \( T_R + T_N \), should be as high as possible.
When defining a detectable change as the change in dc-level of the output voltage equal to the standard deviation of its ac-component, the minimum detectable noise temperature change of an ideal radiometer (i.e. \( G \) is constant) is expressed as

\[
\Delta T = \frac{T_R + T_N}{\sqrt{B\tau}}
\]  

(4.2)

with

\[
\tau = \text{integration time of the integrator},
\]

\[
B = \text{(i.f.) bandwidth of the radiometer},
\]

Eq.4.2 shows that it is possible to obtain a high sensitivity (low minimum detectable signal) by choosing a large integration time. In practice, \( G \) fluctuations should be taken into account as well and the minimum detectable noise temperature change of the total power radiometer is

\[
\Delta T = (T_R + T_N)\sqrt{\frac{1}{\sqrt{B\tau}} + \left(\frac{\Delta G}{G}\right)^2}
\]  

(4.3)

with

\[
\Delta G \quad \text{the standard deviation of the gain fluctuation}.
\]

The radiometer stability and measurement performance is improved by making it less sensitive or insensitive to \( T_N \) and \( G \) variations. In this way it is possible to distinguish \( T_R \) from \( T_N \) and \( G \) fluctuations. This is done in the Dicke radiometer (insensitive to \( T_N \) fluctuations) and the noise injection radiometer (insensitive to \( G \)- and \( T_N \) fluctuations).

### 4.1.2 Dicke radiometer

The Dicke radiometer [1,2,3] (Fig.4.4) is a total power radiometer, of which the stability is improved by eliminating its \( T_N \) dependence. This is achieved by rapidly switching the radiometer input between \( T_R \) and the known temperature of a reference load \( T_{\text{ref}} \) and comparing the outputs.

\[
V_{\text{out1}} = (T_R + T_N)G
\]  

(4.4)

during one half period and

\[
V_{\text{out2}} = (T_{\text{ref}} + T_N)G
\]  

(4.5)

during the other half.
Provided the switch period is much shorter than the integration time, the output of the radiometer is

\[ V_{out} = V_{out1} - V_{out2} = (T_R - T_{ref})G \]  \tag{4.6}

and the \( T_N \) dependence is eliminated.

![Block diagram Dicke radiometer](image)

*Figure 4.4* Block diagram Dicke radiometer.

Apart from elimination of the \( T_N \) dependence, the influence of variations in \( G \) is reduced because in Eq.4.6 the signal of interest is \( T_R - T_{ref} \) instead of \( T_R + T_N \). The minimum detectable noise temperature change is now reduced to

\[
\Delta T = \sqrt{\left(\frac{T_R + T_N}{\sqrt{B \tau / 2}}\right)^2 + \left(\frac{T_{ref} + T_N}{\sqrt{B \tau / 2}}\right)^2 + \left[\frac{\Delta G}{G}\right]^2 (T_R - T_{ref})^2} \tag{4.7}
\]

### 4.1.3 Noise injection radiometer

The stability of the Dicke radiometer is further improved by making its output independent of the gain of the radiometer receiver. This is achieved by adding such an amount of noise \( T_{in} \) to antenna noise \( T_R \), that the resultant noise temperature \( T_R' \) is equal to the temperature of the reference load \( T_{ref} \) (Fig.4.5). Note that for conversion of \( T_R' \) into \( T_b \) in the noise injection radiometer [1,2,3,4], losses in the wave-guide coupler should be taken into account as well. (section 4.4).
Figure 4.5 Block diagram of noise injection radiometer.

\[ V_{out} = (T_R' - T_{ref}) G = 0 \]  \hspace{1cm} (4.8)

and

\[ T_R' = T_R + T_{in} \equiv T_{ref} \]  \hspace{1cm} (4.9)

\( T_{in} \) is generated by injecting a variable number of noise pulses from a known noise source.

\[ T_{in} = NQ \]  \hspace{1cm} (4.10)

where

\( N \) = number of noise pulses injected by the noise source,

\( Q \) = quantum of noise.

Eq.4.9 shows that the stability of the radiometer only depends on the stability of \( T_{ref} \) and \( T_{in} \). It is independent of radiometer gain, receiver fluctuations and mismatches in the noise injection feedback loop and ensures a high long-term stability. Provided that variations in the noise quantum \( Q \) can be neglected, the minimum detectable signal is given by

\[ \Delta T = 2 \frac{T_R + T_{in} + T_N}{\sqrt{B \tau}} = 2 \frac{T_{ref} + T_N}{\sqrt{B \tau}}. \]  \hspace{1cm} (4.11)
4.2 Functional description Rescom radiometer

Fig.4.6 shows the block diagram of the Rescom noise injection radiometer [1]. Note that measurements are still sensitive to loss/temperature fluctuations in the antenna. The next section shows how these fluctuations are taken into account when calculating $T_b$ from $T_R$.

Figure 4.6 Block diagram radiometer.

Antenna (A)
The antenna consists of an offset reflector and a dual-band feed assembly. It is designed in such a way that side lobes and consequently noise pick up from the surroundings is very low. The main lobes in the two 20 and 30 GHz bands are nearly equal. Feed horn and diplexer are protected from precipitation by placing them above the reflector. The antenna is surrounded by an enclosure that is painted white to avoid heating by solar irradiation. A heater keeps the temperature of the reflector surface above 0°C in order to avoid build-up of snow or ice. Heater, blower and Peltier elements, regulating the temperature in the receiver box, are controlled by the distribution box.

Receiver (R)
Each receiver consists of a front end, an i.f. section and a detector. They are placed in an insulated, temperature-regulated box. Internal temperatures are in the range of 40°C ±1°C. Receiver temperatures are regulated by Peltier elements, which are directly connected to the front-end mounting plates.
**Front-end**
Each front-end consists of a noise injection assembly, a radiometer switch, a waveguide band pass filter, an isolator and the down-converter assembly. The noise injection assembly consists of a noise diode and a coupler. Via the coupler, the noise signal is injected in the antenna line. The switch alternatively connects the down converter to the reference load and the antenna plus noise injection output. The band pass filter rejects harmful out of band RF signals. The down converter converts the RF signal into an IF signal. It consists of a mixer, an IF amplifier and a local oscillator.

**I.f. section and detector**
The detector contains the necessary circuits to amplify, detect and digitise the difference between the antenna-plus-injected noise signal and the reference load noise signal. First, the 70 MHz IF signal passes a 40 MHz band pass filter and is amplified. The square-law detector produced a base band signal proportional to the difference between the antenna plus injected noise level and the reference load noise level, which is generated and low-pass filtered in a digital integrator. Finally this signal is digitized in the pulse modulator. The output of the modulator controls the noise source to achieve

\[ T_R + NQ = T_{ref}. \]  \hspace{1cm} (4.12)

The antenna temperature follows from

\[ T_R = T_{ref} - NQ. \]  \hspace{1cm} (4.13)

**Computer**
The computer processes measurement data and monitors the radiometer functions. Both antenna noise temperatures and temperature data of important microwave components are collected and transferred to the data handling system. They are used to calculate sky noise temperatures and atmospheric attenuation. The results and status information are presented on the computer’s display and stored on a hard disk.

**Positioner**
The positioner subsystem is used when performing tip-curve calibrations or when changing the elevation angle of the radiometer.
4.3 Conversion of antenna noise into sky noise temperatures

As was already mentioned in section 4.1.3, the temperature that the radiometer measures $T_R$ is not equal to $T_b$, due to side lobe pickup, horn spill over, losses in the antenna and wave-guide parts and mismatch to the receiver input. In this section it is shown how $T_b$ values are retrieved from $T_R$ values [1]. Fig.4.7 shows a more detailed picture of the measurement situation. Note that the temperature at the receiver input $T_R$ in the previous section is given by $T_{A(5)}$. Loss factors in reflector $L_{refl}$, feed horn $L_h$, diplexer $L_{wg,1}$, coupler/switch $L_{wg,2}$, the weight factor $a$, reflection coefficient $r_{ant}$ and horn spill over $G_h$ are determined in the basic calibration (the so called $P_{cb}$ parameters) and saved in a set up file (see Appendices A, B). Temperatures of these components are measured by temperature sensors and saved in separate files (see Appendices A, B).

![Figure 4.7 Measurement situation.](image)

**Basic measurement**

$T_{A(5)}$ is the difference between the temperature of the reference load and the amount of injected noise

$$T_{A(5)} = T_{ref} - T_{in}.$$  \hfill (4.14)

The radiometer software applies a number of corrections to the basic measurements $T_{A(5)}$, as explained below.

**Mismatch Correction**

If the circulator switch is in the position of measuring $T_{A(5)} + NQ$, the noise signal from the reference load is transmitted to the antenna. Because of mismatching of the antenna to the receiver input, part of $T_{ref}$ will be reflected. As a consequence, an extra term $r_{ant} T_{ref}$ is introduced.
Figure 4.8 Mismatching.

\[ T_A^{(5)} + NQ + r_{ant} T_{ref} = T_{ref} \]  

(4.15)

or

\[ T_A^{(5)} = T_{ref} - NQ - r_{ant} T_{ref}. \]  

(4.16)

**Radiometer loss corrections**

The ohmic losses in the antenna and wave-guide parts before the noise injection point influence the received noise \( T_A^{(5)} \). When a noise signal \( T_A^{(n-1)} \) enters a passive component with temperature \( T_{comp} \) and transmission factor \( g \), the output signal \( T_A^{(n)} \) is equal to

\[ T_A^{(n-1)} = T_{comp} g \]

\[ T_A^{(n)} = g T_A^{(n-1)} + (1 - g) T_{comp}. \]  

(4.17)

and \( T_A^{(n-1)} \) is calculated from \( T_A^{(n)} \) via

\[ T_A^{(n-1)} = \frac{T_A^{(n)}}{g} - (\frac{1}{g} - 1) T_{comp} \]  

(4.18)
where
\[ g = \text{transmission factor}. \]

Because the loss \( g \) is always smaller than 1, the loss factor \( L \) is used
\[ L \equiv 1/g \] (4.19)
and
\[ T_A^{(n-1)} = LT_A^{(n)} - (L - 1)T_{\text{comp}}, \] (4.20)

In the correction, loss contributions in the wave-guide coupler and switch \( L_{wg,2} \), the diplexer and wave-guide bend \( L_{wg,1} \), the feed horn \( L_h \) and the metal reflector surface \( L_{\text{refl}} \) are taken into account. The accompanying \( T_A^{(n-1)} \) values are calculated from
\[ T_A^{(4)} = L_{wg,2}T_A^{(5)} - (L_{wg,2} - 1)T_{wg,2} \] (4.21)
\[ L_{wg,2} = \text{loss in wave-guide coupler and switch}, \]
\[ T_{wg,2} = \text{physical temperature of wave-guide coupler and switch, which is taken as} \ T_{wg,2} = T_{\text{ref}}, \]
\[ T_A^{(3)} = L_{wg,1}T_A^{(4)} - (L_{wg,1} - 1)T_{wg,1}, \] (4.22)
\[ L_{wg,1} = \text{loss in the diplexer the wave-guide bend at its input}, \]
\[ T_{wg,1} = \text{physical temperature of diplexer the wave-guide bend, which is taken as} \ T_{wg,1} = T_{\text{ref}}, \]
\[ T_A^{(2)} = L_hT_A^{(3)} - (L_h - 1)T_h, \] (4.23)
\[ L_h = \text{loss in feed horn and its circular-to-rectangular transition}, \]
\[ T_h = \text{temperature, calculated from the temperature of the horn} \ T_{\text{horn}} \text{ and transition} \ T_{\text{trans}} \text{ using} \]
\[ T_h = aT_{\text{horn}} + (1-a)T_{\text{trans}}, \] (4.24)
where the adjustable weighting factor \( a \) is usually set to 0.5
\[ T_A = L_{\text{refl}}T_A^{(1)} - (L_{\text{refl}} - 1)T_{\text{refl}}, \] (4.25)
\[ L_{\text{refl}} = \text{loss in metal reflector surface}, \]
\[ T_{\text{refl}} = \text{temperature of metal reflector surface}. \]
Horn Spill over
Because of horn spill over, the feed horn 'sees' the walls of the enclosure. If absorbing plates are installed in the enclosure, thermal radiation of the absorbers will produce an offset

\[ T_A^{(2)} = G_h T_{abs} + (1 - G_h) T_A^{(1)} \]  

(4.26)

\[ G_h = \text{fraction of the pattern contained in the spill over}, \]

\[ T_{abs} = \text{temperature of the absorbers}. \]

In the absence of absorbers, \( G_h = 0 \), the feed horn will pick up radiation reflected by the metal walls of the enclosure. This radiation will primarily come from the sky and therefore is of the same order of magnitude as \( T_b \) and will not cause an appreciable offset.

Side lobe pick up
Finally the antenna noise temperature should be corrected for side lobe pick up of radiation from ground and sky. The noise pick up by the side lobes depends on the antenna pattern and the intensity of the radiation of the surrounding terrain. The sky noise temperature is calculated from:

\[ T_A = T_{\text{tot}} G_P - T_b (1 - G_P) \]  

(4.27)

\[ G_P = \text{fraction of total power in the side lobes directed towards the ground as function of antenna elevation}. \]

In section 5.3.1 it is verified that the fraction \( G_P \), also often referred to as the 'pattern integration factor' is realistically modelled.

4.4 Calibrations

In order to assure a high measurement accuracy, it is necessary to calibrate the radiometer. Two types of calibration are distinguished: the basic calibration [1] and the tip-curve calibration [1,5]. The basic calibration is performed prior to delivery. It consists of calibration of injected noise per pulse \( Q \) (cold load calibration) and determination of the values of the \( P_{cb} \) parameters (see Appendix A). Tip-curve calibrations are executed on-site and can be used to update \( P_{cb} \) to more accurate values. If only the basic calibration is used, Rescom guarantees an accuracy of 3-5 K on \( T_b \). In case tip-curve measurements are used as well, it is possible to achieve an accuracy of 1 K. This accuracy is only valid if the tip-curves are executed carefully and under favourable conditions.

Cold load calibration (cryogenic load)
Calibration of \( Q \) is necessary, because in practice it is not possible to adjust the noise source output and coupling coefficient so precisely, that \( Q \) can be controlled exactly. When assuming perfect adjustment, \( Q \) is determined from evaluating the extreme situation where \( T_A^{(2)} = 0 \). In that case \( T_i \) is maximal, \( N \) is maximal (i.e. \( N_{max} = 2048 \)) and
\[ Q = \frac{T_{\text{ref}} - T_A^{(5)}}{N} = \frac{313}{2048} \] \[ K. \] But as the adjustment is not perfect, \( Q \) must be calibrated. A correction factor \( k \) is determined from calibration with a cryogenic load. This load is connected to the test port. Fig. 4.10 shows a typical calibration situation.

**Figure 4.10** Calibration measurement situation.

The exact temperature of the cryogenic load \( (T_{CL,\text{exact}}) \) can be read from its calibration certificate. In order to calculate \( Q_{\text{cal}} \), \( T_{CL,\text{exact}} \) must first be corrected for losses and reflections between the test port and the noise generator.

\[
T_{CL,\text{exact}}^{(1)} = \frac{T_{CL,\text{exact}} + (L_{wg,3} - 1)T_{wg,3}}{L_{wg,3}}, \quad (4.28)
\]

\[
T_{CL,\text{exact}}^{(2)} = \frac{T_{CL,\text{exact}}^{(1)} + (L_{wg,4} - 1)T_{wg,4}}{L_{wg,4}}, \quad (4.29)
\]

\[
T_{CL,\text{exact}}^{(3)} = \frac{T_{CL,\text{exact}}^{(2)} + (L_{wg,5} - 1)T_{wg,5}}{L_{wg,5}}, \quad (4.30)
\]

\( Q_{\text{cal}} \) is calculated from \( T_{CL,\text{exact}}^{(3)} \) with

\[
Q_{\text{cal}} = \frac{T_{\text{ref}} - T_{CL,\text{exact}}^{(3)}}{N} \quad (4.31)
\]

and the correction factor \( k \) follows from

\[
k = \frac{Q_{\text{cal}}}{Q}. \quad (4.32)
\]
Diplexer/Input calibration
The diplexer/input circuit consists of a diplexer, an input circuit, a wave-guide switch and a coupler. They are calibrated by measuring the loss of the components in the wave-guide with a Network Analyser before assembly of the front ends.

Antenna calibration
In- factory antenna calibration consists of:

- estimation of the losses in feed horn \( L_h \) and reflector \( L_{\text{refl}} \)
- estimation of the weight factor \( a \)
- determination of antenna characteristics to calculate the horn spill over fraction
- estimation of a correction for side lobe pick up, which is low for elevations above 5°

Tip-curve calibration
Tip-curve calibrations are performed in addition to the basic calibration. They make it possible to adjust \( P_{cb} \) parameters to more accurate values. As it is not possible to determine which of the parameters actually changed, Rescom suggests to change the loss factor in the feed horn \( L_h \), which is supposed to be the dominant factor causing variations in calibration.

During a tip-curve calibration, the elevation angle \( \theta \) of the radiometer is changed in steps from 90° to 25°. Meanwhile, \( T_b \) values are measured. In case of a stratiform atmosphere, changing \( \theta \) implies changing the path length with a factor \( n_a \) and thereby the attenuation \( A \)

\[
A = \int_0^{\theta_0} \frac{x_a}{\sin \theta} \, dx = \frac{1}{\sin \theta} \int_0^{\theta_0} \alpha_z(x) \, dx = n_a A_z
\]

(4.33)

where the number of atmospheres \( n_a = \frac{1}{\sin \theta} \) (which is a measure of the ‘distance’ of the atmosphere, along which the radiometer receives radiation) and the subscript \( z \) indicates zenith, so the relation between \( A \) and \( n_a \) is linear. The \( A \) value at \( n_a = 0 \) \( (A(0)) \), which can be found from extrapolation of the linear relation \( n_a = \frac{1}{\sin \theta} \), indicates whether the calibration of the radiometer is correct (Fig.4.11). \( A(0) \) should be equal to 0 at \( n_a = 0 \), because in that case there is no radiation contribution of the atmosphere. \( A(0) \) can be determined by linearly (Eq.4.33) extrapolating the \( A \) values calculated from \( T_b \) measurements.
**Figure 4.11** Tip-curve measurement and linear extrapolation.

$T_b(0)$ is calculated from $A(0)$ using

$$T_b(0) = (T_{bg} - T_{eff}) \exp^{-0.23025A(0)} + T_{eff}.$$  \hspace{1cm} (4.34)

In literature, $T_b(0)$ values are mostly determined from direct extrapolation of the $T_b$ tip-curve measurements. In that case, the extrapolated value $T_b(0)$ indicates whether the calibration is correct. It is expected to be equal to the cosmic background radiation (2.73 K). This approximation is only valid for low attenuation rates ($T_b \ll T_{eff}$). In that case, the non-linear relation between $T_b$ and $A$ (or $\tau$), with

$$T_b = \int T(x) \alpha(x) e^{-\tau(0,x)} dx + T_{bg} e^{-\tau(0,\infty)}$$  \hspace{1cm} (4.35)

is approximated by

$$A = 10 \log \left( \frac{T_{eff} - T_{bg}}{T_{eff} - T_b} \right)$$  \hspace{1cm} (4.36)

which for small attenuation values can be approximated by the linear relation

$$A = 10 \log \left( \frac{T_{eff} - T_{bg}}{T_{eff} - T_b} \right) \approx 4.343 \frac{T_b - T_{bg}}{T_{eff}}$$  \hspace{1cm} (4.37)
where
\[
\frac{T_b - T_{bg}}{T_{eff}} = \frac{1}{\sin \theta} \left( T_z - T_{bg,z} \right) \quad (4.38)
\]

with
\[
\begin{align*}
T_z & = \text{zenith brightness temperature}, \\
T_{bg,z} & = \text{zenith background temperature}
\end{align*}
\]
or
\[
T_b = T_{bg} + \frac{1}{\sin \theta} \left( T_z - T_{bg,z} \right) \quad (4.39)
\]

In Fig.4.12, the $T_b(0)$ value, calculated from the extrapolation of the linear function described in Eq.4.39, is compared with the value $A_l(0)$, retrieved from the extrapolation of the function described in Eq.4.33, which is converted to $T_b(0)$ by using Eq.4.36.

![Figure 4.12](image)

**Figure 4.12** $T_b(0)$ values calculated from linear interpolation of the attenuation at 31 GHz $A_{31}$ and the brightness temperature at 31 GHz $T_{b31}$ for suitable tip-curves between 1-8-96 and 1-9-1996.
Figure 4.13 $T_b(0)$ values calculated from linear interpolation of the attenuation at 21 GHz $A_{21}$ and the brightness temperature at 21 GHz $T_{b21}$ for suitable tip-curves between 1-8-96 and 1-9-1996.

But as the approximation $T_b \ll T_{\text{eff}}$ is generally not valid (see also Fig.4.12 and 4.13) for frequencies of 20 GHz and above, it is recommended to use $A(0)$ values for recalibration.

If $A(0)$ is not equal to zero while calibration conditions are suitable, it is assumed that this change is caused by a change in $L_h$. Recalibration is obtained by introducing a new $L_h$, which produces a $T_b(0)$ value of 2.73 instead of the $T_b(0)$ value retrieved from the tip-curve measurement.

![Feed horn diagram](image)

**Figure 4.14** Temperatures at reference planes feed horn.
The new $L_h$ is solved from (see Fig. 4.14)

$$L_{h,new} = \frac{T_{A,new}^{(2)} - T_h}{T_{A,tip}^{(3)} - T_h}$$  \hspace{1cm} (4.40)

where $T_{A,tip}^{(3)}$ is calculated from $T_b(0)_{\text{tip}}$ and $L_{h,old}$ and $T_{A,new}^{(2)}$ from the correct $T_b(0)$ value (=2.73 K) using

$$T_A = (1 - G_p)T_b(0) + T_{\text{gnd}}G_p,$$  \hspace{1cm} (4.41)

$$T_A^{(1)} = (T_A + (L_{rfl} - 1)L_{rfl}) / L_{rfl} \text{ (reflector loss)},$$  \hspace{1cm} (4.42)

$$T_A^{(2)} = G_hT_{abs} + (1 - G_h)T_A^{(1)} \text{ (horn spill over)},$$  \hspace{1cm} (4.43)

$$T_A^{(3)} = (T_A^{(2)} + (L_{h,old} - 1)L_h) / L_{h,old}.$$  \hspace{1cm} (4.44)

with

$$T_{\text{gnd}} = \text{ temperature at the ground.}$$

Internal temperatures $T_{rfl}, T_{wg1}, T_{wg2}, T_h$ are measured in real time.

The abovementioned procedure calculates the exact value of $L_h$. In the radiometer manual it is suggested to approximate this by

$$L_{h,app} = L_{h,old}\left(1 + 0.0035(T_b(0)_{\text{tip}} - 2.73)\right),$$  \hspace{1cm} (4.45)

$$L_{h,app} = L_h\left(\frac{A(0)}{10}\right).$$  \hspace{1cm} (4.46)

Fig. 4.15 and 4.16 show that this approximation is not justified for the 31 GHz channel, but is reasonable for the 21 GHz channel.
Figure 4.15 Comparison of the calibration using the new and the approximated determination of the feed loss $L_h$ for $f=31$ GHz.

Figure 4.16 Comparison of calibration using the new and the approximated determination of the feed loss $L_h$ for $f=21$ GHz.
If tip-curve measurements are not evaluated in real-time, measurement data can also be recalibrated afterwards, using the same procedure as the radiometer. For this purpose temperatures of reflector, diplexer, wave-guide bend, wave-guide coupler and switch should be available (see Eq.4.40-4.44). Fig.4.17 and 4.18 show that it is allowed to choose a constant value for these temperatures instead of using real time data.

Figure 4.17 $\Delta T_{21}$ using real-time and constant values for $T_h$ and $T_{refl}$.

Figure 4.18 $\Delta T_{31}$ using real-time and constant values for $T_h$ and $T_{refl}$. 
References

5 Accuracy of radiometer measurements

5.1 Introduction

This chapter evaluates how accurate the radiometer can measure when using tip-curve measurements.

In section 5.2, the accuracy of the tip-curve calibration is evaluated by studying the procedure to analyse the tip-curve data. In section 5.3, the error introduced by side lobe pick up is studied. Finally, the stability of the 21, 31 GHz channels of the radiometer is evaluated using the above-mentioned tip-curve procedure.

5.2 Tip-curve calibrations

In order to study the accuracy of tip-curve calibrations for determining measurement errors in the 21, 31, 51, 53 and 54 GHz channels, an ideal tip-curve for a clear sky standard atmosphere \((T_g=288\ \text{K}, \ P_g=101.3\ \text{kPa}, \ \nu_g=7.5\ \text{g/m}^3)\) has been generated. This is done for a standard ITU-R atmosphere using MPM, for path lengths between 0 and 2.5 Number of Atmospheres (NA) (Eq.4.33). The results are shown in Fig.5.1 and 5.2.

![Figure 5.1 Tip-curve generated for standard ITU-R atmosphere.](image-url)
As can be seen from Fig.5.2, it is not possible to use tip-curves to recalibrate the 53 and 54 GHz channels, because they saturate under lower elevation angles.

**Linear extrapolation of 21/31/51 GHz values**

From Fig.5.1 and 5.2 it can be seen that tip-curves of the 21 and 31 GHz are slightly bent, whereas that of the 51 GHz channel is strongly bent. It is obvious from Fig.5.2 that it is not possible to use linear extrapolation of 51 GHz values to determine $T_b(0)$ which is the parameter determining the absolute radiometer calibration (ideally equal to 2.7 K). For small $T_b$ values on the other hand, it is allowed to approximate the non-linear relation between $T_b$ and the Number of Atmospheres by a linear relation (Eq. 4.37-4.39). To check if values of the 21 and 31 GHz channels in all clear sky situations are small enough to allow linear extrapolation, the dependence of $T_b(0)$ on both $v_g$, $T_g$ and $P_g$ was determined. Results show, that the dependence on $T_g$ and $P_g$ can be neglected, whereas that on $v_g$ not (see Fig.5.3). Therefore, it is not reliable to use linear extrapolation of $T_b$ values for the 21, 31 and 51 GHz channels, even if tip-curves are executed under clear sky conditions.
Figure 5.3 $T_b(0)$ dependence of the 21/31 GHz channels on the amount of vapour at the ground $v_g$.

Procedure using attenuation extrapolation
The relation between the attenuation $A$ [dB] and the number of atmospheres for a horizontally stratified atmosphere is linear. Therefore we investigate if the procedure, described in the block scheme below

```
Convert measured $T_b$ values into $A$ values
Extrapolate $A$ values linearly to determine $A(0)$
Convert $A(0)$ into $T_b(0)$
```

improves the $T_b(0)$ estimate. For the conversion of $T_b$ to $A$, Eq. 4.36 is used. In this formula, $T_{eff}$ is the effective medium temperature. In the consideration below, $T_{eff}$ is calculated from a standard ITU-R atmosphere with MPM. In case of elevation dependence, $T_{eff}$ is calculated at all elevation angles of the tip-curve. Below it is determined how sensitive $T_b(0)$ calculations are to the choice of $T_{eff}$. As $T_{eff}$ is also sensitive to the elevation angle, both the sensitivity to the elevation angle and an error in $T_{eff}$ is evaluated. Table 5.1 shows the difference between using an elevation dependent and an elevation- independent value for $T_{eff}$ (at zenith).
Table 5.1 Sensitivity of $T_b(0)$ to $T_{\text{eff}}$.

<table>
<thead>
<tr>
<th></th>
<th>$T_b(0)_{21}$</th>
<th>$T_b(0)_{31}$</th>
<th>$T_b(0)_{51}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant $T_{\text{eff}}$</td>
<td>2.6534</td>
<td>2.7007</td>
<td>-2.1871</td>
</tr>
<tr>
<td>Elevation dependent $T_{\text{eff}}$</td>
<td>2.73</td>
<td>2.73</td>
<td>2.73</td>
</tr>
</tbody>
</table>

It can be concluded that only for the 21 and 31 GHz channels it is allowed to use an elevation-independent value for $T_{\text{eff}}$. For the 51 GHz channel, it is necessary to use an elevation-dependent value. To evaluate the influence of an error in the estimated value of $T_{\text{eff}}$, $T_b(0)$ is calculated for different errors in $T_{\text{eff}}$ (Fig.5.4) using the abovementioned procedure. It can be concluded that for the 51 GHz, $T_{\text{eff}}$ values should be determined with an accuracy of 1 to 1.5 K, whereas for the 21 and 31 GHz channels, the value of $T_{\text{eff}}$ is not very critical. Since $T_{\text{eff}}$ is directly related to the temperature of the atmosphere at ground level $T_g$, this means that an accuracy of $T_g$ of 1 to 1.5 K is required for the determination of $T_{\text{eff}}$ (Fig.5.5). Calculations show that sensitivity of $T_{\text{eff}}$ to $v_g$ (error of 10 g/m$^3$ is allowed), $P_g$ and cloud thickness (1.5 km) is not very large, so input of measured ground values is not necessary.

The above results, it can be concluded that for the 21 and 31 GHz channels, it is allowed to calculate $T_{\text{eff}}$ from a standard ITU-R standard atmosphere (with $T_g=288.15$, $P_g=101.3$ and $v_g=7.5$) at zenith, whereas for the 51 GHz channel, it is necessary to use ground data as input of the standard ITU-R profiles and an elevation-dependent $T_{\text{eff}}$. 

**Figure 5.4** $T_b(0)$ as a function of the error in the medium ($T_m$) or effective medium ($T_{\text{eff}}$) temperature, with 21 GHz --, 31 GHz ---, 51 GHz -. 

From the above results, it can be concluded that for the 21 and 31 GHz channels, it is allowed to calculate $T_{\text{eff}}$ from a standard ITU-R standard atmosphere (with $T_g=288.15$, $P_g=101.3$ and $v_g=7.5$) at zenith, whereas for the 51 GHz channel, it is necessary to use ground data as input of the standard ITU-R profiles and an elevation-dependent $T_{\text{eff}}$. 

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To check if the above-mentioned procedure also holds in realistic cases, a tip-curve was generated from two radiosonde profiles and the accompanying $T_b(\theta)$ value was calculated. The first represents an average profile, the second a fairly extreme example of a ground inversion. Results can be found in Table 5.2.

![Figure 5.5](image) Temperature profiles of radiosonde, -- average profile, - radiosonde profile with ground inversion.

<table>
<thead>
<tr>
<th>$T_b(\theta)$ according to above procedure calculated from the radiosonde of starday 108 at 6 o’clock</th>
<th>21 GHz</th>
<th>31 GHz</th>
<th>51 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_b(\theta)$</td>
<td>2.7200</td>
<td>2.7238</td>
<td>2.4743</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T_b(\theta)$ according to above procedure calculated from the radiosonde of starday 233 at 6 o’clock</th>
<th>21 GHz</th>
<th>31 GHz</th>
<th>51 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_b(\theta)$</td>
<td>2.5305</td>
<td>2.6731</td>
<td>2.9808</td>
</tr>
</tbody>
</table>

**Table 5.2** $T_b(\theta)$ values calculated from realistic profiles.

It can be concluded that the procedure is valid for realistic cases. This leaves us with the situation of inhomogeneous atmospheres. If the inhomogenities result in a difference in slope, this causes errors in $T_b(\theta)$ determination. As a measure of the inhomogeneity has been taken the standard deviation of the measurements from the linear fit.
If the standard deviation from this fit is larger than a certain criterion, it is assumed that the fit gives unreliable $T_b(0)$ results. The criterion is determined from the $T_b(0)$ offset caused by introducing a deviation in the ideal tip-curve. This is done by generating a random variation from a normal distribution on the ideal tip-curve (at 7 points). The mean of this normal distribution is calculated from the ideal tip-curve and the standard deviation can be changed. In Fig.5.6, the $T_b(0)$ value is determined for 100 random generations with standard deviations of 0.01 and 0.005.

![Figure 5.6](image)

**Figure 5.6** $T_b(0)$ value determined for 100 random generations with standard deviations of 0.01 and 0.005.

Results show that the maximal allowable standard deviation in the $A$ fit is 0.005 in order to obtain a $T_b(0)$ error smaller than 0.5 K. The upper 50 GHz channels (i.e. 53.85 and 54.85 GHz) can be recalibrated by calculating $T_b$ values from a standard atmosphere with ground $T_g$ and $v_g$ fitted with the recalibrated 21, 31 and 51 GHz data. Note that the fit should be within 0.5 K. More accurate results for recalibrating the 50 GHz channels can be obtained by using radiosonde profiles or GPS data to obtain the $v$ profile instead of standard profiles. As both radiosondes and GPS do not measure values of liquid water content ($L$), it is necessary to select clear sky data. This can be done by lidar, radar, video, photos, ceilometer or infrared radiometer. Because of the insensitivity of the calibration of the 21 and 31 GHz channels to $T_{eff}$, it was found to be also possible to do tip-curves in conditions of closed stratus cloud cover. For recalibration of the 51 GHz channel, which is very sensitive to the choice of $T_{eff}$, it is necessary to select clear sky situations from radar and lidar or to gather cloud information such as cloud base or cloud top.
5.3 Side lobe pickup

In this section, the influence of side lobe pick up on \( T_b(0) \) determination is analysed. In the radiometer manual, the contribution by ground pickup was modelled by (Eq.4.27)

\[
T_A = T_g G_p - T_b (1 - G_p).
\]  

with

- \( T_A \): antenna temperature
- \( G_p \): the fraction of energy in the lower hemisphere
- \( T_g \): temperature at the ground
- \( T_b \): brightness temperature

In Eq.5.1, the fraction \( G_p \) of energy in the lower hemisphere is calculated at every elevation angle from measurements of the far field radiation pattern of the radiometer. Fig.5.7 shows this fractional power as supplied by the manufacturer.

![Fractional power as supplied by the manufacturer.](image)

**Figure 5.7** Fractional power as supplied by the manufacturer.

As the effect of pick up at elevations above 5° is low, for \( T_g \) the typical value of 250 K is used in the radiometer manual. Next, the consequence of inaccuracies in \( G_p \) for deriving \( T_b(0) \) is evaluated. For this purpose, \( T_b \) values calculated for an ideal tip-curve (\( T_b = T_A \)) are compared with the one obtained by applying the correction of Eq.5.1 to this tip-curve. The difference in \( A \) when \( T_b(0) \) error was 0.0039 dB. This means that ground pick up introduces a \( T_b(0) \) error of 0.31 K, so it is not necessary to have accurate knowledge of the antenna pattern for calibration of the radiometer.
5.4 Evaluation of the recalibration accuracy and stability of the radiometer

In this section, both the suitability of the tip-curves for recalibration of the radiometer and the stability of the radiometer are evaluated on the basis of data gathered in the tip-curve campaign held from 5-11-97 until 19-1-98. In this campaign, tip-curves were executed both during cloudy, rain and clear sky conditions. Fig.5.9 and 5.10 show \( T_b(0) \) values and standard deviations of the measurements of this campaign.

![Figure 5.9 \( T_b(0) \) values of the measurement campaign.](image)

It is clear from Fig.5.9 that not all tip-curves are suitable for recalibration. In some situations, \( T_b(0) \) errors are very large, which is caused by the high values of the standard deviations of the attenuation fit (see Fig.5.10a and b). Fig.5.10a and b confirm the assumption that there were situations with a large standard deviation. As we already saw in section 5.2, in order to obtain a measurement accuracy of 1 K, it is necessary to select the tip-curves for which the standard deviation of the attenuation fit smaller is than 0.01 dB.
From the photographs that were taken during this campaign, it was determined that 14.22 % percent of the selected tip-curves were executed in clear sky, 76.14 % in cloudy and 9.6 % in rainy situations. This means that a suitable tip-curve does not necessarily have to be executed in a clear sky situation. It was also determined that the standard deviation of tip-curves executed under cloudy conditions were not significantly higher than those executed in clear sky situations. Fig.5.11 shows the standard deviations of tip-curves with a standard deviation smaller than 0.01 dB.
Comparison of 21 and 31 GHz $T_b(0)$ values show that for the selection of a tip-curve, it would be possible to use a criterium of 0.005 for the 31 GHz channel. For the 21 GHz channel it cannot be smaller than 0.01. The difference in criterium is caused by the fact that inhomogenities in the vapour profile are better visible in the 21 GHz channel (Fig.2.7) and therefore have larger values in clear sky conditions. As was already shown in section 5.2, for a standard deviation in the attenuation fit of 0.005 dB, the $T_b(0)$ uncertainty is 0.5 K, whereas a standard deviation of 0.01 dB results in a $T_b(0)$ uncertainty of 1 K. This means that it is possible to calibrate the 31 GHz channel with an accuracy of 0.5 K and the 21 GHz channel with an accuracy of 1 K. Fig.5.12 shows $T_b(0)$ values for tip-curves with a standard deviation smaller than 0.01 dB for the 21 GHz channel and smaller than 0.005 dB for the 31 GHz channel. It confirms that with these tip-curves, it is possible to recalibrate the 21 and 31 GHz channels with an accuracy of, respectively, 1 K and 0.5 K. The variations of $T_b(0)$ around the mean value (indicated by the line) are introduced by the standard deviation in the attenuation fit. The accuracy of the calibration of the 21 GHz channel can be improved by using more than one tip-curve for recalibration.
With respect to the stability of the radiometer, it can be concluded that the calibration of the radiometer changed during the whole period. The calibration of the 31 GHz channel varies between an offset of 1.2 and 2.2 K, whereas for the 21 GHz channel it varies between –1.8 and 0.9 K, so the 31 GHz channel is more stable than the 21 GHz channel.

**Calibration 51 GHz**

For recalibration of the 51 GHz channel, it was necessary to use ground data and to select tip-curves executed in clear-sky situations, because for the 51 GHz the determination of $T_b(0)$ values is very sensitive to the effective medium temperature. Clear sky data were selected by using photographs made during the tip-curve campaign, ground data were supplied by the KNMI. For the evaluation of results, tip-curves with a standard deviation smaller than 0.01 dB were selected. Fig.5.13 shows $T_b(0)$ values and Fig.5.14 accompanying standard deviations of the selected tip-curves.

---

**Figure 5.12** $T_b(0)$ values determined from tip-curve data campaign.
Figure 5.13 $T_{b}(\theta)$ values of the tip-curves of the 51 GHz.

Figure 5.14 Standard deviation of the attenuation fit of 51 GHz tip-curves.
Results show that it is possible to use tip-curves for recalibration of the 51 GHz channel, if ground data and knowledge of the clearness of the atmosphere is available. Fig.5.13 shows that the 51 GHz channel was not stable.

**Calibration of the upper 50 GHz channels for this measurement campaign**

It is not possible to recalibrate the upper 50 GHz (53 and 54 GHz) channels, as they saturate at higher elevation angles. Another way to recalibrate these channels is to use the recalibrated brightness temperatures at 21 and 31 GHz to solve the accompanying atmosphere with the Matched Atmosphere Algorithm and to calculate (MPM) brightness temperatures at 53 and 54 GHz. By comparing calculated and measured brightness temperatures, it is possible to recalibrate the upper 50 GHz channels. Fig.5.15 shows recalibration values of the upper 50 GHz channels, obtained by using the abovementioned procedure.

![Figure 5.15](image.png)

**Figure 5.15** Calibration values for the 53 and 54 GHz channels.

Results show, that profile changes cause changes in calibration, so only differential profile changes can be analysed.
5.5 Conclusions

Tip-curve measurements show that it is only possible to use tip-curve calibrations for recalibration of the 21, 31 and 51 GHz channels, because the upper 50 GHz channels saturate at higher elevation angles. For these 21, 31 and 51 channels, even under clear sky conditions, linear extrapolation of the tip-curve measurements does not give accurate enough results. Extrapolation of attenuation values, on the other hand, does. The procedure to determine $T_b(0)$ in that case is to convert brightness temperature measurements into attenuation values, extrapolate these to find $A(0)$ and reconvert $A(0)$ into $T_b(0)$ again. For the conversion of $T_b$ into $A$, $T_{eff}$ must be calculated. Results show that for the 21 and 31 GHz channels, it is allowed to calculate $T_{eff}$ from a standard ITU-R atmosphere at one elevation angle. Because of their insensitivity to $T_{eff}$, it is also possible to calibrate them under cloudy conditions, as long as the cloudiness is homogeneous.

For the 51 GHz channel on the other hand, it is necessary to select clear sky days and to use ground data (i.e. $T_g$), because the determination of $T_b(0)$ values for the 51 GHz channel is very sensitive to the effective medium temperature. From random simulations it was found that only the $T_b(0)$ values of an $A$ fit with standard deviation smaller than 0.005 dB (0.01 dB at most) give $T_b(0)$ values with the required accuracy of 0.5 K. Because of its sensitivity to $T_{eff}$, it is not possible to calibrate the 51 GHz under cloudy conditions, unless data on clouds are available ($H_{base}, H_{top}$). The 53 and 54 GHz channels can be calibrated by comparison with $T_b$ values, calculated with the Matched Atmosphere Algorithm from the recalibrated 21 and 31 GHz data. This empirical calibration allows only changes in the profile to be analysed.

The suitability of tip-curve measurements and the stability of the radiometer were evaluated on behalf of measurements executed during a tip-curve campaign. It turned out that not all tip-curves can be used for recalibration of the radiometer, if a measurement accuracy of 1 K must be guaranteed. Only tip-curves with a standard deviation of the attenuation fit that is smaller than 0.01 dB are able to guarantee this accuracy. It was found that it is possible to use this criterium for the 21 and 51 GHz channels. For the 31 GHz channel it is possible to use a criterium of 0.005 dB. This means that the 21 GHz channel can be calibrated with an accuracy of 1 K and the 31 GHz with an accuracy of 0.5 K.

The influence of ground pick up on the $T_b(0)$ value was determined as well. It was found that ground pick up introduces at most an error of 0.5 K in $T_b(0)$ value.

The measurement accuracy can be further improved by using more than one tip-curve. For the 31 GHz channel, $T_b(0)$ retrieval from tip-curves is only reliable, in case clouds are thinner than 31 GHz. Tip-curves of the 51 GHz are only reliable under clear sky conditions.

With respect to the stability of the radiometer, it was concluded that it changed during the campaign for all channels.
6 Evaluation of retrieval results

In this chapter, retrieval performance of the different algorithms is evaluated by using the data of the CLARA campaign. Besides radiometer data, radar, lidar, radiosonde and GPS data were used for this evaluation. Radar data were used to determine the height of the top of the cloud and to model the liquid water profile of the cloud from its reflectivity profile. Lidar data were used to determine the base of the cloud and the opacity characteristics of the cloud. GPS and radiosonde data were used for comparison of the amount of water vapour. Events were selected as a representative illustration of retrieval performance of the different algorithms. The evaluation of other events can be found in the articles, mentioned in the list of publications.

6.1 Description of instruments

6.1.1 Global Positioning System (GPS)

GPS is used to estimate the coordinates of points on the surface of the earth. The system consists of a constellation of satellites that transmit at two L band frequencies. If coordinates of the points are known, the amount of water vapour can be retrieved from the propagation delay of GPS signals in the troposphere. The delays were first considered as disturbing parameters, but they later appeared to correlate rather well with microwave radiometer estimates of water vapour.

In section 6.3.4 we compare radiometric, radiosonde and GPS water vapour retrieval. The amount of water vapour is retrieved from the zenith wet delay of the neutral atmosphere. To obtain this value, first the ionospheric delay is removed from the total delay by using the dispersive properties of the ionosphere and the information from the two frequencies of the signals. What is left is the delay of the neutral atmosphere, which is mapped into a zenith delay by so-called mapping functions. The generally - used mapping function of Niell is said to be accurate even for satellite elevations down to 3 ° and depends on the day of the year, latitude and height of the station above the geoid [1].

The zenith delay of the neutral atmosphere can be divided into two: a zenith hydrostatic delay (ZHD), caused by the induced dipole moment and a zenith wet delay (ZWD), caused by the permanent dipole moment of water vapour and liquid water in the troposphere. Errors in the estimate of ZHD will be absorbed in the ZWD and therefore result in an error in the estimate of the amount of water vapour.

The delay of water vapour can be obtained from the ZWD and thus by removing the ZHD from the total delay. To obtain an accuracy of 1 mm for the total amount of water vapour, the accuracy of the surface pressure should be at least 0.3 hPa. The amount of water vapour can be calculated from the wet component as [2]:

\[ V = \Pi \times ZWD \]  

(6.1)

with
\[
\Pi = \frac{10^6}{\rho R_v \left[ \frac{k_1}{T_{med}} + k_2 \right]}
\]  

(6.2)

where

- \(R_v\) is the gas constant,
- \(\rho = 10^3 \text{ kg/m}^3\) is the density of liquid water,
- \(k_1\) and \(k_2\) are physical constants,
- \(T_{med}\) is the medium temperature value.

\(T_{med}\) can be inferred from surface temperature measurements \((T_{med} = 70.2 + 0.72 T_s,\) Bevis et al) or from atmospheric profiles generated by weather models with an accuracy of 2%. It is expected that the largest error source is the initial estimate of ZWD. It was found [1] that vapour retrieval is very sensitive to the choice of the hydrostatic and wet mapping functions. Besides, it is also sensitive to the choice of the network configuration.

In the CLARA campaign, the Active GPS Reference System (AGRS) was used as network. It consists of five permanent GPS stations in the Netherlands, international stations and a GPS data processing and archiving center. For GPS water vapour retrieval, measurements of the receiver situated at Delft were used [3,4].

6.1.2 Lidar and radar cloud boundary measurements

![Figure 6.1 DARR radar of TUDelft.](image)
Figure 6.2 Pictures of RIVM lidar in operation. A) Faculty of Electrical Engineering, TUDelft. The green laser beam is faintly visible against the overcast nighttime skies, B) laser beam leaving the exit window, C) exit window with exit orifice and telescope/receiver assemblies below, and D) laser head, two telescopes and laser beam enclosure with electronics rack in the background.
In the evaluation of the radiometer data, radar reflections are used to determine the cloud top and base. In the CLARA campaign, the Delft Atmospheric Research Radar (DARR) was used for radar measurements. DARR (Fig.6.1) is a 9-cm (3 GHz) Frequency Modulated Continuous Wave (FM-CW) Doppler radar. For small randomly distributed water droplets, the received power is proportional to the diameter to the sixth power. For this reason, the radar is most sensitive to large droplets. Because its wavelength is much longer than that of a typical cloud radar, part of the reflections are caused by clear air turbulence. The cloud top and base are defined as the height at which the measured radar reflectivity decreases to 10 % of its maximum value [5]. The lidar used in the CLARA campaign (Fig.6.2) is a backscatter lidar operating in the near infrared (1064 nm). In general, radar reflection measurements are most sensitive to large droplets, whereas lidar measurements are most sensitive to small droplets. For this reason the lidar is more suitable to detect the base of the cloud. But in case clouds have a large optical depth, the lidar signal is extinguished completely and radar measurements are necessary to determine the top of the cloud. Especially clouds that contain a lot of water are optically too thick for the lidar to ‘see’ the cloud top.

6.2 Data analysis

Calibration during the CLARA campaign
The radiometer data used in the evaluation of the performance of the algorithms are calibrated by selecting the suitable tip-curves performed before and after the three CLARA measurement campaigns. It was found that measurements were accurate up to 1.5 K, which leads to an error of 1-2 mm for water vapour and 0.1-0.2 mm for liquid water for typical cases.

Algorithms
In the evaluation of the performance of the algorithms using 20/30 GHz data it is checked how much vapour and liquid water retrieval differ when using the linear algorithm or the matched atmosphere algorithm (MAA) as described in Chapter 3, with and without additional information. In this way, it is evaluated how local the linearity of the $V/T_{20}/T_{30}$ and $L/T_{20}/T_{30}$ relation is and if it is necessary to tune the atmosphere to obtain linearity in the abovementioned relations. Finally, this local linearity is also used to retrieve temperature profile information (linear perturbation theory) from 20/30 and 51/53/54 GHz data.

Motivation of the data choice
For the evaluation of the retrieval performance of the algorithms, both clear sky (22-11-1996) and cloudy situations (19-4-1996 and 4-9-1996) were selected. By selecting clear sky situations, it is possible to validate vapour retrieval most accurately with GPS and radiosonde measurements and to evaluate the tip-curve calibration accuracy. By analysing cloudy data, the importance of additional information such as radar and lidar measurements for liquid water modelling can be examined. Besides, it can be concluded if vapour and liquid water retrieval are independent.

For the evaluation of the 20/30/50 GHz retrieval, 1-12-1996 was chosen, as radiosonde profiles show elevated inversions and we are interested in the possibility to introduce structure in the temperature profile with the perturbation algorithm.
6.3 Evaluation 20/30 GHz retrieval algorithms

6.3.1 Convergence area of the Matched Atmosphere Algorithm

The 19th of April 1996 was a day, with ground humidity varying between 45 and 85 % and ground temperature between 11 and 18 °C. In Appendix C, satellite images of the weather situation and lidar images of the clouds can be found. When analysing MAA retrieval of the 19th of April 1996, initially, recalibrated measurement points (o in Fig.6.3.a) were lying outside the convergence area (see section 3.3.2) i.e. the convergence area was not in the right position. The convergence area (Fig.6.3a) is the area, generated by changing the humidity profile with the tuning parameter $RH_{ref}$ (section 3.3.2) and the liquid water profile with the base of the cloud. Changes in $RH_{ref}$ occur from left to right (increase in $RH_{ref}$) along side 1 of the convergence area, whereas changes in $H_{base}$ occur from bottom to top along side 2 (decrease in $H_{base}$).

![Figure 6.3a](image)

**Figure 6.3a** Convergence (*) area with $RH_{ref}$ and base of the cloud as tuning parameters.
Obviously, it was not possible to tune the humidity profile in such a way that for a particular ground temperature enough vapour is allowed to match measurements and modelling. This problem was solved (Fig.6.3b) by using an exponential vapour profile \( \exp\left(-\frac{h}{h_0}\right) \), which is tuned by changing the scale height \( h_0 \). In this way, it was possible to get the measurements inside the convergence area. In contrast to the Peter and Kämpfer model, this time, cloud modelling was not done in the vapour profile, as vapour absorption is only very weakly temperature dependent. As a result, the shape of the vapour profile is not so very important for matching the data.

Note, furthermore, that in case the 0°C criterium is used for the determination of the top of the cloud, temperature changes only result in a change of the height of the cloud, but not in differences in the temperature of the cloud and hence in the convergence area. In case radar \( H_{\text{top}} \) and lidar \( H_{\text{base}} \) values are used, changes in temperature will also result in changes in cloud temperature and hence in the convergence area, as the absorption coefficient of liquid water is strongly temperature dependent. It can therefore be concluded that it is not right to use the 0°C criterium in the \( L \) retrieval to determine the height of the top of the cloud. In fact, additional \( H_{\text{top}} \) and \( H_{\text{base}} \) values derived from radar and lidar are required to ensure that the measurements fall inside the convergence area.

**Figure 6.3b** Convergence area with \( h_0 \) and base of the cloud as tuning parameters.
6.3.2 Linearity of the $V(T_{20}/T_{30})$ and $L(T_{20}/T_{30})$ relation

Next, we have evaluated if the $V(T_{20},T_{30})$ and $L(T_{20},T_{30})$ relation is globally linear (linear algorithms) or locally linear (MAA). This is done by comparing retrieval by a linear algorithm (using the MPM spectral model and referred to as LMPM$_{BT}$) with a MAA retrieval (referred to as MAA$_{BT}$), where in both retrievals additional radar $H_{top}$ and lidar $H_{base}$ are used for accurate cloud modelling. Results are shown for the 19th of April 1996.

The difference between MAA$_{BT}$ and LMPM$_{BT}$ is that in MAA$_{BT}$, the exponential decay in the vapour profile ($h_0$) and the concentration ($C$) inside the cloud are tuned, whereas in LMPM$_{BT}$ these values are fixed ($h_0=2$km and $C=0.4$). By tuning it is possible to come so close to the measurements that local linearity holds.

![Figure 6.4](image)

Figure 6.4 Tuned and fixed standard values of water vapour scale height $h_0$, evaluated for the 19th of April 1996.
Figure 6.5 Tuned and fixed standard values of cloud water concentration $C$, evaluated for the 19th of April 1996.

Fig.6.4 and 6.5 show both the matched and the fixed $h_0$ and $C$ values. It can be seen that for liquid water, concentrations $C$ can become much larger than 0.4 and for water vapour, $h_0$ values are also larger than 2.

The next step is to check if global or only local linearity holds. To verify that retrieval difference between $\text{MAA}_{\text{BT}}$ and $\text{LMPM}_{\text{BT}}$ is not caused by interpolation in different planes, retrieval using interpolation in the plane determined by the three nearest points (original implementation, section 3.3.2 and Fig.6.3a and b) is compared with retrieval using extrapolation in the tangential plane, calculated with the linear algorithm for the atmosphere belonging to the point nearest to the measurements. As expected, the difference decreases gradually when refining the grid. By choosing the grid small enough, the error can be neglected.

Fig.6.6 and 6.7 show the relative difference in $V$ and $L$ retrieval between $\text{LMPM}_{\text{BT}}$ and $\text{MAA}_{\text{BT}}$ in % of the $\text{MAA}_{\text{BT}}$ retrieval, caused by the fact that $\text{LMPM}_{\text{BT}}$ uses global and $\text{MAA}_{\text{BT}}$ uses local linearity. Fig.6.6 shows that for $V$ retrieval, global linearity holds, in general.
Figure 6.6 Water vapour difference ($\text{MAA}_{\text{BT}} - \text{LMPM}_{\text{BT}}$) in % of MAA$_{\text{BT}}$, evaluated for the 19th of April 1996.

Figure 6.7 Liquid water difference ($\text{MAA}_{\text{BT}} - \text{LMPM}_{\text{BT}}$) in % of MAA$_{\text{BT}}$, evaluated for the 19th of April 1996.
Fig. 6.7 shows that for $L$ retrieval, differences up to 70% occur, i.e. only local linearity holds. This is as expected, since opposite to the liquid water retrieval, the retrieval of water vapour is not strongly dependent on temperature. The physical explanation for this fact is, that using an average cloud in the linear algorithm is not very realistic.

It was concluded from the evaluation that, in general, the relation $V(T_{20}, T_{30})$ may be considered as globally linear, whereas the relation $L(T_{20}, T_{30})$ is not globally linear. This is especially true in case of thicker clouds. Therefore, for $V$ retrieval, it is allowed to use a linear algorithm, whereas for $L$ retrieval it is strongly recommended to use MAA.

In the next section, several implementations of the MAA are evaluated (with and without additional information) for liquid and vapour retrieval.

6.3.3 Matched atmosphere algorithm with and without additional data

Liquid water

In this section, retrieval results of four different implementations of the MAA algorithm are evaluated. The first implementation (MAA$_{0C}$), uses the 0°C criterium to determine the top of the cloud ($H_{\text{top}}$) [6-7]. The second uses radar $H_{\text{top}}$ values (MAA$_{RT}$) [8,9], the third uses lidar $H_{\text{base}}$ values (MAA$_{LB}$) [10]) and the last uses both lidar $H_{\text{base}}$ and radar $H_{\text{top}}$ values (MAA$_{BT}$) [11,12,13]. Retrieval results are evaluated for measurements of the 4th of September 1996.

![Graph of cloud base and top height](https://via.placeholder.com/150)

**Figure 6.8** Differences in measured (i.e. lidar/radar) and tuned cloud base and cloud top height, evaluated for the 4th of September 1996.
The 4th of September 1996 was a day, with ground humidity varying between 63 and 94 % and ground temperature between 13 and 17 °C. In Appendix C, satellite images of the weather situation and lidar images of the clouds can be found. In both MAAOC and MAART, the base of the cloud is the tuning parameter. In MAALB, the top of the cloud is the tuning parameter and in MAABT the concentration $C$ inside the cloud is the tuning parameter. Fig. 6.6 shows radar $H_{\text{top}}$, lidar $H_{\text{base}}$, 0°C $H_{\text{top}}$ values and tuned $H_{\text{base}}$ and $H_{\text{top}}$ values as calculated with the abovementioned algorithms. Only times with clouds below 3 km and a clear, continuous cloud boundary were selected.

Results (Fig. 6.8) show that the top of the cloud as estimated by the 0°C criterion is much higher than values given by the radar. Consequently, the tuned base of the cloud is much too high. Besides, the daily trend of the top of the cloud, introduced by the daily temperature variations, is not present in the observations. Tuned values of $H_{\text{top}}$ and $H_{\text{base}}$ of MAALB and MAART correlate well with radar $H_{\text{top}}$ and lidar $H_{\text{base}}$ values.

![Figure 6.9](image)

**Figure 6.9** Concentration inside the cloud.
Fig. 6.9 shows the concentration fluctuations inside the cloud if lidar values for the base and radar values for the top of the cloud are used. Results show that concentrations can differ significantly from 0.4. For interpretation of the results, note on the other hand that errors in cloud-boundary estimation (of approximately 100 meter) result also in errors in concentration estimation (of approximately 0.25). Fig. 6.10 shows the relative difference between liquid water values estimated with MAA0C and, respectively, MAA LB, MAA BT and MAA RT for the period considered. It can be seen that the error of MAA0C introduced by too high $H_{top}$ values results in an underestimation of liquid water retrieval of approximately 20-30 %. The underestimation is explained by the fact that the absorption coefficient of liquid water increases for lower temperatures (section 3.2.2), resulting in too low liquid water values at low temperatures (too high $H_{top}$ values). Note that in case of thicker clouds, errors will be even larger.

When comparing retrieval performance of MAA RT, MAA LB and MAA BT, in general the differences in retrieval are very small compared to the average offset of 20 %. It can be concluded that it is most important to determine the height of the cloud (whether provided by radar or lidar). As it is expected that lidar $H_{base}$ values are more reliable than radar $H_{top}$ values (see 6.2), lidar $H_{base}$ values are preferred.

As the temperature of the cloud base (or top) is important to derive accurate $L$ values, the use of an infrared radiometer was considered as well. Unfortunately, it is only possible to use the infrared radiometer to determine the temperature of the base of the cloud in case the cloud is optical thick in the wavelength interval of the infrared radiometer, and even then a correction has to be made for the atmosphere between the instrument and the cloud. Therefore it is recommended to use lidar $H_{base}$ or radar $H_{top}$ rather than infrared radiometer data.

Fig. 6.11 shows differences in vapour retrieval with respect to the MAA0C retrieval. It illustrates the conclusion that differences are not significant, i.e. additional values of radar $H_{top}$ and/or lidar $H_{base}$ do not significantly improve vapour retrieval. More publications on the use of radar/ lidar data and sensor synergy from the CLARA and CLARE’98 campaign can be found in [14-21].
Figure 6.10 Difference in liquid water retrieval with respect to $\text{MAA}_{0C}$ in % of $\text{MAA}_{0C}$ evaluated for the 4\textsuperscript{th} of September 1996.

Figure 6.11 Difference in water vapour retrieval with respect to $\text{MAA}_{0C}$ in % of $\text{MAA}_{0C}$, evaluated for the 4\textsuperscript{th} of September 1996.
6.3.4 Validation of water vapour and liquid water retrieval in clear sky situation

**Figure 6.12a** Comparison radiometric MAA and linear MPM, radiosonde and GPS water vapour retrieval.

**Figure 6.12b** Comparison of radiometric MAA and linear CCIR, radiosonde and GPS water vapour retrieval [3].
Retrieval of clear sky data is illustrated using data of the 22\textsuperscript{nd} of November 1996. Fig.6.12 compares radiometric MAA and Linear Algorithm (LA) vapour retrieval, using different implementations of the linear algorithm, with GPS and radiosonde vapour retrieval. Fig.6.12a shows linear vapour retrieval if the MPM spectral model is used for the calculation of the coefficients. Fig.6.12b shows the same retrieval but now using the CCIR spectral model to calculate coefficients. Coefficients are calculated for different standard atmospheres (see legend). From Fig.6.12a, it can be seen that in case the MPM spectral model is used, the choice of the atmosphere is not very relevant. This is also expected, as water vapour absorption is very insensitive to temperature i.e. to profiling. Fig.6.12b on the other hand shows a larger retrieval difference for different atmospheres, which is introduced by the inaccurate modelling of the absorption coefficient in the CCIR spectral model. From comparison of Fig.6.12a and 6.12b it can be seen that by using different spectral models, an offset in radiometric retrieval is introduced, which is of the order of the difference between radiometric and GPS water vapour retrieval. Note that for GPS and radiometric retrieval also different spectral models are used. For GPS retrieval the spectral model is contained in the mapping function. It is a simplified model, comparable to radiometric retrieval using the CCIR model. It was found that GPS retrieval is very sensitive to small changes in mapping functions [1]. The difference between GPS and radiometric vapour retrieval may therefore be explained by the different spectral models that are used.

**Liquid water**

Data for measurements of 22 December 1996 are recalibrated by searching for brightness temperature change at 21 and 31 GHz, such that the minimum liquid water value $L$ for this clear sky day is zero. It was found that $T_{20}$ and $T_{30}$ values have to be changed by +1 K and –1.4 K with respect to the tip-curve recalibration.

![Figure 6.13](image_url)  
**Figuur 6.13** Radiometric MAA liquid water retrieval.
Fig. 6.13 shows MAA liquid water values for this clear sky day after the abovementioned recalibration. It can be concluded that the calibration of the radiometer was confirmed, as the estimated recalibration accuracy of the tip-curve calibration is approximately ±1 K (see section 4.5).

6.4 Evaluation of algorithms for 20/30/50 GHz retrieval

As we are interested in the possibility to study structure changes in the temperature profile with the perturbation algorithm, for an evaluation of retrieval performance the 1st of December 1996 was chosen, because radiosonde temperature profiles of this day show elevated inversions. Of this day, the periods between 0:00 and 2:00 UTC and between 5:00 and 6:00 UTC were selected, because for these times, clouds lower than 3 km occurred and cloud boundaries changed gradually in time. More details on the weather situation can be found in Appendix C.

21, 31, 51, 53 and 54 GHz measurements were analysed by calculating a new set of weighting functions every 10 minutes for the atmosphere, solved with MAALB. During the ten minutes, weighting functions were assumed to be constant, as it is expected that during this time, profile changes are small and the linear approximation of the radiative transfer equation, treated in section 2.3, is valid.

Brightness temperature changes were evaluated with respect to the first measurement of each averaging period, so at t=0:00 UTC, the change in temperature profile is zero. In this way, errors in brightness temperature changes, introduced by long-term variations in calibration, were avoided. Solved profiles were averaged over 10 minutes, in order to smooth them. The averaging was done in overlapping intervals, i.e. from 0 to 10 minutes, from 5 to 15 minutes etc.

Below, it is evaluated how much water vapour and liquid water retrieval are improved, when the perturbation algorithm is used. First, temperature profile retrieval is evaluated [22, 23]. Fig. 6.14 shows the evolution of the retrieved temperature profile changes. Continuous lines are the retrieved profiles, the dashed line is the difference between the 6:00 UTC temperature radiosonde profile and the standard temperature profile used as a start profile. During this six hour period, an elevated inversion developed.

Fig. 6.14 shows that it is possible to introduce structure in the temperature profile; more particularly, it is possible to retrieve the elevated inversion. Furthermore, it was found that retrieved profile changes converge to radiosonde temperature profile changes, for heights lower than 1.5 (max 2 km), for which the algorithm is able to solve temperature changes.
Figure 6.14 Evolution of the retrieved temperature profile change between t=0:00 and 2:00 and t=5:00 and 6:00 UTC. The blue line is the solved temperature profile, averaged over the last 10 minutes before t=6:00 UTC.

Figure 6.15 Liquid water retrieval relative to liquid water retrieval of the original MAA.
Figure 6.16 Retrieved cloud heights with the MAA_{0C} and MAA_{LB} algorithm.

Figure 6.17 Water vapour retrieval relative to water vapour retrieval of the original MAA.
Next, it is evaluated how much $L$ retrieval is improved when the retrieved temperature profiles are used. In order to compare the improvement obtained by adding lidar information with the improvement obtained from more accurate temperature profiling inside the cloud, both the results of MAALB and perturbation retrieval are plotted.

In the evaluation of liquid water retrieval, the temperature inside the cloud is modelled by a standard (red line), radiosonde (black line) and retrieved (magenta line) temperature profile, respectively. The red line in Fig.6.15 shows the relative improvement by using lidar information (MAALB) instead of the $0^\circ$C criterium (MAA0C) to determine the height of the cloud (as shown in Fig.6.16). The offset is negative, because the lidar cloud is colder than the $0^\circ$C cloud (Fig.6.16) and colder clouds contain less water. Lidar cloud base information can be expressed in terms of temperature changes, by multiplying the adiabatic temperature change per km with the change the height of the base of the cloud. In our example, this is equal to $(0.6/0.7 \ \text{km} \times 6.5) \sim 4 \ \text{K}$.

The black dotted line in Fig.6.15 shows the reference for $L$ retrieval. To obtain the reference, the relative difference in total liquid water content between the standard and the radiosonde temperature profile (dashed line in Fig.6.14) was determined. This difference is positive, because the slope of the radiosonde temperature profile inside the cloud is larger than that of the standard (adiabatic) temperature profile ($-10 \ \text{K/km}$ instead of $-6.5 \ \text{K/km}$), so more water condensates. As a result, the cloud contains more liquid water. The magenta line shows the difference in liquid water, between using the standard and the retrieved temperature profile to model the temperature inside the cloud. Perturbation values are smaller than reference values, because the slope of the retrieved profile inside the cloud is smaller than that of the radiosonde temperature profile. It can be concluded that by using a more accurate shape of the temperature profile, liquid water retrieval is improved. This result cannot be retrieved from lidar information, because lidar data can only give information on an offset in temperature inside the cloud. It cannot account for changes in liquid water retrieval, introduced by differences in slope of the temperature profile.

Finally, it was evaluated how much $V$ retrieval is improved by the retrieved temperature profile changes. Fig.6.17 shows the relative difference between MAALB and MAA0C water vapour retrieval (centre line) and the vapour change, retrieved with the perturbation algorithm, relative to the start value of every averaging period (upper line).

As reference (lower line) was taken the relative difference between vapour retrieved with the radiosonde temperature profile and vapour retrieved with a standard profile. All retrievals show no significant improvement, because water vapour retrieval is not very sensitive to temperature profiling. It appears that more accurate temperature profiling slightly improves vapour retrieval. However since the differences are small, this is hardly significant.
6.5 Conclusions

In this chapter, 3 representative days of the CLARA campaign were selected, to illustrate the retrieval performance of the algorithms. It was found that it is sufficient to use the linear algorithm for water vapour retrieval. For liquid water retrieval on the other hand, the linear algorithm is not suitable, because it uses an average cloud, whereas cloud modelling should be done more accurate. Using linear retrieval can even result in negative amounts of liquid water. Accurate cloud modelling is very important, as liquid water absorption is very sensitive to temperature. One way to improve cloud modelling and liquid water retrieval, is by using lidar base- or radar top of the cloud. In this way, the modelled cloud has the right temperature. Liquid water retrieval can be further improved, by introducing 50 GHz data to retrieve a more accurate temperature profile inside the cloud. Chapter 7 gives more details on the abovementioned conclusions.
References

7 Conclusions

In this thesis, the accuracy of water vapour ($V$) and liquid water ($L$) retrieval from radiometer measurements is evaluated. It was seen that this accuracy depends both on the accuracy of the retrieval algorithms and on the accuracy of the radiometer measurements.

First, retrieval performance of linear and non-linear algorithms, using 20/30 GHz or 20/30/50 GHz radiometer data, has been evaluated. In the linear algorithms, $V$ and $L$ are assumed to depend linearly on 20/30 GHz brightness temperature measurements. In the non-linear Matched Atmosphere Algorithm (MAA), the atmosphere is modelled by the Peter and Kämpfer relative humidity profile, a Slobin liquid water profile and an ITU-R temperature and pressure profile, from which brightness temperatures are calculated. By using tuning parameters, (the reference humidity $R_{\text{H_ref}}$ for the relative humidity profile and the base of the cloud $H_{\text{base}}$ for the liquid water profile) it is possible to generate a look-up table with different $V$- and $L$-values and accompanying brightness temperatures. With this look-up table, the $V$- and $L$-values belonging to certain 20/30 GHz measurements can be directly calculated.

Furthermore, the improvement when using additional lidar/radar and/or 50 GHz data has been evaluated. Lidar and radar data are used to retrieve information on the boundaries of the cloud. 50 GHz data are used to retrieve information on the temperature profile inside the cloud. By using more physical information of the cloud in the non-linear MAA, it is possible to improve $L$- (and $V$-) retrieval.

Next, the accuracy of the radiometer calibration procedure was evaluated. It was seen that for 20/30 GHz radiometer measurements, an accuracy of 1 K can be obtained when tip-curve calibrations are used. For the calibration of the 50 GHz radiometer it is impossible to use tip-curve calibrations, since the measurements saturate under lower elevation angles. For this reason an alternative solution has been found.

Below, conclusions on the performance of retrieval algorithms and calibration procedures are summarized.

7.1 20/30 GHz retrieval

Linearity

Liquid water

For accurate liquid water retrieval, it is necessary to use the non-linear Matched Atmosphere Algorithm (MAA) instead of linear retrieval algorithms. Retrieval results show that especially for liquid water retrieval, only local linearization holds, i.e. that it is necessary to use the MAA to get a modelled point near to the measurements from which it is allowed to extrapolate linearly.

Water vapour

For water vapour on the other hand, in most cases it is sufficient to use a linear retrieval algorithm.

Analysis of clear-sky data shows that by using different spectral models an offset in radiometric retrieval is introduced, which is of the order of magnitude of the difference between radiometric and GPS water vapour retrieval. The difference between GPS and radiometric vapour retrieval can therefore be explained by the different spectral models that are used.
Additional radar and lidar information

Liquid water
The 0°C criterion to determine the top of the cloud derives values different from the real top of the cloud and introduces an unrealistic dynamic behaviour. Retrieval results show that this can lead to errors in the amount of liquid water of approximately 20-30%. The error depends on the type of cloud.
This is caused by the fact that by using this criterion, the temperature inside the cloud is always the same. As the absorption coefficient of liquid water is very strongly temperature dependent, this introduces large errors in liquid retrieval. For an accurate L-retrieval, it is necessary to use radar and/or lidar to derive cloud boundaries.
It was shown that differences between using lidar base of the cloud ($H_{\text{base}}$) and radar top of the cloud ($H_{\text{top}}$) or using one of the two only become significant for thicker clouds.
If a choice has to be made between lidar and radar data to give a cloud parameter, the lidar cloud base is more reliable than the radar cloud top.

Water vapour
For water vapour on the other hand, additional radar $H_{\text{top}}$ and/or lidar $H_{\text{base}}$ information does not improve retrieval performance.

Convergence of the Matched Atmosphere Algorithm
In order to have a large enough convergence area (this is the area in which $V$- and $L$-values, belonging to brightness temperature measurements are looked up) in the Matched Atmosphere Algorithm, it was necessary to replace the Peter and Kämpfer relative humidity profile, tuned by $RH_{\text{ref}}$, by an ITU-R profile, tuned by the scale height $h_0$.

Tuning parameters in the matched Atmosphere Algorithm
Tuned $H_{\text{base}}$ and $H_{\text{top}}$ values, retrieved from the MAA, agree rather well with radar and lidar derived values.

7.2 20/30/50 GHz retrieval

Adding 50 GHz information
By adding 50 GHz radiometer measurements, it is possible to retrieve structure such as inversion in the temperature profile. Introducing 50 GHz information improves liquid water and water vapour retrieval.

Adding lidar measurements
For an optimal water vapour and liquid water retrieval performance, it is necessary to combine 20/30/50 GHz radiometer with lidar cloud base measurements.

Temperature profiling
By more accurate temperature profiling, it is possible to improve liquid water retrieval. Temperature profile changes different from an offset can only be retrieved with 50 GHz data. With lidar data, only a fixed estimate of the temperature inside the cloud can be obtained.
Het 'Atmospheric Water' project

Het in deze thesis beschreven onderzoek is uitgevoerd in het kader van het Atmospheric Water (Atwater) project. Doel van dit project is het ontwerpen van een meetconfiguratie, in combinatie met een radiometer, waarmee zo nauwkeurig mogelijk hoeveelheden waterdamp in de atmosfeer en vloeibaar water in een wolk kunnen worden bepaald. De TU/e radiometer heeft meegedaan aan een tweetal intensieve wolkenmeetcampagnes: de CLouds And RAdiation campagne (CLARA) in 1996 en het Cloud Lidar And Radar Experiment (CLARE’98) in 1998. In beide campagnes speelden radar-, lidar- en radiometermetingen een belangrijke rol bij het verkrijgen van informatie over wolkenparameters. In de laatste jaren is onderzoek aan wolken steeds belangrijker geworden in atmosfeer- en klimaatonderzoek. Om bestaande wolkenmodellen te valideren zijn metingen aan wolken, zoals met radiometer, radar en lidar, nodig. Uit radiometermetingen kan de totale hoeveelheid vloeibaar water worden bepaald, uit lidar-data de basis van de wolk en uit radar data de top van een wolk. Bij de analyse van de radiometerdata kwam na verloop van tijd duidelijk naar voren dat een grote nauwkeurigheid in retrieval van geïntegreerde hoeveelheden waterdamp en vloeibaar water slechts op twee manieren tot stand kon komen: door nauwkeurige metingen d.w.z. door een juiste calibratie van de radiometer en door het ingeven van zoveel mogelijk ‘real-time’ fysische informatie over de atmosfeer. Vandaar dat in dit proefschrift naast de implementatie van de algoritmes ook relatief veel aandacht is besteed aan de behandeling van de calibratie van de radiometer.

Uit evaluatie van retrieval-resultaten van 20/30 GHz radiometermetingen bleek, dat als er in de spectrale modellering veel gebruik wordt gemaakt van statistische informatie, waterdamp en vloeibaar water retrieval erg onnauwkeurig zijn. Indien gebruik wordt gemaakt van ‘real-time’ gronddata en, beter nog, meer realistische profielen (b.v. door gebruik te maken van informatie over de hoogte van de wolk, zoals verkregen uit lidar-metingen), dan wordt de retrieval een stuk nauwkeuriger. Naast 20/30 GHz radiometermetingen, zijn ook 50 GHz metingen geanalyseerd. Daaruit kwam naar voren, dat het inderdaad mogelijk is om uit 50 GHz data temperatuur-profiel informatie op te lossen. Indien 20/30/50 GHz metingen worden gecombineerd, kan zowel informatie van het wolkenprofiel als het waterdamp- en het temperatuur profiel worden verkregen. Uit evaluatie van retrieval-resultaten kan worden geconcludeerd dat, in geval van een nauwkeurige calibratie, de optimale meetconfiguratie bestaat uit een combinatie van een 20/30/50 GHz radiometer en een lidar, waarbij grondtemperatuur en relatieve vochtigheid worden geregistreerd.
### Appendix A

**Factory calibration values**

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<th>Description</th>
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</tr>
<tr>
<td>2</td>
<td>21.3 Frequency receiver no 2 (0.0 if nonexisting)</td>
</tr>
<tr>
<td>3</td>
<td>23.8 Frequency receiver no 3 (0.0 if nonexisting)</td>
</tr>
<tr>
<td>4</td>
<td>0.0 ******* Not used *******</td>
</tr>
<tr>
<td>5</td>
<td>2 Noise scale 0=none, 1=0..75, 2=0..150, 3=0..300 K</td>
</tr>
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<td>6</td>
<td>2 Atten. scale 0=none, 1=0..0.9, 2=0..3, 3=0..12 dB</td>
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<td>7</td>
<td>3 Vapour scale 0=none, 1=0..30, 2=0..60, 3=0..120 mm</td>
</tr>
<tr>
<td>8</td>
<td>1 Liquid scale 0=none, 1=0..3, 2=0..6, 3=0..12 mm</td>
</tr>
<tr>
<td>9</td>
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<td>22</td>
<td>40.0 Receiver, temp. regulator nominal temp, in deg.C</td>
</tr>
<tr>
<td>23</td>
<td>47.0 Receiver-noise temp high limit, deg.C</td>
</tr>
<tr>
<td>24</td>
<td>-30.0 Receiver-peltier block temp low limit, deg.C</td>
</tr>
<tr>
<td>25</td>
<td>80.0 Receiver-peltier block temp high limit, deg.C</td>
</tr>
<tr>
<td>26</td>
<td>-55.0 Receiver-cold load temp low limit, deg.C, def -40.0</td>
</tr>
<tr>
<td>27</td>
<td>-10.0 Receiver-cold load temp high limit, deg.C</td>
</tr>
<tr>
<td>28</td>
<td>10.0 Scaling frequencies - min value</td>
</tr>
<tr>
<td>29</td>
<td>35.0 Scaling frequencies - max value</td>
</tr>
<tr>
<td>30</td>
<td>60.0 Internal data logging rate in sec., def 60, 0=disable</td>
</tr>
<tr>
<td>31</td>
<td>29.7 First scaling frequency (disable = 0.0)</td>
</tr>
<tr>
<td>32</td>
<td>0.0 External sensor logging rate in sec., def 15, 0=disable</td>
</tr>
<tr>
<td>33</td>
<td>19.8 Second scaling frequency (disable =0.0)</td>
</tr>
<tr>
<td>34</td>
<td>0.0 ******* Not used *******</td>
</tr>
<tr>
<td>35</td>
<td>11 Radiometer serial number</td>
</tr>
<tr>
<td>36</td>
<td>0.0 Plot speed at startup, 0=no plot, 1, 4, 16 sec</td>
</tr>
<tr>
<td>37</td>
<td>0.0 ******* Not used *******</td>
</tr>
<tr>
<td>38</td>
<td>15.0 Reflector heater, low limit in deg.C for start</td>
</tr>
<tr>
<td>39</td>
<td>40.0 Horn heater, low limit in deg.C for start</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>30.0</td>
<td>Horn blower, high limit in deg.K for start</td>
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<td>45.0</td>
<td>Reflector blower, low limit in deg. C to stop</td>
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<td>Media temperature, Frequency no 2 (0.0 if nonexisting)</td>
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<td>Media temperature, Frequency no 3 (0.0 if nonexisting)</td>
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<td>1.040</td>
<td>Reflector loss, Frequency no.2B</td>
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<td>Horn spillover, Frequency no.2B</td>
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<td>TC calib.-mode: 0=forward[90.25],1=backward[90..155] degrees</td>
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<td>TC calib.-plot: 0=Noise Plot, 1=Attenuation Plot</td>
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<td>TC calib.-scans [1..10]</td>
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<td>2400</td>
<td>Radiometer link baud rate setting (1200, 2400, 4800 or 9600 baud)</td>
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<tr>
<td>5</td>
<td>Interrupt no for IEEE-488 service routine, default 5</td>
</tr>
<tr>
<td>8</td>
<td>IEEE-488 host address</td>
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<td>Zero pointing for Elevation, ticks (stand alone: 2325 960917 LDK)</td>
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<tr>
<td>3695</td>
<td>Zero pointing for Azimuth, ticks</td>
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<tr>
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<td>dummy mode 0=no, 1=13, 2=20/30, 3=20/20/30-AZ, 4=20/20/30+AZ</td>
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<td>integration time (legal values: 1.0,2.0,4.0,8.0,16.0,32.0)</td>
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<td>azimuth motor control: active (1.0), off (0.0)</td>
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<td>elevation motor control: active (1.0), off (0.0), manual gear (2.</td>
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<td>Scaling factor for Elevation, deg. pr tick 0.1</td>
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<td>Def.aux curve</td>
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<td>Minimum step size for antenna scan, degrees</td>
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<td>Reflection coefficient for ref. temp. (int. load) (31.7 GHz)</td>
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<td>Loss in feed horn (21.3 GHz)</td>
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<td>137</td>
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<tr>
<td>138</td>
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Appendix B Example of a data (*.dat) file

Column overview

1998    year
3       month
10      day
0       hour
5       min
44      sec
0       acquisition mode
68      integration time
0.3981481E-02    sampling time
90.000  elevation
9.500    azimuth
16.5    tb_31
10.8    tb_21
19.4    tb_23
0.0     acquisition mode
1       integration time
1       sampling rate (sec)
0.001   extsens_1
0.000   extsens_2
0.001   extsens_3
0.002   extsens_4
-50.58  30GHz channel cold load_int sensor
-50.58  30GHz channel cold load_ext sensor
11.69   30GHz channel test port
40.41   30GHz channel noise source
40.66   30GHz channel reference load
10.37   30GHz channel Peltier element
-50.58  20GHz channel cold load_int sensor
-50.58  20GHz channel cold load_ext sensor
12.01   20GHz channel test port
39.02   20GHz channel noise source
39.59   20GHz channel reference load
8.52    20GHz channel Peltier element
38.63   temperature diplexer $T_{wgl}$
41.63   temperature horn $T_h$
36.60   temperature wave guide transition $T_{wgl}$
7.48    temperature antenna enclosure $T_{abs}$
32.79   temperature antenna reflector blower
14.47   temperature antenna reflector $T_{refl}$
0       time_tag, time offset from(hour-min-sec as known from the start record)
Recalibration if \( L_h \) is not updated immediately

\[
T_A^{(4)} = T_A^{(5)} L_{wg2} - (L_{wg2} - 1)T_{wg2} \\
T_A^{(3)} = T_A^{(4)} L_{wg1} - (L_{wg1} - 1)T_{wg1} \\
T_A^{(2)} = L_{h,new} T_A^{(3)} - (L_{h,new} - 1)T_h \\
T_A^{(1)} = (T_A^{(2)} - G_h T_{abs})/(1 - G_h) \\
T_A = L_{rfl} T_A^{(1)} - (L_{rfl} - 1)T_{rfl} \\
T_S = (T_A - T_{gnd} G_P)/(1 - G_P)
\]
Appendix C

19th of April
22nd of November
1st of December
List of symbols and abbreviations

*a1-a6* spectroscopic parameters

*b1-b6* spectroscopic parameters

*A* attenuation [dB]

*B* bandwidth [Hz]

*c* speed of light ($2.998 \times 10^8$ m/s)

*C* tuning parameter, depending on the type of cloud

*C1* parameter for hydrosopic aerosols, depending on the particular location

*CLARA* CLouds And Radiation

*CLARE’98* Cloud Lidar And Radar Experiment ‘98

*D* standard dropsize distribution

*DARR* Delft Atmospheric Research Radar

*e* partial water vapour pressure profile [kPa]

*es* water vapour saturation pressure [kPa]

*E* electric field strength [V/m]

*E0* amplitude of *E*

*f* frequency [Hz;GHz]

*F* complex shape function [GHz$^{-1}$]

*FM-CW* Frequency Modulated Continuous Wave

*GPS* Global Positioning system

*h_p* Planck’s constant = $6.626 \times 10^{-34}$ [Js]

*h_0* scale height

*H_top* height of the top of the cloud [km]

*H_base* height of the base of the cloud [km]

*I(x)* intensity at height *x* [W/(m$^2$ sr Hz)]

*I_b* spectral brightness of a black body [W/(m$^2$ sr Hz)]

*I_bg* background radiation intensity entering the atmosphere

*k* Boltzmann’s constant ($1.38 \times 10^{-23}$ [J/K])

*k_0* wavenumber [1/m]

*L* integrated amount of liquid water [mm]

*LCCIR* linear algorithm using the CCIR spectral model to determine the constants

*LMPM_{BT}* linear algorithm using the MPM spectral model and lidar base and radar top of the cloud

*mR* empirically determined constant with respect to rainrate

*MAA* Matched Atmosphere Algorithm

*MAA_{BT}* MAA with lidar base and radar top of the cloud

*MAA_{LB}* MAA with lidar base of the cloud

*MAA_{RT}* MAA with radar top of the cloud

*MAA_{0C}* MAA using 0C criterium to determine the top of the cloud

*MPM* Millimeter wave Propagation Model

*n* complex refractive index of the atmosphere in formula

*n* refractive index [ppm]

*n_0* non dispersive part [ppm]

*n'* real dispersive part [ppm]

*n''* imaginary dispersive part [ppm]

*N* refractivity [ppm]
\[ N = (n-1)10^6 \]
\[ N_0 = (n_0-1)10^6 \]
\[ N' = n'10^6 \]
\[ N'' = n''10^6 \]
\[ N''_L \] moist air resonance contributions
\[ N''_d \] dry air nonresonant spectra
\[ N''_c \] water vapour continuum spectrum
\[ N''_w \] suspended water-droplet-refractivity
\[ N''_R \] rain approximation
\[ N_A \] Number of Atmospheres
\[ p \] pressure [kPa]
\[ P \] barometric pressure [kPa]
\[ P_g \] pressure at the ground [kPa] (\( P_g = 101.325 \) kPa under ITU-R conditions)
\[ P_r \] noise power [W]
\[ r \] distance [km]
\[ R \] rain rate [mm/hour]
\[ RH \] relative humidity [%]
\[ RH(0) \] relative humidity at the ground [%]
\[ RH_{ref} \] value of the RH profile from 1.5 km above the ground to the base of the cloud (\( H_{base} \)) and from \( H_{top} \) to 1.5 km above that point. If there is no cloud \( H_{base} = H_{top} \)
\[ S \] line strength [kHz]
\[ t \] time [sec]
\[ T_{init} = T_g - 6.5x \] [K]
\[ T \] absolute (physical or kinetic) temperature [K]
\[ T_b(x) \] brightness temperature [K] of the incident radiation at height \( x \)
\[ T_b(0) \] brightness temperature if number of atmospheres \( NA = 0 \)
\[ T_{bg} \] background radiation temperature [K]
\[ T_g \] temperature at the surface (\( T_g = 288.15 \) K under standard ITU-R conditions)
\[ T_{eff} \] effective medium temperature [K]
\[ T_{med} \] medium temperature [K]
\[ T_{20} \] brightness temperature at 20 GHz [K]
\[ T_{30} \] brightness temperature at 30 GHz [K]
\[ T_{50} \] brightness temperature at 50 GHz [K]
\[ x \] height [km] (\( h<12km \))
\[ x_o \] total atmospheric height [km]
\[ x_g \] height of the surface of the atmosphere [km]
\[ V \] integrated amount of vapour [mm]
\[ v_g \] amount of water vapour at the ground
\[ w \] liquid water density profile [g/m³]
\[ w_o \] dry mass concentration of hydroscopic aerosols [g/m³]
\[ w_A \] hydroscopic aerosols [g/m³]
\[ w_c \] contribution of clouds [g/m³]
\[ ZHD \] zenith hydrostatic delay
\[ ZWD \] zenith wet delay
\[ \alpha(x) \] absorption coefficient [Np/m]
\[ \beta \] phase dispersion [rad/sec]
\[ \gamma \] pressure broadened width
\[ \delta \] pressure -induced interference
\[ \varepsilon \] intensity emitted by the layer [W/(m² sr Hz)]
\[ \varepsilon \] permittivity
\[ \varepsilon'' \] imaginary part of the permittivity for liquid water
\[ \varepsilon' \] real part of the permittivity for liquid water
\[ \theta \] inverse temperature \(300/T\) [K⁻¹]
\[ \upsilon \] delay dispersion [psec/km]
\[ \omega \] angular frequency [rad/sec]
\[ \rho_{\text{base}} \] saturated water vapour density at the base of the cloud [g/m³]
\[ \rho_{\text{top}} \] saturated water vapour density at the top of the cloud [g/m³]
\[ \tau \] optical thickness [Np]
Acknowledgements

I would like to thank everybody who supported me in writing this thesis. In the first place, I would like to thank the ones that are dearest to me, my boyfriend Nicola, my parents Joop en Koos, brothers Joost en Jeroen, their wives Mojca and Susanne and my two little cousins Luka en Sašo.

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Last but not least, I would like to say to Gert Brussaard, Matti Herben, André van Lammeren, Leo Ligthart en Frans Sluijter, who paid extra attention to my thesis, that I am very grateful for all the time they invested in improving its quality.
Stellingen

1) Lineaire algoritmes zijn geschikt om de hoeveelheid waterdamp in de atmosfeer te bepalen, maar voor een goede schatting van de hoeveelheid water in een wolk is fysische modellering noodzakelijk (Dit proefschrift).

2) Kennis van de hoogte van de wolk, zoals bepaald uit lidar of radar data, is van essentieel belang voor een nauwkeurige bepaling van de hoeveelheid vloeibaar water in een wolk (Dit proefschrift).

3) Met een 50 GHz radiometer kan de bepaling van atmosferisch water worden verbeterd, doordat structuren in het temperatuurprofiel kunnen worden gedetecteerd (Dit proefschrift).

4) De 3 GHz radar is wel geschikt voor het bepalen van de hoogte van de wolk, maar niet voor een verbetering van de wolkenprofilering (Dit proefschrift).

5) Sensorsynergie is meer dan met meer dan één sensor meten.

6) Geluk is als een wolk: als het over je komt werpt het meteen een schaduw op je.

7) Elke vergissing is het begin van een ontdekking.

8) De radar voert een dialoog met de atmosfeer, de radiometer is slechts toehoorder.

9) De kat (felis ochreata domestica) is een sociaal dier. Zij weet precies hoe ze alles van je gedaan krijgt.

10) Wolken en mensen voldoen aan een soort Heisenberg principe: hoe gedetailleerder je ze probeert te modelleren, hoe verder je van de waarheid af zit.

11) Mannen weten veel, vrouwen begrijpen veel.

12) Muziek is een machtig instrument: het bespeelt hart en ziel.
Curriculum Vitae


Suzanne Jongen was born at the 25th of March in Venlo. After receiving her Gymnasium Diploma at the St.Thomascollege in Venlo, she started her studies in Physics at the Catholic University in Nijmegen, which she successfully finished in 1992. Next, she decided to study for a degree as a teacher physics. She received her degree in 1993 and taught 4 years at several secondary schools. In 1996 she started her PhD at the faculty of Electrical Engineering at the Eindhoven University of Technology in Eindhoven. Recently, she started as a researcher at TNO Bouw in the field of renewable energy, more particularly PV panels.