Development of methods for the numerical error correction of machine tools

Confidential final project report

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Abstract

Inaccuracies in the use of machine tools are mainly caused by errors of form geometry, changes of the geometry due to finite stiffness and, most important, changes of the geometry due to thermal effects. The aim of this project is the on-line improvement of the accuracies of machine tools by the adaptation and extension of the correction methods, used in the field of coordinate measuring machines. Although establishing general principles, the project will mainly deal with a five axis milling machine for which an error model will be developed and its effectiveness demonstrated.

A general model has been developed, capable of describing the geometric error structure of any machine, composed of rotary and linear elements in an arbitrary serial configuration. This general model has been elaborated to an individual model, which describes the error structure of the investigated five-axis milling machine for both the geometry form errors and the finite stiffness errors.

To describe the thermo-mechanical errors of the machine tool, both an analytical and an empirical approach have been investigated. The empirically determined model directly relates temperature variations to deformations of the machine's structure using only 16 temperature sensors. The analytical model describes the deformation as a functional dependence of its temperature, using large finite elements. Although more sensors are needed, this model has only to be determined once for a machine type.

Besides direct measurement techniques, also a new indirect technique has been applied to calibrate the machine tool. Therefore, a totally new test workpiece has been developed. With this workpiece both the geometric and thermally induced errors can be determined.

All developed models are implemented in the controller of the milling machine. This has resulted in a software error compensation, which can operate in a real-time environment and corrects the errors during normal operation. Validation experiments showed that the errors of form geometry and due to finite stiffness effects can be reduced with 81%. The most important thermo-mechanical error, the zero point drift, has been reduced with 71% by the empirical model and 67% by the analytical model. Validations with the developed test-workpieces showed an overall accuracy improvement of 80%.
Acknowledgement

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1. Introduction

1.1 General introduction

For many years now, machine tools play an important role in the manufacturing of complex products. One can observe a tendency towards higher accuracy at all levels of production. As the driving forces behind higher accuracy [1] could be mentioned the requirement for:

- better performance and reliability of the products;
- miniaturization and integration of product-parts for weight and space saving purposes;
- automatic assembly;
- active noise reduction of accurate parts in gearboxes etc;
- response to increased accuracy in other fields, for example electronics.

Forced by these requirements, the performance of machine tools is continuously being improved. The classical way to increase the performance of a machine tool is to enhance the behaviour of the mechanical structure. In terms of geometric behaviour this implies the aim at faultless movements of the carriages, exact squareness between the guides and no finite stiffness effects of the elements. For the improvement of the thermal behaviour various experiments with thermal control [2], thermal invariant structures [3] and compensating heat sources [4, 5, 6] are carried out. However these methods of improvement are costly and the physical limits will soon be achieved. For this reason, new techniques to improve the overall behaviour of machine tools, are being developed. With the aid of computer technology it is possible to compensate for the errors existing in machine tools instead of avoiding them. However, this error correction method requires a very thorough investigation of the machine tool's behaviour and the factors influencing it. This is the fundamental reason for the initiation of a research project, supported by BCR, to develop methods for the numerical error correction of machine tools.
1.2 Description of the project

When the basic function of a machine tool is reviewed, it can be expressed as the transformation of rough material into a usable product. If the product has specified dimensions and tolerances, the function of the machine tool is to generate the product with accompanying specifications within a given time and at acceptable costs. A number of influences acting on this transformation are responsible for exceeding the tolerances of the final product. An overview of the main influences that may disturb the final product accuracy, is depicted in figure 1.1 [7].

Fig. 1.1 Sources of errors in machine tools.
In this BCR-project we have chosen to concentrate on the most influential error sources. Together these errors contribute to more than seventy percent of the resulting error of a machine tool [8, 9]. These error sources can be described as:

- **Geometric errors due to imperfect movements of the carriages.**
  In the production of the guides the manufacturer tries to avoid unwanted motions of the carriages, for example rotations. But even with the present production techniques the limits of achievable accuracy are restricted and therefore the carriage will show certain deviations from the perfect behaviour. These deviations will manifest themselves as position and orientation errors of the tool with respect to the workpiece.

- **Geometric errors due to finite stiffness of the elements of the machine.**
  One should make a distinction between the static and dynamic stiffness of a machine tool. The static stiffness of the machine is important when the machine is loaded with heavy workpieces, when heavy machine parts are moving while machining or when large cutting forces are to be expected. In this case the bending of machine parts will result in a deviation in the position of the tool with respect to the workpiece, and as a result this will introduce an error in the dimensions of the product. The dynamic stiffness contributes merely to the surface roughness of the product and less to the dimensional properties. Since these influences are an order of magnitude smaller, it will not be taken into account.

- **Thermal behaviour of the machine's structure.**
  Under the influence of several internal and external heat sources a machine tool will demonstrate a certain thermal behaviour with, due to expansion of the individual elements of the machine, an accompanying distortion. The resulting errors in the position and orientation of the tool are at least of the same order of magnitude as the errors caused by geometric errors [10].
The main goal of this project is the improvement of the accuracy of a commercially available machine tool by the implementation of an appropriate correction algorithm into the control system. The error correction should comprise the most significant geometric errors, the finite stiffness errors and also reduce the effect of the thermal behaviour of the machine's structure. This goal requires a systematic investigation of the main error sources in multi axis machine tools. Therefore the available experience in research on accuracy of coordinate measuring machines has been adapted and extended to machine tools.

In order to achieve the aims of the project the following contractors participated in the project:

- Eindhoven University of Technology, Metrology Laboratory (TUE);
- Physikalisch Technische Bundesanstalt (PTB);
- Philips Industrial Electronics, Machine Tool Controls (Philips);
- Maho Aktiengesellschaft (Maho).

Together these partners have developed methods to model and describe the influence of a certain error source on the accuracy of a machine tool. Based on this research a correction algorithm has been developed. This error correction has been implemented on a five axis milling machine, supplied by Maho, to validate the developed methods.

In order to achieve the aims of the project, three workpackages have been defined:

- First, a geometric error model had to be developed and validated, which is also capable to describe the finite stiffness errors;
- Secondly, methods had to be developed for the description and analysis of the thermally induced errors;
- Finally, an error correction had to be developed and implemented in a five axis milling machine to correct for the geometric, finite stiffness and thermo-mechanical errors.
1.3 Final project report

The project has been initiated on November, 1st 1989 and was finished on December, 31st 1992. In this final report, the project results will be discussed.

A general survey of the project will be given in chapter 2. Besides the time-schedule, also the general project agreements are discussed in this chapter.

In Chapter 3, the geometric error model and its validation will be discussed extensively. The mathematical model will be presented, capable of determining the resulting error of the tool with respect to the workpiece, as a function of the individual geometric errors.

Two different methods to determine the geometric error components are investigated. First the error components are measured directly, using laserinterferometer, levelmeters and displacement sensors. Secondly, special test-workpieces have been designed to determine the geometric errors. Both techniques will be presented.

Based on the direct measurements, the geometric error model will be used to simulate holeplate measurements. In order to validate the model, this simulation will be compared with the actual measurements of a holeplate.

In Chapter 4, the developed thermo-mechanical error models are discussed. First, the empirical method will be presented. This approach implies the simultaneous measurement of the actual thermal distribution of the machine's structure together with the resulting relative displacement of the tool. By application of statistical analysis techniques, a relationship is determined that describes the displacement of the tool in relation with the temperature distribution on the machine tool. This empirical model has been validated by comparison of simulations with verification measurements.

Besides the empirical method an analytical method has been investigated. Therefore, the machine structure has been divided into large segments, comparable to finite element methods (FEM). The deformation of each segment is modelled by changes of length and bending of a body with a stationary temperature field and a homogeneous expansion coefficient. The total deformation is the sum of the deformations of the modelled segments.
With the developed test-workpieces the thermo-mechanical errors of the machine tool were determined. Based on these experiments, a proper comparison of both thermo-mechanical models could be carried out. The result of this comparison will be discussed including a verification with the actual determined drift.

An error correction has been developed and implemented in software of the controller of the milling machine. Using this correction system, the modelled errors are compensated on every position of the machine. The developed software correction system will be discussed in Chapter 5. The effectiveness of the compensation and thereby the modelling methodology has been verified. The geometric and finite stiffness error correction has been verified with the holeplate method. With the developed test-workpieces the effectiveness of the entire error correction has been verified. The results of this validation are presented in Chapter 5.

Finally, in the last chapter, conclusions will be drawn with respect to the developed methods and the software correction system.
2. General project description

2.1 Introduction
The main task of this BCR-project was the improvement of the accuracy of a commercially available machine tool. Initially, the project goal was to achieve an accuracy enhancement by 70% with real-time software error correction. Therefore, the most influential error sources will be investigated: the basic geometric errors, thermal behaviour of the machine's structure and finite stiffness effects. Based on this research a correction algorithm will be developed and implemented. The effectiveness of this correction algorithm has been validated on the milling machine, supplied by Maho for this research.

In order to achieve the aims of the project the following tasks are carried out by the partners involved in the project:

- **Classification of multi-axis machine tools**
  A survey of the different types of five axis CNC milling machines will be made in order to establish the origins and characteristics of their related errors.

- **Bibliographical studies**
  A bibliographical study will be made to establish the characteristics of currently available error models, methods of temperature correction, related measurements and methods of parameter extraction from measurements of workpieces.

- **Agreements and standardizations for the exchange of data**
  In order to make the exchange of information possible a standard data format will be defined, measurement programs and interfaces will be developed and an error model will be chosen.

- **Geometric error modelling including finite stiffness**
  A general error model capable of modelling the geometric error structure of an arbitrary five-axis milling machine will be developed. This model will be evaluated by determining all geometric error components with direct measurements or by using artifacts. Simulations will be carried out to verify this error model. Also the finite stiffness effects of the machine tool caused by moving parts, cutting force and the weight of workpieces will be considered.
The development of thermomechanical models
In order to investigate the thermomechanical behaviour of a machine tool, both a statistical and an analytical approach will be investigated. For both approaches measurement techniques will be developed to determine the thermally induced errors. Based on measurements, both methods will be evaluated, verified and compared. The possibility of a combination of both models will be investigated.

Real-time software error correction
A concept for a real-time error correction will be developed. This concept will be implemented in the hard- and software of the controller of the milling machine. The concept will be verified by milling workpieces with and without error correction. These workpieces will be evaluated to demonstrate the improved accuracy obtained with the real-time correction. Finally the uncertainties associated with the error correction method will be evaluated.

Not all partners have contributed the same amount of work to a certain workpackage, especially while the experience of the partners differs. Both PTB and TUE have experience in the research on the accuracy of coordinate measuring machines and machine tools. Philips has knowledge in the field of controlling machine tools. Maho has experience in the field of machine tools and related errors. Therefore a distinction had to be made between the four partners, concerning the research to be carried out. A complete summary of these workpackages is given in table 2.1. Within this table, also a time-schedule is given.

Two milestone reports have already been delivered. In the first milestone report, the geometric error model has been discussed extensively. The main topic of the second milestone report was the development of thermo-mechanical models. In this final report a complete overview will be presented of all workpackages, completed in the project.
Table 2.1 Workplan of the project, including a time-schedule.

### 2.2 Classification

During the first project-meeting, all partners agreed that a classification should be developed to simplify the identification of significant errors. Therefore a separation is made between machine types with the same kind of errors (type dependant errors).

A first approach, made by Maho and Philips [11], identified the most important type dependant geometric and thermally induced errors of machine tools. Using this approach and some relevant literature [12] TUE made a proposal, which has been accepted by all partners as the final version.

In this classification [13] a distinction is made between the basic machine and the additional rotary axes. Because rotary axes are available as an option, the three linear axes constitute the basic machine.
In all cases the rotary axes will be added to this basic machine and therefore they will not interfere with the kinematic chain of linear axes. Also the spindle of a milling machine can be defined as a rotary axis. However, the spindle will not be included in this classification, since the errors introduced by the spindle are not machine type dependent.

**Basic machine**

The basic machine has three perpendicular linear axes. These linear axes represent the kinematic chain of the basic machine. One end of this kinematic chain supports the tool, at the other end the workpiece can be positioned. The base of the machine is situated within this kinematic chain. As measurements mostly are referred to the base of a machine, a distinction between two chains can be made:

- **A** - chain: this chain supports the tool;
- **B** - chain: this chain supports the workpiece.

In figure 2.1 an example of a milling machine with its kinematic chain representation is depicted.

*Fig. 2.1 Example of A- and B-chain in a knee-type milling machine.*
<table>
<thead>
<tr>
<th>Number of axes in:</th>
<th>Classification</th>
<th>Some of the class dependent errors during traverse of the axis without process forces</th>
<th>Examples of machine types</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-Chain</td>
<td>B-Chain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1 | 2 | Class IIA: B-Chain: V → H  
    A-Chain: H | Table guide  
    Ram | Bending of:  
    = F(Table, Weight/Load)  
    = F(Ram) | Knee-type milling machine |
| | | Class IB: B-Chain: H → H  
    A-Chain: V | Table guide | Bending of:  
    = F(Table, Weight/Load) | Fixed-bridge milling machine |
| 2 | 1 | Class IIA: B-Chain: H  
    A-Chain: V → H | Table guide  
    Column  
    Ram | Bending of:  
    = F(Table, Weight/Load)  
    = F(Ram)  
    = F(Ram) | Fixed-column milling machine |
| | | Class IIB: B-Chain: H  
    A-Chain: H → V | Table guide  
    Column/bridge | Bending of:  
    = F(Table, Weight/Load)  
    = F(Ram guide) | Fixed-bridge or travelling column machine |
| 3 | 0 | Class IIIA: A-Chain: H → H → V | Bridge guide  
    Bridge  
    Ram | Bending of:  
    = F(Bridge)  
    = F(Ram guide) | Travelling bridge milling machine |
| | | Class IIIB: A-Chain: H → V → H | Column guide  
    Column  
    Ram | Bending of:  
    = F(Column)  
    = F(Ram)  
    = F(Ram) | Travelling column milling machine |

→: Sequence in kinematic chain  
H: Horizontal axis of motion  
V: Vertical axis of motion  
F(E): Function of position of element  

Table 2.2 Classification of basic machine types.
As a machine consists of three axes, four classes can be discriminated with a different number of axes present in a particular chain. Within these classes two groups are distinguished, where the construction of the machine causes another type dependent error structure. In table 2.2 the resulting classification is presented.

**Rotary axes**

Apart from three linear axes, a five axis milling machine consists of two rotary axes with perpendicular axes of rotation. Taking two rotary axes it is possible to distinguish three classes with a different number of rotary axes present in a chain. In the next table 2.3 this classification is shown.

<table>
<thead>
<tr>
<th>Number of rotary elements in:</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-Chain 0 B-Chain 2</td>
<td>Class 1</td>
</tr>
<tr>
<td></td>
<td>Workpiece table can rotate about vertical axis and swivel about horizontal axis</td>
</tr>
<tr>
<td>A-Chain 1 B-Chain 1</td>
<td>Class 2</td>
</tr>
<tr>
<td></td>
<td>Workpiece table can rotate about vertical/horizontal axis and tool holder can swivel about horizontal axis</td>
</tr>
<tr>
<td>A-Chain 2 B-Chain 0</td>
<td>Class 3</td>
</tr>
<tr>
<td></td>
<td>Tool holder can swivel about both horizontal and vertical axis</td>
</tr>
</tbody>
</table>

*Table 2.3 Classification of the additional rotary tables*

With the described classification a distinction can be made between different types of five axis milling machines that manifest the same kind of errors (i.e. type dependent errors). With the knowledge of the type dependent errors the definition of a calibration setup and the error structure of the machine are significantly simplified. As the classification distinguishes between different number of axes present in the A- and B-chain, it becomes less difficult to model the geometric error structure of the milling machine by using this classification.
2.3 Bibliographical studies
A bibliographical study has been carried out to establish the characteristics of currently available error models, methods of temperature correction, related measurements and methods of parameter extraction from measurements of workpieces. In order to simplify the traceability of a certain article, TUE developed a database-system, capable of containing all relevant literature [14].

The literature study gives a good indication on the state of the research activities, carried out at other institutes. As for the research topics of this BCR-project, the results of the literature study will be presented in the paragraphs, dealing with a corresponding subject.

2.4 Agreements and standardizations for the exchange of data
During the first year of the project PTB extended the VDA-data format, as defined in DIN 66301 [15], for the exchange of temperature and geometric data. PTB supplied all partners with a program (written in Turbo Pascal) and a manual [16] to support this data-format.

Before one can develop error models, a definition for the errors has to be chosen first. In this paragraph the chosen definition will be presented.

*Geometric error definition*
In general machine tools, in particular milling machines, possess three perpendicular linear axes of movement. In addition to these three axes, rotary axes can be mounted on the machine tool. The three linear axes form a cartesian coordinate system which allows the tool to be positioned at any place within the range of the axes. Normally the linear movements along an axis of this system are performed by a carriage-guide system. In the ideal situation the spatial position of the tool can be determined by the positions of the carriages, presented by the regarding measuring systems. However, due to the imperfect geometrical shape of the guides of a machine tool, the carriages will display erroneous movements. These erroneous movements will result in an error of the location of the tool with respect to the workpiece. In order to classify these erroneous movements we first present some kinematic principles.
Basically a body possesses six degrees of freedom that determine its location (i.e. position and orientation) in space [17, 18]. These degrees of freedom are built up out of three translations and three rotations. Consequently a body can reveal six sources of error which result in another position and orientation of the body than expected. As a carriage of a machine tool is basically a body in space with five degrees of freedom suppressed, this theory does also apply to these elements. Application of this theory to a linear carriage-guide system implicates that, due to imperfections in the shape of the guide, the carriage will display straightness errors, rotations about all three axes and an error in the position along the guide. In figure 2.2 a linear carriage-guide system is depicted with its possible erroneous movements.

![Carriage-guide system with possible geometric errors.](image)

In order to avoid wrong interpretation of each geometric error a definition is required of the nomenclature of the erroneous movements. In principle any definition suits the purpose but for uniformity reasons all partners agreed to use the German standard, lied down in VDI 2617, Blatt 3 [19]. This standard uses three characters to identify the individual geometric errors. Hereby the first, lower-case, character represents the axis of movement, for instance "x" for the guide in figure 2.2. The second, lower-case, character represents the type of geometric error i.e. translation "t", rotation "r". The last, lower-case, character represents the axis along which, or rotation about which, the geometric error is acting. For example, if this notation is applied to the rotation of a carriage of the X-guide about the Y-axis, the geometric error source is denoted as "xry".
It must be noted that the geometric error of the carriage in the direction of movement (xtx, yty or ztz) does not find its cause in the guide but in the measuring system attached to the guide. Thereby, in principle, it is not a geometric error. However, it will be treated as an error in the geometry for simplicity reasons.

For the notation of the squareness errors, the reference axis is represented by the first character. The second character represents the type of error: "w" for squareness errors. The last character represents the other axis. For example, the squareness error between the X- and the Z-axis is denoted as "xwz".

Also rotary elements are liable to erroneous movements. Similar to linear axes a definition of the individual errors is necessary. In figure 2.3 a rotary element is depicted with its geometric errors. This definition is again accordingly to the German standard VDI 2617, Blatt 3. The identification of rotary axes of machine tools is defined in DIN 66217 and ISO 841 [20, 21]. According to this definition the rotary axis around the X-axis is denoted as the A-axis, around the Y-axis as the B-axis and around the Z-axis as the C-axis. In conjunction with the above presented definition of geometric errors for linear axes, the geometric errors of rotary axes are also denoted by three characters. Hereby the first, lower case, character represents the rotary axis of movement ("a", "b" or "c"), the second, lower-case, character represents the nature of the error (i.e. "t" or "r"). The last, lower-case, character represents the axis along which, or rotation about which, the geometric error is acting (i.e. "x", "y" or "z"). As an example, the scale error of a rotary element, which provides the rotation around the Z-axis, is denoted as "crz".

Fig. 2.3 Rotary axis with its geometric errors.
As the coordinate frame rotates along with the rotary element, additional definitions of the directions of the X-, Y- and Z-axis are necessary. In this milestone report is chosen to let the direction of the X-axis of the coordinate frame attached to the rotary element, coincide with the X-direction of the machine coordinate frame when the rotary element is at its zero position. If the rotary element starts its movement, the coordinate frame XYZ rotates along with the rotary element. This definition is necessary to avoid problems in the definition of direction if two serial rotary elements are applied.

**Definition for the thermo-mechanical errors**

The thermally induced errors are referred to the reference situation of the machine tool. As the investigated machine tool is placed in a temperature stabilized room with a temperature of 23 ± 0.5 °C, this temperature is accepted as the reference temperature for all measurements. This reference situation is characterized by a complex temperature situation (thermal reference) and by a geometric error situation (geometric reference). The geometric reference situation of the machine tool is specified by the presented set of error parameters. As the error sources of the thermally induced deformations can be coordinated to the error components of the geometric error model, the presented error definition can be applied.

The model for the zero point drift takes a key position within the thermo-mechanical model, because it is a global description of all relevant machine tool deformations between the reference point of the workpiece and the reference point of the tool. Therefore, the zero point drift generally is the major error component of the thermo-mechanical behaviour. In conjunction with the above presented definition of geometric errors, the zero point drift is also denoted by three characters. Hereby the first, lower case, character is always a "n", The second, lower-case, character represents the nature of the error (i.e. "t" or "r"). The last, lower-case, character represents the axis along which, or rotation about which, the error is acting (i.e. "x", "y" or "z"). As an example, the zero point drift in X-direction, is denoted as "ntx".
3. Geometric error model

3.1 Introduction
One of the major influences that determine the accuracy of multi axis machines are systematic errors in the geometry of the carriages i.e. geometric errors. The main problem in this field is to determine a relationship that describes the systematic error in the location (i.e. position and orientation) of the tool, in dependence of the position of the machine's carriages. Many studies have assessed the problem of describing this relationship [22 - 28]. The applied methods range from correlation models, to trigonometric analysis, to 'error matrix' representations. However, in recent reports a tendency towards the use of rigid body kinematics can be observed. This method yields, in case of a three axis machine, a description of the location error of the tool as a linear combination of 21 measurable geometric errors.

In the next paragraph a general methodology will be described for the construction of a model, which relates the various geometric errors to the location error of the tool. This methodology, developed by TUE [29, 30], will be applied to the investigated five axis milling machine. As a final step in the development of the individual model, the relation between the geometric errors and the position of the carriages has to be obtained. This requires an extensive performance evaluation of the machine tool. In paragraph 3.2, the more or less conventional method is presented, to measure the error components using laserinterferometer, levelmeters and displacement transducers. In paragraph 3.4, the determination of the geometric errors with test workpieces will be explained.

Before the model can be accepted as a tool for software error correction, this model has to be verified. Therefore, holeplate measurements will be simulated, using the individual model, and verified with the actual holeplate measurements. This verification will be discussed in paragraph 3.3. Based on this comparison, conclusions will be drawn with respect to the developed model.
3.2 The modelling system
In the modelling of the error structure of multi axis machine tools, several levels can be specified. At the top of the modelling system stands the general model [31]. This general model relates errors in the location of the tool, with respect to the workpiece, to errors in the location of coordinate frames attached to successive components of the machine (i.e. the geometric errors).

Elaboration of the general model for a machine tool type yields the type dependent model. This model contains the common properties of the error structures belonging to machine tools of the same type. Thereby the type dependent model must be placed below the general. In the optimal situation the type dependent model can be also used to store type dependent errors, for example finite stiffness effects and thermal behaviour. However, as the division into type dependent and individual errors is a complex problem, the practical significance of this error classification is still limited. The result of the modelling methodology is the so-called individual model. This model describes the error structure of an individual machine tool at a certain time and place. With this model the machine's accuracy can be unambiguous assessed and improved by software error correction.

The general model
The general model can be applied to multi axis machine tools, composed of rotary and linear elements in an arbitrary serial configuration. It relates errors between the actual and nominal location of the tool (with respect to the workpiece), to errors in the location of coordinate frames attached to successive components of the machine. Such errors describe the difference between the nominal and actual geometry of machine parts enclosed by two frames. The number and position of the coordinate frames is chosen such that there is one kinematic element, i.e. carriage-guide system, between each two frames. This choice is adequate for application of this methodology to machine tools, coordinate measuring machine and robots.

Starting with the global coordinate system 0 attached to the machine tool's foundation, the orthogonal frames are successively numbered. As depicted in figure 3.1, a prefix is added to this number. This identifies the corresponding frame as being part of kinematic chain 'a' from foundation to tool, or chain 'b' from foundation to workpiece. This differentiation of the kinematic chain into two separate chains is made for convenient assessment of the geometric errors [32, 33]. Two additional frames 'wp' and 'tl' are introduced, which are attached to workpiece and tool respectively.
An actual machine tool possesses errors in the relative location of subsequent frames, as well as in the location of the tool with respect to the last frame 'an' of the kinematic chain. Because none of the contemporary multi axis machines show an absence of Abbe offsets, the relevant errors in the relative location of two subsequent frames are not limited to those in the moving direction of the enclosed kinematic element (i.e. scale errors).

Consequently all possible errors of a guide as defined in paragraph 2.4 have to be taken into account. For the location of frame k with respect to frame k-1 this implies:

- translational errors $\mathbf{e}_{kx}, \mathbf{e}_{ky}, \mathbf{e}_{kz}$ along the x, y and z axes of frame k respectively.
- angular errors $\mathbf{e}_{kx}, \mathbf{e}_{ky}, \mathbf{e}_{kz}$ about the x, y and z axes of frame k respectively.

In figure 3.2 an example of a two-dimensional carriage-guide system is depicted with the two coordinate frames and the related errors.
The nomenclature of the individual errors in the general model differs from the definition as presented in paragraph 2.4. The reason for this departure are the lengthy formulas that arise in the process of developing a general description of an error structure. With the above presented notation, addition and multiplication of a variable number of terms becomes relatively easy to summarize. However, once the general model is worked out for a particular machine tool, the individual geometric errors will be nominated accordingly to the definitions of paragraph 2.4.

In the analysis of the effect of angular errors on the machine tool's accuracy, linearisation is applied, i.e. \( \cos(e) = 1 \) and \( \sin(e) = e \). Since the absolute values of these errors are relatively small for the target group of machine tools, this approximation is valid. Application of this approximation yields additive and commutative properties for the various errors. Also higher order effects will be ignored. This approximation is valid, since the difference between the actual and nominal machine structure usually does not significantly change the active arm of angular errors and the direction in which the various errors act.

In order to obtain a convenient description, which also provides intuitive insight in the basic error relationships, the angular and translational errors between tool and workpiece, can be summarized in the \( 6 \times 1 \) vector \( \mathbf{E}_t \). This vector can be expressed as similarly denoted errors in the relative location between successive frames:

\[ \begin{align*}
\mathbf{E}_t & = \begin{bmatrix}
\mathbf{r}_{x_k} \\
\mathbf{r}_{y_k} \\
k-1 \mathbf{e}_{kx} \\
k-1 \mathbf{e}_{ky} \\
k-1 \mathbf{e}_{kz}
\end{bmatrix} \\
& = \begin{bmatrix}
\mathbf{r}_{x_{k-1}} \\
\mathbf{r}_{y_{k-1}} \\
k-1 \mathbf{e}_{kx} \\
k-1 \mathbf{e}_{ky} \\
k-1 \mathbf{e}_{kz}
\end{bmatrix} + \begin{bmatrix}
\mathbf{e}_{kx} \\
\mathbf{e}_{ky} \\
\mathbf{e}_{kz}
\end{bmatrix}
\end{align*} \]
\[
\begin{align*}
wpE_{tl} &= \sum_{k=1}^{m} (tlF_{bk} E_{bk} k) + \sum_{k=1}^{n} (tlF_{ak} E_{ak} k) + anE_{tl} \\
\text{where: } wpE_{tl} &= [wp\epsilon_{tx}' wp\epsilon_{ty}' wp\epsilon_{tz}' wp\epsilon_{tx}' wp\epsilon_{ty}' wp\epsilon_{tz}']^T \\
\text{This vector represents the errors of the tool with respect to the workpiece, defined in the tool-frame.} \\
k-1E_k &= [k-1\epsilon_{kx}' k-1\epsilon_{ky}' k-1\epsilon_{kz}' k-1\epsilon_{kx}' k-1\epsilon_{ky}' k-1\epsilon_{kz}']^T \\
\text{This vector represents the errors in the location of frame } k \text{ with respect to frame } k-1, \text{i.e. the geometric errors of the kinematic element } k. \text{ The errors are estimated with measurement results, using piecewise polynomials.} \\
ulF_k &= \begin{bmatrix} t\text{lR}_k & 0 \\ (t\text{l}^k \times tlR_k) & tlR_k \end{bmatrix}, \text{ (6 x 6) matrix} \\
\text{This matrix, the so-called F-matrix, denotes the effect of the errors } k-1E_k, \text{ acting between the elements } k-1 \text{ and } k, \text{ on the resulting error between tool and workpiece.} \\
\text{Here } tl^k \times tlR_k \text{ denotes a 3 x 3 matrix whose columns contain the vector cross product of vector } tl^k \text{ with the respective columns of matrix } tlR_k. \\
\text{Note that the errors } wpE_{tl} \text{ are defined in the nominal tool coordinate system. For correction purposes it is necessary to transform these errors to the machine coordinate system. This can be implemented in relation [3.1], by either premultiplying each of the 3 x 3 sub-matrices of } ulF_k \text{ with the appropriate orientation transformation } wpR_{ul}, \text{ or by back transformation of the resulting error to the direction of the machine's axes.}
\end{align*}
\]
Elaboration of the general model to the type dependent model

The methodology described will be applied to the investigated five axis milling machine (figure 3.3).

Fig. 3.3 Maho 700S five axis milling machine

Fig. 3.4 Kinematic representation of the five axes milling machine
This machine tool consists of one horizontal linear element and one rotary element in chain 'a' from foundation to tool. Chain 'b' from foundation to workpiece consists of two linear elements, one vertical and one horizontal, and one rotary element with a vertical axis of rotation. In the first stage of the modelling process, coordinate frames are located in the workpiece, the tool and in the centroid of each joint. In figure 3.4 the kinematic representation of this milling machine is depicted. Note that the length of the tool is implemented in the model by a variable named 'L'.

Application of relation [3.1] and abbreviation of 'cos(q)' and 'sin(q)' to 'cq' and 'sq' respectively, yields the following expression for the errors \( \text{wp} \) and \( \text{wp} \) in the orientation and position of the tool coordinate frame with respect to the workpiece coordinate frame:

**Orientation errors**

\[
\text{wp}^a \text{tl} = a^2 \text{tl} + \begin{bmatrix}
cqa2 & sqa2 & 0 \\
-sqa2 & cqa2 & 0 \\
0 & 0 & 1
\end{bmatrix} a^1 \text{tl} + \begin{bmatrix}
cqa2 & sqa2 & 0 \\
-sqa2 & cqa2 & 0 \\
0 & 0 & 1
\end{bmatrix} a^0 \text{tl} - \begin{bmatrix}
cqa2 & sqa2 & 0 \\
-sqa2 & cqa2 & 0 \\
0 & 0 & 1
\end{bmatrix} b^1 \text{tl}
\]

\[
= \begin{bmatrix}
cqa2 & sqa2 & 0 \\
-sqa2 & cqa2 & 0 \\
0 & 0 & 1
\end{bmatrix} b^1 \text{tl} - \begin{bmatrix}
cqa2 & sqa2 & 0 \\
-sqa2 & cqa2 & 0 \\
0 & 0 & 1
\end{bmatrix} b^2 \text{b3}
\]

\[3.5\]

**Position errors**

\[
\text{wp}^a \text{tl} = a^2 \text{tl} + \begin{bmatrix}
140. & sqa2 & -140. & cqa2 & -(200+L) & cqa2 & sqa2 & 0 \\
140. & cqa2 & 140. & sqa2 & 0 & -sqa2 & cqa2 & 0 \\
(200+L) & cqa2 & (200+L) & sqa2 & 0 & 0 & 0 & 1
\end{bmatrix} a^1 \text{tl} + \begin{bmatrix}
(805-qa1) & sqa2 & -(805-qa1) & cqa2 \\
(805-qa1) & cqa2 & (805-qa1) & sqa2 \\
(-210sqa2) & -(210cqa2-(200+L)) & cqa2 & (210sqa2)cqa2-(210cqa2-(200+L)) & sqa2 \\
210cqa2-(200+L) & cqa2 & sqa2 & 0 \\
-210sqa2 & -sqa2 & cqa2 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} a^0 \text{tl}
\]

\[3.6\]
3.3 Determination of the errors

In order to complete the individual model, which describes the error structure of an individual machine tool at a certain time and place, the error components of the machine tool have to be identified. These error components will be measured directly, using instruments as laser interferometer, levelmeters and displacement sensors. With these measurements not only the geometric errors will be measured, but also the change in geometric errors due to finite stiffness effects. The measurement setups and the obtained results will be discussed in this section.

One of the problems in the assessment of the geometric errors is the physical impossibility to measure in the center of the elements, as accordingly to the definition of the position of the coordinate frames is required. This implies for the obtained translational errors that a correction to the center of the elements is necessary. The correction value is determined by the position of the measurement i.e. the influence of rotational errors on the measured displacement. In figure 3.5 a two-dimensional example is depicted for the measurement of $x_{tx}$. In this case, the bare measurement data have to be corrected with the influence of the rotation $\varepsilon$ (i.e. $\varepsilon.D$) in order to obtain the true error $x_{tx}$, defined in the center of the carriage.

![Frame of measurement](image)

**Fig. 3.5 Example of a measurement of $x_{tx}$ and the effect of rotations**
Software for automation of the measurements
As the number of measurements, required for the purpose defined in this project, is very large, dedicated software is developed. This software has basically two main tasks:

1) Control of the position of the machine tool;
2) Collecting and storing of measurement results.

In order to perform task 1, an interface has been accomplished with the milling machine. This interface uses the RS232 port of an IBM-compatible computer for data transport. The communication with the milling machine runs under a specified protocol which is developed by Philips. With this interface the computer can control almost any feature of the machine tool. This allows convenient programming of, for instance, the positioning of the axis for measurement purposes.

Fig. 3.6 Scheme with measurement instruments and interfaces
The second task of the program is to gather the results from the installed measurement devices. As most modern instruments are available with an IEEE-488 interface, the communication and installation facilities have been developed for all necessary measurement devices as a laser interferometer, electronic level meters, temperature measurement equipment and inductive displacement transducers. With this interface facility, the computer aided setup and read out of these instruments becomes relatively simple. Besides the IEEE interfacing, the program is developed to have communication facilities through RS232 and keyboard input to accommodate any measurement situation that might occur. In figure 3.6 a schematic overview is presented of the applied interfaces and the used measurement instruments. All the underlying described measurements are carried out by application of this software package.

**Measurements of the linear axes**

With the aid of the above presented program all 21 geometric errors present in the system of the three linear axes have been determined. The measurement sequence of these measurements is defined over the entire range of the respective axis of movement with a measurement step of 10 mm. All measurements are carried out back and forth over the range of the axis. Underlying some measurement results are presented. An overview of the calibration procedure is presented in Appendix A.

All measurement equipment, used to determine the geometric error components, is calibrated at Metrology Laboratory of Eindhoven University, which is certified by the Dutch Calibration Organization (NKO-014) [34].

For the measurement of the rotation error $x_{rz}$ a laser interferometer is used. The measurement uncertainty associated with this instrument is less than 0.2 arcsec. The measurement setup is depicted in figure 3.7. The reference interferometer is mounted to the ram of the milling machine while the retroreflector is connected to the workpiece table, which performs the movement in X-direction.

The results of this measurement are presented in figure 3.8. These results directly reflect the rotation error between the coordinate frames attached to the X- and Y-axis respectively. This rotation error is not only composed of the geometric error of the X-guide, caused by the imperfect guide, but also of the finite stiffness effect of the guide. When the workpiece table is moving from $X=0$ to $X=700$ mm, the X-axis will bend (rotation around the Z-axis). This yields a rotation error, almost linear dependent on the position of the X-axis (figure 3.8).
A laser interferometer is also used to measure the scale errors of the milling machine. The measurement uncertainty of this instrument is less than $(0.05 + 0.5*L) \mu m$ $(L$ in $m)$. The measurement setup, to determine the scale error of the $Z$-axis ($ztz$), is schematically depicted in figure 3.9. The interferometer is mounted on the workpiece table, while the retroreflector is connected to the ram of the machine tool. The influences of the rotation $zrx$ and $zry$ have to be eliminated from the obtained measurement results.
This yields the error $\text{zTz}$ of the coordinate frame positioned in the centroid of the Z-carriage. Execution of the described measurement yields the bare, uncorrected results that are graphically depicted in the first part of figure 3.10.

Fig. 3.9 Measurement setup for zTz

Fig. 3.10 Error zTz versus position of the Z-carriage
In the results of ztz a clear form of hysteresis can be observed. This is not caused by the hardware of the machine, but purely by a reproducing temperature field over the Z-scale. In figure 3.11 the temperature on three positions of the Z-scale is depicted. These temperatures were obtained during the measurement of ztz. The second graph in figure 3.10 represents the error ztz corrected for the effect of the changes in the temperature of the Z-scale. The hysteresis has disappeared clearly.

![Temperature of the Z-scale during measurement of ztz](image)

**Fig. 3.11 Temperature of the Z-scale during measurement of ztz**

Analysis of the cause of the thermal problem yields the conclusion that the hydraulic installation warms up the Z-scale by radiation. In order to avoid this problem it is therefore advisable to isolate the scale of the Z-axis.

**Measurements of the rotary axes**

The Maho milling machine contains not only three linear axes, but also two rotary axes. In order to complete the individual model, also the geometric errors induced by these two rotary tables have to be determined.
For the measurement of the linearity error of the B-axis, an optical polygon and an autocollimator is used. The polygon has a measurement uncertainty less than 1 arcsec. The autocollimator introduces a measurement uncertainty less than 0.5 arcsec. The measurement is carried out by positioning the B-axis with a step that equals the angle between two succeeding planes of the polygon. With the autocollimator the difference between the orientation of the two planes is measured. Using a polygon, which has 12 sides, together with the autocollimator, it is possible to measure the position error of the B-axis over the whole range (360°) with steps of 30°.

With the above presented measurement method it is possible to obtain the position error with steps as large as the angle between two successive planes of the polygon. In order to determine the position error between these steps another method is used. A very accurate rotary table is placed upon the B-axis of the milling machine. This rotary table introduces an uncertainty less than 0.5 arcsec. In the center of this table a plane mirror is placed. The measurement sequence is carried out by rotating the B-axis of the milling machine a randomly chosen angle. Using the rotary table, the plane mirror is rotated back to the zero position, which is measured by the autocollimator.

Based on the found, small position errors of rotation tables and the results from research carried out at other institutes [35] it can be concluded that rotation tables have a very high accuracy, compared to the other error sources in general and the geometric errors of the linear axes in particular. The results of the indirect measurement techniques, using a ballplate, confirm this conclusion (Appendix A.2.2).

**Measurements of the finite stiffness effects**

With the above mentioned methods, all error components can be determined, caused by imperfect movements of the guides. In addition to these errors a number of forces act on the machine's structure. These forces cause the structure to deform and consequently disturb the actual position of the tool. While machining a workpiece three basic types of forces [36] are present:

1) forces induced by the cutting process;
2) gravity forces acting on the machines components;
3) gravity forces acting on the workpiece (workpiece load).
Errors due to forces of type 1
The type 1 forces act directly between the tool and the workpiece, causing the tool holder to deflect and thereby introducing errors in the dimensions of the product.

In order to calculate the magnitude of deflections caused by cutting forces, first these cutting forces have to be estimated.

Allowing a roughness of the milled product with a maximum $R_1$-value of 10 $\mu$m, the geometry of the cutting tool yields a maximum allowable feed per tooth of the cutter of 0.2 mm. The cutting forces induced under this condition and practical process parameters, have a magnitude of 500 - 1000 N. Considering a machine tool stiffness of $10^8$ Nm$^{-1}$, a maximum deflection of 10 $\mu$m may be expected, which equals the $R_1$-value. To mill a product very accurate, not only the geometry of the product should be very accurate, but also the roughness of the milled surface should be very small. Taking into account that the stated $R_1$-value is a theoretical value, based on the tools geometry only (in practice the $R_1$-value will be larger), it can be concluded that a correction for the deflection will not significantly improve the accuracy of the milled product, which is deteriorated by the bad surface roughness.

Thus, taking into account the huge effort required to model the errors due to cutting forces and the relative low improvement of the workpiece's accuracy, the errors due to these forces will not be investigated any further during this project.

Therefore, in order to achieve a very accurate product, it is advisable to mill a workpiece in at least two steps (strongly advised by Maho):

1) Most of the material is removed during milling the workpiece with large depths of cut and a large feed;
2) To achieve the desired dimensions, the workpiece is finished by milling with small depths of cut and a small feed.

During the finishing process the static cutting force will be much smaller than the above made estimation. Combined with the high stiffness of the machine tool, the cutting force will not cause a significant deflection.
**Errors due to forces of type 2**

The type 2 forces are caused by movement of the large masses, e.g. of the workpiece table. As a result of the gravity forces acting on these masses, the geometric errors of one guide are dependent of the position of one or more other guides. An example of this effect is depicted in figure 3.12. If the machine type indicates a certain finite stiffness effect due to movement of masses, the measurement and model of the individual geometric errors should be organized to comprehend these effects.

![Fig. 3.12 Possible effect of the movement of a carriage on the geometric error of a guide due to finite stiffness.](image)

The finite stiffness error of a axis, dependent on the position of the axis itself, is actually included in the model. This error is determined during the geometry measurements of the milling machine, but the geometric error and the finite stiffness error are not separable. A separation, however, is not necessary as the resulting error can be modelled perfectly in the developed geometric error model. This concept is similar to the "hardware-correction" practiced by many machine tool manufacturers.

The finite stiffness error, as depicted in figure 3.12, can be determined separately as this error is dependent on the position of another axis. E.g. the error yrz is measured with the workpiece table on the positions X=0, X=350 and X=700.

The measurement results from similar experiments showed no significant change in the geometric errors due to these finite stiffness effect.

Although the presented model and estimation techniques can cope with such errors, it was not necessary to model the geometric errors as a function of the position of another axis.
Errors due to forces of type 3

The type 3 forces are dependent of the weight of the workpiece put on the machine tool. The effect of these forces is that the machine parts will bend and thereby cause a change in the geometric errors.

![Fig. 3.13 Measurement setup for xrx, xry and xrz](image)

![Fig. 3.14 Rotation errors xrx, xry and xrz versus position of the X-carriage](image)
In order to investigate this change in geometric errors a load is placed upon the table of the milling machine. During movement of the X-axis three rotations are measured (figure 3.13). The results of this measurement (figure 3.14) show a significant change in the geometric errors of the X-axis due to a workpiece load.

In order to determine if the geometric errors of the Y-axis are affected by a workpiece load, two geometric errors are measured on three different X-positions (see also point 2). It turned out that the error components of the Y-guide do not significantly change, due to different positions of the X-axis.

3.4 Simulation and validation of the error model
TUE developed a dedicated software package, which enables the calculation of the error vector on any position within the range of the modelled machine, using the individual model. This error vector is calculated with respect to the workpiece coordinate frame, taking into account the dimensions of the tool. In order to facilitate simulations, the error vector can be determined as a function of the position of all axes. Thus it is possible to evaluate the geometric performance of the modelled machine, within its range. Therefore, an indirect measuring method has been applied: the holeplate.

This holeplate is developed to calibrate coordinate measuring machines. In order to verify the real-time error correction, the holeplate will be applied to the Maho milling machine, by using the milling machine as a measuring machine.

The holeplate is a two-dimensional object. The artifact consists of a grid of holes, with an equal mutual distance. By measuring these holes, and knowing the deviation from the nominal mutual distance, it is possible to evaluate the performance of the error models, with respect to the geometric and finite stiffness errors.

The artifact is placed upon the machine in three different planes: XY-, XZ- and YZ-plane, so that the different locations of the holeplate constitute the sides of a cube. Normally, this setup is applied to determine all 21 geometric error components of a machine consisting of three linear axes [52]. Hence, when the holeplate is measured in these three planes, all modelled errors are verified.
To simulate the measurements of the holeplate, the difference between the error vector with respect to the reference hole and the evaluated hole is determined. The program can calculate this error vector for each hole, as it would be measured by the milling machine. These simulations enable direct comparison with the actual measurements carried out on the Maho milling machine under research. But, before comparing the results, the actual measurements have to be corrected for the expansion of the scales and the expansion of the holeplate. Also the alignment error of the holeplate along the axes of the milling machine has to be corrected. After these corrections, direct comparison with the simulations is possible. This comparison has been carried out for all holeplate measurements.

In figure 3.15 both the corrected measurement results and the simulations are depicted, for the holeplate measurement the XY-plane.

Fig. 3.15 Result of the holeplate measurement, together with the residual between measured result and simulation
Before drawing conclusions with respect to the made comparison, the uncertainty of the residuals has to be calculated:

\[2S_{\text{residual}}^{xy} = 6.8 \, \mu \text{m}\]  \[\text{[3.6]}\]
\[2S_{\text{residual}}^{xz} = 6.7 \, \mu \text{m}\]  \[\text{[3.7]}\]
\[2S_{\text{residual}}^{yz} = 12.7 \, \mu \text{m}\]  \[\text{[3.8]}\]

This uncertainty is determined by calculating the positive square root of the variance of the result, multiplied by a factor \(k = 2\). The variance of the result is given by adding the variances corresponding to the different uncertainty components, multiplied by the squares of relevant partial derivatives [37]. In order to get an upper limit estimation, the maximum value for the partial derivatives is substituted, considering the application.

Taking into account this upper limit estimation, it can be concluded that for the comparison, depicted in figure 3.15, there is no significant difference between the model and the actual error structure of the milling machine. However, the uncertainty associated with the squareness measurements is 1.5 arcsec, which yields a large contribution to the uncertainty of the residuals. The comparison of the holeplate measurements with the simulations show a systematic effect in the residuals, which could be caused by a squareness error. Therefore, it can be concluded that more accurate squareness measurements probably will yield a better model.

The results, shown in figure 3.15, are based on a geometric error model, which has been completed in April 1991. The measurements are carried out before April 1991, on a relatively new machine. In August 1992, similar experiments are carried out, to determine the influence of wear on the geometric errors. It turned out that the machine had to be calibrated again, because the errors had significantly changed. Using the calibration results, new simulations have been carried out.
Fig. 3.16  Result of the holeplate measurement, together with the residuals between measured result and simulation, carried out in August 1992.

In figure 3.16, the results are shown of a similar experiment as depicted in figure 3.15. This graph clearly shows that the geometric error structure has totally changed. It turned out that the guidings of the milling machine are worn, caused by movements of the guidings. In December 1992, verification measurements have been carried out, which showed that the geometric error structure has slightly changed. The maximum residual at that time between model and simulation was about 10 μm. Due to these results, a calibration period of approximately one year is advised.

Simulations of holeplate measurements based on the finite stiffness errors
In addition to the geometric errors, the actual position of the tool can be disturbed by a number of forces acting on the machine's structure. In paragraph 3.4, three different basic types of forces are distinguished. However, it is only necessary to evaluate the significance of the forces of type 3 (workpiece load), as forces of type 1 will not cause a significant deflection during finishing and forces of type 2 are not separable from the basic geometric errors.
In order to verify the error model, simulations of the holeplate measurements are compared with actual measurements with different workpiece loads. In figures 3.17 and 3.18, the results of the comparison between the actual holeplate measurement and its simulation is presented for two different loads. The presented comparison is carried out in the XY-plane, where the largest influence of the investigated error is found. Three different states are depicted. The solid line represents the actual measured deviation. The dashed line represents the residue when the measured deviation is compensated for the geometric errors. The dash-dotted line shows the residue when the measured error is compensated for both the geometric errors and the finite stiffness errors due to a workpiece load. The finite stiffness compensation is determined by interpolation between the measured errors with and without a workpiece load.

Taking into account the upper limit estimation for the uncertainty of the residuals ($2S = 6.8 \, \mu m$), it can be concluded that there is no significant difference between the model and the actual error structure of the milling machine under different loads. Thus, it can be concluded that the developed error model is correct.

**Fig. 3.17** Validation of the error model for geometric and finite stiffness errors with a workpiece load of 119 kg.
3.5 Determination of the geometric errors with test workpieces

Nowadays practiced methods to determine machine tool errors do neither meet all requirements of machine tool manufacturers nor of machine tool users. Because on the one hand complete and reliable test results are expected but on the other hand the tests should be carried out quickly and easy. These methods are distinguished into direct and indirect measuring methods. Direct methods give the most complete picture of the machine tool errors but they have the following important disadvantages:

- To measure the error parameters of machine tools, expensive measuring equipment (e.g. laser-interferometer) and qualified personnel is needed;
- A lot of time is needed to measure a complete set of error parameters. Periodical interruptions of manufacturing for such tests are not accepted by machine tool users;
- The conditions of the measurements are different from the operating conditions of the machine tool.

Fig. 3.18 Validation of the error model for geometric and finite stiffness errors with a workpiece load of 267 kg.
Indirect methods, e.g. machine simple test workpieces are commonly applied to check the accuracy periodically under operating conditions. The measured workpiece features, e.g. length, parallelism and squareness, enable a machine tool user to judge the accuracy of the milling process under special working conditions. But this method is not sufficient to analyze the error sources in the machine tool, because the machine tool errors are superimposed on the workpiece geometry and in general cannot be separated.

Also the deformations of the machine tool due to thermal effects lead to changes of the working accuracy. Therefore, it is important to test the machine tool geometry in different thermal situations.

Besides these requirements of machine tool manufacturers and users, it was also necessary to enable the verification of thermo-mechanical models for milling machines in this project. Therefore, a method is needed which enables the determination of the errors as they really occur during long sequences of machining. As none of the known methods is suitable to meet the mentioned requirements PTB decided to develop a new method.

This measuring method should combine the following features:

1) Separation of all important error parameters as they are present in a real manufacturing process:
   • Geometric errors due to imperfect movements of the linear axes and the rotary table (24 error components as functions of the axial positions);
   • Five errors of squareness due to the imperfect orientation of the axes (including the B-axis);
   • Backlash errors;
   • Geometric errors due to the machine control.

2) Determination of thermo-mechanical errors during several hours of operation:
   • Changes of geometric errors;
   • Zero point drift.

3) High degree of reliability of the test results (real operating conditions);

4) Low costs.
**Principles of the Method**

The requirements 1), 2) and 3) can only be met by test workpieces. The basic idea of the introduced method is to machine test workpieces in such a way that each of the error components can be separated from the geometric features of a single workpiece or from a set of workpieces.

**Pict. 3.1** Test workpiece to determine all error components of linear and rotary axes
In picture 3.1 a test workpiece is depicted, which is designed to determine the errors of the linear axes. Circular tracks are integrated for a test of the rotary table. The structure of this workpiece is based on the square and the circular grid shown in figure 3.19. The grids are generated by points (in 3 coordinates) which are measured at the machined tracks by a coordinate measuring machine.

Fig. 3.19  Geometric features of a test workpiece.

To determine all 21 error components of 3 linear axes it is necessary to machine a set of 4 test workpieces in the center position of each axis (figure 3.20). To determine the error behaviour during long-term operation the finishing sequence for the square grid is repeated for several operating situations. The workpiece in figure 3.21 is sufficient to sample 12 error situations during more than 3 hours. For each of these situations the 21 error components can be determined by the evaluation of a complete workpiece set.
Fig. 3.20  Test workpiece orientation (coordinates VDI 3255).

Fig. 3.21  Test workpiece drawing.
Some of the error components can be separated directly from a single workpiece feature. Partly, different machine error components are imaged to the same feature of a workpiece. In this case the error components must be separated by coupling the features of different test workpieces. Thus, the error components can be determined correctly only if the test workpieces are machined under equal thermal conditions which means:

- Starting the operation with the thermally balanced "cold" machine;
- With the same spindle speed;
- With the same operation intervals.

Disturbances which could introduce errors to the determination of the 21 error components of a three axis machine, are suppressed by the following steps:

1) A zero point drift of the machine tool causes a deviation which is superimposed to the linear motions of the axes and to the geometric features of the workpiece. The chosen sequence of machining avoids significant distortion of the square grids due to zero point drifts while machining the tracks of a single grid (Appendix A.2.3);

2) The elastic behaviour of the workpiece material or the machine tool and control errors have no effect to the workpiece features because of the symmetrical finishing process;

3) Distortions of the plane grid due to clamping forces are suppressed by elastic zones in the clamping areas;

4) By a low depth of cutting (0.2 mm) and an optimized cutting speed the roughness is very small.

Additionally to the 21 error components the following errors can be determined:

- The overrun (lag) errors in the control of the linear axes are sampled on tracks which are milled with an angle of 45° as shown in figure 3.21;
- The back lash errors of the linear axes are sampled on tracks next to the corners of the workpieces;
- The errors of the rotary table (B-axis) are sampled on tracks with a circular structure (figure 3.22).
As the range of the C-axis is limited to ± 60°, the axes of the holes and tracks of the test workpieces have an angle of 30° referred to the coordinate system of the workpiece (only in the YZ-plane).
All these additional errors are determined for the thermally balanced "warm" machine tool to avoid disturbances due to a zero point drift. This assumption is valid while the error components of the rotary table can be regarded as temperature invariant due to the constructive symmetry. In figure 3.18 the circular grid is depicted, together with the error features angle, roundness and flatness. From these features the following error components of the B-axis can be determined:

1) Three linear error components:
   - One axial component;
   - Two radial components.

2) Three rotary error components:
   - One angular positioning error;
   - Two components of camming.

3) The squareness errors of the rotary axis with respect to the cartesian axes.

These error components are a function of the position of the B-axis. The errors are sampled in steps of $22.5^\circ$ within a range of $720^\circ$.

**Results**

The error components have been analyzed by an evaluation program. In a first step the measured coordinates of the workpieces are imported into the program. Secondly, the thermal expansion of the workpieces during machining and measuring is corrected by the measured temperature data.

Finally, all coordinates are referred to the required values and can be displayed and plotted in 1D-, 2D- or 3D-graphics. Users of this program can choose the following graphic options for the visual judgment of the workpiece geometry or for a comparison of different workpieces:

1) Shapes and positions of different workpiece contour lines (figure 3.23);
2) Comparison of bore positions (2D) in different error situations of the milling machine (figure 3.24);
3) W-coordinates in different error situations of the machine tool (figure 3.25).

This part of the program is a comfortable tool for periodical quick checks of the machine tool accuracy with single test workpieces.
Fig. 3.23  Error in Z-direction, extracted from the test workpiece (the position is defined along the X-axis).

Fig. 3.24  Comparison of the bore positions in 1st and 12th error situation to the ideal positions.
Fig. 3.25  Error in W-direction (perpendicular to workpiece) in the 1st and 12th error situation.

Fig. 3.26  Squareness error zwy, plotted as a function of the elapsed time.
The next step of the evaluation is to calculate the workpiece features (straightness, length, parallelism, squareness and flatness) from the plane grid. The grid is represented by lines which are calculated by a standard regression algorithm (least square fitting).

Finally, the 21 error components and the zero point drift are separated from the workpiece features of a complete workpiece set. All error components can be plotted as functions of time, e.g. the squareness error zwy (figure 3.26). In figure 3.26 is clearly depicted how important it is to determine the machine tool errors during a long-term operation. Only in this way the accuracy of a milling machine can be judged extensively. Using this feature of the evaluation program it is possible to locate thermally induced deformations in the machine structure. Then, the test-workpiece method has become a very appropriate method to develop and verify a thermo-mechanical model.
4. Thermo-mechanical error models

4.1 Introduction
In order to determine an empirical relationship between the thermal distribution and the deformation of the machine tool, a measurement set-up is built, which is based on a commonly applied principle [38,39]. For correction purposes, the principal interest is not the deformation of each machine component, but the displacement of the tool with respect to the workpiece. Therefore, the measurement set-up is designed to obtain the displacement of the tool holder in three orthogonal directions, and two relevant rotations. The measurement of the thermal distribution is carried out by extensive temperature measurement equipment. In the next paragraph the different parts of the measurement set-up will be described. Furthermore the obtained results will be discussed.

From the measurements of the thermal behaviour a relationship is determined that describes the displacement of the tool in dependence of the thermal rise of several temperature sensors attached to the machine tool. This relationship has been obtained by application of statistical analysis techniques. In paragraph 4.3 the applied techniques will be discussed.

Besides an empirical thermo-mechanical model, also an analytical model has been developed. This analytical approach describes the deformation of a machine tool as a functional dependence of its temperature distribution [40]. The aim of modelling is the determination and (on-line) correction of thermally induced displacements between a workpiece and the tool of a 5-axes milling machine. The displacements are calculated by the geometric parameters of the machine structure and the temperatures which are measured on the machine's surface. In paragraph 4.4, the developed model is presented.

Both models have been applied to describe the thermo-mechanical behaviour of the Maho 700S milling machine. The efficiency of both methods has been determined with a totally different and independent method: by using test workpieces. Thus, a proper comparison of both methods has been possible.

In the last paragraph of this chapter a comparison of both models will be presented, including a discussion of the important features of both methods. The results of the comparison will be presented. This will be completed with an overview of the efficiency of both models.
4.2 Empirical measurement setup

A cylinder is mounted in the tool holder of the machine; on the workpiece table a base is mounted with five contactless eddy current displacement transducers. These transducers allow to measure the displacement of the tool holder with respect to the workpiece in three directions simultaneously. The displacement transducers are calibrated separately in the actual measurement set-up, which showed an uncertainty of less than 1 μm over the full range, after application of a correction formula for systematic errors. Also the influence of temperature changes has been investigated [41]. The results show that temperature does not affect the reading of the displacement transducers significantly.

The transducers are linked to an amplifier (Hottinger Baldwin DMC9012A) that is capable of reading all signals simultaneously.

The obtained signals from the amplifier are sent to a PC by means of IEEE interface. In figure 4.1 the measurement set-up for the determination of the displacement is depicted, as it is applied in the XZ-plane.

![Diagram of measurement set-up](image)

**Fig. 4.1 Scheme of set-up for displacement measurement**

Distributed over the machine's structure a number of Pt-100 temperature sensors are mounted. Each temperature sensor is calibrated with an uncertainty of less than 0.1 °C, in a range from 15 to 60 °C, which is sufficient for this purpose [42].
In figure 4.2 the position of each temperature sensor on the machine tool is depicted. The choice of the position of each sensor is determined after careful consideration of previous experiments at Maho's laboratories and a thorough discussion with the project partners [41].

From previous measurements and bibliographical studies [43], it appeared that the main spindle bearings, together with the main drive, are the most important internal heat sources. This implies for the measurement strategy that the influence of the heat induced by the drives of the carriages on the thermal distortion of the machine tool, will not be taken into account in this study.

For carrying out a measurement the surface of the cylinder is positioned on 0.5 mm of the transducers. In this position the machine tool is loaded with a spindle speed over a specified time. During the measurement the displacement of the tool holder and the temperature distribution of the machine tool are obtained every 60 seconds.
The programming of the machine tool and the collection of the measurement data are carried out automatically by a computer. Therefore the software package, as described in chapter 3, is extended. This software has basically three tasks:

1) positioning of the axes so that a measurement can be carried out;
2) control of the load sequence of the machine tool;
3) collecting and storing of measurement results.

The measurement strategy
The described set-up can be applied for measuring the displacement of the tool holder in three orthogonal directions. In order to obtain a practical impression of the thermal behaviour of the machine tool a measurement strategy is developed. With this measurement strategy it is possible to obtain a thorough impression of the thermal behaviour of the machine tool. Hereby various loads of the spindle speed can be applied.

This strategy contains basically the following items:

1) The position of the measurement is limited to three positions along the Z-axis, i.e. the ram extracted, centered and retracted;
2) The positions of the X- and the Y-axis remain constant during the measurements;
3) The position of measurement remains constant during the entire measurement sequence;
4) All measurements are initiated from the same reference state;
5) No cooling liquid is applied during these measurements.

For the position of the Z-carriage the first statement has lead to the coordinates: 62, 314 and 566 mm. The number of measurement positions is chosen such that, with the continuity of the deformation taken into account, a parabolic deformation can be determined.

The most significant heat sources are located in the Z-axis and the heat produced in the Z-axis does not flow into the column of the machine [41]. When the drift of the tool would be influenced by the position of the X-axis, a heat flow towards the column should be expected. The same yields for the Y-axis; the whole B-chain is not disturbed by the heat generated in the Z-axis.
This conclusion has been verified by measurements on different X- and Y-positions.

Fig. 4.3 Measured displacement in Y- and Z-direction on several positions of the X-axis

Fig. 4.4 Measured drift on different Y-positions.
The measurements, depicted in figure 4.3, have been carried out on three different positions of the X-axis (X=150, 343 and 550 mm). A maximum difference of 10 μm is found between two measurements. This range can be considered as the repeatability of the thermal behaviour.

The same investigation has been carried out for the Y-axis. The measurement results, depicted in figure 4.4, show that also for different positions of the Y-axis no significant influence on the measured drift can be found.

The strange behaviour of the drift measured in X-direction, depicted in figure 4.5, is caused by the spindle stop which occurs in the DIN-spectrum after 90, 240 and 390 minutes (see figure 4.7). Although the cylinder has been machined on the milling machine, still some roundness errors are left. Especially, while the cylinder has been exchanged with other tools. This effect seems to appear randomly. However, it depends on the orientation of the spindle, which is pure random when the spindle is stopped. The roundness errors will not have effect on the measurement data gathered under operating conditions. While the measurement data is gathered with a frequency of 1200 measurements per second and averaged over these 1200 measurements, the roundness errors are eliminated numerically.

**Limitations of the measurement strategy**

Although the presented measurement strategy supplies a large amount of information concerning the actual displacement of the tool holder, some limitations are inherent to the set-up and will be discussed below.

**Environment**

All measurements are relative to one specific thermal reference situation. This implies that the milling machine must be placed in a temperature stabilized environment in order to get repeatable results.

**Cooling liquid**

The use of cooling liquid causes difficulties in the collection of the measurement data using the applied eddy current displacement transducers. Therefore, the measurements are carried out without using cooling liquid.
However, the developed workpiece method is a very appropriate method to determine the thermal behaviour of the machine tool with cooling liquid. Therefore, workpieces have been milled with and without cooling liquid. The results of these experiments show that the temperature distribution on the machine tool does not significantly change when cooling liquid is applied. Only one temperature sensor gives another reading: sensor 53. This sensor is located on the position where the cooling liquid is pumped into the machine. Therefore, it is concluded that the cooling liquid has no influence on this behaviour.

Moving carriages
The measurements are carried out with fixed carriages. This does not reflect the normal situation of operation when the carriages move in order to generate a workpiece. While moving the carriages, the thermal behaviour of the machine tool might differ from the fixed situation. However, the movement of the carriages does not induce structural heat sources compared to the main sources. Therefore the effect of moving carriages on the thermal behaviour is restricted to a varying heat flow. The investigation of this effect is not carried out in this study. A main practical problem of implementing moving carriages into the model, is the choice of sequence for the movements. If this sequence seriously effects the thermal behaviour the modelling of this behaviour becomes highly complicated.

Cutting process
The effect of the heat induced by the cutting process is not taken into account by the proposed measurement set-up. However, the influence of the cutting process on the thermal behaviour on the total machine structure is regarded to be negligible in finishing processes if the structure is insulated from falling chips [44].

This assumption has been verified by milling test workpieces. These experiments show that the heat induced by the cutting process is negligible during finishing.
Measurement results

With the measurement set-up described a number of measurement cycles are carried out, applying a spindle speed as a thermal load to the machine tool. In figure 4.5 the results of a measurement with a spindle speed of 5000 rpm for 6 hours, followed by a spindle stop of 8 hours, is presented. Only the displacement of the cylinder is depicted. For the X- and Z-direction the displacements are measured by the lower transducers of the measurement set-up.

As the standard available temperature compensation of the machine tool operates on the information of the temperature of the spindle head, the temperature variation of the spindle head is marked out on the abscissa. This choice enables us to draw conclusions with respect to the suitability of one temperature sensor. On the ordinate the measured displacement of the tool holder is plotted.

![Diagram of spindle head displacement](attachment:image)

**Fig. 4.5** Results of a drift measurement in Y- and Z-direction

Load situation: \( n = 5000 \text{ rpm for 6 hours, } n = 0 \text{ for 8 hours} \)

Solid line: warming up; Dashed line: cooling down

Thick marks: time indication every 30 minutes
The solid part of the graph represents the warming up time, whereas the dashed part represents the cool down period. The thick marks in the graphs represent the elapsed time with a thick mark placed each 30 minutes. During this measurement the spindle was positioned in the center of the working space.

With the results depicted in figure 4.5 the necessity of an extended thermal model can be clearly demonstrated. If a temperature rise on the spindle head of approximately 25 °C is measured, the displacement in Y-direction can be 33 as well as 38 μm.

The difference is determined by the warming up and cooling down of the machine tool. In the Z-direction this effect is even larger and the predicted displacement at a temperature rise of 25 °C can even range as much as 60 μm. This difference, that may look like a clear form of hysteresis, is caused by the thermal behaviour of the machine tool.

These measurement results are obtained with one specific load (5000 rpm). In practice, changing the load situation causes differences in displacement up to 100 μm of the tool holder at a specific temperature rise of the spindle head.

From these observations we can conclude that the use of one temperature sensor for correction of thermal behaviour is absolutely insufficient.

Besides loading the machine tool with a spindle speed of 5000 rpm, measurements are carried out with loads over the entire range of possible speeds (0-6300 rpm). Figure 4.6 shows some results with a different load. Note that the choice of the abscissa is not restricted to a particular temperature sensor on the machine tool, but in all graphs we have chosen for the temperature of the spindle head. Some measurements show a discontinuity in the displacement measurement at the switch-point from warming up to cooling down (in the graph: the point where solid line changes to dashed line). This effect is caused by the run-out error of the spindle: during warming up this run-out error is averaged by the rotating spindle, contrary to cooling down, when the spindle has been stopped.
Drift of the tool holder with a load of 1500 rpm for 6 hours

Although loading the machine tool with a constant spindle speed provides a large amount of information concerning the behaviour of the machine tool, practical situations often display a rapidly changing load situation. To obtain an impression of the machine tool's thermal behaviour in practical situations, it is loaded with a spindle speed spectrum as defined in DIN 8602. This spectrum consists of spindle speeds varying from zero to maximum spindle speed \( n_{\text{max}} \), with steps of 25% of \( n_{\text{max}} \). For the machine tool under investigation \( n_{\text{max}} \) is 6300 rpm. Each load situation is maintained for 15 minutes. A graphical presentation of this spectrum is depicted in figure 4.7. The displacement of the tool against the elapsed time is depicted in figure 4.8.

All presented results are obtained in the center of the working space. The same series of measurements are repeated for different Z-coordinates of the machine i.e. with the Z-carriage extracted and retracted. These measurements yield a different behaviour than those obtained in the center. An example of the displacement in Y-direction, measured on different positions of the Z-carriage, is depicted in figure 4.9. Here a load sequence is applied of 5000 rpm for 6 hours and no load for 8 hours. The solid line represents the warm up period, whereas the dashed line depicts the cool down period.
Fig. 4.7  **DIN 8602 spectrum**

Fig. 4.8  **Drift of the tool holder under load of the DIN-spectrum**
Fig. 4.9 Drift of the tool holder on different Z-positions

All the described measurements reflect the displacement of the tool holder relative to a specific reference situation. This implies that no historic parameters are included in the modelling technique. In the next paragraph we will discuss the development of a thermal model based on these measurements. It should be noted here that we will confine us to the description of the static thermal behaviour i.e. the displacement of the tool holder is related to the current thermal state of the machine tool, no historic information is used in the modelling.

4.3 Empirical model

With the measurement set-up as described in paragraph 4.2, a large number of measurements are carried out. From these measurements representative samples have been extracted to create two different data sets. One data set to serve as input and one data set for verification of the derived model.
**Verification data set (dset3)**

The verification data set (dset3) is composed of loads which cover the entire range of spindle speeds. The samples are built up by the following sequences ('n' represents the spindle speed in rpm):

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Load situation</th>
<th>No-load situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>n=1500, 6 hours</td>
<td>n=0, 8 hours</td>
</tr>
<tr>
<td>2</td>
<td>n=3000, 6 hours</td>
<td>n=0, 8 hours</td>
</tr>
<tr>
<td>3</td>
<td>n=4000, 6 hours</td>
<td>n=0, 8 hours</td>
</tr>
<tr>
<td>4</td>
<td>n=5000, 6 hours</td>
<td>n=0, 8 hours</td>
</tr>
<tr>
<td>5</td>
<td>n=6000, 6 hours</td>
<td>n=0, 8 hours</td>
</tr>
<tr>
<td>6</td>
<td>DIN 8602</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>Product simulation</td>
<td>--</td>
</tr>
</tbody>
</table>

*Table 4.1 Verification data set (dset3)*

Measurement 6 loads the machine with the spindle speed spectrum defined in DIN 8602. The measurement sequence 'Product simulation' is built up by the following spindle speeds and no-load periods:

- n=4000, 10 min; n=0, 1 min; n=5500, 30 min; n=0, 1 min;
- n=3000, 5 min; n=0, 1 min; n=6000, 10 min; n=0, 5 min.

This sequence is repeated twice.

**Modelling data set (dset4)**

For the above described data set, it takes at least seven days to complete all experiments. As these experiments have to be carried out for different Z-positions and both horizontal and vertical spindle, it is desirable to reduce number of experiments. Therefore, a new measurement sequence has been defined, which covers all these sequences.

Within 24 hours, the machine is loaded with randomly chosen spindle speeds for randomly chosen time intervals. In order to achieve a representative spectrum, for a milling machine during normal operation, the following restrictions have been chosen:

- the time interval is chosen between 5 and 45 min;
- the spindle speed is chosen between 800 and 6300 rpm.

The resulting spindle speed spectrum is depicted in figure 4.10.
In order to complete the modelling data set, a measurement sequence is added, which represents the machine tool in a thermally stabilized situation (steady state). Therefore, the milling machine is loaded with a spindle speed of 6000 rpm during 24 hours. Also a cooling down period of 24 hours has been taken into account. The results of this experiment are depicted in figure 4.11.

**Fig. 4.11 Random spindle speed spectrum**

**Fig. 4.12 Steady state experiment, with a load of 6000 rpm**
Together these two experiments complete the modelling data set. Both experiments can be carried out in three days. This can be shortened, as the milling machine stabilizes after 20 hours during warming up and 8 hours during cooling down.

As a measurement is taken every minute, the total sample contains 4320 measurements. Investigation of the obtained results reveal a continuously changing thermal behaviour. Therefore, it is allowed to reduce the amount of data by taking every 2nd measurement into the sample from the random spectrum; and every 5th measurement from the steady state experiment. This results in a data set of 1296 measurements, whereas each observation contains the values of the measured displacement in three directions, and 40 temperatures (39 attached to the machine tool and the air temperature of the room).

These measurement samples were taken on three different Z-positions in order to obtain a thorough description of the drift in dependence on the position of the Z-axis.

The statistical modelling methodology
The goal of this part of the study is to determine a relationship between the relevant temperature sensors and the measured displacement of the tool holder, based on empirical obtained measurement results. The temperature sensors must be chosen such that the predictive value of the model is optimal.

Several approaches are carried out in order to obtain the relation between relevant temperature sensors and the displacement. First, preliminary calculations using least squares fitting procedures are performed. With an intuitive selection of the temperature sensors this method displayed fairly good results. However, as the theoretical importance of a particular temperature sensor is unknown, a methodology for relevance detection is desired. A possible approach is the use of statistical criteria for elimination and calculation of the optimal model. Therefore the data sets are examined with the use of a software package for statistical analysis of data (SAS). This software package offers a wide range of utilities for regression analysis [45].
Regression analysis.

Taking a data set and a direction of the displacement, the task of the regression procedure is to determine:

1) The temperature sensors relevant for the description of the displacement;
2) An estimate of the coefficients of the predictors;
3) A statement on the confidence interval for future predictions on new data.

As there is more than one regressor, this is a multiple regression problem. A general notation of this problem is:

\[ \hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_K x_K + \varepsilon \]  \[ \text{[4.1]} \]

with:

\( \hat{y} \): the estimated response variable (in this case the displacement);
\( x_1, \ldots, x_K \): the regressors in the total model (in this case the temperatures on various locations of the machine tool);
\( \beta_0, \beta_1, \ldots, \beta_K \): the unknown parameters of the model;
\( \varepsilon \): the part of the results that are not explained by the model.

The first condition requires that the best suitable model can be extracted from the total model, based on all available temperature sensors that are attached to the machine tool. Because evaluating all possible regression models - i.e. the models with one sensor, the models with all sets of two sensors, etc - is computationally very extensive, various methods have been developed for evaluating only a small number of subset regression models by either adding or deleting regressors one at a time [46]. These methods are generally referred to as stepwise-type procedures. They can be classified into three categories:

1) Forward selection;
2) Backward elimination;
3) Stepwise regression, which is a combination of 1) and 2).
Although the mentioned selection techniques gave good results [50], the presented forward, backward and stepwise regression techniques highly depend on the chosen cut-off values, SLE and SLS. These values are changed when the predictive capacity of the model is not good enough or when the number of sensors in the model is too high.

As this is done rather intuitively, another technique has been applied to determine the temperature sensors, relevant for the description of the displacement.

As already stated, it computationally very extensive to evaluate all possible regression models. However, taken the restriction that the number of sensors should be very small, the number of possible regression models can be reduced enormously when the maximum number of sensors is limited.

Using SAS software it is possible to determine the best regression model, given a fixed number of regressors. The regressors will be chosen such that the predictive capacity of the model is optimal for the modelling data set. This will be judged by the value of $C_p$, which has been introduced by Mallows [45]:

$$C_p = \frac{SS_{E_p}}{MS_E} + 2p - n$$ [4.2]

With:
- $SS_{E_p}$: Sum of squares of the error for a model with $p$ parameters;
- $MS_E$: Mean square of the error for the full model;
- $p$: Number of parameters;
- $n$: Number of observations.

The described technique can determine the optimal model for the given data set. However, the residual between actual results and model will always be smaller with a larger number of regressors [47]. Therefore, including more sensors will always give a better $C_p$-value. Unfortunately, this will not always result in a better model.

When the number of sensors is increased, the predictive capacity of the model will decrease, caused by nonsense relations (called "Over-fitting").
In order to avoid this, the efficiency of the model is calculated for the verification data set (dset3) as a function of the number of sensors included in the model by SAS. The efficiency of the model is given by:

\[
\delta = \left[ 1 - \frac{\sum_{i=1}^{n} | \Delta_i^m - \Delta_i^s |}{\sum_{i=1}^{n} | \Delta_i^m |} \right] \times 100
\]

With: \( \Delta_i^m \): Measured drift [\( \mu \text{m} \)]

\( \Delta_i^s \): Simulated drift [\( \mu \text{m} \)]

\( \delta \): Efficiency [-]

**Results**

The goal of the procedure is defined as the determination of an adequate subset model for prediction of the displacement of the tool holder, based on measured temperature drift.

After careful analysis of the problems, inherent to application of regression analysis, a modelling methodology has been developed according to the following sequence:

1) Calculate the best model (lowest \( C_p \)) given a fixed number of sensors;
2) Plot residuals against predicted values and look for systematic patterns;
3) Check for individual outliers with the studentized residuals;
4) Check for leverage points with the Cook's statistics [46];
5) Check the normality of the residuals;
6) Apply principal component analysis to the relevant predictors and perform regression with those principal components with an eigenvalue larger than one;
7) Transform the regression coefficients of the principal components back to the original predictors;
8) Calculate the efficiency (relation [4.3]) of the model, using the validation data set (dset3);
9) Repeat steps 1-8 for 1 till 10 sensors, for each axis;
10) Plot efficiency as a function of the number of sensors in the model;
11) Chose the optimal model.
With the developed modelling procedure, three measurement samples (obtained at three different positions of the Z-axis) have been evaluated for two different planes. This yields 18 sets of parameters linked to 18 sets of temperature sensors (2 planes with 3 times a set in X-, Y- and Z-direction).

In the next figures 4.12 and 4.13 the efficiency of the model, based on the measurements in the XZ-plane, is depicted as a function of the number of sensors applied in the model. In figure 4.13 the efficiency of the model for the modelling data set is shown. It clearly shows a better efficiency when the number of sensors is increased, as already mentioned. In figure 4.13 the efficiency of the model is depicted for the verification data set. One can clearly observe that only one sensor in X gives already good results. When the number of sensors is increased above 5, the efficiency of the model suddenly decreases. In both Y- and Z-direction only one sensor results even in a negative efficiency e.g. the model would worsen the machine instead of improving it.

![Graphs showing efficiency of the model in X, Y, and Z directions](image)

**Fig. 4.12** Efficiency of the model based on the measurements in the XZ-plane. The dotted line represents the efficiency when all sensors are added to the model.
Fig. 4.13 Efficiency of the model for the verification data set in the XZ-plane. 
The dotted line represents the efficiency when all sensors are added to the model.

The same analysis has been carried out in the XY-plane.

Based on the above described analysis, the following 'optimal' model has been chosen:

**XZ-plane:**
- X-axis: 1 sensor;
- Y-axis: 9 sensors;
- Z-axis: 8 sensors;

**XY-plane:**
- X-axis: 2 sensors;
- Y-axis: 2 sensors;
- Z-axis: 4 sensors.

Together, only 16 different temperature sensors are needed to model the drift of the milling machine in an appropriate way.

The parameters are applied for prediction of the drift of the tool holder on new measurements; not included in the modelling or in the verification data set. In the figures 4.14 and 4.15 the results of the developed model are presented.
**Fig. 4.14**  Measured and simulated drift in X-direction.
*Experiment carried out in the XZ-plane (load: DIN 8602)*

**Fig. 4.15**  Measured and simulated drift in Y-direction.
*Experiment carried out in the XY-plane (load: 6000 rpm)*
A strong improvement can be observed in the behaviour of the machine tool. Especially the stationary states are described in an accurate way. The model has difficulties in describing transient states of the milling machine [48]. Another modelling strategy, for example by including history in the model, could yield a better model for transient states. However, the small residuals and the small number of necessary sensors validate the conclusion that this model can be used for the final version of the software error compensation.

4.4 Analytical model

Analytical approaches to describe thermally induced errors of machine tools have been developed with little success in the past. The reasons for the failure of these efforts are mainly:

1) The simplifications of the used error models;
2) The attempts to include thermo-dynamic modelling (unstable parameters of heat transfer).

All analytical approaches are a more or less simplified descriptions of deformations of a machine geometry due to its currently measured temperature distribution. Prior error models consider the thermal expansion and almost neglect thermally induced bending in the machine structure. However on machine tools with wide working ranges the bending in the carriages and guideways cause significant errors.

Therefore, a thermo-mechanical modelling principle for machine tools had to be developed which considers all significant deformations. The aim is to correct the tool position during machining as a function of the on-line measured temperatures and for arbitrary tool positions. The thermo-mechanical model should be applied for the Maho 700S five-axes milling machine. This model should consider the type dependent parameters of the machine structure.

A machine type dependent analytical model

The model is based on three relations between the temperature distribution of a body and its deformation [49]:

1) The expansion \( \delta x \) of a body is a function of the temperature change \( \delta T(x) \) referred to a temperature distribution in a reference situation:

\[
\delta x = \alpha \cdot \int \delta T(x) \, dx
\]  

\[ [4.4] \]
2) A temperature gradient $G_x = \frac{dT}{dx}$ causes a spherical bending of a body with a curvature $\beta$ (figure 4.16) referred to the reference situation;

3) A differential gradient e.g. $dG_x/dy = \frac{d^2T}{(dydx)}$ causes a torsion as shown in figure 4.17.

Fig. 4.16  *Deformation for the case of a linear temperature distribution.*
To apply these basic relations for a machine tool, its structure is divided into segments corresponding to the finite element method (FEM). For each of the segments the deformations (expansion, bending and torsion) are determined by the referring temperature distribution. The displacement of the tool relative to the workpiece is calculated from the superposed deformations of each segment and from the constellation of the segments. The temperature distribution of the machine tool is measured with a total number of 30 to 40 sensors on the machine tool surface. To enable machining with a tool that is attached to the milling head (C-axis) or alternatively to the horizontal spindle two constellations of the machine tool are possible. The two constellations generally have a different behaviour of the thermally induced tool displacement. The geometric parameters of the constellations are defined along the kinematic chain between the workpiece and the tool.
As shown in figure 4.18 the kinematic chain (dot line) is divided into machine components. The expansion is modelled for the following components:

- Scales;
- Vertical and horizontal spindle;
- Tool.

Expansion and bending is modelled for the machine components:

- X-table;
- Y-slide;
- column;
- ram;
- milling head.

Torsions are modelled for the machine components:

- column;
- ram.
The thermo-mechanical model considers details in the constructive design:

- The structure of the machine components (e.g. length);
- The constellation of machine components (e.g. fastening of the scales);
- The geometric features of the links between the machine components (e.g. supporting surface of the slides and length of the guideways).

Expansions are calculated by an integration of the temperature within the machine segments along the reference path (dot line) in figure 4.18. The temperature is described as a linear or quadratic function along the reference path. High attention for the correct modelling is paid to the most significant heat sources (spindle bearings) and the heat drains. The heat due to the friction in the spindle bearing is conducted into the machine structure and into the tool. The expansion of the spindle between the bearing and the tool tip is approximated by a linear interpolation between the temperature at the bearing and the workpiece. This approach is suitable for the milling head and the horizontal spindle with or without using a coolant.

Bending is calculated by integration of the temperature gradients within the machine segments along the reference path. The gradients are described as linear functions along the reference path. In figure 4.19 an example is depicted for a tool displacement caused by a gradient $G_x = f(y)$ in the column. The model considers the curvature in the reference path as well as the resulting angle between the machine components which are substituted in the diagram by the levers $H_x$ and $H_y$. For each machine component the degrees of freedom for bending is listed in the chart in figure 4.19. For gradients that have no effect on the tool position because of the symmetry of the machine the chart contains blank elements. The chart does not include errors due to an infraction of the Abbe-principle as shown in figure 4.20.

The torsion of a machine component is calculated from differential gradients in the structure. In figure 4.21 the deformations of the Z-slide (ram) is depicted due to the differential gradients $dG_y/dx$ and $dG_x/dy$ along the reference path in Z-direction. The resulting torsion is the change of the inclination $\alpha$ which corresponds in this case to a change of the C-position of the milling head.
Fig. 4.19  Error components due to thermal gradients in the machine components.
Fig. 4.20  Error of length due to thermal gradients

\[ e_x = -a \cdot h \cdot \frac{1}{Z_{rel}} \int_z G_x(z) \, dz \]

Fig. 4.21  Deformation due to differential gradients.

Gradients:
\[ G_y(x) = -\frac{dG_y}{dx} \cdot x, \quad G_x(y) = -\frac{dG_x}{dy} \cdot y \]

Torsion of a plane with an inclination \( \alpha \):
\[ \alpha = \tan \frac{y}{x} \]

Change of inclination due to torsion:
\[ \varphi(\alpha) = \varphi(G_x) \sin \alpha + \varphi(G_y) \cos \alpha \]
\[ = \alpha \cdot \frac{1}{2} \left( z_2^2 - z_1^2 \right) \left( \frac{dG_x}{dy} \cdot \sin \alpha - \frac{dG_y}{dx} \cdot \cos \alpha \right) \]
Integration of the thermo-mechanical model into the geometric error model

The aim of an integration is to describe the geometric errors as temperature dependent functions. This means to relate local deformations in the machine structure to the error components of the geometric model including the zero point drift.

The model for the zero point drift takes a key position within the thermo-mechanical model because it describes tool displacements as the result of all relevant deformations in the machine structure along the kinematic chain between the workpiece and the tool. Therefore the zero point drift in general is the major error of the thermo-mechanical behaviour.

The zero point drift is independent of the error components of the linear axes. It is defined as a displacement of the tool referred to the workpiece in the reference position \( X_{\text{ref}}, Y_{\text{ref}}, Z_{\text{ref}} \) of the linear axes and for arbitrary positions of the B-axes. The zero point drift consists of the linear components \( ntx, nty, ntz \) and the rotational components \( nrx, nry, nrz \), calculated for a tool with a reference length \( L_{\text{ref}} \) (see chapter 2.4). For different tool sizes the inclination of the tool axis \( (nrx, nry, nrz) \) causes additional displacements of the tool tip.

These statements lead to a basic equation that describes the zero point drift as a function of the variables:

- Change of temperature distribution \( \delta T \) (referred to the reference situation);
- C-axis position.

The parameters of the zero point drift are:

- Reference positions \( X_{\text{ref}}, Y_{\text{ref}}, Z_{\text{ref}} \);
- Reference tool length \( L_{\text{ref}} \):

\[
\bar{N} = \begin{bmatrix} \sum e_x \\ \sum e_y \\ \sum e_z \end{bmatrix} = f(\delta T, c, X_{\text{ref}}, Y_{\text{ref}}, Z_{\text{ref}}, L_{\text{ref}}) \tag{4.5}
\]
To calculate thermally induced tool displacements for a general position \( X,Y,Z \) the local deformations in the machine structure are coordinated to the error components of the geometric error model:

- Errors of position due to expansions of the scales and due to bending of the slides and guideways (figure 4.20);
- Straightness errors, squareness errors, yaw and pitch due to bending of slides and guideways;
- Roll due to torsions of the column and the ram.

The geometric error components of the B- and C-axis can be regarded as invariant with temperature changes because of the symmetric constructive design.

The following two examples show how the error components are calculated from the local temperature distributions in the machine structure referred to the reference situation.

1) The straightness error \( y_{tx} \) of the Y-axis due to a gradient \( G_x(y) \) (figure 4.20) is:

\[
y_{tx}(y) = -\alpha \cdot \int_{Y_{ref}}^{Y} G_x(y) \, dy^2
\]

\[
= -\frac{1}{2} \cdot \alpha \cdot (Y_{ref}^2 - Y^2) \cdot G_x(y)
\]

2) The yaw error \( y_{rz} \) of the Y-axis is:

\[
y_{rz}(y) = \alpha \cdot \int_{Y_{ref}}^{Y} G_x(y) \, dy
\]

\[
= \alpha \cdot (Y_{ref} - Y) \cdot G_x(y)
\]

**Verification and improvement of the analytical model**

The analytical model was verified for the Maho 700S milling machine. The thermo-mechanical errors were determined with 8 test workpieces, which were machined in 4 positions and under different operating conditions (with \( n=6000 \) rpm and according to DIN8602).
During machining the temperatures at the machine tool and at the test workpieces were measured and recorded. The algorithms of the thermo-mechanical model were implemented in a simulation module which is part of the evaluation program for the test workpiece. The simulation program calculates the zero point drift and the changes of the most significant error components of the linear axes as functions of the recorded temperature data according to the above described rules. For the verification of the model the zero point drift was defined in the centre point of the workpiece. The calculated drift components were compared with the errors ntx, nty, ntz which were determined by the test workpieces (e.g. figure 4.20 and figure 4.21). In this way the algorithms of the simulation program were tested and corrected step by step.

Furthermore the comparison was used to analyze the thermo-mechanical behaviour of the machine structure in detail. The tool displacements due to local deformations were calculated for different positions of the C-axis and under different thermal conditions. Based on a large number of evaluated error situations it was possible to verify and improve the machine type dependent model.

The comparison of measured and simulated zero point drift shows good results of the model for an operation with constant spindle speed and stationary temperature states. For transient states due to changes of the spindle speed, the calculated errors follow the measured error with a significant delay. The reason is the limited heat conductivity between a heat source in the machine tool and a temperature sensor on the machine tool surface. This causes a filter effect and time delay in the measured temperature behaviour. The temperature differences between the heat sources and the sensor cause transient errors of the modelled deformation (figure 4.22). Thus, the efficiency of the aimed error correction depends on the operating conditions of the machine tool. This effect can be reduced if the sensors are positioned closer to the heat sources.
Fig. 4.22 Transient error behaviour of the temperature correction.

To judge the results quantitatively the efficiency of the simulations was calculated as the ratio of the simulated error to the measured error (see equation 4.4). For the simulation of the most important thermo-mechanical errors (zero point drift, errors of positions, errors of squareness) an average efficiency of the analytical model $> 70\%$ was achieved.
4.5 Synthesis of both the statistical and analytical model

Two totally different models have been developed: a statistical model and an analytical model. Both models have their advantages and disadvantages in the way they are developed and applied. In this paragraph an overview is given of the major differences between both methods. The following items will be discussed:

1) Effort to create a model;
2) Utilization;
3) Preconditions;
4) Sensitivity for disturbances;
5) Efficiency.

4.5.1 Effort to create a model

Analytical model:

Comprehensive experience is required to set up the analytical model. All model parameters which are specific for the machine type have to be determined e.g. the kinematic chain (reference path) is to be defined by the kinematics and some construction parameters of the machine tool. The temperature distribution on the machine tool has to be studied first to achieve optimum positions of the sensors.

A clear advantage of the analytical model is that both construction parameters and sensor positions have to be determined only once for a specific machine type. However, sensor reduction is not possible.

Statistical model:

The model, which is fully based on experimental data, can be achieved with a small effort. The modelling procedure does not take more than a few days. Also the amount of necessary experiments is limited to two days per position. To accomplish a full model 6 positions are needed (3 positions in 2 different planes).

The number of sensors can be optimized with respect to the desirable accuracy. Good results can be achieved with a small number of sensors. However, special precautions have to be made to reduce the chance of incorrect models for not verified machine states (by "over-fitting").
4.5.2 Utilization

Analytical model

The analytical method is developed for arbitrary positions of the C-axis (milling head) and arbitrary tool lengths. With these parameters the method considers a wide range of geometric constellations of the tool tip. Thus, under specific operation conditions some of the calculated deformations in the machine tool are of no significance for the displacement of the tool but have to be determined for a general application. According to the large number of calculated deformations in the machine structure and the different error components of the linear axes the number of temperature sensors for the analytical method is presently a minimum of 30, dependent on the machine type.

Statistical model

Verification measurements have shown that the thermal behaviour highly depends on the positions of the Z-axis. The other axes have no significant influence. As the Z-axis has been modelled on three positions, the model is very well capable of describing the thermal behaviour on all positions in the working volume.

The model has limited capabilities in describing the distortion of the milling machine when using both the horizontal and vertical spindle. This is mainly due to the used measurement set-up; not to the model.

A selection technique has been developed to reduce the number of sensors. With this technique it is possible to relate the number of necessary sensors to the desired efficiency of the model. The amount of sensors can be reduced enormously with this method. A very accurate model can be realized with 16 temperature sensors.

4.5.3 Preconditions

Analytical model

In the analytical error model the measured temperatures on the surface of the machine tool structure represent deformations in a more or less wide range between the sensors. Transient effects due to quick temperature changes in the heat sources cause a delay in the error compensation which depends on the sensor positions. Thus, frequent changes in the spindle speed reduce the efficiency of the error compensation. Application of more temperature sensors or modelling the transient effects would improve the model.
The analytical method at the present state already considers arbitrary axial positions and tool lengths and is independent of the ambient temperature. The reference situation to which thermo-mechanical changes in the machine tool structure are referred can be chosen arbitrarily. This reference situation, however, should be stable, as the geometric errors have to be determined in this reference situation. Then, the absolute accuracy of the measured temperatures is of no significance for the accuracy of the compensation, because thermally induced errors are relative errors, e.g. relative to the geometric errors, determined in the arbitrary reference state.

**Statistical model**

*Static modelling*

The current thermal model is static, i.e. no influence of history is taken into account. By implementing historic parameters into the thermal model, one should be able to model the response to quick variations in the thermal situation. As the ability to determine the thermal behaviour of a machine tool in a not-controlled environment is also dependent on the implementation of historic parameters, this extension of the modelling methodology is strongly recommended.

*Environment*

Like the analytical model all measurements are relative to one specific thermal reference situation. This implies that the milling machine must be placed in a temperature stabilized environment in order to get repeatable results. Similar to the analytical model, this reference situation can be chosen arbitrarily, as the drift of the tool holder is based on a linear combination of temperatures. This is valid when the thermal and thermo-mechanical behaviour of the machine tool is linear.

*Cooling liquid*

Workpieces have been milled with and without cooling liquid. The results of these experiments show that the temperature distribution on the machine tool does not significantly change when cooling liquid is applied. Therefore, it is concluded that the cooling liquid has no influence on this behaviour.

*Moving carriages*

The measurements are carried out with fixed carriages. This does not reflect the normal situation of operation when the carriages move in order to generate a workpiece. While moving the carriages, the thermal behaviour of the machine tool might differ from the fixed situation.
However, the movement of the carriages does not induce structural heat sources compared to the main sources. Therefore the effect of moving carriages on the thermal behaviour is restricted to a varying heat flow due to the changing structure of the machine.

Cutting process
The effect of the heat induced by the cutting process is not taken into account by the proposed measurement set-up. However, the influence of the cutting process on the thermal behaviour off the total machine structure is regarded to be negligible in finishing processes if the structure is insulated from falling chips. This has been verified by milling test workpieces. These experiments show that the heat induced by the cutting process is negligible during finishing.

4.5.4 Sensitivity for disturbances

Analytical model
According to the number of temperature sensors the probability for a correction failure due to a hardware error is higher for the analytical method. There is no experience from a continuous duty but it is expected that there is no change in the thermo-mechanical behaviour of the machine tool. Though, a change in the thermal behaviour due to aging e.g. of the bearings may shift the optimum positions of the temperature sensors. Thus, the temperature distributions in the machine structure should be checked periodically under equal operating conditions.

Statistical model
The model will be influenced by bad temperature readings. The influence of these disturbances depend on the magnitude of the disturbance and on the absolute value of the coefficients of the model. Therefore, principle components have been applied to lower the absolute values of the coefficients. This will improve the prediction power of the model if during actual operation a temperature is measured wrongly or a situation occurs that is not included in the model. Principle Components analysis is capable of accomplishing this effect without a structural loss of predictive power.
When the thermo-mechanical relation between temperature and distortion significantly changes due to aging, periodic tests are needed. In order to analyze this effect, the model has been exercised on experimental data from one year ago. The results, depicted in figure 4.23, 4.24 and 4.25, show that the model, based on recent measurement data, has lost some of its predictive power. However, the thermo-mechanical behaviour of the machine tool is still being improved. Although the found influence of aging is relatively small, it is recommended to carry out periodic tests to ensure a correct thermo-mechanical model.

Fig.4.23 Statistical model verified on old data
Experiment carried out one year before the experiments, used in the modelling data set (load: DIN 8602)
Fig. 4.24  Statistical model verified on old data set
Experiment carried out one year before the experiments, used in the modelling data set (load: DIN 8602)

Fig. 4.25  Statistical model verified on old data set
Experiment carried out one year before the experiments, used in the modelling data set (load: DIN 8602)
4.5.5 Efficiency of both models with test-workpieces

Before a comparison of the efficiency of both models can be carried out, the efficiency of the statistical model has to be determined, using the same experimental data as the analytical model. Therefore, the efficiency of the statistical model will also be based on the results of the test workpieces.

During the milling process, the temperature distribution is measured every minute. As the resulting time span to machine one track is about 8 minutes, the acquired temperatures are averaged over this period of time. Using these averaged temperatures, the statistical model estimates the drift of the tool holder. These calculated results are compared with the actual drift, extracted from the workpieces.

Both models have been verified in the most frequently used positions of the working volume (center position). In the second milestone report [48] a complete overview is presented of the verification of both models. In figure 4.26a and 4.26b the results of the comparison in X-direction for \( n=6000 \) is depicted. A very good similarity between modelled and actual drift is shown.

![Validation with test workpieces](image)

**Fig. 4.26a** Statistical model verified with test workpieces

*Experiment carried out in the XY-plane (load: 6000 rev/min)*
Fig. 4.26b  Analytical model verified with test workpieces
Experiment carried out in the XY-plane (load: 6000 rev/min)

Fig. 4.27a  Statistical model verified with test workpieces
Experiment carried out in the XZ-plane (load: DIN 8602)
Fig. 4.27b  Analytical model verified with test workpieces

Experiment carried out in the XZ-plane (load: DIN 8602)

The model has also been verified in the XZ-plane with a spindle load according to the DIN 8602 spectrum. In figure 4.27a and 4.27b the results of this experiment is depicted in Z-direction.

A complete overview of the predictive power of both models is given in tables 4.2 and 4.3. The efficiency (equation [4.3]) of both models is calculated for every comparison and for all axes.

As the efficiency highly depends on the absolute values of the measured drift, also the maximum residual $\varepsilon$ is given in these tables.
### Efficiency in the XY-plane

<table>
<thead>
<tr>
<th>Axis</th>
<th>Statistical model</th>
<th>Analytical model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{6000}$</td>
<td>$\varepsilon_{DIN}$</td>
</tr>
<tr>
<td>$X$</td>
<td>4 $\mu$m</td>
<td>5 $\mu$m</td>
</tr>
<tr>
<td>$Y$</td>
<td>25 $\mu$m</td>
<td>13 $\mu$m</td>
</tr>
<tr>
<td>$Z$</td>
<td>3 $\mu$m</td>
<td>7 $\mu$m</td>
</tr>
</tbody>
</table>

Table 4.2 Efficiency of both analytical and statistical models in the XY-plane

- $\varepsilon_{6000}$, $\varepsilon_{6000}$: Efficiency and maximum residual for the workpiece milled 6000 rev/min
- $\delta_{DIN}$, $\varepsilon_{DIN}$: Efficiency and maximum residual for the workpiece milled with DIN 8602

### Efficiency in the XZ-plane

<table>
<thead>
<tr>
<th>Axis</th>
<th>Statistical model</th>
<th>Analytical model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{6000}$</td>
<td>$\varepsilon_{DIN}$</td>
</tr>
<tr>
<td>$X$</td>
<td>10 $\mu$m</td>
<td>8 $\mu$m</td>
</tr>
<tr>
<td>$Y$</td>
<td>6 $\mu$m</td>
<td>6 $\mu$m</td>
</tr>
<tr>
<td>$Z$</td>
<td>6 $\mu$m</td>
<td>13 $\mu$m</td>
</tr>
</tbody>
</table>

Table 4.3 Efficiency of both analytical and statistical models in the XZ-plane

- $\varepsilon_{6000}$, $\varepsilon_{6000}$: Efficiency and maximum residual for the workpiece milled with 6000 rev/min
- $\delta_{DIN}$, $\varepsilon_{DIN}$: Efficiency and maximum residual for the workpiece milled with DIN 8602
When the zero point drift in the XY-plane is examined in Y-direction, both models have a very good efficiency in describing this error. However, the maximum remaining error is still 25 µm for the statistical model and 19 µm for the analytical model. The good efficiency is caused by the large actual drift (129 µm). Both models are capable of reducing this zero point drift with more than 100 µm!

The opposite is found in Z-direction. Although both models have a bad efficiency for the DIN-workpiece, the remaining error is very small. In figure 4.28a and 4.28b the verification of both models is depicted in Y-direction for the DIN-workpiece milled in the XZ-plane. It clearly shows a small zero point drift. Although both models give a very bad efficiency, the absolute value of the residual is still limited.

![Graph](image)

**Fig. 4.28a**  Statistical model verified with test workpieces  
*Experiment carried out in the XZ-plane (load: DIN 8602)*

Both models show very good results for stationary states. The zero point drift can be described very accurately. The statistical model realizes an averaged efficiency of 77% in the XZ-plane and 86% in the XY-plane. The analytical model realizes a similar averaged efficiency of 67% in the XZ-plane and 75% in the XY-plane.

For transient states, both models show a smaller efficiency. This is caused by quick temperature changes.
In order to achieve a more accurate model for these transient states further investigation is necessary. The statistical model can be extended by including history in the model; the analytical model can be extended with an experimental part, which should effect higher efficiency for the transient states without any additional sensors.

The statistical model can achieve an averaged efficiency of 71%, the analytical model 59%; whereas the averaged efficiency of the analytical model is deteriorated by the bad efficiency for the transient state in Y-direction in the XZ-plane.

With both models it is possible to describe the thermo-mechanical behaviour of the milling machine. Application of both models will result in a significant improvement of the accuracy of the milling machine. As the analytical and the statistical model show an almost similar efficiency, both models will be applied for the final version of the real-time software error correction.
5. Real-time software error correction

5.1 Introduction
The primary goal of this project is defined as the enhancement of machine tool accuracy with real-time time software error correction. For this purpose a correction algorithm has been developed and implemented into the control system. The correction algorithm is based on the measurements of the behaviour of the machine tool and is split into the following three parts:

1) The basic geometric errors;
2) The finite stiffness errors due to workpiece loads;
3) The thermal behaviour of the machine tool.

The first part of the correction algorithm, i.e. the geometric error correction, is dependent on the reference state of the machine tool and does not change rapidly. In general the compensation values remain constant while machining.

The latter two parts are dependent on the actual state of the machine tool and can change rapidly. Thus, the correction values must be updated while operating the machine tool. Therefore, the architecture of the real-time error correction should be designed with the possibility to change the correction parameters during operation.

In the next sections we will present the development of the error correction system, capable of the compensation for the above mentioned errors. In order to compensate the geometric and finite stiffness errors, the developed individual model (paragraph 3.3) will be applied. To compensate the thermal behaviour of the machine tool two different methods can be applied. These analytical and empirical models showed no significant difference in their modelling capacity to describe the thermo-mechanical behaviour of the machine tool. Therefore, it will be possible to evaluate the real-time error correction, based on each of these models.

Finally, verification measurements have been carried out to show the effectiveness of the real-time error correction and thereby the modelling methodology. The geometric and finite stiffness error correction has been verified with holeplate measurements. With the developed test-workpiece method the effectiveness of the entire error correction system has been verified including the thermo-mechanical behaviour. The results of these validations are presented in the last part of this chapter.
5.2 Software correction system

For the correction of the geometric errors the mathematical model, as developed by TUE (chapter 3), will be applied. With the known error sources, the resulting workpiece-to-tooltip error can be calculated at any position of the machine's carriages. However, as the position of the carriages can change rapidly, the calculation of the correction terms must be performed every cycle of the control systems set point generation. This implies that every $15$ ms a new correction vector must be available to the motion controller. As the machine tool motion controller operates in a real-time environment, no possibilities exist to extend this time.

First version of the real-time software error correction

As mentioned above, the available calculation time in the machine tool controller is limited and experience with real-time software error correction for the developed models was lacking. Therefore, first an error correction has been developed to correct the most significant errors, without major alterations in the machine tool controller [48].

The control system was utilized to store a total of six correction tables. In order to determine which error to compensate, the workpiece-to-tooltip error has been simulated along a line, parallel to the machine's axis, in the center of the production volume. This simulation yields nine error functions that can be applied for correction purpose. From these nine functions the "linearity errors" ($xtx, yty, ztz$) are chosen and the three largest "straightness errors" ($zty, ytx, xty$).

Philips has implemented this preliminary error correction together with the first correction for the thermally induced errors. For checking the effectiveness of the error compensation on the Maho milling machine, the holeplate method and the spindle drift measurement setup have been applied. These verifications showed a good accuracy improvement with this simplified correction method [48]. However, the software error correction can achieve a significant better accuracy improvement, when the full model would be implemented in the controller. As timing problems did not occur, a totally new concept has been developed which facilitates the evaluation of the full model.
Final version of the real-time software error correction

The final version of the real-time error correction software developed in this project uses the full geometric model with 18 compensation functions. Hence, it is capable of correcting both the translational and rotational error terms for the X-, Y- and Z-axes. These error terms can be updated during normal operation of the machine tool to allow changes in the error terms, e.g., due to changes in the thermal state of the machine or workpiece load. The correction has been implemented in a Philips CNC 3000 F700 Numeric Control.

Global description

The real-time error correction system is depicted in figure 5.1. A program running on an external PC reads the sensor data and computes the 18 translational and rotational error terms by adding the error contributions of the temperature and workpiece load to the geometry errors. The thermo-mechanical error model used is either the empirical error model or the analytical error model.

![Diagram of the real-time error correction system](image)

*Fig. 5.1 Overview of the real-time error correction system.*
The error terms are stored in tables. These tables are sent to the CNC every 60 seconds. The CNC reads the values of the translation and rotation error terms corresponding to the axis positions. With the aid of the full geometric error model, including tool length and orientation, the CNC computes the resulting workpiece to tooltip errors in the X, Y and Z positions. The corrections for these errors are added to the position readings of the measuring scales.

Detailed description

5.2.1 The model for the effect of error terms on workpiece-to-tooltip position

The model of the machine tool developed at the TUE describes how the error terms xtx, xty, ..., xrx, xry, ..., zrz affect the workpiece-to-tooltip position, depending on the axis positions, machine construction parameters and tool length and orientation. The workpiece-to-tooltip errors can be written as:

\[
\begin{align*}
\text{wp}^{e_{\text{lx}}} &= f_1 + f_2 + f_3 + g_1 + h_1 \\
\text{wp}^{e_{\text{ly}}} &= f_4 + f_5 + f_6 + g_2 + h_2 \\
\text{wp}^{e_{\text{lz}}} &= f_7 + f_8 + f_9 + g_3 + h_3
\end{align*}
\]

With, for the MAHO 700S (with fixed lengths in mm):

\[
\begin{align*}
f_1 &= 95*xry - 1190*xrz - xtx \\
f_2 &= 235*yry - 1122.5*yrz - Y*yrz - ytx \\
f_3 &= 210*zrz - 215*zry + Z*zry + ztx \\
f_4 &= -95*xrx + 340*xrz + X*xrz - xty \\
f_5 &= -235*yrx - yty \\
f_6 &= 215*zrx - Z*zrx + zty \\
f_7 &= 1190*xrx - 340*xry - X*xry - xtz \\
f_8 &= 1122.5*yrx + Y*yrx - ytz \\
f_9 &= -210*zrx + ztx \\
g_1 &= -Y*xrz - Z*(xrx+yry) \\
g_2 &= Z*(xrx+yrx) \\
g_3 &= Y*xrx
\end{align*}
\]
In the equations [5.1] to [5.18], the X, Y and Z positions are defined relative to the Machine Reference Point, and L_x, L_y, L_z the three tool vector components, constituted by the tool's length and orientation.

Here, the contributions to the correction terms are separated into three types:

1) \( f_i \) - functions of one position only, e.g., ztx, but also -215*zry and Z*zry
2) \( g_i \) - functions of one position, multiplied by a position on a different axis, e.g., Z*yry
3) \( h_i \) - functions of one position, multiplied by a tool vector component, e.g., -L_z*zry.

5.2.2 Functional partition of the real-time error correction system

The machine model described in the previous subsection is used in the real-time software error correction system. Figure 5.2 depicts the real-time correction system in relation to its environment, with the input and output data.

The inputs are:

- The nominal geometric error functions;
- The temperature and workpiece load data to calculate the error terms;
- The information of axis positions, machine parameters and actual tool.

The outputs are:

- The compensations for the X, Y and Z axis;
- The values of the compensations for display on the CNC's screen.
Fig. 5.2  Data and control context diagram of the real-time software error correction system.

Fig. 5.3  Highest level data and control flow diagram of the real-time software error correction system.
Figure 5.3 shows the functionality of the real-time software error correction system. Process 1 in this diagram is the process on the external PC. It reads the nominal geometric errors and the sensor data, computes the error terms (Subsection 5.2.3) and sends the error tables (Compensation_table) to the CNC. The error tables (Subsection 5.2.4) are the interface between the external PC and the CNC. Process 3 reads the tables and stores them in the CNC. Process 2 computes the values of the tool vector components (Subsection 5.2.5) Process 4 computes the X, Y and Z compensations (Subsection 5.2.6). Process 5 outputs the calculated compensations to the motion control system (Subsection 5.2.7). Process 6 shows the compensations on the CNC's screen.

Processes 1 and 3 are activated every 60 s, processes 4 and 5 every 15 ms, process 6 every 200 ms, and process 2 when the tool's length and/or orientation changes.

5.2.3 Calculation of the error terms
The translational and rotational error terms xtx, xty, ..., ztz, xrx, xry, ..., zrz are composed of the nominal geometric errors of the machine tool, a contribution from thermal distortion and a contribution from distortion due to workpiece load. The nominal geometric errors are stored on the external PC. Every 60 seconds, the PC reads the temperature sensors on the machine tool and computes the thermal contributions to the translational and rotational error terms. These contributions are added to the geometric errors. In order to save computing time on the CNC, the error terms fi are calculated on the PC. The terms fi and the rotational terms xrx, xry, ..., zrz necessary to compute gi and hi are stored in tables which are sent to the CNC every 60 seconds. More details on the geometric and thermo-mechanical error correction are given in paragraph 5.3 and 5.4, respectively.

5.2.4 The error tables
The error terms fi and the terms xrx, ..., zrz are discretised on a fixed (in this case, 10 mm) grid. This yields an approximation to the error term as shown in figure 5.4.
Fig. 5.4 Grid of point $p_j$ on axis $p$ and discretisation of the error terms.

With $prq: p \in \{X, Y, Z\}$ and $q \in \{X, Y, Z\}$, which results in 9 rotational errors according to the definition presented in paragraph 2.4.

- $f_i$: Compensation function (equation 5.4 to 5.12)
- $f_{ij}$: Compensation value $j$ for compensation function $f_i$
- $p_{prq}$: Compensation value $j$ for compensation function $prq$

The discretised values of the terms $f_i$ and $xrx, ..., zrz$ are stored in 6 error tables:

1) A table for $f_1$, $f_4$, $f_7$ (functions of $X$);
2) A table for $f_2$, $f_5$, $f_8$ (functions of $Y$);
3) A table for $f_3$, $f_6$, $f_9$ (functions of $Z$);
4) A table for $xrx$, $xry$, $xrz$ (functions of $X$);
5) A table for $yrx$, $yry$, $yrz$ (functions of $Y$);
6) A table for $zrx$, $zry$, $zrz$ (functions of $Z$).

Every 60 seconds, these error tables are read by the correction software running on the CNC. To allow fast access, they are copied to three internal correction tables: one table for the functions of $X$, one for those of $Y$, and one for those of $Z$, respectively.
5.2.5 The tool vector

In the formulas for the workpiece-to-tooltip errors in Subsection 5.2.1, the three tool vector components \( L_x, L_y \) and \( L_z \) appear. The values of these components depend on the tool's length and its orientation, i.e., horizontal or vertical milling and, in the latter case, the position of the C axis:

For the vertical spindle:
\[
\begin{align*}
L_x &= \sin(aC)*(200+L); \\
L_y &= \cos(aC)*(200+L); \\
L_z &= 0.
\end{align*}
\]

For the horizontal spindle:
\[
\begin{align*}
L_x &= 0; \\
L_y &= 150; \\
L_z &= L-149.
\end{align*}
\]

With:
- \( aC \): the angle between the Y axis and the vertical milling head at a position C of the C-axis;
- \( L \): the tool length of the current tool as stored in the tool memory.

The tool vector may need to be recalculated when a tool change occurs, when switching from horizontal to vertical milling v.v., or when the C-axis position changes. In the former two cases, it suffices to recalculate the tool vector after a tool change. In the latter case, the tool vector should be recalculated after any change of the C-axis position. In principle, this requires the evaluation of sine and cosine terms every 15 ms, which would mean an unwanted heavy load of the CNC's processor. Hence it was decided to adapt only the tool vector when the C-axis would have reached an arbitrary fixed position; reaching of such a fixed position can be indicated to the CNC by programming of an M10 function (C-axis clamp). Milling with fixed C-axis positions covers most of the applications that require high accuracy.

5.2.6 Computation of the compensations

Every 15 ms, the internal correction tables are searched for the error terms \( f_i \) and \( p_{jq} \) that correspond to the actual X-, Y- and Z-axis positions (figure 5.4). From the actual positions and the terms \( p_{jq} \) the functions \( e_{ij} \) are computed; from the current tool vector components and the terms \( p_{jq} \) the functions \( h_{ij} \) are computed (cf. Subsection 5.2.1). Next, the workpiece to tooltip errors are calculated with the formulas of Subsection 5.2.1.
The Maho 700S is a machine in which movements in the X- and/or Y-axes change the position of the workpiece with respect to a fixed reference frame, whereas movements in the Z-axis change the position of the tool with respect to a fixed reference frame. Therefore, the compensations for the X- and Y-axes are:

\[
\begin{align*}
X_{\text{cmp}} &= e^{t \cdot u_x} \\
Y_{\text{cmp}} &= e^{t \cdot u_y}
\end{align*}
\]

whereas the compensation for the Z-axis is:

\[
Z_{\text{cmp}} = e^{t \cdot l_z}
\]

These compensations must be added to the measuring scale readings of the X-, Y- and Z-axes.

5.2.7 The compensation dosing system

The compensations \(X_{\text{cmp}}, Y_{\text{cmp}}, Z_{\text{cmp}}\) are functions of place and time. Since both the place and the time dependencies are discretised (10 mm grid, 60 s temperature monitoring, respectively), spatial and temporal changes of several resolution steps (\(\mu m\)) at a time may occur in these functions (figure 5.5). In case of such a spatial step, limit cycles or "forbidden areas" around a grid point result; in case of a temporal step, a visible edge in a workpiece will appear. A compensation dosing system is added to the real-time software error correction to smooth those steps.

![Fig. 5.5 Temporal and spatial smoothing of compensation output.](image-url)
In the real-time correction developed for the Maho 700S milling machine, it appeared that with a gridpoint spacing of 10 mm, no spatial steps larger than 1 μm occurred. Therefore, no special measures for spatial dosing needed to be taken, and only a temporal dosing system has been added. It causes the actual compensation for an axis to be changed by +/- 1 μm at most in n*15 ms, so that any temporal compensation steps are smoothed.

5.3 Geometric error correction
As presented in the previous paragraph, an error correction has been developed, which uses the full geometric error model with 18 compensation functions. Hence, it is capable of a correction for both the translational and rotational error terms for the X-, Y- and Z-axis. These error terms can be updated during normal operation of the machine to allow changes in the error terms, e.g. due to changes in the thermal state of the machine or workpiece load.

The geometric error correction is based on the individual model, as developed by TUE (paragraph 3.2). The parameters of the individual model are stored on an external PC. The correction terms are determined on this PC, by evaluation of the individual model.
In order to minimize the number of calculations in the machine tool controller the contributions to the correction terms are separated into three types: f-, g- and h-functions (Section 5.2.1). In order to save computing time, the f-functions are calculated on the external PC and directly downloaded to the controller. The g- and h-functions are evaluated in the controller, using the compensation functions prq, which are stored in tables (figure 5.4).

f-functions
The f-functions are determined by simulation, using the full geometric error model (including the finite stiffness model). For computation in the controller, expressing positions with respect to the Machine Reference Point is favoured. Therefore, the workpiece-to-tooltip errors are simulated along a line through the Machine Reference Point (X=690, Y=490, Z=590) and parallel to the machine’s axes. The compensation functions are set to zero in this Machine Reference Point, which means that the machine is not compensated when it is positioned on the Machine Reference Point.
The simulation yields nine error functions that can be applied for correction purposes. In figure 5.6 the compensation function \( f_4 \) is depicted. This compensation function corrects the straightness error \( x_{ty} \), between the tool and workpiece, when the X-axis is moved from 0 to 700 mm with the Y- and Z-axis on the Machine Reference Point.

**Fig. 5.6  Compensation function \( f_4 \) (simulation of \( x_{ty} \) with respect to the Machine Reference Point)**

**g- and h-functions**
Besides these nine translational compensation functions, which are a function of the position of one axis, also the contributions of the g- and h-functions have to be determined. These functions describe the compensation terms, dependent on the position of more than one axis and on the tool length (L). These functions are evaluated by the controller, using the prq compensation tables (figure 5.4). Again, simulations are carried out to determine the nine compensation tables with respect to the Machine Reference Point.
Special precautions had to be taken with respect to the squareness errors. The contributions of these squareness errors is also evaluated during the simulation of the nine translational f-functions. However, the squareness errors are modelled as a constant rotational error (the rotational errors start with an offset). In order to avoid that the squareness error is corrected twice, the rotational compensation functions have to be set to zero in the Machine Reference Point. Then, the contribution of the squareness errors is eliminated.

With this simulation, a total of nine compensation tables are created and sent to the controller of the machine tool. An example of one of these functions is depicted in figure 5.7 (the rotational error zrz).

As the available calculation time is limited, numerical multiplications or divisions have to be avoided. Therefore, the values of the functions prq are given in units of $2^{-21}$ radians ($\approx 0.098355$ arcsec). This is done to adapt the range and resolution of the rotation terms to the ones of the table entries; the choice of the factor $2^{-21}$ allows a fast conversion (by bit-shift) to the units of the compensations.
The geometric errors due to imperfect movements of the carriages will not change rapidly. Several calibrations over a long period of time showed that a calibration period of one year is advisable (paragraph 3.4). Within this period, it is allowed to store the parameters of the geometric error model (on the external PC).

**Finite stiffness errors**

Besides the errors in the geometry of the machine tool, also the finite stiffness error have to be corrected. From these errors, only the errors due to a workpiece load (type 3) have to be corrected (paragraph 3.3).

The developed error model, as presented in paragraph 3.2, is capable of modelling these finite stiffness errors. Therefore, the finite stiffness correction will be based on this model. Whenever, a workpiece is placed upon the milling machine, the geometric errors change, dependent on the weight of the workpiece. Experiments have shown that only the errors of the X-axis change significantly; the workpiece load has no influence on the geometric errors of the other axes. Based on the workpiece load the additional geometric errors are calculated and added to the matching nominal geometric errors. With these error functions, the compensation terms are determined as described above and sent to the machine tool controller.

As the update of the software correction parameters takes place every 60 sec, it is not allowed to start a milling process within 60 sec after the workpiece has been placed upon the machine. Also the determination of the workpiece zero point should be carried out after 60 sec. Otherwise, a large error will occur in the position of the workpiece zero point. This is not a real limitation:

1) After a workpiece is placed upon the machine, it has still to be fastened, which takes most of the times more than 60 sec.

2) When the machine is running in mass production, with batches of similar products, the fastening of the workpiece can be carried out within these 60 sec. However, in mass production, the weight of the products is constant, as the products are similar. After fastening the first product, the correct finite stiffness compensations have to be determined. As the correction parameters are updated every 60 sec, this will take a maximum of 60 sec. After the compensation values have been determined once, the finite stiffness compensation can be treated like the geometric compensation. This means that during the production of the whole batch of products, the compensations values are kept constant.
Scale and workpiece expansion

The geometric errors are modelled at a reference temperature of 20°C. However, both the milling machine and the workpiece will not have a temperature of 20°C during normal operation. Therefore, the influence of this temperature difference has to be compensated. This temperature influence is part of the thermo-mechanical behaviour of the machine tool, which is actually described in the next paragraph. However, as both the expansion of the scales and the workpiece have a significant influence [48], a verification of the developed method is not possible without correction for these expansions. Therefore, the compensation method for these errors sources is discussed in this paragraph.

The X-, Y- and the Z-scales are provided with three temperature sensors; one on both edges and one in the middle. It is assumed that there is a constant gradient between two of these three sensors; this results in an error which is a quadratic function of the position between the two sensors (figure 5.8).

Again the errors are modelled additionally: the expansion of the X-scale is added to function \( f_1 \), the expansion of the Y-scale to function \( f_2 \), and the expansion of the Z-scale to function \( f_3 \). The expansion coefficient of the rulers is determined by PTB: \( \alpha_{\text{ruler}} = 9.75 \, \mu m/\text{m}^°\text{C} \).

![Diagram of temperature sensors and compensation function](image)

*Fig 5.8  Compensation due to temperature gradient along the Y measuring scale*
Also the expansion of the workpiece is compensated. Therefore, the temperature of the workpiece has to be measured. Usually, this is carried out with a temperature sensor attached to the workpiece. However, when a workpiece is milled with cooling liquid, it is also possible to measure the temperature of this cooling liquid, as the temperature of the cooling liquid is equal to the temperature of the workpiece [48]. For the experiments, carried out in this project, always a temperature sensor was attached to the workpiece.

Assumed was a homogeneous temperature distribution in the workpiece: thus, the temperature of the workpiece could be measured with one sensor. The expansion is modelled linear using an expansion coefficient $a$, which can be entered into the correction system as a parameter. The expansion is determined with respect to the workpiece reference point. Again, the compensation terms are treated additionally. As the temperature of the scales and the workpiece vary in time, the compensation functions are evaluated and sent to the controller every 60 sec.

*Verification of the real-time software error correction*

In order to validate the efficiency of the developed real-time software error correction system, verification experiments have been carried out. First, the geometric and finite stiffness error correction has been verified. This verification could be carried out with laser measurements. However, these experiments can only be carried out in one dimension. Software correction for errors which have effect in two dimensions, like the squareness errors, can only be validated with laser measurements along a diagonal within the machine's working volume. Thus, a proper verification would result in a large number of experiments. Therefore, an indirect measuring method has been applied to validate the efficiency of the real-time software error correction: the holeplate.

This holeplate is developed to calibrate coordinate measuring machines. In order to verify the real-time error correction, the holeplate will be applied to the Maho milling machine, by using the milling machine as a measuring machine. This is possible as the real-time error correction is not only able to improve the positioning accuracy of the machine tool, but also capable of a compensation of the obtained measurement results.
The holeplate is a two-dimensional object. The artifact consists of a grid of holes on equal distances. By measuring these holes, and knowing the deviation from the nominal distance, it is possible to evaluate the performance of the error correction, with respect to the geometric and finite stiffness errors.

The artifact is placed upon the machine in three different planes: XY-, XZ- and YZ-plane, so that the different locations of the holeplate constitute the sides of a cube. Normally, this setup is applied to determine all 21 geometric error components of a machine consisting of three linear axes [52]. Hence, when the holeplate is measured in these three planes, all compensated geometric and finite stiffness errors are verified.

The holeplate has been measured twice on the machine tool. Once, without any correction, and once with the full error correction, including the compensation for the scale and workpiece expansion. In figure 5.9 the results of the validation in the XZ-plane are depicted.

![Holeplate experiment XZ-plane](image)

**Fig. 5.9** Validation of the real-time error correction with holeplate measurements. The holeplate is measured in the XZ-plane, with and without real-time software error correction.

Similar experiments have been carried out in the XY-plane (figure 5.10) and the YZ-plane (figure 5.11).
Fig. 5.10 Validation of the real-time error correction with holeplate measurements. The holeplate is measured in the XY-plane, with and without real-time software error correction.

Fig. 5.11 Validation of the real-time error correction with holeplate measurements. The holeplate is measured in the YZ-plane, with and without real-time software error correction.
These three holeplate experiments, implicitly check the developed real-time software error correction for the different machine configurations. For the experiment in the XZ-plane, the vertical milling head is applied. The holeplate is measured with the probe in the horizontal spindle in the XY-plane. In the YZ-plane, the vertical milling head is applied, but with the C-axis rotated 60° (picture 5.1).

![Location of the hole plate for testing the geometric error compensation.](image)

The validation experiments show an impressive accuracy improvement of the machine tool with the developed real-time software error correction for the geometric and finite stiffness errors. In the next table (table 5.1), an overview is depicted of the achieved efficiency, together with the maximum remaining error. The efficiency is calculated as the ratio of the remaining error with correction to the remaining error without correction. The efficiency is calculated over the entire experiment. Thus, all measurements of the holes are included in this ratio. The maximum remaining error is the maximum absolute error found during the experiment. This maximum error is given to get a more complete impression of the real-time error correction: a good efficiency can be achieved with large errors. However, a small efficiency is calculated when the absolute errors, without correction, are already very small. But, this does not mean that the performance of the real-time error correction is bad.
Table 5.1 Efficiency of the real-time error correction:
Validation of the geometric and finite stiffness error correction

$\delta$: Efficiency of the model
$\varepsilon$: Maximum remaining error (2-D).

<table>
<thead>
<tr>
<th>Plane:</th>
<th>XY-plane</th>
<th>XZ-plane</th>
<th>YZ-plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency $\delta$</td>
<td>87%</td>
<td>65%</td>
<td>92%</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{without}}$</td>
<td>80 $\mu$m</td>
<td>25 $\mu$m</td>
<td>109 $\mu$m</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{with}}$</td>
<td>11 $\mu$m</td>
<td>11 $\mu$m</td>
<td>11 $\mu$m</td>
</tr>
</tbody>
</table>

In table 5.1 the achievements of the real-time error correction are clearly shown. The efficiency of the model is far beyond the goal of 70%. Only in the XZ-plane, the efficiency is 65%. However, this is mainly due to the small errors present in this plane. For each plane, the maximum error is 11 $\mu$m. This means that, with software error correction, no errors are found larger than 11 $\mu$m, which is an impressive performance.

As already discussed in paragraph 3.4, this maximum residual could even be smaller, when the validation had been carried out immediately after the calibration. Now, the validation has been carried out a half year after the calibration. In this period, the geometric errors slightly changed due to wear. This has deteriorated the achievements of the real-time software error correction a little. However, with a calibration period of one year, the maximum residual will be around 10 $\mu$m, whereas the maximum error in the machine tool without correction is more than 100 $\mu$m!

Besides the holeplate experiments, also the developed test-workpiece method (paragraph 3.5) is suitable to check the geometric part of the real-time software error correction. However, during the milling of the first stage, the temperatures inside the machine tool change. Thus, there will always be an interference between the thermo-mechanical behaviour and the geometric errors of the milling machine.
In the next three figures (figure 5.12, 5.13 and 5.14), the accuracy improvement is depicted, extracted from test-workpieces. Figure 5.12 shows the errors in the position of the holes in one direction, when the workpiece is milled with and without correction. In figure 5.13, a two dimensional representation of the error in the location of the holes is given. In figure 5.14, the accuracy improvement in Y-direction is presented for a workpiece milled in the XZ-plane. In next paragraph, a complete discussion of the validation with test-workpieces of the real-time error correction will be presented.

![Graph](image)

**Fig. 5.12**  Errors of position in X-direction: Corrected/Uncorrected
Fig. 5.13  Errors in the position of the holes in X- and Z-direction: Corrected/Uncorrected

Fig. 5.14  Errors in Y-direction: Corrected/Uncorrected
With the developed real-time software error correction, an overall accuracy improvement is achieved with an efficiency of 81% for the geometric and finite stiffness errors. Thereby, the maximum remaining error will be around 11 μm, when a calibration period of one year is considered. In the next paragraph, the real-time error correction of the thermo-mechanical behaviour will be discussed. The real-time software correction for these errors will be validated with test-workpieces.

5.4 Thermo-mechanical error correction

Besides the finite stiffness and geometric errors, also an error correction for the thermo-mechanical behaviour has been developed. To describe this behaviour of the machine tool two different models have been derived. As the verification experiments, based on simulations, showed similar results for both models, the real-time software error correction will be carried out for each models. In this paragraph, the results of both methods will be presented.

In paragraph 5.2, the applied software correction system is discussed. This system is partly implemented on the machine tool controller and partly on an external PC. In this way, an arbitrary number of measurement equipment can be applied without major changes to the hardware of the machine, for example to measure the temperature distribution on the machine tool. A more significant advantage of this architecture is that any developed model can be evaluated in the real-time error correction. The only restriction is that the software correction parameters are updated every 60 sec. However, as the temperatures on the machine tool do not change that rapidly, this is no significant problem for the correction of the thermo-mechanical behaviour.

In paragraph 3.3 and 4.2, a dedicated software package is described, which has been applied for all experiments. This software package has been extended with an error correction module, which enables the implementation of both the empirical and analytical model.
Software interface

In order to enable the implementation of both models, a software interface has to be defined, so that both the software package and the software correction module can exchange data by this interface.

As the software package has been written in Turbo Pascal, the module, which will determine the thermally induced errors as a function of the temperature distribution on the machine tool, should also be written in this language.

The thermo-mechanical error correction module should carry out two procedures. First, an initialization has to be carried out to determine a reference situation. Secondly, after this initialization, the model can be evaluated every time interval desired.

1) Initialization:
In order to accomplish the initialization the following parameters are necessary:
• Temperature:
  Initial temperature distribution on the machine tool has to be known.
• Workpiece zero point:
  The workpiece expansion is calculated with respect to the workpiece zero point. This zero point has to be passed to the module.
• Horizontal or vertical spindle:
  It is possible to mill with either the horizontal or vertical spindle. As the machine structure is different for each configuration, this information has to be passed to the unit.
• Initialization flag:
  This flag indicates that the initialization procedure has to be carried out.
2) Evaluation:
The following parameters are necessary to evaluate the model:

- Temperature:
  The temperature distribution on the machine tool has to be known.

- Horizontal or vertical spindle:
  It is possible to mill with either the horizontal or vertical spindle.
  As the machine structure is different for each configuration, this information has to be passed to the unit.

- Initialization flag:
  This flag indicates that the initialization procedure has not to be carried out.

The thermo-mechanical correction module returns the correction parameters to the software package. As both models treat this compensation differently, the returned results are presented in the following subsections.

Both the analytical and empirical real-time error correction have been validated with test-workpieces. A total of nine workpieces have been milled on the machine tool. Three workpieces without correction; three workpieces with a correction based on the analytical model; and three workpieces with a correction based on the empirical model. In the next two subsections the thermo-mechanical error correction based on both models will be presented, together with the results of the validation experiments.

5.4.1. Analytical model
PTB implemented a correction module which applies the analytical model to calculate the most significant thermomechanical errors of the Maho 700S milling machine and the workpiece expansion. The correction module has been tested with three test workpieces: one workpiece in the XY-plane and two workpieces in the XZ-plane of the milling machine. The thermally induced errors are calculated from the temperature changes referred to the temperature distribution at the start of the machining process.
The following error components were corrected:

1) The zero point drift;
2) The thermally induced changes of the error components of the linear axes (position errors, squareness errors);
3) The thermal expansion of the workpiece.

The calculated zero point drift considers all significant deformations in the machine structure between the workpiece and tool. The correction of thermally induced errors of position considers expansion of the scales and the bending of the slides. The correction of the squareness errors consider the bending of and between the slides.

The correction procedure is divided into an initialization sequence and a correction sequence:

1) At first the initialization sequence is started with the machining process. At this time the current temperature distribution at the machine structure is defined as the reference situation and stored in the program. For the initialization the module reads the mentioned parameters: tool length and orientation, temperatures (reference situation) and workpiece position.

The thermo-mechanical errors (output) are zero in the reference situation. At the end of the initialization procedure the correction sequence is active.

2) The correction sequence reads the current temperatures and calculates all mentioned errors. The thermal expansion of the workpiece is considered in the position errors. Thus the following output parameters are available for the correction system:

• Components of the zero point drift:
  -ntx (tool drift in X-direction);
  -nty (tool drift in Y-direction);
  -ntz (tool drift in Z-direction).

• Change of position errors:
  -xtx (factor of X-scale);
  -yty (factor of Y-scale);
  -ztz (factor of Z-scale).
- Change of squareness errors:
  - zwx (angle between Z- and X-axis);
  - zwy (angle between Z- and Y-axis);
  - ywx (angle between X- and Y-axis).

In order to obtain full compatibility between the thermo-mechanical error model and the real-time error correction the zero point drift is calculated in two steps:

1) The zero point drift is determined for the center point of the test workpieces (see chapter 4.3).

2) In a second step the drift components are transformed to the reference position \( X_{\text{ref}}, Y_{\text{ref}}, Z_{\text{ref}} \) for which the 21 error components are defined. The calculation of an arbitrary tool position \( x,y,z \) considers that the thermally induced changes of the error components and the zero point drift are superimposed.

**Validation**

A total of six test workpieces have been milled on the machine tool. Three were machined without correction and 3 test workpieces were machined with a geometric error correction and the analytical thermo-mechanical error correction. The workpiece errors of the corrected milling process are compared to the workpiece errors of the uncorrected process. The thermally induced relative changes (maximum-minimum) of the workpiece errors and the resulting average efficiency of the correction were determined. The results and plots are created using the evaluation program for test workpieces. In Appendix B.1, the complete package of verification experiments is presented.

The measured components of the zero point drift for the corrected and the uncorrected machine tool are compared in figure 5.15, 5.16 and 5.17 for respectively the drift in X-direction, the drift in Y-direction and the drift in Z-direction. The geometric error correction has no effect on the efficiency of the drift correction, as the drift is determined relatively to the start time. In the next table (table 5.2) the performance of the analytical thermo-mechanical error correction is presented for the zero point drift.
Fig. 5.15  Zero point drift in X-direction: Corrected/Uncorrected
XY-plane, N=DIN-Spectrum.

Fig. 5.16  Zero point drift in Y-direction: Corrected/Uncorrected
XZ-Plane, N=DIN-Spectrum.
Fig. 5.17 Zero point drift in Z-direction: Corrected/Uncorrected XZ-plane, N=6000 rpm.

<table>
<thead>
<tr>
<th>Efficiency of the real-time error correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero point drift:</td>
</tr>
<tr>
<td>efficiency δ</td>
</tr>
<tr>
<td>residual ε_{without}</td>
</tr>
<tr>
<td>residual ε_{with}</td>
</tr>
<tr>
<td>ntx</td>
</tr>
<tr>
<td>nty</td>
</tr>
<tr>
<td>ntz</td>
</tr>
<tr>
<td>69 %</td>
</tr>
<tr>
<td>72 %</td>
</tr>
<tr>
<td>59 %</td>
</tr>
<tr>
<td>37 μm</td>
</tr>
<tr>
<td>95 μm</td>
</tr>
<tr>
<td>124 μm</td>
</tr>
<tr>
<td>12 μm</td>
</tr>
<tr>
<td>24 μm</td>
</tr>
<tr>
<td>30 μm</td>
</tr>
</tbody>
</table>

Table 5.2 Efficiency of the analytical thermo-mechanical error correction:

Validation of the zero point drift error correction

δ: Efficiency of the model

ε: Maximum remaining error.
Similar to the validation of the geometric error correction, the error in the position of the holes for the corrected and uncorrected machine tool can be compared. In figure 5.18 the linearity error is depicted as a function of the elapsed time. The error is presented with the first order coefficient, which has been obtained by a linear least square fit through the positioning errors in the holes.

![Graph](image)

**Fig. 5.18** Error of position in X-direction: Corrected/Uncorrected XY-plane, N=DIN-Spectrum.

The positioning error is represented as a function of the elapsed time by its first order coefficient, obtained by a linear least square fit through the errors in the location of the holes.

Only the additional compensation due to temperature variations of the workpiece and scales are considered. Therefore, the errors of position can be characterized with the first order coefficient. Similar to the validation of the geometric error correction, the averaged efficiencies and the maximum remaining error of the real-time error correction based on the analytical thermo-mechanical model are calculated for these workpiece and scale expansions (table 5.3).
Table 5.3 Efficiency of the analytical thermo-mechanical error correction:

<table>
<thead>
<tr>
<th>Position error:</th>
<th>xtx</th>
<th>yty</th>
<th>ztz</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency δ</td>
<td>62 %</td>
<td>-9%</td>
<td>65 %</td>
</tr>
<tr>
<td>residual ε\text{without}</td>
<td>60 µm/m</td>
<td>12 µm/m</td>
<td>45 µm/m</td>
</tr>
<tr>
<td>residual ε\text{with}</td>
<td>16 µm/m</td>
<td>13 µm/m</td>
<td>23 µm/m</td>
</tr>
</tbody>
</table>

Table 5.3  Efficiency of the analytical thermo-mechanical error correction:

\( \delta \): Efficiency of the model

\( \varepsilon \): Maximum remaining error.

The measured squareness errors of the workpieces for the corrected and uncorrected machine tool are compared in figure 5.19. Thermally induced squareness errors between Y- and X-axis (ywx), between Z- and X-axis (zwx) and between Z- and Y-axis (zyw) are represented by different workpiece features:

- ywx and zwx correspond to the squareness error present in the workpiece, milled respectively in the XY- and the XZ-plane;
- zyw cannot be determined from workpiece feature of the milled workpieces but changes of zyw correspond to the changes of orientation of the workpiece plane (w-direction) as shown in figure 5.20.
Fig. 5.19  Squareness error: Corrected/Uncorrected
XZ-plane, N=6000 rpm.

Fig. 5.20  Drift of workpiece orientation: Corrected/Uncorrected
XZ-Plane, N=6000 rpm.
Similar to the position errors, the efficiencies of the analytical thermo-mechanical correction are calculated separately from the geometric error correction, which is superimposed (table 5.4).

<table>
<thead>
<tr>
<th>Efficiency of the real-time error correction</th>
<th>ywx</th>
<th>zwx</th>
<th>zwy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squareness error:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>efficiency $\delta$</td>
<td>-91%</td>
<td>72%</td>
<td>22%</td>
</tr>
<tr>
<td>residual $\epsilon_{\text{without}}$</td>
<td>14 $\mu$rad</td>
<td>79 $\mu$rad</td>
<td>160 $\mu$rad</td>
</tr>
<tr>
<td>residual $\epsilon_{\text{with}}$</td>
<td>22 $\mu$rad</td>
<td>23 $\mu$rad</td>
<td>68 $\mu$rad</td>
</tr>
</tbody>
</table>

Table 5.4  
Efficiency of the analytical thermo-mechanical error correction:

Validation of the squareness error present in the workpiece.

$\delta$: Efficiency of the model (see equation 4.3)

$\epsilon$: Maximum remaining error.

The corrected and uncorrected workpiece geometries are depicted in figure 5.21 for the errors in Y-direction and in figure 5.22 in X- and Z-direction. The absolute errors of the axial vector components with respect to the required workpiece geometry are determined. From these vector components the worst case which is the maximum absolute error is given for each workpiece in table 5.5, together with the achieved efficiencies.

<table>
<thead>
<tr>
<th>Efficiency of the real-time error correction</th>
<th>XY, DIN</th>
<th>XZ, 6000</th>
<th>XZ, DIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workpiece geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>efficiency $\delta$</td>
<td>68%</td>
<td>73%</td>
<td>71%</td>
</tr>
<tr>
<td>residual $\epsilon_{\text{without}}$</td>
<td>117 $\mu$m</td>
<td>131 $\mu$m</td>
<td>61 $\mu$m</td>
</tr>
<tr>
<td>residual $\epsilon_{\text{with}}$</td>
<td>29 $\mu$m</td>
<td>39 $\mu$m</td>
<td>29 $\mu$m</td>
</tr>
</tbody>
</table>

Table 5.5  
Efficiency of the analytical thermo-mechanical error correction:

Validation of the errors in the workpiece geometry.

$\delta$: Efficiency of the model (see equation 4.3)

$\epsilon$: Maximum error present in the workpiece.
Fig. 5.21  Errors in Y-direction: Corrected/Uncorrected
XZ-plane, N=6000 rpm.

Fig. 5.22  Errors in X- and Z-direction: Corrected/Uncorrected
XZ-Plane, N=6000 rpm.
The results show that the efficiency of the thermo-mechanical error correction depends on the scale of the corrected error. The most significant machine tool errors which are the zero point drift components can be corrected with high efficiency. Even a low efficiency leads to significant reduction of the maximum error (see results of ntz).

The position errors of the machine tool and of the workpiece expansion are corrected sufficiently for the X- and Z-direction. The thermally induced changes of the position error \( y_{txy} \) are to small (\(< 15 \, \mu m/m\)) to be corrected in this case. However, in general the correction of the position errors is important for all directions to correct the workpiece expansion during machining because the workpiece temperature varies.

The squareness error \( z_{wxy} \) and \( z_{wy} \) are corrected sufficiently (figure 5.19 and 5.20). The reduction of the maximum squareness error of 90 \( \mu \text{rad} \) means a reduction of the maximum workpiece error of 20 \( \mu m \) (for a workpiece length of 220 mm). The thermally induced changes of squareness error \( y_{wxy} \) are not significant (\(< 16 \, \mu \text{rad}\)) and cannot be corrected with the used algorithms.

The results show that the zero point drift and thermally induced changes of the error components of the linear axes are significant for the machining accuracy and can be corrected for the most part using an analytical model of the machine tool.

### 5.4.2 Empirical model

Contrary to the analytical model, the empirical model is based on a model of the zero-point drift. While, for correction purposes, the principal interest is not the deformation of each machine component, but the displacement of the tool with respect to the workpiece.

However, the drift of the tool holder is dependent on the position of the Z-axis. This is caused by a significant change in the error components of the Z-axis. Therefore, the empirical model is determined on three different Z-positions. The compensation values between these gridpoints are obtained by linear interpolation. By modelling the zero-point drift dependent on the position of the Z-axis, the error components of the Z-axis are implicitly modelled.
Like the analytical thermo-mechanical correction module, the correction sequence for the empirical model is divided into an initialization sequence and a correction sequence:

1) The initialization is started with the machining process. At this time the temperature distribution is defined as the reference situation and stored in the program. The initialization does not consider the expansion of the workpiece and scales, as these are compensated with respect to a reference temperature of 20°C, which is mostly not equal to the temperature at initialization.

2) The correction sequence reads the current temperatures and calculates the drift of the tool holder on three different Z-positions. This will result in a zero-point drift, which is a function of the position of the Z-axis:
   - Components of the zero point drift:
     - \( n_{tx} \) (tool drift in X-direction as a function of Z);
     - \( n_{ty} \) (tool drift in Y-direction as a function of Z);
     - \( n_{tz} \) (tool drift in Z-direction as a function of Z).

Besides the zero-point drift, also the scale and workpiece expansion is calculated (see paragraph 5.3). These expansions are described in the translational errors \( x_{tx}, y_{ty} \), and \( z_{tz} \).

- Workpiece and scale expansion:
  - \( x_{tx} \) (function of X);
  - \( y_{ty} \) (function of Y);
  - \( z_{tz} \) (function of Z).

**Validation**

A total of six test workpieces have been milled on the machine tool to test the empirical error correction. Three were machined without correction and three test workpieces were machined with the geometric error correction and the empirical thermo-mechanical error correction. The workpiece errors of the corrected milling process are compared to the workpiece errors of the uncorrected process. Similar to the geometric error correction, the maximum remaining error in the workpiece and the resulting average efficiency of the correction were determined. The results and plots are created using the evaluation program for test workpieces. In Appendix B.2, the complete package of verification experiments is presented.
The measured components of the zero point drift for the corrected and the uncorrected machine tool are compared in figure 5.23, 5.24 and 5.25 for respectively the drift in X-direction, the drift in Y-direction and the drift in Z-direction. The geometric error correction has no effect on the efficiency of the drift correction, as the drift is determined relatively to the start time. In the next table (table 5.6) the performance of the empirical error correction is presented for the zero point drift.

Fig. 5.23  Zero point drift in X-direction: Corrected/Uncorrected XY-plane, \( N = \text{DIN-Spectrum} \).

<table>
<thead>
<tr>
<th>Efficiency of the real-time error correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero point drift:</td>
</tr>
<tr>
<td>efficiency ( \delta )</td>
</tr>
<tr>
<td>residual ( \varepsilon_{\text{without}} )</td>
</tr>
<tr>
<td>residual ( \varepsilon_{\text{with}} )</td>
</tr>
</tbody>
</table>

Table 5.6  Efficiency of the empirical error correction:

- Validation of the zero point drift error correction
- \( \delta \): Efficiency of the model
- \( \varepsilon \): Maximum remaining error.
Fig. 5.24  Zero point drift in Y-direction: Corrected/Uncorrected XZ-Plane, N=DIN-Spectrum.

Fig. 5.25  Zero point drift in Z-direction: Corrected/Uncorrected XZ-plane, N=6000 rpm.
Similar to the validation of the geometric error correction, the errors in the position of the holes for the corrected and uncorrected machine tool are compared. In figure 5.26 the position error is depicted as a function of the elapsed time. The error is presented with the first order coefficient, which has been obtained by a linear least square fit through the positioning errors in the holes.

![Graph showing error of position in X-direction: Corrected/Uncorrected XY-plane, N=DIN-Spectrum](image)

**Fig. 5.26**  
Error of position in X-direction: Corrected/Uncorrected  
*XY-plane, N=DIN-Spectrum.*  
The positioning error is represented as a function of the elapsed time by its first order coefficient, obtained by a linear least square fit through the errors in the location of the holes.

Only the additional compensation due to temperature variations of the workpiece and scales are considered. Therefore, the errors of position can be characterized with the first order coefficient. In the following table the averaged efficiencies of the real-time error correction based on the empirical thermo-mechanical model are depicted for these workpiece and scale expansions (table 5.7).
Efficiency of the real-time error correction

<table>
<thead>
<tr>
<th>Position error:</th>
<th>xtx</th>
<th>yty</th>
<th>ztz</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency $\delta$</td>
<td>72 %</td>
<td>-15 %</td>
<td>79 %</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{without}}$</td>
<td>60 $\mu$m/m</td>
<td>12 $\mu$m/m</td>
<td>45 $\mu$m/m</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{with}}$</td>
<td>58 $\mu$m/m</td>
<td>14 $\mu$m/m</td>
<td>40 $\mu$m/m</td>
</tr>
</tbody>
</table>

Table 5.7 Efficiency of the empirical error correction:

Validation of the error of position

$\delta$: Efficiency of the model

$\varepsilon$: Maximum remaining error.

The maximum remaining error in X- and Z-direction is very large, despite the good efficiency. This is caused by the results from the workpiece milled in the XZ-plane with $n=6000$ rpm. During the milling process of the first two stages in this workpiece, the temperature sensor was not attached properly to the workpiece. Hence, the temperature was not measured correctly, which resulted in a wrong workpiece expansion correction.

The measured squareness errors of the workpieces for the corrected and uncorrected machine tool are compared in figure 5.27. Thermally induced squareness errors between Y- and X-axis ($ywx$), between Z- and X-axis ($zwx$) and between Z- and Y-axis ($zwy$) are represented by different workpiece features:

- $ywx$ and $zwx$ correspond to the squareness error present in the workpiece, milled respectively in the XY-plane and the XZ-plane;
- $zwy$ cannot be determined from workpiece feature of the milled workpieces but changes of $zwy$ correspond to the changes of orientation of the workpiece plane (w-direction) as shown in figure 5.28.
Fig. 5.27  Squareness error: Corrected/Uncorrected
XZ-plane, N=6000 rpm.

Fig. 5.28  Drift of workpiece orientation: Corrected/Uncorrected
XZ-Plane, N=6000 rpm.
Similar to the position errors, the efficiencies of the empirical thermo-mechanical correction are calculated separately from the geometric error correction, which is superimposed (table 5.8).

### Table 5.8 Efficiency of the empirical error correction:

<table>
<thead>
<tr>
<th>Squareness error:</th>
<th>ywx</th>
<th>zwx</th>
<th>zwy</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency $\delta$</td>
<td>-80 %</td>
<td>31 %</td>
<td>20 %</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{without}}$</td>
<td>14 $\mu$rad</td>
<td>79 $\mu$rad</td>
<td>160 $\mu$rad</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{with}}$</td>
<td>21 $\mu$rad</td>
<td>46 $\mu$rad</td>
<td>120 $\mu$rad</td>
</tr>
</tbody>
</table>

$\delta$ : Efficiency of the model  
$\varepsilon$ : Maximum remaining error.

The corrected and uncorrected workpiece geometries are depicted in figure 5.29 for the errors in Y-direction and in figure 5.30 in X- and Z-direction. The absolute errors of the axial vector components with respect to the required workpiece geometry are determined. From these vector components the worst case which is the maximum absolute error is given for each workpiece in table 5.9, together with the achieved efficiencies.

### Table 5.9 Efficiency of the empirical error correction:

<table>
<thead>
<tr>
<th>Workpiece geometry</th>
<th>XY, DIN</th>
<th>XZ, 6000</th>
<th>XZ, DIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency $\delta$</td>
<td>80 %</td>
<td>88 %</td>
<td>70 %</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{without}}$</td>
<td>117 $\mu$m</td>
<td>131 $\mu$m</td>
<td>61 $\mu$m</td>
</tr>
<tr>
<td>residual $\varepsilon_{\text{with}}$</td>
<td>24 $\mu$m</td>
<td>32 $\mu$m</td>
<td>22 $\mu$m</td>
</tr>
</tbody>
</table>

$\delta$ : Efficiency of the model  
$\varepsilon$ : Maximum error present in the workpiece.
Fig. 5.29  Errors in Y-direction: Corrected/Uncorrected
XZ-plane, N=6000 rpm.

Fig. 5.30  Errors in X- and Z-direction: Corrected/Uncorrected
XZ-Plane, N=6000 rpm.
The results show that a very good efficiency can be achieved with the empirical thermo-mechanical error correction. The most significant machine tool errors which are the zero point drift components can be corrected with an averaged efficiency of 71%. The maximum error found in the zero-point drift of the test workpiece is 24 μm.

The expansion of the workpiece and scales are corrected sufficiently. A low efficiency is achieved by the correction for the squareness errors present in the workpiece. However, a squareness error has a contribution to the resulting error in two dimensions. With the empirical model, only the position of the Z-axis is considered. Therefore, the squareness errors are only partly compensated. A small accuracy improvement is found in the squareness errors zwx and zwy. Both squareness have a contribution to the resulting error, which is dependent on the position of the Z-axis.

The thermally induced changes of squareness error ywx are not compensated at all. However, the variations in ywx are very small. A change in squareness of 14 μrad results in a maximum error of 3 μm in the workpiece (which has a maximum length of 220 mm). Errors of this magnitude are too small to detect with the presented method.

Considering the efficiency of the error correction based on the empirical model combined with the geometric and finite stiffness error correction, a superb accuracy improvement can be found. Milling the test-workpieces with and without correction showed an average accuracy improvement of 80%. The maximum remaining error of 131 μm present in the workpiece milled without correction is reduced to 32 μm for the workpiece with correction (table 5.9). These results are far beyond the project goal, to achieve an accuracy improvement of 70% with real-time software error correction.
6. Conclusions

The main task of this BCR-project was the improvement of the accuracy of a commercially available machine tool. Initially, the project goal was to achieve an accuracy enhancement by 70% with real-time software error correction. In order to accomplish this goal the most influential error sources have been investigated: basic geometry, thermal behaviour of the machine's structure and finite stiffness effects.

In order to describe the geometric errors of a machine tool a general model has been derived. This general model, developed by TUE, relates the errors in the location of the tool, with respect to the workpiece, to errors in the location of coordinate frames attached to successive components of the machine. With this general model it is not only possible to model a specific machine tool, like the investigated Maho milling machine, but it can be applied to any multi axis machine, composed of rotary and linear elements in an arbitrary serial configuration.

The general model has been elaborated to the individual model, which describes the geometric error structure of the Maho milling machine. Therefore all geometric error components have been measured, using the developed direct measuring techniques.

Also the finite stiffness errors have been investigated. The finite stiffness errors, dependent on the position of the axes itself, are already included in the individual geometric error model. The finite stiffness errors due to cutting forces turned out to be negligible when the workpiece is finished with a low feed and small cutting depths, which is necessary to achieve the desired surface roughness. The additional error to be investigated was the bending of machine parts due to a workpiece load on the machine tool table, which showed that only in the X-axis a significant finite stiffness errors occurs.

The finite stiffness errors are modelled as a change in the geometric errors dependent on the workpiece load. Thus, the individual geometric error model can be applied to describe these finite stiffness errors.
In order to describe the thermo-mechanical behaviour of a machine tool, two different approaches have been investigated: an empirical and an analytical approach.

An empirical model has been developed, which describes the relation between the thermal distribution and the drift of the tool holder, based on experimental data. For this purpose a measurement set-up has been developed, yielding simultaneous information on the drift of the tool holder and the thermal distribution of the machine tool. This information has been analyzed by statistical methods and a relationship between the displacement and the measured relevant temperatures is obtained, with a minimum number of sensors. The modelling procedures, including the necessary measurements, do not take more than a few days.

An analytical model has been developed, which describes the deformation of a machine tool as a functional dependence of its temperature distribution, using large finite elements. Comprehensive experience and a large effort is required to complete the analytical model. However, as the model is machine type dependent, this has to be carried out only once for a machine type.

Besides the already mentioned direct measurement techniques, which constitute a complete calibration technique, a totally new method has been developed to calibrate a machine tool by milling test-workpieces. With these workpieces both the geometric and the thermally induced errors of a machine tool can be determined. Four of these workpieces are sufficient to accomplish a complete calibration. A software package has been developed to support the evaluation of these workpieces.

The test-workpieces are also applied to verify both thermo-mechanical models in the most frequently used positions in the working volume (center position). The verification has been carried out by milling four workpieces in two different planes, with different process conditions (which results in stationary and transient states). From every workpiece the three components of the drift of the tool holder have been verified with the predictions. During these experiments, similar efficiencies have been found for both models. As the empirical and analytical model have their own advantages, each model has been implemented in the real-time software error correction.
All developed models are implemented in a real-time software error correction. This correction will enable a compensation using the full geometric model. This results in 18 compensation tables. Also the thermally induced errors, the expansion of the scales and the finite stiffness errors due to a workpiece load are included in the software error correction. As these errors change during normal operation, the software correction system has been developed with the possibility to update the correction parameters.

An external PC determines the machine's status, recalculates the parameters and sends the compensation tables to the machine tool controller. This sequence is repeated every 60 sec. This open architecture has been developed to enable the implementation of both thermo-mechanical models. Also the communication with the measurement equipment could easily be implemented. The software correction system itself runs in a real-time environment. Hence, the errors can be compensated during every machine movement.

The performance of the real-time error correction for geometric and finite stiffness errors has been validated with holeplate measurements. A holeplate has been measured on the machine tool with and without correction and in different planes. The results of these experiments clearly show the performance of the developed error correction: an efficiency of 81% is achieved! Thereby, the maximum remaining error will not exceed 11 μm, when a calibration period of one year is considered.

The accuracy improvement of the milling machine with the thermo-mechanical error correction, has been validated by milling the developed test workpieces with and without error compensation.

A total of nine workpieces have been milled:
- Six workpieces in the XZ-plane;
  - Three workpieces with a spindle speed as defined in DIN 8602; one with the analytical model; one with the empirical model; one without correction to show the improvement.
  - Three workpieces with a constant spindle speed of n = 6000 rpm; one with the analytical model; one with the empirical model; one without correction to show the improvement.
• Three workpieces in the XY-plane.
  - Three workpieces with a spindle speed as defined in DIN 8602; one with the analytical model; one with the statistical model; one without correction to show the improvement.

The evaluation of these workpieces showed a significant accuracy improvement with real-time error correction. The most significant machine tool error, which is the zero point drift, is compensated with an averaged efficiency of 71% for the empirical model and 67% for the analytical model. Also a maximum remaining error is found of respectively 24 µm and 30 µm. A real significant difference in the modelling capacity is not found between the two thermo-mechanical models. So, the decision, which model to use, highly depends on the demands of the application and the user. However, with both methods a very good accuracy improvement can be achieved.

The performance of the entire software error correction is also validated with test workpieces. Considering the efficiency of the error correction based on the empirical model combined with the geometric and finite stiffness error correction, a superb accuracy improvement can be found. Milling the test-workpieces with and without correction showed an averaged accuracy improvement of 80%. The maximum remaining error of 131 µm present in the workpiece milled without correction is reduced to 32 µm for the workpiece with correction.

When, instead of the empirical model, the analytical model is applied an averaged efficiency of 71% is achieved, with a reduction of the maximum remaining error to 39 µm.

These results are better than the project goal, to achieve an accuracy improvement of 70% with real-time software error correction.

It can be stated that the participants of this project have proved that, with the developed real-time software error correction, an important step has been made to achieve high accuracies on 'normal' machine tools, without special adaptations to the hardware.

Therefore, the real-time software error correction can be considered as the ultimate tool in the future design of machines. Nowadays, special attention is paid to the construction of the machine to achieve a high accuracy. In the near future most attention must be paid to the reproducibility of the machine. Then, software correction will take 'precision' far beyond the current borders.
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A. Two reference procedures to calibrate multi axis machine tools

The next two different calibration procedures have been developed to calibrate machine tools:

1) Direct measurement procedure;
   During this calibration, the error components of the machine tool are measured directly and separately.

2) Indirect measurement procedure.
   A calibration with indirect measurement procedures is carried out by milling or measuring dedicated artifacts on the machine tool. With these experiments the resulting tooltip-to-workpiece error is determined. Artifacts with special designed features enable the extraction of the error components.

In the next two paragraphs, both calibration methods will be presented.

A.1 Direct measurement procedures
Underlying the measurement setups for the calibration procedure with direct measurement techniques are presented. In this project, these measurement techniques are actually applied to determine the geometric, finite stiffness and thermally induced errors. The measurement data is gathered using the software package described in paragraph 3.2. All data is available on floppy and therefore easy to assess.
In order to ensure the traceability of the calibration procedure, all measurement equipment should be calibrated. In this project, the measurement equipment is calibrated by the Metrology Laboratory of Eindhoven University, which is certified by the Dutch Calibration Organization [NKO-014]. The applied instruments are:

- HP 5528 laserinterferometer with accompanying optics, air-sensor and temperature sensors;
- Wyler electronic levelmeters;
- Hilger Watts autocollimator;
- Hilger Watts polygon;
- Ceramic square;
- Inductive displacement transducers;
- Sipp rotary table.
- Pt-100 temperature sensors, attached to a Keithley scanner and digital multimeter;
- Contactless eddy current displacement transducers, connected to a Hottinger Baldwin DMC9012A amplifier.

First, the direct measurement techniques are presented, which can be applied to determine the geometric errors. With similar measurement setups, the finite stiffness errors of the machine tool can be measured, when the machine is supplied with additional load. Secondly, the measurement setup to determine the thermo-mechanical behaviour of the machine tool is described.

**Measurements of the X-axis**

For the measurement of the scale error $x_{tx}$, a laserinterferometer is used with accompanying linear optics. The measurement uncertainty associated with this option of the laserinterferometer is less than $(0.05 + 0.5*L) \mu m$ (L in m).

The measurement setup is depicted in figure A.1. The interferometer is mounted to the ram of the machine tool, while the retroreflector is connected to the workpiece table, which performs the movement in X-direction. The influences of the rotation $x_{ry}$ and $x_{rz}$ have to be eliminated from the obtained measurement results. This yields the error $x_{tx}$ of the coordinate frame positioned in the centroid of the X-carriage.
For the measurements of the rotation of the X-axis, the laser interferometer, with accompanying angular optics, is used for determination of $\text{x}_\text{ry}$ and $\text{x}_\text{rz}$. This option of the laser interferometer introduces an uncertainty less than 0.2 arcsec ($1 \text{ arcsec} = 4.8\times10^{-6} \text{ rad}$). The error $\text{x}_\text{rx}$ is determined by a set of electronic levelmeters. The electronic levelmeters have an uncertainty less than 0.5 arcsec.

The measurement setup for $\text{x}_\text{rx}$ is depicted in figure A.2. The reference levelmeter is mounted on the ram of the machine tool, thereby eliminating the effect of rotation of the overall machine structure, while the measurement levelmeter is placed on the workpiece table, which performs the movement in X-direction.
The rotation error does not depend on the position of measurement, so the obtained results directly reflect the rotation error between the coordinate frames attached to the X- and Y-axis respectively.

The measurement setups for $x_{ry}$ and $x_{rz}$ are depicted in figure A.3 and A.4. The reference interferometer is mounted to the ram of the machine tool while the retroreflector is connected to the workpiece table, which performs the movement in X-direction. The results of these measurements directly reflect the rotation errors between the coordinate frames of the X- and Y-axis respectively.

Fig. A.3 Measurement setup for $x_{ry}$

Fig. A.4 Measurement setup for $x_{rz}$
Measurements of the Y-axis

For the measurement of the scale error \( y_{ty} \) the laserinterferometer is applied with accompanying linear optics. The measurement setup is depicted in figure A.5. The interferometer is mounted to the ram of the machine tool, while the retroreflector is placed on the workpiece table, which performs the movement in \( Y \)-direction. The influences of the rotation \( y_{rx} \) and \( y_{rz} \) have to be eliminated from the obtained measurement results. This yields the error \( y_{ty} \) of the coordinate frame positioned in the centroid of the \( Y \)-carriage.

![Measurement setup for \( y_{ty} \)](image)

Fig. A.5 Measurement setup for \( y_{ty} \)

Several instruments are applied to measure the rotation errors of the \( Y \)-axis. First, for the determination of the error \( y_{rx} \), the laserinterferometer is applied. The setup for this measurement is depicted in figure A.6. The interferometer is mounted to the ram of the machine tool, while the retroreflector is placed on the workpiece table, which performs the movement in \( Y \)-direction. The obtained results directly reflect the rotation error between the frame attached to the \( Y \)-axis and the machine coordinate frame.
Fig. A.6 Measurement setup for \( y_{rx} \)

Fig. A.7 Measurement setup for \( y_{ry} \)
The measurement setup for $y_{ry}$ is depicted in figure A.7. While this measurement cannot be carried out easily by a laser interferometer nor a set of levelmeters, two direct straightness measurements are performed. The straightness measurement is performed by movement of a straight-edge, in this case a calibrated surface of a squareness reference block, along a displacement transducer. The reading of the transducer is a measure of the error $y_{tz}$ with a contribution of $y_{ry}$.

By choosing the setup as depicted in figure A.7 the effect of the error $y_{ry}$ in the measurement result will reverse sign between measurement 1 and 2. With the necessary displacement in $X$-direction between the measurements the change in active arm is known. The results of these measurements have to be corrected for the effect of the error $x_{rx}$ on the orientation of the straight-edge. This error results in a displacement $Y \times x_{rx}$ which is not caused by the $Y$-axis.

Taking the above procedure the error $y_{ry}$ can be calculated as:

$$y_{ry} = \frac{ytz(2) - ytz(1) - Y \times x_{rx}}{x(1) - x(2)} \text{ [Rad]} \quad [A.1]$$

with:
- $ytz(2)$ and $ytz(1)$ are the uncorrected results of the straightness measurement;
- $x(1)$ and $x(2)$ are the positions of the $X$-carriage during the respective straightness measurements;
- $Y$ is the position of the $Y$-carriage during the straightness measurements.

The measurement setup for $y_{rz}$ is depicted in figure A.8. The reference interferometer is mounted to the ram of the machine tool, while the retroreflector is connected to the workpiece table, which performs the movement in $Y$-direction. Also these results directly reflect the rotation errors between the coordinate frame attached to the $Y$-axis and the machine coordinate frame.
Measurements of the Z-axis

For the measurement of the scale error $ztz$, the laser interferometer is applied with accompanying linear optics. The measurement setup is depicted in figure A.9. The interferometer is mounted on the workpiece table, while the retroreflector is connected to the ram of the machine tool. The influences of the rotation $zrx$ and $zry$ have to be eliminated from the obtained measurement results. This yields the error $ztz$ of the coordinate frame positioned in the centroid of the Z-carriage.
Several instruments are applied to measure the rotation errors of the Z-axis. First, for the determination of the error $z_{rx}$, the laser interferometer is applied with the angular optics. The setup for this measurement is depicted in figure A.10. The interferometer is placed on the workpiece table while the angular retroreflector mounted on the ram of the machine tool. The obtained results directly reflect the rotation error between the machine coordinate frame and the frame attached to the Z-axis.

![Fig. A.10 Measurement setup for $z_{rx}$](image)

![Fig. A.11 Measurement setup for $z_{ry}$](image)
The measurement setup for $z_{ry}$ is depicted in figure A.11. This measurement is also carried out by a laser interferometer and angular optics. The setup is similar to the measurement setup, applied for the determination of $z_{rx}$, except the optics are rotated about the $Z$-axis over ninety degrees.

The measurement setup for $z_{rz}$ is depicted in figure A.12. The reference levelmeter is mounted on the workpiece table, while the measurement levelmeter is connected to the ram of the machine tool. Also these results directly reflect the rotation errors between the coordinate frame attached to the $Z$-axis and the machine coordinate frame.

![Diagram of measurement setup for $z_{rz}$]

Fig. A.12 Measurement setup for $z_{rz}$

**Straightness and squareness measurements**

The straightness measurements are carried out with a ceramic square and inductive displacement sensors. The results of these measurements can be used for both the analysis of the straightness errors and the analysis of the squareness errors of the milling machine. The straightness error is defined as the difference between the actual data and its least-square. As the straightness measurements are carried out during the determination of the squareness errors, the measurement setups for both the straightness and squareness measurements are presented together.
The squareness error between the carriages is determined by measurement of a ceramic reference block. This block is calibrated on a 3D measuring machine and possesses a squareness error of +1.7 arcsec. The measurements are carried out using inductive displacement transducers.

To measure the squareness error two straightness measurements are carried out. To calculate the squareness error, the pure straightness error will be eliminated by Least-Square fitting.

Squareness error between the X- and Z-guide

For this measurement the reference block is placed on the machine table. The block is supported on three points. The block is aligned along the X-axis to secure that the displacement transducers remain in their calibrated range (0-2 mm). First, the displacement transducer is mounted on the ram of the machine tool and the reference side of the block is measured, yielding the error xtz and the alignment error of the block (figure A.13).

![Fig. A.13 Measurement setup of xtz (top view of machine tool)](image)

Secondly, the other side of the block is measured yielding the error ztx and an error that includes both the alignment error of the block, the squareness error in the block and the squareness error of the machine tool (figure A.14).
The measurements indicate an alignment error (\(\alpha\)); the second measurement yields an alignment error (\(\beta\)). The total uncorrected out of squareness, defined as the actual angle included by the guides minus 90 degrees, is in this case calculated by: -\(\beta\)-\(\alpha\). However, the squareness error of the block, i.e. 1.7 arcsec, must be added to this result to achieve the total squareness error of the Z-axis with respect to the X-axis.

**Squareness error between the X- and Y-guide**

Again the reference block is supported by the machine table and aligned along the X-axis. First, the displacement transducer is mounted on the ram of the machine tool and the reference side of the block is measured, yielding the error xty and the alignment error of the block (figure A.15).
Secondly, the other side of the block is measured yielding the error $ytx$ and an error that includes both the alignment error of the block, the squareness error in the block and the squareness error of the machine tool (figure A.16).

![Figure A.16 Measurement setup of $ytx$ (front view of machine tool)](image)

From these measurements the squareness error can be determined, similar to the squareness error in the XZ-plane.

**Squareness error between the Y- and Z-guide**

Again the reference block is supported by the machine table. The block is now aligned along the Z-axis. First, the reference side of the block is measured, yielding the error $zty$ and the alignment error of the block (figure A.17).

![Figure A.17 Measurement setup of $zty$ (side view of machine tool)](image)
Secondly, the other side of the block is measured yielding the error $y_{tz}$ and an error that includes both the alignment error of the block, the squareness error in the block and the squareness error of the machine tool (figure A.18).

Fig. A.18 Measurement setup of $y_{tz}$ (side view of machine tool)

Out of these measurements the squareness error can be determined, similar to the squareness error in the XZ-plane.

Measurements of the rotary axes
The Maho milling machine consists not only of three linear axes, but also two rotary axes. In order to complete the individual model, also the geometric errors induced by these two rotary table have to be determined.

For the measurement of the linearity error of the B-axis, a optical polygon and an autocollimator are applied. The polygon has a measurement uncertainty less than 1 arcsec. The autocollimator introduces a measurement uncertainty less than 0.5 arcsec. The measurement is carried out by positioning the B-axis with a step that equals the angle between two successive planes of the polygon. With the autocollimator the difference between the orientation of the two planes is measured. Using a polygon, which has 12 sides, together with the autocollimator, it is possible to measure the position error of the B-axis over the whole range ($360^\circ$) with steps of $30^\circ$. 
With the above-presented measurement method it is possible to obtain the position error with steps as large as the angle between two successive planes of the polygon. In order to determine the position error between these steps another method is used. A very accurate rotary table is placed upon the B-axis of the milling machine. This rotary table introduces an uncertainty less than 0.5 arcsec. In the center of this table a plane mirror is placed. The measurement sequence is carried out by rotating the B-axis of the milling machine a randomly chosen angle. Using the rotary table, the plane mirror is rotated back to the zero position, which is measured by the autocollimator.

Both measurement methods are applied to the B-axis of the Maho milling machine. To measure the position error with the first method the polygon is place in the center of the B-axis, as depicted in figure A.19. The autocollimator is placed outside the milling machine on a stable tripod, which necessitates to check the zero point when the measurement has been finished. In the same figure the measurement setup for the second method is depicted. Here a SIPP rotary table is placed in the center of the B-axis. A plane mirror is placed upon this table. The autocollimator is also placed outside the milling machine.

![Fig. A.19 Both the measurement setups for bry.](image-url)
Also the position error of the C-axis of the Maho milling machine is measured. As the C-axis rotates around the Z-axis of the milling machine, which is a horizontal axis, it was not possible to connect the SIPP rotation table to this axis. Therefore this axis is only measured with the first method. In order to get more information on the position error of the intermediate positions the measurement was carried out with different start positions.

The measurement setup for the position error crz is depicted in figure A.20. The polygon is placed upon a rotation table, which is connected to the C-axis of the milling machine. This rotation table is used for the different start positions. In order to create a suitable measurement setup, an optical square is placed on the table of the milling machine. The autocollimator is placed outside the milling machine on a stable tripod. Also with this setup, the zero point has to be checked.

Fig. A.20 Measurement setup for crz
Calibration of the thermo-mechanical behaviour with direct measurement techniques

For calibration purposes, the principal interest is not the deformation of each machine component, but the displacement of the tool with respect to the workpiece. Therefore, the measurement setup is designed to obtain the displacement of the tool holder in three orthogonal directions, and the two relevant rotations.

In the tool holder of the machine tool a cylinder is mounted. On the workpiece table a base is mounted with five contactless eddy current displacement transducers. These transducers allow to measure the displacement of the tool holder with respect to the workpiece in three directions and two rotations simultaneously. The displacement transducers are calibrated separately in the actual measurement set-up, which showed an uncertainty of less than 1 μm over the full range, after application of a correction formula for systematic errors. Also the influence of temperature changes has been investigated [41]. The results show that temperature does not affect the reading of the displacement transducers significantly.

The transducers are linked to an amplifier (Hottinger Baldwin DMC9012A) that is capable of reading all signals simultaneously. From the amplifier the obtained signals are sent to a PC by means of IEEE interface. In figure A.21 the measurement setup for the determination of the displacement is depicted, as it is applied in the XZ-plane.

![Fig. A.21 Scheme of set-up for displacement measurement](image-url)
For a calibration it suffices to measure the drift of the tool holder. For correction purposes, however, it is also necessary to measure the temperature distribution on the machine tool. Therefore, distributed over the machine's structure a number of Pt-100 temperature sensors are mounted. Each temperature sensor is calibrated with an uncertainty of less than 0.1 °C, in a range from 15 to 60 °C, which is sufficient for this purpose [42].

![Fig. A.22](image)

**Fig. A.22  Position of the temperature sensors on the machine tool**

In figure A.22 the positions of the temperature sensors on the machine tool are depicted. The choice of the position of each sensor should be determined after measurement of local heat sources, for example by infrared measurements.

For carrying out a measurement the surface of the cylinder is positioned on 0.5 mm of the transducers. In this position the machine tool is loaded with a spindle speed over a specified time. During the measurement the displacement of the tool holder and the temperature distribution of the machine tool are obtained every 60 seconds. The programming of the machine tool and the collection of the measurement data are carried out automatically by a coupled computer.
Although loading the machine tool with a constant spindle speed provides a large amount of information concerning the behaviour of the machine tool, practical situations often display a rapidly changing load situation. To obtain an impression of the machine tool's thermal behaviour in practical situations, it is loaded with a spindle speed spectrum as defined in DIN 8602. This spectrum consists of spindle speeds varying from zero to maximum spindle speed ($n_{\text{max}}$), with steps of 25% of $n_{\text{max}}$. For the machine tool under investigation $n_{\text{max}}$ is 6300 rpm. Each load situation is maintained for 15 minutes. A graphical presentation of this spectrum is depicted in figure A.23.

Fig. A.23  DIN 8602 spectrum
A.2 Indirect measurement procedures

To calibrate the machine tool, two different indirect measurement procedures are developed and applied in this project. First, dedicated artifacts are measured on the machine tool for the extraction of all geometric errors. Secondly, special test-workpieces are designed and applied to determine both the geometric and the thermally induced errors.

A.2.1. Ballplate method to determine the parametric errors of a three axis machine

A reference object based method had been developed and tested by PTB earlier to this project which yields the 21 geometrical parametric errors of a cartesian 3-axis machine (machine tool or coordinate measuring machine). Precondition is that the machine's error behaviour can be described by a rigid body error model.

Ball- or boreplate (requirements)

The plate should have reference elements (balls or cylindrical bores) arranged in an equidistant raster. But it is sufficient if the plate is provided with reference elements along its periphery on all four sides. The reference elements must lay in the neutral plane of the plate and be measurable on their equator (i.e. in the neutral plane) from both sides of the plate. Measurements are done in six measuring cycles (in four different positions) in the machine's working space (figure A.24). These positions represent four cross sections of the working space employing five different probe styli for the measurement.

Measurement procedure

With the plate parallel to the three coordinate planes, two measurements each are performed so that different effective distances of the plate's plane from the coordinate axes are obtained, either by shifting the plate parallel to itself or by using two different probe stylus lengths.

In each plate position and with each of the five styli, two complete measurements are performed: a spiral or circular sequence is used, beginning with the first sphere in the zero point and continuing in the positive X-direction of the plate. Each sphere is measured with five points, one on the pole in stylus direction, and four on the equator in the plate's plane. After having measured all spheres once, the spheres are measured a second time in each position, now progressing in the opposite direction. By this procedure drifts in the machine can partly be eliminated.
The coordinates are reported in the plate's coordinate system according to its calibration certificate. Stylus directions and lengths as well as the positions of the first ball/bore center in the machine's coordinate system are reported for each measurement (table A.1). The six measurements are given identifications according to the following scheme:

11 - XY-plane, stylus in -Z direction, plate at positive end of Z-range;
12 - XY-plane, stylus in -Z direction, plate at negative end of Z-range;

21 - XZ-plane, stylus in -Y direction, plate in middle of Y range;
22 - XZ-plane, stylus in +Y direction, plate in middle of Y range;

31 - YZ-plane, stylus in -X direction, plate in middle of X range;
32 - YZ-plane, stylus in +X direction, plate in middle of X range.

![Diagram](image.png)

*Fig. A.24 Required ball plate positions to determine a complete set of geometrical parametric errors of a three axis machine.*
<table>
<thead>
<tr>
<th>Plate orientation</th>
<th>XY-plane</th>
<th>XZ-plane</th>
<th>YZ-plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position code</td>
<td>11</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>(X_0/\text{mm})</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>(Y_0/\text{mm})</td>
<td>25</td>
<td>25</td>
<td>300</td>
</tr>
<tr>
<td>(Z_0/\text{mm})</td>
<td>50</td>
<td>550</td>
<td>25</td>
</tr>
<tr>
<td>Probe length (X_p/\text{mm})</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Probe length (Y_p/\text{mm})</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Probe length (Z_p/\text{mm})</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table A.1*  *Mask in evaluation program to enter data of plate set up*

![Diagram](image)

**Fig. A.25**  *Example of overlapping plate positions*

**Evaluation**

From the six measurement runs the 18 error functions and the three squareness errors are determined. The obtained density of datum points corresponds to the plate's raster spacing.
The dimensions of the plate raster have to match with the three cross sections of the measuring volume to be calibrated. If this is not possible, the plate may be measured in several overlapping positions in order to cover the whole measuring range (figure A.25).

The accuracy of the method is determined by the stability/reproducibility of the machine and by the degree of validity of the rigid body model for the machine.

For more information, there exists a DKD-guideline on the calibration of coordinate measuring machines [53] and the user instructions for the evaluation program "KALKOM" available from PTB.

A.2.2 Calibration of a rotary table by a ballplate method
The errors of a rotary table can be determined by measuring a ball plate in as many angular positions of the rotary table as desired [1] as datum points. The measurement is done using the axes of motion and the probing system of the coordinate measuring machine or of the milling machine on which the table is installed.

Pict. A.1 Ball plate during calibration of a rotary table
Ballplate (requirements)
The ballplate needs not be calibrated. In order to achieve the best possible results, the ballplate should have 12 or more spheres arranged in an equally spaced angular array (on a circle). Each sphere must be accessible for probing from all sides except for one pole. This pole is used to fix the ball to the surface of the plate (figure A.26).

Measurement procedure
All the ball centers are measured by probing at least five points: four with a 90° spacing on the equator and one on the pole. The equator plane is parallel to the plane in which all the ball centers lay. The sequence is to measure in the same position of the cartesian axis system all balls by incrementing the rotary table by the angular ball spacing from 0° up to 360°. This is first carried out for the cartesian axis system positioned in the vicinity of the first ball (ball "No.1") of the plate. Next all ball positions (respectively all angular positions of the table which correspond to a ball position) are measured for the cartesian axis system positioned in the vicinity of the second ball. The same procedure is carried out for the third ball and so on, until one has commenced once with each of the balls. In each case the first ball measured remains ball "No.1" of the plate.

If measurements with a finer angular spacing than that of the ball plate shall be carried out, the machine's cartesian axes and the rotary table are incremented only a fraction of the plate's angular spacing between each 360° rotation of the table (which itself is performed with the full increments of the ball spacing).

Evaluation
The evaluation is carried out for all parametric errors of the rotary table:
- angular position error;
- axial slip;
- run out in X-direction;
- run out in Y-direction;
- camming angle around the X-axis;
- camming angle around the Y-axis.
The principle of the evaluation is based on the fact that the sums of all errors of angular position of the balls as well as of all angular positions of the rotary table are zero. Thus, averaging over all ball positions, which are measured in one point of the cartesian axis system, one obtains the machine's errors at that location. Knowing the machine's errors for all ball positions, one can determine the true coordinates of the ball centers in the plate coordinate system. Finally: knowing the machine's error vectors and the ball coordinates, it is possible to calculate the searched for rotary table's errors. First these errors are available as error vectors for all ball center position for all angular positions of the rotary table. These error vectors are then transformed in the above six parametric errors.

In case one needs to measure more angular positions than balls are available on the ballplate, the errors of the machine are interpolated between two ball positions. The rest of the evaluation is done in the same way as described above.

For more information, there exists a user instruction for the evaluation program "DTKAL" available from PTB.

The ballplate calibration method has been applied to the B-axis of the investigated Maho milling machine. The results of this calibration is depicted in the next two figures (figure A.27 and A.28). These results confirm the conclusion, based on the direct measurement techniques, that the errors of the B-axis are not significant. Hence, a software error correction for the errors of the B-axis has not been implemented.
Fig. 4.27  Calibration results of the B-axis with ballplate measurements.

Angular position error, camming angle around the X-axis, camming angle around the Y-axis
Run out in X-direction, Run out in Y-direction, Axial slip

H.-C. Schaub

PTB

5.32
A.2.3 Procedure to calibrate a five axis milling machine with test-workpieces

*Calibration of the linear axes*

This calibration procedure consists of finishing and measuring the workpieces and the evaluation of the measured data. To determine the errors of the linear axes four test workpieces are machined successively in the same (central) positions $x, y, z$ (tolerance: $\pm 20$ mm) and different orientations of the tool. The planer jack is required to adapt the zero point position of each workpiece to the tool position. Accordingly, the workpiece coordinate system $u, v, w$ is attached to the center of each workpiece with defined orientations (figure A.29).

*Fig. A.29  Workpiece positions*
If the working range of the C-axis is limited to less than +/-90° the geometric parameters for the workpieces in the YZ-plane are adapted to the angle of the tool axis. This adaptation has no effect to the principles of the evaluation method. For a correct error determination the finishing conditions of the workpieces have to be equal:

- Finishing is started at the thermally balanced "cold" machine.
- The machine tool errors are determined during more than 3 hours of operation.
- The spindle speeds are equal for each workpiece. Figure A.30 shows two examples of spindle speeds which are defined as operation conditions A and B:
  A) Machining on the continuously operating machine (warming up with constant spindle speed);
  B) Machining with changes of spindle speed (DIN-spectrum).

![Spindle speed graphs](image)

**Fig. A.30 Operation conditions A and B.**

The calibration procedure for the determination of the linear axes was implemented on a Maho 700S milling machine with the C-position of the milling head limited to +/-60°. These limits are considered for the geometry of the workpieces which are machined in the YZ-plane. The geometries of the test workpieces are shown in the drawings in last part of this appendix.
All workpieces are prefinished from cylindrical disks (Dural AlCuMg 2) with a diameter of 480 mm. This size enables to calibrate an axial range of 230 mm. Drawings 16.1, 16.2, 18.1 and 18.2 show the prefinished test workpieces. The reference tracks (drawing 18.1) are already finished. The reference tracks are necessary to measure the workpiece position before the finishing process is started.

The cutting parameters for the finishing process are chosen to achieve low cutting forces, low tool wear and low temperature changes in the workpieces. For dural aluminum a cutting depth of 0.2 mm and a feed of 500 mm/min are optimal. Coolant is advantageous for low temperature changes.

The finishing procedure of each workpiece is divided into 12 milling cycles and additional milling sequences which are optional. The 12 milling cycles correspond to 12 situations of the machine tool in which the error components of the linear axes are determined. Each milling cycle is divided into three sequences in which the 28 bores in the test workpiece and 28 tracks between the bores are finished. To avoid distortion in the square grid of the workpiece, due to a zero point drift, the bores are milled in two sequences of counterwise directions (figure A.31).

Fig. A.31  Milling directions
Two tracks of each bore are machined during each of the sequences (drawing 16.4.2). The average position of the four tracks of a bore represent the coordinates U and V of the bore position. In the third sequence the tracks between the bores which represent the W-coordinates are milled (drawing 16.4.3).

After completing the 12 finishing cycles, the following additional tracks are machined optionally:

- "Backlash error tracks";
  The backlash errors are sampled on parallel tracks (drawing 16.4.6). The errors due to a time delay in the axial control are separated from the distance of the parallel tracks.
- "Lag error tracks".
  The lag error tracks are sampled on parallel tracks next to the reference areas (drawing 18.1). The error due to a reversion of the axial motion before the milling step is separated from the distance between the parallel tracks.

The geometrical parameters of the finishing programs vary with the orientation (four positions: +YZ, -YZ, XZ, XY) of the workpieces. The process parameters for the different orientations of the workpieces are defined as follows:

1) Workpiece machined in the "+YZ"-position:
   - Angular position of milling head: C = +60°;
   - Geometry: drawing 18.3 (see page 189);
   - The following tracks are milled:
     - Square grid tracks (12 error situations);
     - Lag error tracks;
     - Backlash error tracks.

2) Workpiece machined in the "-YZ"-position:
   - Angular position of milling head: C = -60°;
   - Geometry see drawing 18.3 (see page 189);
   - The milling process contains the same tracks as in 1).
3) Workpiece machined in the "XZ"-position:
   - Angular position of the milling head: C = 0°;
   - Geometry see drawing 17.3 (see page 195);
   - The following tracks are milled:
     - Square net tracks (12 error situations);
     - Lag error tracks;
     - Backlash error tracks.

4) Workpiece machined in the "XY"-position:
   - Tool axis horizontal (parallel to Z-axis);
   - Geometry see drawing 17.3 (see page 195);
   - The following tracks are milled:
     - Square net tracks (12 error situations);
     - Lag error tracks;
     - Backlash error tracks.

During finishing the workpiece temperature is measured periodically and stored.

After the finishing the test workpieces are measured on a coordinate measuring machine. At the beginning and at the end of measuring procedure the workpiece temperature is measured. These temperature data and the required geometry of the workpiece are stored in a parameter file for the evaluation procedure.

The evaluation is done by the program WASTE which is structured as shown in figure A.32. The menu-structure of the program helps the user to create the parameter files, to define a set of test workpieces and in general to step through the evaluation procedure easily.
Fig. A.32  Evaluation program.

1) In a first evaluation step the temperature data of the specified workpiece are imported and filtered.

2) In a second step the measured geometry data are imported into the program. The thermal expansion of the workpiece is corrected with the imported temperature data (referred to 20°C). The required workpiece geometry is determined by the evaluation program using the workpiece parameters.

3) In a third evaluation step the workpiece errors (straightness, length, parallelism, squareness, flatness and zero point drift) are calculated by a comparison of required and measured data.

4) The machine tool errors are calculated if a complete set of workpieces (4 positions) is evaluated.
Figure A.33 and figure A.34 show how the error components of the linear axes are imaged to the workpiece features. Some of the error components can be separated directly from a single workpiece, e.g. yawing xry (figure A.33) is calculated from parallelism in the XZ-plane. Other error components cannot be separated from a single workpiece because the resulting errors are superimposed. In this case the error components are calculated from a combination of errors of different workpieces as shown in figure A.34. The results are stored in files and can be plotted graphically and numerically.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Straightness</th>
<th>Straightness</th>
<th>Parallellity</th>
<th>Squareness</th>
<th>Flatness</th>
<th>Length</th>
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Fig. A.33  Imaging of error components on plane test workpieces.
The calibration procedure for a rotary table (B-axis) is an optional part of the described machine tool calibration. The process is started after the finishing of the 12 milling cycles on the "warm" machine to avoid errors due to a zero point drift. The tracks can be integrated into the test workpiece of the XZ-plane as shown in drawing 16.3. The drawing shows pockets milled in a circular grid which is divided into steps of 22.5°. The pockets are milled in 32 positions of the B-axis within a range of 720°. To integrate all tracks into the shown workpiece the pockets are positioned along two circles with a radius of \( R = 49 \text{ mm} \) and \( R = 79 \text{ mm} \) and in different levels of the grid as shown in drawing 16.3.1. In each position of the B-axis two pockets are milled along the \( U = 0 \) axis and two pockets are milled along the \( V = 0 \) axis. Each position of a pocket is represented by four tracks which are machined in an axial motion. This milling sequence is repeated for 32 positions of the B-axis.

Furthermore eight concentric ring-like tracks are machined with a radius of about \( R = 100 \text{ mm} \). All concentric tracks are milled in continuous motion of the B-axis in ranges of 0°... 360° and 360°... 720°. Each range is represented by a single ring. To separate the errors of flatness the 8 rings (drawing 16.3.5, page 191) are milled in different tool positions X,Z.
After measuring the workpiece on a coordinate measuring machine the following error components of the B-axis are separated in an evaluation program:

- Angular position error (bry);
- Radial and axial runout (btx, btz, bty);
- Camming angle (brx, brz).

<table>
<thead>
<tr>
<th>Workpiece features (Radius r, Position b)</th>
<th>B-Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rotational error</td>
</tr>
<tr>
<td></td>
<td>btx</td>
</tr>
<tr>
<td>R₁(r₁,b₁)</td>
<td>x</td>
</tr>
<tr>
<td>R₂(r₂,b₂)</td>
<td>y</td>
</tr>
<tr>
<td>R₃(r₃,b₃)</td>
<td>z</td>
</tr>
</tbody>
</table>

Fig. 4.35   Error components of the rotary table, extracted from test workpieces
To calculate these errors a reference plane is generated from the measured geometry of the circular grid. The errors are referred to this reference plane. Figure A.35 shows how the error components are imaged to the workpiece features. The calculation of the error components of the rotary table is done in a single step of the evaluation program. A correction of thermal expansion of the workpiece is not necessary.

**Technical drawings**

In the next part technical drawings are presented of the different types of test workpieces.
Schnitt A-B

8 Stufen 0.2mm

8 Ringe 2.5mm = 20mm

\( \sqrt{Rz\ 16} \)

Detail 5

Temp.-Prüfwerkst.

Ringspuren

DIN 7168 mittel

DIN ISC 1302

PTB

16.3.5
Schnitt A-B

Detail 2

--- | --- | --- | --- | --- | ---
5.3 | DIN 7168 | mittel | DIN 1302 | | 

1991 Datum | Name | Benennung
--- | --- | ---
| | Erb. | LuGst.
| | Spr. | Lock
| | Norm |

Temp.-Prüfwerkst.
Aussenspur

PTB

Zeichnungsnr. 16.4.2

Boll
Schnitt A-B

Mitte des Fräsers:
U = 32 mm
V = -2 mm
W = 0

Detail 1

Detail 2

Detail 3

Temp.-Prüfwerkst.
bohren und fräsen

Teil 18.3
B. Verification of the thermo-mechanical error correction with test-workpieces

Two totally different approaches have been investigated to describe the thermo-mechanical behaviour of machine tools. An empirical model has been developed, which describes the relation between the thermal distribution and the distortion based on experimental data. Secondly, an analytical model has been developed, using large finite elements.

These thermo-mechanical models have been implemented in the real-time error correction, besides the geometric and finite stiffness error correction. The efficiency of each error corrections has been determined with a totally different and independent method: by using test workpieces.

In this Appendix a complete overview is given of the verification of both models. For the verification a total of nine workpieces have been milled:

- Six workpieces in the XZ-plane;
  - Three workpieces with a spindle speed as defined in DIN 8602; one with the analytical model; one with the empirical model; one without correction to show the improvement.
  - Three workpieces with a constant spindle speed of \( n = 6000 \) rpm; one with the analytical model; one with the empirical model; one without correction to show the improvement.
- Three workpieces in the XY-plane.
  - Three workpieces with a spindle speed as defined in DIN 8602; one with the analytical model; one with the statistical model; one without correction to show the improvement.

In the next part, the performance of the real-time error correction will be presented, based on the analytical model; in the following part, the performance of the real-time error correction, based on the empirical model. In both cases, the verification has been carried out with the same geometric and finite stiffness error correction.
B.1 Verification of the thermo-mechanical error correction, based on the analytical thermo-mechanical model

**Fig. B.1**
Zero Point Drift: uncorrected/corrected
XY-Plane, N=OIN-Spectrum, Efficiency = 63%

**Fig. B.2**
Zero Point Drift: uncorrected/corrected
XY-Plane, N=OIN-Spectrum, Efficiency = 80%
**Fig. B.3**
Zero Point Drift: uncorrected/corrected
XY-Plane, N=DIN-Spectrum, Efficiency = -47%

**Fig. B.4**
Zero Point Drift: uncorrected/corrected
XZ-Plane, N=6000 rpm, Efficiency = 62%
Fig. B.5  Zero Point Drift: uncorrected/corrected
XZ-Plane, N=6000 rpm, Efficiency= 52%

Fig. B.6  Zero Point Drift: uncorrected/corrected
XZ-Plane, N=6000 U/min, Efficiency= 73%
Fig. B.7
Zero Point Drift: uncorrected/corrected
XZ-Plane, NaDIN-Spectrum, Efficiency = 50%

Fig. B.8
Zero Point Drift: uncorrected/corrected
XZ-Plane, NaDIN-Spectrum, Efficiency = 75%
**Figure 8.9**
Zero Point Drift: uncorrected/corrected
XZ-Plane, N=DIN-Spectrum, Efficiency = 54%

**Figure 8.10**
Error of Position: uncorrected/corrected
XY-Plane, N=DIN-Spectrum
Fig. B.11  Error of Position: uncorrected/corrected
XY-Plane, N=DIN-Spectrum

Fig. B.12  Error of Position: uncorrected/corrected
XZ-Plane, N=6000 rpm
Fig. 8.13
Error of Position: uncorrected/corrected
XZ-Plane, N=6000 rpm

Fig. 8.14
Error of Position: uncorrected/corrected
XZ-Plane, N=DIN Spectrum
Fig. B.15  
Error of Position: uncorrected/corrected  
XZ-Plane, N=DIN-Spectrum

Fig. B.16  
Error of Squareness: uncorrected/corrected  
XY-Plane, N=DIN-Spectrum
**Error of Squareness: uncorrected/corrected**

XZ-Plane, N=5000 rpm

**Drift of Workpiece Orientation: uncorr. / corrected**

XZ-Plane, N=5000 rpm
Fig. B.19

Error of Squareness: uncorrected/corrected
XZ-Plane, NeDIN-Spectrum

Fig. B.20

Drift of Workpiece Orientation: uncorrected/corrected
XZ-Plane, NeDIN-Spectrum
Fig. 8.21

Errors in Y-direction: uncorrected/corrected
XZ-Plane, N=6000 rpm, time (rel.) = 205 min

Fig. 8.22

Errors in X- and Z-direction: uncorrected/corrected
XZ-Plane, N=6000 rpm, time (rel.) = 205 min
Fig. 8.23

Errors in Y-direction: uncorrected/corrected
XZ-Plane, N=DIN-Spectrum, time (rel.) = 200 min

Fig. 8.24

Errors in X- and Z-direction: uncorrected/corrected
XZ-Plane, N=DIN-Spectrum, time (rel.) = 200 min
Fig. B.25

Errors in Z-direction: uncorrected/corrected
XY-Plane, N=DiN-Spectrum, time (rel.) = 200 min

Fig. B.26

Errors in X- and Y-direction: uncorr./corrected
XY-Plane, N=DiN-Spectrum, time (rel.) = 200 min
B.2 Verification of the thermo-mechanical error correction, based on the empirical thermo-mechanical model.

**Fig. B.27**
Zero point drift: uncorrected/corrected
XY-plane, NaDI spectrum, Efficiency = 75%

**Fig. B.28**
Zero point drift: uncorrected/corrected
XY-plane, NaDI spectrum, Efficiency = 78%
Fig. B.29
Zero point drift: uncorrected/corrected
XY-plane, NaDIN-Spectrum, Efficiency = 25%

Fig. B.30
Zero Point Drift: uncorrected/corrected
XZ-Plane, N=6000 rpm, Efficiency = 73%
Zero Point Drift: uncorrected/corrected
XZ-plane, N=6000 rpm. Efficiency = 37%

Zero Point Drift: uncorrected/corrected
XZ-plane, N=6000 rpm. Efficiency = 82%
Fig. B.33
Zero Point Drift: uncorrected/corrected
XZ-Plane, N=DIN-Spectrum, Efficiency = 87%

Fig. B.34
Zero Point Drift: uncorrected/corrected
XZ-plane, N=DIN-Spectrum, Efficiency = 50%
Fig. B.35  
**Zero Point Drift**: uncorrected/corrected  
XZ-Plane, N=OIN-Spectrum, Efficiency = 62% 

Fig. B.36  
**Error of Position**: uncorrected/corrected  
XY-plane, N=OIN-Spectrum
Fig. B.37
Error of Position: uncorrected/corrected
XY-plane, N=50,000 rpm

Fig. B.38
Error of Position: uncorrected/corrected
XZ-plane, N=6000 rpm
Fig. B.39  Error of Position: uncorrected/corrected  
XZ-plane, N=5000 rpm

Fig. B.40  Error of Position: uncorrected/corrected  
XZ-Plane, N=DIN-Spectrum
Fig. B.41  
Error of Position: uncorrected/corrected  
XZ-Plane, N=DIN-Spectrum

Fig. B.42  
Error of squareness: uncorrected/corrected  
XY-Plane, N=DIN-Spectrum
**Fig. B.43**  
Error of squareness: uncorrected/corrected  
XZ-Plane, N=6000 rpm

**Fig. B.44**  
Drift of workpiece orientation: uncorrected/corrected  
XZ-Plane, N=6000 rpm
**Fig. B.45**

Error of Squareness: uncorrected/corrected
XZ-plane, NaDIN-Spectrum

**Fig. B.46**

Drift of Workpiece orientation: uncorrected/corrected
XZ-plane, NaDIN-Spectrum
Fig B.47

Errors in Y-direction: uncorrected/corrected
XZ-Plane, N=6000 rpm, time (rel.) = 205 min

Fig B.48

Errors in X- and Z-direction: uncorrected/corrected
XZ-Plane, N=6000 rpm, time (rel.) = 205 min
Errors in Y-direction: uncorrected/corrected
XZ-Plane, N=DIN-Spectrum, time (rel.) = 200 min

Errors in X- and Z-direction: uncorr./corrected
XZ-Plane, N=DIN-Spectrum, time (rel.) = 200 min
Fig. B.51

Errors in Z-direction: uncorrected/corrected
XY-Plane, NeDIN-Spectrum, time (rel.) = 200 min

Fig. B.52

Errors in X- and Y-direction: uncorrected/corrected
XY-Plane, NeDIN-Spectrum, time (rel.) = 200 min