Sound Radiation of Resilient Train Wheels with SYSNOISE and BARD

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Introduction

This report describes the work done in a project to calculate the radiated sound of resilient train wheels with the software package BARD (Basic Axisymmetric Radiation Design by Kuijpers (1996)). The calculations are performed with data that are obtained with the Finite Element Package ANSYS for a specific range of frequencies. The radiated sound is calculated with BARD and compared with results that are obtained with SYSNOISE and TWINS (Train Wheel Interaction Noise Software).
Chapter 1

Vibration of Train Wheels

The vibration of a train wheel can be classified as vibration in radial, axial and circumferential direction ($r$, $z$ and $\theta$). Modes can be described by the number of sine waves in radial and circumferential directions, which result in nodal diameters and nodal circles respectively. In figure 1.1 the dotted lines represent nodal diameters and the dotted circle represents a nodal circle. A special case occurs with the VSG wheel where the modes of the inner and outer wheel (called the web and the tire) are mainly uncoupled.

1.1 Previous research

The section NSTO (Nederlandse Spoorwegen, Technisch Onderzoek) investigates the sound production of train wheels. Special attention is paid to the new resilient VSG train wheel, that consist of an inner and outer wheel, which are connected by rubber elements. Numerical simulations predict that this wheel will radiate less sound than a standard ICM train wheel, that consists of only one part. The radiated sound was predicted with the acoustic software packages SYSNOISE and TWINS (Train Wheel Interaction Noise Software), see van Lier (1996).
Both the ICM and the VSG wheel are modeled in ANSYS in a cylindrical coordinate system \((r, \theta, z)\) with three dimensional hexahedral elements. Only a half wheel is modeled by applying a plane of symmetry. The mesh (written in a .cdb-file) and the calculated resonant frequencies and mode shapes (written in a .rst-file) are used as input for SYSNOISE.

SYSNOISE claims to predict the radiated sound very accurately, provided that the surface vibrations are accurately known. The ANSYS mesh is converted to an ‘envelope’ mesh that consists only of surface elements (1858 quadrangular elements with 2255 nodes for the ICM wheel). The BEM nodes of SYSNOISE are chosen to coincide with the FEM nodes of ANSYS if possible. Else they are interpolated. The radiated sound of the train wheels is also calculated with TWINS. The following assumptions are made:

- The geometry is rotationally symmetric. This implies that two dimensional axisymmetric Fourier elements can be used with an order \(n\) equal to the number of nodal diameters. Appendix B shows that an approximation of order \(n\) will be sufficient.
- The sound radiated by the radial and torsional wheel vibrations are dominated by motions of the tire (the outer VSG wheel).

Radiation efficiency

The radiation efficiency of train wheels was studied by van Lier (1996). At low frequencies, the sound radiation from the wheel depends strongly on the number of nodal diameters \(n\) in a given mode shape. In practice, most resonant frequencies of massive train wheels are higher than their corresponding critical frequencies, resulting in \(\sigma = 1\) for most nodes. The TWINS model expects the VSG wheel to have more resonant frequencies below their corresponding critical frequencies than the ICM wheel, because the resonant frequencies are lowered due to the insertion of the rubber blocks.

The values \(\sigma\) of the radiation efficiency for ICM wheels and for axial tire and web modes of VSG wheels calculated by SYSNOISE and TWINS agree very well. The radiation efficiency of radial and torsional tire modes calculated by SYSNOISE are structurally higher than those predicted by TWINS. The SYSNOISE results are mistrusted by van Lier (1996) because the influence of the slit between the web and the tire parts on the sound radiation has been neglected. This however does not explain the differences between the SYSNOISE and TWINS results, because in both models used by van Lier (1996) there is no slit present.

Conclusions

The conclusions made by van Lier (1996) are the following:

- The radiation efficiencies \(\sigma\) of all modes of the ICM wheel and all axial tire and web modes of the resilient VSG wheel calculated with SYSNOISE agree well with the corresponding values calculated with TWINS: deviations are less than 3 dB for all modes.
- The \(\sigma\)-values of the radial and torsional tire modes of the VSG wheel calculated with SYSNOISE are structurally higher (10-20 dB) than the corresponding TWINS results. Two reasons for these deviations are an overprediction of the modal critical frequencies in TWINS and an overprediction of the \(\sigma\)-values in SYSNOISE by neglect of the sound radiated by the slit between the web and the tire parts. Setting all \(\sigma\)-values of the radial and torsional tire modes equal to 1 improves the result but is seen as an ad-hoc adjustment.
1.2 Discussion

Van Lier (1996) expects that the slit between the inner and outer wheel has some influence on the radiated sound, especially for radial and torsional vibration modes (with axial modes the slit reduces the sound radiating surface and will lead to a reduction of the radiation efficiency). The compression and rarefaction of the air in the slit should form a dipole with the sound radiating outer surface of the web or the tire. The critical frequency of a dipole can be approximated by:

\[ f_c = 1.41 \frac{c}{\sqrt{\pi S}} \]  

(1.1)

with \( c \) the speed of sound and \( S \) a surface. Considering the complexity of the vibrations of the wheel simple monopole or dipole models have little meaning. The radial vibration modes result in regions of compression and rarefaction depending on the mode of vibration. A compression dipole and a rarefaction dipole can act together as a quadrapole, which needs very high frequencies for efficient radiation. Two monopoles or dipoles that have opposite phase can result in an acoustic short-circuit. The critical frequency below which this occurs is dependent on the distance between these elementary sources of sound, and is therefore dependent on the mode shape (number of nodal diameters). Considering this it is very difficult to predict the efficiency of sound radiated by compression and rarefaction of air in the slit. The effects of the slit are investigated in the next chapter where a comparison is given between two models; one with and one without slit.
Chapter 2

Modeling the VSG train wheel

An ANSYS finite element model of the VSG train wheel has been made. The ANSYS calculations are performed on a model of a half train wheel, with symmetry boundary conditions in the plane $y = 0$. The rubber blocks are modeled as matrix elements, connecting the inner and outer wheel. A modal analysis with ANSYS has produced 44 modes with frequency lower than 5 kHz. The results of the ANSYS calculations are written in an 'rst'-file, containing the resonant frequencies and mode shapes (normalized with respect to the mass matrix). This file is used to describe the boundary conditions in the SYSNOISE model.

2.1 Model of the VSG wheel in SYSNOISE

In the SYSNOISE calculations two different models (meshes) are used. Since the rubber blocks where modeled as matrix elements, they are not present in the mesh. The mesh in figure 2.2 shows the slit between the inner and outer wheel. The air in this slit is subject to compression and rarefaction and may therefore have a contribution to the radiated sound. This effect is investigated by comparing the results of this model with results of a model where the slit is filled by solid elements (figure 2.1). The same boundary conditions apply to both models.

Figure 2.1: Detail of the VSG wheel without a slit.

Figure 2.2: Detail of the VSG wheel with a slit.
2.2 Model of the VSG wheel in BARD

The software package BARD (Basic Axisymmetric Radiation Design) is based on the Boundary Element Method and predicts the sound generated by geometrically axisymmetric structures. The boundary conditions are not required to be axisymmetric, but are described as a Fourier series in the circumferential direction.

The mesh that BARD needs to describe the geometry of the train wheel is only a cross-section of the model that is used in SYSNOISE. Two models are used: one without and one with a slit. The (non-axisymmetric) boundary conditions from the ANSYS calculations must be translated to a Fourier series of boundary conditions that can be applied to the axisymmetric structure. The meshes that are used in BARD are plotted in figures 2.4 and 2.3.

Figure 2.3: BARD model of the VSG wheel without a slit.

Figure 2.4: BARD model of the VSG wheel with a slit.
Chapter 3

Computations

3.1 SYSNOISE computations

Table 3.1 contains the radiation efficiencies for both models for the first ten modes. The radiation coefficients for the model with the open slit are generally lower than those for the model without the slit (except for the ninth mode). For axial modes it is easy to understand that this holds because of a smaller surface that is oscillating in axial direction. For radial modes there are much larger differences (smaller for mode 4 and higher for mode 9). The higher radiation efficiency of mode 9 can possibly be ascribed to compression and rarefaction of air in the slit. Since it radiates sound very efficient the frequency is assumed to be above the critical frequency. This effect is not present in mode 4 which is possibly due to an acoustic short-circuit.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Mode type</th>
<th>ND</th>
<th>Radiation Efficiency σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Without slit</td>
</tr>
<tr>
<td>1</td>
<td>148.942</td>
<td>Axial Tire</td>
<td>1</td>
<td>0.004</td>
</tr>
<tr>
<td>2</td>
<td>202.718</td>
<td>Axial Tire</td>
<td>0</td>
<td>0.385</td>
</tr>
<tr>
<td>3</td>
<td>351.566</td>
<td>Axial Tire</td>
<td>2</td>
<td>0.062</td>
</tr>
<tr>
<td>4</td>
<td>424.317</td>
<td>Radial Tire</td>
<td>1</td>
<td>0.339</td>
</tr>
<tr>
<td>5</td>
<td>527.126</td>
<td>Axial Web</td>
<td>1</td>
<td>0.910</td>
</tr>
<tr>
<td>6</td>
<td>563.822</td>
<td>Radial Tire</td>
<td>2</td>
<td>0.531</td>
</tr>
<tr>
<td>7</td>
<td>584.310</td>
<td>Axial wheel</td>
<td>0</td>
<td>0.758</td>
</tr>
<tr>
<td>8</td>
<td>618.912</td>
<td>Axial Web</td>
<td>2</td>
<td>1.140</td>
</tr>
<tr>
<td>9</td>
<td>832.721</td>
<td>Radial Tire</td>
<td>3</td>
<td>0.643</td>
</tr>
<tr>
<td>10</td>
<td>977.613</td>
<td>Axial Tire</td>
<td>3</td>
<td>0.524</td>
</tr>
</tbody>
</table>

Table 3.1: Radiation efficiencies for the first ten modes for both models. The mode type refers to axial and radial modes of the Tire (the outer wheel) and the Web (the inner wheel). ND is the number of nodal diameters.

3.2 BARD computations

To compute the radiation efficiencies of the axisymmetric model the boundary conditions must be provided as Fourier coefficients for all nodes. Routines have been written in MATLAB to select all nodes (slave nodes) in the three dimensional mesh that have the same radius and z-coordinate as the nodes in the cross-sectional two
dimensional mesh (master nodes). For each node in the two dimensional mesh a set of nodes results that is sorted on the angle $\theta$. When the velocity boundary conditions are known for all these nodes the boundary conditions on the master nodes can be computed as velocity Fourier coefficients by applying an FFT analysis for each master node. With these Fourier coefficients and the two dimensional mesh the radiation coefficients are computed with \textsc{bard}. Since the velocities could not be extracted in Cartesian coordinates, the normal velocities (scalar) at the nodes were used. This leads to a small overestimation of the radiation efficiency as will be explained in the next section.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Mode type</th>
<th>ND</th>
<th>Radiation Efficiency $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148.942</td>
<td>Axial Tire</td>
<td>1</td>
<td>0.004</td>
</tr>
<tr>
<td>2</td>
<td>202.718</td>
<td>Axial Tire</td>
<td>0</td>
<td>0.385</td>
</tr>
<tr>
<td>3</td>
<td>351.566</td>
<td>Axial Tire</td>
<td>2</td>
<td>0.062</td>
</tr>
<tr>
<td>4</td>
<td>424.317</td>
<td>Radial Tire</td>
<td>1</td>
<td>0.339</td>
</tr>
<tr>
<td>5</td>
<td>527.126</td>
<td>Axial Web</td>
<td>1</td>
<td>0.910</td>
</tr>
<tr>
<td>6</td>
<td>563.822</td>
<td>Radial Tire</td>
<td>2</td>
<td>0.531</td>
</tr>
<tr>
<td>7</td>
<td>584.310</td>
<td>Axial wheel</td>
<td>0</td>
<td>0.758</td>
</tr>
<tr>
<td>8</td>
<td>618.912</td>
<td>Axial Web</td>
<td>2</td>
<td>1.140</td>
</tr>
<tr>
<td>9</td>
<td>832.721</td>
<td>Radial Tire</td>
<td>3</td>
<td>0.643</td>
</tr>
<tr>
<td>10</td>
<td>977.613</td>
<td>Axial Tire</td>
<td>3</td>
<td>0.524</td>
</tr>
</tbody>
</table>

Table 3.2: Radiation efficiencies for the first ten modes for the VSG wheel without slit calculated with \textsc{sysnoise} and \textsc{bard}. The mode type refers to axial and radial modes of the Tire (the outer wheel) and the Web (the inner wheel). ND is the number of nodal diameters.

### 3.3 Comparison of \textsc{sysnoise} and \textsc{bard} results

The main difference in computation of radiation efficiencies and output power for the three dimensional \textsc{sysnoise} model and the two dimensional \textsc{bard} model is the computation time. Computations with the three dimensional model with about 2000 nodes takes about half an hour whereas computations with the two dimensional modes with only 52 nodes takes about 15 seconds.

The radiation efficiencies agree very well. The \textsc{bard} results are a little higher because normal velocities were prescribed. On edge nodes the normal velocity is therefore prescribed in two directions which can result in the situation illustrated in figure 3.1.

For vibrations purely in vertical direction the vertical velocity on the edge node is the normal velocity of the top element. It is also used as the horizontal velocity of the left element. This problem can be overcome by using the real velocities in Cartesian coordinates and taking the dot product with the normal vector of the element. The edge nodes have to be duplicated to have a node on the top and the left element.

### 3.4 More computations

Computations were also performed on the model with the open slit. In this case the differences in the \textsc{sysnoise} and \textsc{bard} results for the radial modes are larger.
Figure 3.1: When an edge of an object moves in one direction (e.g. vertical), the edgenode should be duplicated. One of the nodes on the edge has zero velocity (used in the SYSNOISE model). If the edge node is not duplicated the normal velocity will be applied in two directions and will introduce an overestimation of the radiation efficiency (used in the BARD model).

For the axial modes the BARD results are a little higher than the SYSNOISE results because of the error that is made on edge nodes. Mode 4 predicts a negative radiated power. This error is possibly due to internal resonance which can be overcome by applying chief points (see Kuijpers (1996)).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Mode type</th>
<th>$ND$</th>
<th>SYSNOISE</th>
<th>BARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>148.942</td>
<td>Axial Tire</td>
<td>1</td>
<td>0.0036</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>202.718</td>
<td>Axial Tire</td>
<td>0</td>
<td>0.273</td>
<td>0.223</td>
</tr>
<tr>
<td>3</td>
<td>351.566</td>
<td>Axial Tire</td>
<td>2</td>
<td>0.043</td>
<td>0.064</td>
</tr>
<tr>
<td>4</td>
<td>424.317</td>
<td>Radial Tire</td>
<td>1</td>
<td>0.027</td>
<td>-0.024</td>
</tr>
<tr>
<td>5</td>
<td>527.126</td>
<td>Axial Web</td>
<td>1</td>
<td>0.727</td>
<td>0.623</td>
</tr>
<tr>
<td>6</td>
<td>563.822</td>
<td>Radial Tire</td>
<td>2</td>
<td>0.504</td>
<td>0.167</td>
</tr>
<tr>
<td>7</td>
<td>584.310</td>
<td>Axial wheel</td>
<td>0</td>
<td>0.597</td>
<td>0.602</td>
</tr>
<tr>
<td>8</td>
<td>618.912</td>
<td>Axial Web</td>
<td>2</td>
<td>0.862</td>
<td>0.718</td>
</tr>
<tr>
<td>9</td>
<td>832.721</td>
<td>Radial Tire</td>
<td>3</td>
<td>15.835</td>
<td>12.024</td>
</tr>
<tr>
<td>10</td>
<td>977.813</td>
<td>Axial Tire</td>
<td>3</td>
<td>0.419</td>
<td>0.588</td>
</tr>
</tbody>
</table>

Table 3.3: Radiation efficiencies for the first ten modes for the VSG wheel with slit calculated with SYSNOISE and BARD. The mode type refers to axial and radial modes of the Tire (the outer wheel) and the Web (the inner wheel). $ND$ is the number of nodal diameters.

To illustrate the correspondence of the SYSNOISE and BARD results, post processing results are given for mode 4 and 7 of the VSG wheel without slit.
Figure 3.2: Mode 4 of the VSG wheel without slit at 424.317 Hz (Radial outer wheel mode). Results (top) are obtained from SYSNOISE calculations. Results (bottom) are obtained from BARD calculations and exported for SYSNOISE post processing.
Figure 3.3: Mode 7 of the VSG wheel without slit at 584.310 Hz. The inner and outer wheel vibrate axially in opposite directions. Results (top) are obtained from SYSNOISE calculations. Results (bottom) are obtained from BARD calculations and exported for SYSNOISE post processing.
Chapter 4

Conclusions and recommendations

In this chapter some conclusions and recommendations are given.

4.1 Conclusions

- The model of the VSG wheel without and with slit are both not a correct model since the slit is filled for two thirds with rubber blocks. The model without slit (slit is closed by elements) will therefore predict the radiation efficiency too high for axial modes. Compression and rarefaction of air in the slit is not considered in the model without slit. The model with slit underestimates the radiation efficiency but the compression and rarefaction effects of the air in the slit in practice will be smaller than predicted by this model.

- The results obtained with BARD correspond very well with the SYSNOISE results, especially for the model without slit. The BARD results are structurally higher which is a result of an error that is made on edge nodes as described in chapter 3.

- The reduction of computation time with the use of axisymmetric models with Fourier coefficients as boundary conditions is enormous. SYSNOISE analyses on the three dimensional model of the VSG wheel take about half an hour, whereas BARD only needs approximately 15 seconds. The extra computation time to select the master and slave nodes and to compute the Fourier Coefficients is approximately 1 minute (in MATLAB, can be optimized by translation to C++ code).

4.2 Recommendations

- The BARD results can be made more accurate when the scalar product of the three dimensional velocities and the normal vector is used. Edge nodes have to be duplicated in this case.
Bibliography


Appendix A

The Boundary Element Method

SYSNOISE (NIT (1993)) is a FEM/BEM software package for acoustic problems. It can perform FEM and BEM calculations in the time and frequency domain. The problem can be formulated in a direct or indirect way and solved with a collocational or variational solution method. Both interior and exterior problems can be solved. The user can also choose to solve a problem coupled or uncoupled. The unknown variables can be on the nodes or on the elements.

From now on we restrict to uncoupled radiation problems. In an uncoupled problem the structural vibrations and the acoustic field are uncoupled. A vibrating surface excites the surrounding medium. The train wheel sound radiation problem as described in this report is an uncoupled problem. The acoustic field has no influence on the vibration of the wheel. The vibrations are entered as velocity boundary conditions and the sound field can be determined.

A.1 Theory

The wave equation can be written as (see Ciskowski & Brebbia (1991)):

$$\nabla^2 p' - \frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} = 0 \quad (A.1)$$

where $p'(x, t)$ is the acoustic pressure fluctuation and $c$ is the speed of sound. For harmonic solutions $p' = p e^{j\omega t}$ with $\omega$ the angular frequency, the wave equation yields the Helmholtz equation:

$$\nabla^2 p + k^2 p = 0 \quad (A.2)$$

with $k = \omega/c_0$ the wave number. The normal velocity $v_n$ on a surface $S$ with normal coordinate $n$ is given by:

$$v_n = \frac{1}{j\omega \rho_0} \frac{\partial p}{\partial n} \quad (A.3)$$

This problem can be solved with one of the following boundary conditions:

1. Dirichlet boundary condition:
   prescribe the pressure amplitude $p = p_r$, for example $p = 0$ at a free surface.

2. Neumann boundary condition:
   prescribe the normal velocity amplitude $v_n = v_r$, for example $v_n = 0$ at a rigid surface.
3. Robin boundary condition:
   prescribe the normal impedance \( Z_n = p/v_n = Z_r \), for example at an absorbing surface.

Uncoupled radiation problems are defined by equations (A.2) and (A.3). Often the normal velocity at the surface of a vibrating structure is given as a Neumann boundary condition.

### A.2 Boundary Element Method

Sound sources are modeled with Green’s functions \( G(\mathbf{x}, t ; \mathbf{y}, \tau) \), defined as the solution of the wave equation at position \( \mathbf{x} \) and time \( t \), caused by a pulse transmitted at position \( \mathbf{y} \) and time \( \tau \). In Fourier space a Helmholtz formulation can be used:

\[
\nabla^2 G + k^2 G = -\delta(\mathbf{x} - \mathbf{y})
\]  
(A.4)

The general solution for three dimensional waves to this equation is:

\[
G(\mathbf{x}|\mathbf{y}) = \frac{e^{-jk|\mathbf{x} - \mathbf{y}|}}{4\pi|\mathbf{x} - \mathbf{y}|}
\]  
(A.5)

With Green’s Third Identity the Boundary Integral Equation can now be derived:

\[
C(\mathbf{x}) \cdot p(\mathbf{x}) = \int_S \left[ p(\mathbf{y}) \frac{\partial G(\mathbf{x}|\mathbf{y})}{\partial n} - G(\mathbf{x}|\mathbf{y}) \frac{\partial p(\mathbf{y})}{\partial n} \right] dS
\]  
(A.6)

where the coefficient \( C(\mathbf{x}) \) depends on the position \( \mathbf{x} \) according to:

\[
C(\mathbf{x}) = \begin{cases} 
0 \text{ for } \mathbf{x} \text{ outside the acoustic medium}, \\
1 \text{ for } \mathbf{x} \text{ inside the acoustic medium}, \\
\frac{1}{2} \text{ for } \mathbf{x} \text{ on the smooth surface } S \text{ of the acoustic medium}.
\end{cases}
\]  
(A.7)

Now the boundaries have to be discretized and a choice must be made for the element distribution and the interpolation functions within each element. After that the equations can be rewritten as:

\[
A p = B \frac{\partial p}{\partial n}
\]  
(A.8)

in an implicit formulation that describe the pressure \( p \) on the node points on the surface \( S \). Points in the radiated sound field must fulfil:

\[
p(\mathbf{x}) = A(\mathbf{x}) p - B(\mathbf{x}) \frac{\partial p}{\partial n}
\]  
(A.9)

which is an explicit formulation. This equation is used in postprocessing to calculate the sound pressure field.

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Appendix B

Determination of Fourier Coefficients

In this appendix the procedure to determine Fourier Coefficients is described. The model user for the BEM computations in SYSNOISE is based on a mesh of a half VSG train wheel in ANSYS format. The boundary conditions for the BEM computations are the displacements and velocities for each eigenmode computed in ANSYS. The data in the ANSYS result-file is in binary format, therefore SYSNOISE is used to translate these data to text format. At this point it was only possible to export the normal velocities on the nodes in a SYSNOISE data-file. This results in a small overestimation of the radiated sound as explained in chapter 3.

B.1 Master and slave nodes

Fourier Coefficients can be used to prescribe non-axisymmetric boundary conditions on an axisymmetric model. The VSG wheel is an axisymmetric model with non-axisymmetric boundary conditions. The three dimensional model can be reduced to a two dimensional model by defining master and slave nodes. The master nodes are chosen as the envelope of the nodes at \( y = 0 \) and \( x > 0 \). The resulting coordinates in the two dimensional model are the \( x \) and \( z \) directions.

The three dimensional model (with \( r \), \( \theta \) and \( z \) coordinates) contains the slave nodes. For each master node with coordinates \( x_i, z_i \), the slave nodes are selected that have the same radius \( r_j \) and height \( z_j \) within a given tolerance.

\[
\text{if } \text{abs}(r(j)-r_m(i))<\text{tol} \& \text{abs}(z(j)-z_m(i))<\text{tol}
\]

For each master node a column of slave nodes results. These columns are sorted on the coordinate \( \theta \). The permutations are stored in a matrix.

The boundary conditions are sorted with the permutation matrix to result in a matrix where each column corresponds with a master node and contains the normal velocities for increasing coordinate \( \theta \). Since the model of the VSG wheel contains only a half wheel the data are copied in reverse order and added to all columns to obtain columns that correspond with the contour of a complete circle. To make sure the signal is exactly periodic there is only one value for \( \theta = \pi \) and the point \( \theta = 2\pi \) is omitted (equals \( \theta = 0 \)).
B.2 Fourier Transform

The Fourier Coefficients $c_k$ and $s_k$ describe the (complex) amplitude of the (complex) cosine and sine with $k$ periods in circumferential direction according to equation (B.1).

$$V(\theta) = \sum_{k=0}^{N} c_k \cos(k \theta) + s_k \sin(k \theta) \quad (B.1)$$

These coefficients can be determined from the columns with boundary conditions with a Fast Fourier Transform algorithm. The four columns that result are $S_{re}$, $S_{im}$, $C_{re}$ and $C_{im}$ containing the real and imaginary parts of the sine and cosine terms of equation (B.1).

These Fourier Coefficients are written to a file that is read in a user-routine in BARD. In the user-routine the user can specify which modes have to be used in the computations. For the first 10 modes of the VSG wheel the maximum number of periods in circumferential direction that is observed is five. Therefore only modes 0 to 5 have to be used in the calculations. The coefficients for higher modes should be approximately zero as can be seen in figure B.1.

Figure B.1: Imaginary Cosine coefficients for mode 9. Only the first four coefficients are significant. Mode 9 is an axial tire mode with 3 nodal diameters.