Designing a contoured beam antenna

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Designing a contoured beam antenna

by

M.H.A.J. Herben
DESIGNING A CONTOURED BEAM ANTENNA

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Summary
This report describes the design of a contoured beam reflector antenna. The region on earth to be illuminated is transformed to a spherical coordinate system with a geostationary satellite as origin. The radiation pattern of a defocused paraboloid (symmetrical as well as offset configuration) has been studied.

The important magnitudes required for the design, $\theta_{3dB}$, BDF and $(F/D)_{min}$ are determined. A method has been developed of finding suitable excitation coefficients. Examples are shown for the Benelux and for Great Britain-Ireland. The power to be delivered to a contoured beam antenna is compared with that to be delivered to a conventional parabolic reflector antenna to illuminate the same region. It is checked whether the calculated power distribution meets the WARC specification. The frequency dependence of the contoured beam radiation pattern has been investigated. Finally, attention is paid to the aperture distribution, the spillover, the beam efficiency and the gain of a contoured beam antenna.

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1. **General introduction**

High gain spacecraft antennas will in the future employ contoured beams, i.e. beams giving a sharply contoured footprint on earth. With such antennas it is possible to concentrate the majority of the transmitted power on board the satellite uniformly over a defined area and to minimize the power outside that area. Dion et al. [1] designed a variable coverage satellite antenna consisting of a lens and a feed cluster. Duncan et al. [2] proposed, by using reflector antennas, to illuminate a defined region with a composite beam consisting of spot beams, each spot beam being combined with the adjacent spotbeam(s) at their -3dB contours. The design discussed in this report uses some of Duncan et al.'s ideas, but with narrower contours [3], [4]. Moreover, practical circumstances with respect to the launch vehicle to be used have been taken into consideration. This means that the maximum diameter of the reflector should be less than 2.5 metres if the Ariane rocket is used and 4.5 metres if Space Shuttle is used. The frequency applied is 20 GHz.
2. The coordinate transformation

2.1 Introduction
A circularly parabolic reflector antenna with only one feed will have a main beam with circularly symmetrical properties if the feed, too, has these properties. If we replace this main beam by a circular cone with the vertex at the satellite the cone pointing to earth, the area $A_1$ of earth and cone will generally not be circular (Fig. 2.1). On the other hand, the area $A_2$ on a sphere that has its centre at the vertex of the cone, will be a circle. In order to simplify calculations regarding a certain region of the earth (e.g. the Benelux), a coordinate transformation has to be carried out. This transformation enables us to determine for a point $P$ on earth with known longitude and latitude the coordinates $\theta$ and $\phi$ of a spherical coordinate system around the satellite (Fig. 2.2). In this way the contour of the region on earth is found transformed on a sphere with radius 1 and its centre $S$ which represents the satellite. The corresponding region can be covered with (circular) spotbeams which touch each other at their -3 dB contours (Fig. 2.3). We suppose that the satellite antenna axis points to the "centre" $O$ of the contour. The angle between this direction and the equatorial plane is $\theta_0$. In the present chapter equations will be derived to fulfil the coordinate transformation.

2.2 Transformation equations
Fig. 2.4 shows the latitude $\alpha$-longitude $\beta$ system of the earth with centre $M$ and a $\lambda$, $\theta'$, $\phi'$ satellite coordinate system. The satellite is assumed to be in a geostationary orbit above a point $A$ on the equator. Point $A$ is the reference point for our longitude $\beta$. In other words, the longitude in this point is supposed to be 0. The latitude $\alpha$ is the ordinary geographical latitude. For the signs of $\alpha$ and $\beta$ see Fig. 2.4. First of all we have to determine the $\theta'$, $\phi'$ coordinates of a point $P$ on earth by a given longitude and latitude.

The angle $\gamma$ is given by

$$\cos \gamma = \cos \alpha \cdot \cos \beta$$  \hspace{1cm} (2.1)

Using the cosine equation for triangle MSP we obtain
\[ PS^2 = MP^2 + MS^2 - 2 \, MP \cdot MS \cdot \cos \gamma \]
\[ = a^2 + r'^2 - 2 \, ar' \cos \gamma \]
\[ = a^2 \left( 1 + \frac{R^2}{a^2} - 2 \frac{R}{a} \cos \gamma \right) \]
\[ PS = a \sqrt{\frac{R^2}{a^2} - 2 \frac{R}{a} \cos \gamma + 1} \]  
(2.2)

with:

- \( a \) : radius of the earth,
- \( r' \) : distance from the centre \( M \) of the earth to the origin \( S \) of the satellite coordinate system,
- \( R = \frac{r'}{a} \)

The sine equation for triangle \( MSP \) results in

\[ \sin \frac{\theta'}{MP} = \sin \frac{\gamma}{PS} \]  
(2.3)

With eq. (2.1), (2.2) and (2.3) we obtain for \( \sin \frac{\theta'}{MP} \)

\[ \sin \frac{\theta'}{MP} = \frac{\sqrt{1 - \cos^2 \alpha \cos^2 \beta}}{\sqrt{R^2 - 2R \cos \gamma + 1}} \]  
(2.4)

To calculate the angle \( \phi' \) we introduce a new coordinate system \( x'_a, y'_a \) with \( M \) as origin. The \( x'_a, y'_a \) axes are tangential to the \( x', y' \) axes respectively (Fig. 2.5).

\( MP_1 \) is the projection of \( MP \) on the plane \( V \). \( P, P_1, P_2 \) is the projection of \( PQ \) on the plane \( V \) and \( PQ \) is tangential to the plane \( V \).

Hence

\[ P, P_1, P_2 = MP_1, PQ = a \sin \alpha. \]  
(2.5)

\( MP_1 \) is the projection of \( QR \) on the plane \( V \) and \( QR \) is tangential to the plane \( V \).

Hence

\[ MP_1 = QR = a \cos \alpha \sin \beta. \]  
(2.6)

Therefore

\[ \tan \phi' = \frac{MP_1}{MP_1} = \frac{\sin \beta}{\tan \alpha} = \tan \phi' \]  
(2.7)
Suppose that the antenna points to a point \( O (\alpha = \alpha_0', \beta = 0) \) on earth. We now define a new satellite coordinate system \( x, y, z \) (Fig. 2.6). The new coordinates are given by

\[
\begin{align*}
    x &= \sin \theta \cos \phi = \sin \theta' \cos \phi' \cos \theta_0 - \cos \theta' \sin \theta_0 \\
    y &= \sin \theta \sin \phi = \sin \theta' \sin \phi' \\
    z &= \cos \theta = \cos \theta' \cos \theta_0 + \sin \theta' \cos \phi' \sin \theta_0
\end{align*}
\]  

(2.8)

With the cosine equation on triangle MSP the distance satellite - earth \( SP \) is found to be

\[
SP = r' \cos \theta' - \sqrt{a^2 - r'^2(1 - \cos^2 \theta')}
\]  

(2.9)

with (using eq. 2.8)

\[
\cos \theta' = \left[ \frac{\cos \theta - \sin \theta \cos \phi}{\sin \theta_0 \cos \theta_0} \right] / \left[ \tan \theta_0 + \cotan \theta_0 \right]
\]  

(2.10)

2.3 Calculations

The coordinate transformation described in this chapter has been worked out for the contours of the Benelux and that of Great Britain - Ireland. As the contours are relatively small, the opening angle at the satellite bounding the contours is very small as well (\( \cos \theta \approx 1 \)); therefore, the contours may be mapped in one plane. Fig. 2.7 shows the transformed contour of the Benelux.

The satellite axis points to a point \( O \) on earth with geographical longitude \( 5^\circ \) E.L.; \( \theta_0 = 7.44^\circ \).

Fig. 2.8 represents the transformed contour of Great Britain - Ireland. In this case the satellite axis is directed to a point \( O \) on earth with geographical longitude \( 4.25^\circ \) W.L.; \( \theta_0 = 7.65^\circ \).

The following numerical values have been used:

\[
\begin{align*}
    a &= 6378 \text{ km} \\
    r' &= 42162 \text{ km} \\
    R &= 6.61
\end{align*}
\]
3. **Excitation coefficients of the feeds**

3.1 **Introduction**

Let us now illuminate a region transformed to the \( \theta, \phi \) coordinate system by an antenna with more than one feed. If the excitation coefficients of all feeds are equal, the power distribution over the illuminated area will in general not correspond to the requirements. This is caused by interference between the spotbeams. Therefore, we have to modify the excitation coefficients in such a way that the resulting power distribution approximates the desired power distribution as well as possible. We shall introduce optimization points, i.e. the points for which the desired power is defined. In this chapter a method of calculating the excitation coefficients will be developed.

3.2 **Calculation of the excitation coefficients**

Suppose we have \( n \) feeds and \( m \) optimization points. Define the far field in point \( SP_m, \theta_m, \phi_m \) caused by the \( n \)th feed of the reflector antenna as

\[
E_n(SP_m, \theta_m, \phi_m) = ER_n(SP_m, \theta_m, \phi_m) + jEI_n(SP_m, \theta_m, \phi_m)
\]

(3.1)

with

\[
ER_n(SP_m, \theta_m, \phi_m) : \text{the real part of } E_n(SP_m, \theta_m, \phi_m)
\]

\[
EI_n(SP_m, \theta_m, \phi_m) : \text{the imaginary part of } E_n(SP_m, \theta_m, \phi_m)
\]

The total far field in the point \( SP_m, \theta_m, \phi_m \) is

\[
E(SP_m, \theta_m, \phi_m) = ER(SP_m, \theta_m, \phi_m) + jEI(SP_m, \theta_m, \phi_m)
\]

(3.2)

with

\[
ER(SP_m, \theta_m, \phi_m) : \text{the real part of } E(SP_m, \theta_m, \phi_m)
\]

\[
EI(SP_m, \theta_m, \phi_m) : \text{the imaginary part of } E(SP_m, \theta_m, \phi_m)
\]

The complex excitation coefficient of the \( n \)th feed is

\[
COEF_n = COEFR_n + j \cdot COEFI_n
\]

(3.3)

with
We shall also define matrices \( \text{ReE} \) and \( \text{ImE} \) as

\[
\begin{bmatrix}
\text{ReE} \\
\text{ImE}
\end{bmatrix} =
\begin{bmatrix}
ER_1(SP_1', \theta_1', \phi_1') & \cdots & ER_n(SP_1', \theta_1', \phi_1') \\
\vdots & \ddots & \vdots \\
ER_1(SP_m', \theta_m', \phi_m') & \cdots & ER_n(SP_m', \theta_m', \phi_m')
\end{bmatrix}
\begin{bmatrix}
EI_1(SP_1', \theta_1', \phi_1') & \cdots & EI_n(SP_1', \theta_1', \phi_1') \\
\vdots & \ddots & \vdots \\
EI_1(SP_m', \theta_m', \phi_m') & \cdots & EI_n(SP_m', \theta_m', \phi_m')
\end{bmatrix}
\]

(3.4)

According to the complex arithmetic, the relation between Eq. 3.1, 3.2, 3.3 and 3.4 is given by

\[
\begin{bmatrix}
\text{ReE} \\
\text{ImE}
\end{bmatrix}
\begin{bmatrix}
\text{COEF}_1 \\
\text{COEF}_n
\end{bmatrix} =
\begin{bmatrix}
ER(SP_1', \theta_1', \phi_1') \\
\vdots \\
ER(SP_m', \theta_m', \phi_m')
\end{bmatrix}
\begin{bmatrix}
EI(SP_1', \theta_1', \phi_1') \\
\vdots \\
EI(SP_m', \theta_m', \phi_m')
\end{bmatrix}
\]

(3.5)

The desired total far field in an optimization point \( SP_m', \theta_m', \phi_m' \) is

\[
EO(SP_m', \theta_m', \phi_m') = EOR(SP_m', \theta_m', \phi_m') + j EOI(SP_m', \theta_m', \phi_m')
\]

(3.6)

with

\[
EOR(SP_m', \theta_m', \phi_m') : \text{the real part of } EO(SP_m', \theta_m', \phi_m')
\]

\[
EOI(SP_m', \theta_m', \phi_m') : \text{the imaginary part of } EO(SP_m', \theta_m', \phi_m')
\]

To calculate the excitation coefficients we have to solve
\[
\begin{bmatrix}
\text{ReE} & \text{-ImE} \\
\text{ImE} & \text{ReE}
\end{bmatrix}
\begin{bmatrix}
\text{COEFR}_1 \\
\text{COEFR}_n
\end{bmatrix}
- \begin{bmatrix}
\text{EOR}(\text{SP}_1, \theta_1, \phi_1) \\
\text{EOR}(\text{SP}_m, \theta_m, \phi_m)
\end{bmatrix} = \vec{0}
\]
(3.7)

With
\[
E_n'(\text{SP}_m, \theta_m, \phi_m) = E_n(\text{SP}_m, \theta_m, \phi_m) \cdot e^{jkSP_m}
\]
and
\[
EO'(\text{SP}_m, \theta_m, \phi_m) = EO(\text{SP}_m, \theta_m, \phi_m) \cdot e^{jkSP_m}
\]

Equation (3.9) becomes
\[
\begin{bmatrix}
\text{ReE}' & \text{-ImE}' \\
\text{ImE}' & \text{ReE}'
\end{bmatrix}
\begin{bmatrix}
\text{COEFR}_1 \\
\text{COEFR}_n
\end{bmatrix}
- \begin{bmatrix}
\text{EOR}'(\text{SP}_1, \theta_1, \phi_1) \\
\text{EOR}'(\text{SP}_m, \theta_m, \phi_m)
\end{bmatrix} = \vec{0}
\]
(3.9)

Equation (3.9) will have one exact solution if the number of feeds \(n\) is equal to the number of optimization points \(m\). Generally \(m > n\).

In this case we have to determine the least squares or the minimax solution of Equation (3.9).

The least squares solution of \(Ax - b = \vec{0}\) with \(A\) a known \(m' \times n'\) matrix, \(b\) a known \(m'\) vector and \(x\) an unknown \(n'\) vector is given by

\[
\min \sum_{i=1}^{m'} (b_i - \sum_{j=1}^{n'} A_{ij} x_j)^2
\]
(3.10)

The minimax solution is

\[
\min \max_i | b_i - \sum_{j=1}^{n'} A_{ij} x_j |
\]
(3.11)
In the next chapters we shall make use of the least squares solution. The phase of $EO(S_{m}^{P}, \theta_{m}, \phi_{m})$ is so chosen that the destructive interference between individual spotbeams is minimum.

4. **Calculation of the contoured beam antenna pattern**

4.1 **Introduction**

First the radiation pattern of a defocused parabolic reflector antenna has to be determined to calculate the radiation pattern of a contoured beam antenna. In this chapter we shall use a scalar theory to calculate the radiation pattern. The symmetrical parabolic reflector antenna will be considered. Because of the blockage in such a system, the same will be done for an offset configuration. The important magnitudes required for the design $\theta_{3dB}$, $RDF$ and $(F/D)_{min}$ are determined. After that we shall design contoured beam antennas for Great Britain-Ireland and the Benelux.

4.2 **The symmetrical parabolic reflector antenna**

4.2.1 **The defocused symmetrical parabolic reflector antenna**

Fig. 4.1 shows the geometry of a defocused symmetrical parabolic reflector antenna. According to Ruze [5] the scalar far field $E(R, \theta, \phi)$ is given by

$$E(R, \theta, \phi) = \frac{je^{-jkR}}{\lambda R} g(\theta, \phi)$$

(4.1)

with

$$g(\theta, \phi) = \int_{0}^{D/2} \int_{0}^{2\pi} f(r, \phi') e^{jkR} e^{jkrA \cos (\phi - \beta)} r dr d\phi'$$

(4.2)

$$A^2 = u^2 - \frac{2uv_s^2}{M(r)} \cos \phi + \frac{u_s^2}{M(r)^2}$$

(4.3)

$$\tan \beta = \frac{u \sin \phi}{u \cos \phi - u_s/M(r)}$$

(4.4)

$$u = \sin \theta \quad ; \quad u_s = \epsilon_x/F$$

(4.5)

$$\epsilon = \epsilon_x x' + \epsilon_y y' + \epsilon_z z'$$

(4.6)

$$\epsilon_z = (\epsilon_x^2 + \epsilon_y^2) / 2F \text{ (Petzval surface [5], [6])}$$

(4.7)
\[ M(r) = 1 + \left(\frac{r}{2F}\right)^2 \]  

(4.8)

In case \( f(r, \phi') = f(r) \) and \( \varepsilon_x = \varepsilon_t \cos \alpha, \varepsilon_y = \varepsilon_t \sin \alpha \) equation (4.2) becomes

\[ g(\theta, \phi) = 2\pi \int_0^{D/2} f(r) J_0(krA) rdr \]  

(4.9)

with

\[ x^2 = u^2 - \frac{2uu_s}{M(r)} \cos (\phi - \alpha) + \frac{u_s^2}{M(r)^2} \]  

(4.10)

\[ \tan \beta = \frac{u \sin (\phi - \alpha)}{u \cos (\phi - \alpha) - u_s/M(r)} \]  

(4.11)

\[ u_s = \varepsilon_t / F \]  

(4.12)

\[ \varepsilon_z = \varepsilon_t^2 / 2F ; \quad \varepsilon_t = \sqrt{\varepsilon_x^2 + \varepsilon_y^2} \]  

(4.13)

The approximations \( \varepsilon_t \ll 1 \) and \( \cos \theta = 1 \) have been introduced. In our calculations we shall use the aperture distribution function [7]

\[ f(r) = q + (1 - q) \left(1 - (2r/D)^2\right)^2 \]  

(4.14)

Fig. 4.2 shows the computed radiation patterns of a symmetrical parabolic reflector antenna \((f(r) = 1, \ D/\lambda = 510, \ F/D = 0.67, \ \phi = 0)\) with several feed displacements along the negative x axis given by \( \varepsilon_x = k \times 0.75 \lambda \) (k: number of beamwidths scanned).

In this figure the power is normed to the power \( P_0 \) radiated in the \( \theta = 0 \) direction by the same antenna with a focused feed.

From the figure we see that
- The left sidelobes become higher (coma),
- The right sidelobes become lower,
- The first right sidelobe disappears completely for a given feed displacement \((k = 6)\),
- The 3 dB beamwidth is almost constant,
- The maximum gain decreases little.
4.2.2 The 3 dB beamwidth

We have seen in section 4.2.1, that the field strength \( |g(\theta, \phi)| \) is given by

\[
|g(\theta, \phi)| = |g(\theta)| = |2\pi \int_0^{D/2} f(r) J_0(krA)rdr|.
\]

(4.15)

Under focused condition this becomes

\[
|g(\theta)| = |2\pi \int_0^{D/2} f(r) J_0(kr \sin \theta)rdr| = |2\pi \left( \frac{D}{2} \right)^2 \int_0^1 f(r') J_0 \left( r' \frac{\pi D}{\lambda} \sin \theta \right) r'dr' |.
\]

(4.16)

with \( r' = 2r/D \).

(4.17)

To calculate \( \theta_{3dB} \) we have to solve

\[
|g( \frac{\pi D}{\lambda} \sin (\theta_{3dB}) )| - |g(0)| \times 10^{-0.15} = 0.
\]

(4.18)

For small angle \( \theta_{3dB} \) the general solution becomes

\[
\sin \theta_{3dB} = \theta_{3dB} = \text{const} \frac{\lambda}{D}.
\]

(4.19)

Fig. 4.3 shows const as a function of \( p, q \) being a parameter.

4.2.3 Beam deviation factor

The beam deviation factor BDF is defined as [5]

\[
\text{BDF} = \frac{\sin \theta_m}{\tan \theta_s}.
\]

(4.20)

with

\[
\tan \theta_s = \frac{\epsilon_t}{F}.
\]

(4.21)

The angle \( \theta_m \) is the \( \theta \) belonging to \( |g(\theta)|_{\text{max}} \).
For small feed displacements \( \frac{c}{d} \ll 1 \) the BDF is given by [5]

\[
BDF = \frac{1}{\int_{0}^{1} f(r')r'\frac{M(r')}{r'}dr'}
\]

with

\[
M(r') = 1 + \left( \frac{r'D}{4F} \right)^2.
\]

Fig. 4.4 shows BDF as a function of \( F/D \), \( p \) and \( q \) being parameters.

For a large \( F/D \) (plane mirror) the BDF approximates 1.

4.2.4 Relation between the minimum \( F/D \), the diameter of a feed and the aperture distribution

As we have seen in Chapter 1, the feeds of a contoured beam antenna are so arranged that the -3 dB contours of adjacent feeds touch each other in the far field.

We now define \( (F/D)_{\text{min}} \) as the \( F/D \) of a parabolic reflector in which the feeds touch each other as do the accompanying -3 dB contours in the far field. For this \( (F/D)_{\text{min}} \) we find

\[
(F/D)_{\text{min}} = d \frac{\text{BDF}}{\lambda \text{ const}}
\]

where \( d \) is the diameter of the feed, which is mostly between 0.75 \( \lambda \) and 1.5 \( \lambda \) [2].

In Fig. 4.5 \( (F/D)_{\text{min}} \) has been represented as a function of \( d/\lambda \), \( p \) and \( q \) being parameters.

4.2.5 The contoured beam antenna pattern

Fig. 4.6a shows the radiation pattern of a symmetrical parabolic reflector antenna with 19 equally excited feeds arranged hexagonally and located on the Petzval surface. It appears from this figure that the power distribution over the illuminated part of the sphere around the satellite is not very uniform (-6.5 dB in the \( \theta = 0 \) direction).

Fig. 4.6b illustrates the radiation pattern again but now with improved excitation coefficients. The ripple is lower than 0.5 dB. From these two figures we see that it is very useful to optimize the excitation coefficients.

Fig. 4.7a and 4.8a show the normalized power distribution over Great Britain-Ireland using a symmetrical parabolic reflector antenna with
reflector diameter $D = 2.5 \text{ m}$ and $D = 4.5 \text{ m}$. The total number of feeds is 7 and 14, the optimization points (see also Figs. 4.7.b and 4.8.b) are 15 and 29 respectively.

An attempt to design also a contoured beam antenna for the Benelux failed because of the $D = 4.5 \text{ m}$ limit. However, as soon as it is possible to launch larger reflector diameters, it will also be possible to illuminate the Benelux with an appropriate contoured beam antenna. In the meantime we have designed a contoured beam antenna with a 10-metre diameter reflector for the Benelux.

In Fig. 4.9.a the normalized power distribution over the Benelux has been represented using a symmetrical parabolic reflector antenna with reflector diameter $D = 10 \text{ m}$, illuminated by 18 feeds.

The $-3 \text{ dB}$ contours of the individual spotbeams and the 36 optimization points are mapped in Fig. 4.9.b.

All computed powers are normed to the power $P_0 \text{ (dB)}$. This power is related to the power radiated in the $\theta = 0 \text{ direction}$ by a similar antenna with one feed in the focus, the excitation coefficient being equal to 1.

4.3 The offset parabolic reflector antenna

4.3.1 The defocused offset parabolic reflector antenna

The scalar far field $E(R, \theta, \phi)$ of a defocused offset fed parabolic reflector antenna with the geometry as shown in Fig. 4.10 illuminated by a feed with illumination function $G_f'(\psi, \xi)$ is given by [8].

$$E(R, \theta, \phi) = \frac{j \omega \mu}{2\pi R} e^{-jkR} \left[ \frac{P}{2\pi} \right]^{1/2} e^{-2jkF} g(\theta, \phi)$$

(4.25)

with

$$g(\theta, \phi) = \frac{2\pi}{\rho'} \int_{0}^{\rho} \int_{0}^{\psi} \frac{G_f'(\psi, \xi)}{2\pi} e^{-jk(\rho' - \rho, \xi - 2F)} \rho'^2 \sin \psi \sin \xi \text{ d}\psi \text{ d}\xi.$$  

(4.26)

One has to determine $\vec{p}, \vec{a}_R$, $\rho'$ and $G_f'(\psi, \xi)$ to calculate the far field.

The scalar product $\vec{p}, \vec{a}_R$ is found to be

$$\vec{p}, \vec{a}_R = \rho (\sin \psi \cos \xi \cos \psi_0 + \cos \psi \sin \psi_0) \sin \theta \cos \phi$$

$$-\rho \sin \psi \sin \xi \sin \theta \sin \phi$$

$$-\rho (\cos \psi \cos \psi_0 - \sin \psi \cos \xi \sin \psi_0) \cos \theta.$$  

(4.27)
The distance \( p' \) from feed to reflector is

\[
\rho' = \left[ (\rho (\sin \psi \cos \xi \cos \psi_0 + \cos \psi \sin \psi_0) + \mathbf{e}_x \right]^2 + \\
\left[ \mathbf{e}_y - \rho \sin \psi \sin \xi \right]^2 + \\
\left[ \mathbf{e}_z - \rho (\cos \psi \cos \psi_0 - \sin \psi \cos \xi \sin \psi_0) \right]^2 + \\
\left( \rho \sin \psi \sin \xi \right)^2.
\] (4.28)

Using the approximation \( \frac{\mathbf{e}}{\rho} \ll 1 \), Eq. (4.28) yields

\[
\rho' = \rho + \mathbf{e}_x (\sin \psi \cos \xi \cos \psi_0 + \cos \psi \sin \psi_0) \\
- \mathbf{e}_y \sin \psi \sin \xi \\
- \mathbf{e}_z (\cos \psi \cos \psi_0 - \sin \psi \cos \xi \sin \psi_0). 
\] (4.29)

The displacement vector \( \mathbf{e} \) can also be defined in the \( x'', y'', z'' \) coordinate system

\[
\mathbf{e} = \mathbf{e}_{x''} = \mathbf{e}_{y''} = \mathbf{e}_{z''}. 
\] (4.30)

The relation between \( \mathbf{e}_{x''}, \mathbf{e}_{y''}, \mathbf{e}_{z''} \) and \( \mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z \) is

\[
\mathbf{e}_x = \mathbf{e}_{x''} \cos \psi_0 + \mathbf{e}_{z''} \sin \psi_0 \\
\mathbf{e}_y = -\mathbf{e}_{y''} \\
\mathbf{e}_z = \mathbf{e}_{x''} \sin \psi_0 - \mathbf{e}_{z''} \cos \psi_0. 
\] (4.31)

Substitution of (4.31) in (4.29) yields

\[
\rho' = \rho + \mathbf{e}_{x''} \sin \psi \cos \xi \mathbf{e}_{y''} \sin \psi \sin \xi + \mathbf{e}_{z''} \cos \psi. 
\] (4.32)

The exponential power in the integrand (Eq. 4.26) now becomes (without the constant factor \( -jk \))

\[
\rho' - \rho \mathbf{e}_R - 2 F = \rho + \mathbf{e}_{x''} \sin \psi \cos \xi + \mathbf{e}_{y''} \sin \psi \sin \xi + \mathbf{e}_{z''} \cos \psi \\
- \rho (\sin \psi \cos \xi \cos \psi_0 + \cos \psi \sin \psi_0) \sin \theta \cos \phi \\
+ \rho \sin \psi \sin \xi \sin \theta \sin \phi \\
+ \rho (\cos \psi \cos \psi_0 - \sin \psi \cos \xi \sin \psi_0) \cos \theta \\
- 2F.
\]
Introducing the approximation \( \cos \theta = 1 \) and using the relation
\[
\rho = \frac{2F}{(1 + \cos \psi_0 \cos \psi - \sin \psi_0 \sin \psi \cos \xi)},
\]
Eq. (4.33) may be written as
\[
\rho' = \rho - 2F = \epsilon_+ \sin \psi \cos \xi + \epsilon_- \sin \psi \sin \xi + \epsilon_0 \cos \psi
\]
\[
- \rho (\sin \psi \cos \xi + \cos \psi \sin \psi_0) \sin \theta \cos \phi
\]
\[
+ \rho \sin \psi \sin \xi \sin \theta \sin \phi.
\]
(4.34)

To calculate \( G_{\xi}'(\psi, \xi) \), we define a new shifted coordinate system \( \psi', \xi' \) (see Fig. 4.11).

In this system the cosine illumination function of the feed is given by
\[
G_{\xi}'(\psi', \xi') = G_{\xi}(\psi') = 2(n+1) \cos^n \psi',
\]
\[
0 < \psi' < \frac{\pi}{2}
\]
\[
= 0 \quad \quad \psi' > \frac{\pi}{2}
\]
(4.35)

The relation between \( \psi, \xi \) and \( \psi', \xi' \) coordinates is
\[
\rho' \sin \psi' \cos \xi' = \rho \sin \psi \cos \xi + \epsilon_+ \sin \psi \sin \xi + \epsilon_0 \cos \psi
\]
\[
\rho' \sin \psi' \sin \xi' = \rho \sin \psi \sin \xi + \epsilon_- \cos \psi \sin \theta \cos \phi
\]
\[
\rho' \cos \psi' = \rho \cos \psi + \epsilon_0 \sin \theta \sin \phi
\]
(4.36)

Eq. 4.35 and Eq. 4.36 yield
\[
G_{\xi}'(\psi, \xi) = 2(n+1) \left\{ \frac{1}{\rho'} (\rho \cos \psi + \epsilon_0 \sin \theta \sin \phi) \right\}^n
\]
(4.37)

Fig. 4.12 shows the normalized power and phase of \( g(\theta, 0) \) and \( g(\theta, \frac{\pi}{2}) \) with different feed displacements along the \( x', x'' \) and \( y'' \) axis.

To get an impression of the total radiation pattern some radiation patterns are mapped for all values of \( \phi \) (see Fig. 4.13).

In this figure the power is normed to the power \( P_0' \) radiated in the \( \theta = 0 \) direction by the same antenna with a focused feed. The decrease in the maximum gain caused by defocusing seems to depend on \( \psi_0 \) and \( \psi_a \). Therefore, 20 log \( (|g(\theta, 0)|)_{\max} - P_0' \) and 20 log \( (|g(\theta, \frac{\pi}{2})|)_{\max} - P_0' \) have been calculated as functions of \( k \) for several \( \psi_0', \psi_a \) combinations (Fig. 4.14).

From these figures we can see that
- For feed displacements in the x", y" plane (offset focal plane [9]) BDF is a constant. The BDF belonging to a feed displacement along the x' axis is smaller than the BDF belonging to feed displacements along the y' (y") axis.

- The 3 dB beamwidth is almost constant for feed displacements along the x" and y" axis.

- The radiation patterns for feed displacements along the y" axis are in good agreement with the radiation patterns of a defocused symmetrical parabolic reflector antenna.

- For feed displacements along the x" axis we see a rather fast degradation of the radiation pattern. We have no deep zeros any longer, the main lobe gets a 'shoulder'.

- The decrease in the maximum of 20 \log \left( |g(\theta,0)| - P_0 \right) is almost equal to the decrease in the maximum of 20 \log \left( |g(\theta, \frac{\pi}{2})| - P_0 \right).

The larger the offset angle, the faster the decrease in the maximum by a given \psi_a.

- With regard to the main beam we notice that under the focused condition there exists a phase plane, which comprises the y axis and which is at an angle to the x axis, depending on \psi_0 (in case \psi_0 = 0 this angle is 0) [10]. If we displace the feed along the x' axis the phase plane will shift but the angle to the x axis remains the same. However, if we displace the feed along the x" axis the phase plane will almost coincide with the phase plane under the focused condition.

4.3.2 Power n of the cosine illumination function

Pagones [8] has derived an expression for the gain factor g of an offset parabolic reflector antenna irradiated by a feed with a cosine illumination function. He found

\[ g = 2^{(n+1)} \left( \frac{\cos \psi_0 + \cos \psi_a}{\sin \psi_a} \right)^2 \left( \int_0^{\psi_a} \cos \frac{n}{2} \psi \left( \frac{\sin \psi}{\cos \psi_0 + \cos \psi} \right) d\psi \right)^2 \]

(4.38)

He also found that g has a maximum as a function of n at a given \psi_0 and \psi_a.
In the following we shall choose the power \( P \) at a given \( \phi_0 \) and \( \phi_a \) in such a way that \( g \) is maximum under the focused condition. Fig. 4.15 shows \( g_{\max} \) as a function of \( \phi_a, \phi_0 \) being a parameter. For the \( \phi \) belonging to \( g_{\max} \) as a function of \( \phi_a \) see Fig. 4.16.

### 4.3.3 The 3 dB beamwidth

From Eq. 4.26 and \( \rho \sim D \) we obtain

\[
g(\theta, \phi) = \int_0^\infty \int_0^\infty \frac{\psi_a}{\psi} \left[ \frac{G_f(\psi)}{\rho} \right]^2 \exp \left( j \frac{\pi D}{\lambda} \sin \theta \text{ function} (\psi, \xi, \phi, \psi_0, \psi_a) \right) \rho^2 \sin \psi \, d\rho \, d\xi
\]

To calculate \( \theta_{3\, \text{dB}} \), we have to solve

\[
|g\left( \frac{\pi D}{\lambda} \sin \left( \frac{\theta_{3\, \text{dB}}}{\lambda} \right), \phi \right) - g(0)| \cdot 10^{-0.15} = 0
\]

The solution appears to be almost independent of \( \phi \).

For a small angle \( \theta_{3\, \text{dB}} \) the general solution becomes

\[
\sin \theta_{3\, \text{dB}} = \text{const} \frac{\lambda}{D}.
\]

Fig. 4.17 shows const as a function of \( \phi_a, \phi_0 \) being a parameter.

### 4.3.4 Beam deviation factor

The BDF of an offset parabolic reflector antenna is given by [11].

\[
\text{BDF} = F \sin \theta_m \sqrt{\frac{\varepsilon_y^2}{\varepsilon_x'^2} + \varepsilon_y'^2}.
\]

The \( x_1, y_1 \) plane is the offset focal plane.

Fig. 4.18 shows BDF as a function of \( \phi_a, \phi_0 \) being a parameter. It has been found in lit. [9] that for a given set of \( \phi_a \) and \( \phi_0 \) the ratio of the BDF to the \( F/D \) is constant for a given \( \phi_a \) and a given edge illumination. This means that the BDF of an offset reflector can be calculated from that of a front fed reflector with the same subtended angle using the following equation.
Thus

\[
\frac{(BDF)_{\text{offset}}}{(F/D)_{\text{offset}}} = \frac{(BDF)_{\text{sym}}}{(F/D)_{\text{sym}}} \tag{4.43}
\]

we have:

\[
(BDF)_{\text{sym}} = (BDF)_{\text{offset}} \frac{\cos \psi + 1}{\cos \psi + \cos \psi_0} \tag{4.44}
\]

We calculated \((BDF)_{\text{sym}}\) from \((BDF)_{\text{offset}}\) using Eq. (4.44). Fig. 4.19 shows this \((BDF)_{\text{sym}}\) as a function of \(\psi_a\).

4.3.5 Relation between the maximum subtended angle \(\psi_{a_{\text{max}}}\) and the diameter \(d\) of a feed.

According to Eq. 4.24 the relation between \((F/D)_{\text{min}}\) and the diameter of a feed \(d\) is given by:

\[
\frac{d}{\lambda} = \text{const} \frac{(F/D)_{\text{min}}}{BDF} \tag{4.45}
\]

In Section 4.3.4 we have seen that the ratio of BDF and F/D is independent of \(\psi_0\) at a given \(\psi_a\). From Fig. 4.17 it is seen that \(\text{const}\) is almost independent of \(\psi_0\). The result is that \(d/\lambda\) is only a function of \(\psi_a\).

Fig. 4.20 shows \(\psi_{a_{\text{max}}}\) as a function of \(d/\lambda\).

4.3.6 Phase of the desired total far field \(E_0'(SP_m', \theta_m', \phi_m')\)

In Section 4.3.1 we have seen that with regard to the main beam there exists a phase plane parallel to the y axis, and which is at an angle to the x axis depending on \(\psi_0\) (in case \(\psi_0 = 0\) this angle is 0). In the case of 2 feeds (Fig. 4.21) we have to choose the phase of \(E_0'(SP_m', \theta_m', \phi_m')\) in such a way that in point \(SP_1, \theta_1, \phi_1\) the phase of \(E_1'(SP_1, \theta_1, \phi_1')\) is equal to the phase of \(E_2'(SP_1, \theta_1, \phi_1')\). In other words, the phase planes of spotbeams 1 and 2 should coincide. Otherwise destructive interference will occur.

4.3.7 The contoured beam antenna pattern

Fig. 4.22a shows the normalized power distribution over Great Britain - Ireland, using an offset parabolic reflector antenna with reflector diameter \(D = 4.5\) m. The number of feeds and optimization points are 14 and 29 respectively (see also Fig. 4.22b). We notice that this power distribution is almost equal to the power distribution found in section 4.2.5 (Fig. 4.8).
5. Further investigation of some properties of the contoured beam antenna

5.1 Introduction
In this chapter the power delivered to the contoured beam antenna is compared with that delivered to a conventional antenna to obtain the same power density in the illuminated region in both cases. To check whether the calculated power distribution meets the WARC specifications we shall calculate the sidelobes of the contoured beam antenna pattern. After that the frequency dependence of the radiation pattern will be studied. Finally, attention will be paid to the aperture distribution, the spill-over, the beam efficiency and the gain of a contoured beam antenna [12].

5.2 Comparison of contoured beam antennas with conventional reflector antennas
The overall gain $G(0,0)$ of a reflector antenna, irradiated by a feed with a cosine illumination function is given by

$$G(0,0) = \frac{\pi P}{P_{T}}$$

with Eq. 4.38

$$G(0,0) = \left(\frac{\pi D}{\lambda}\right)^2 \frac{\cos \psi_0 + \cos \psi_a}{\sin \psi_a} \left(\int_{0}^{\psi_a} \cos \frac{n}{2} \left(\frac{\sin \psi}{\cos \psi_0 + \cos \psi}\right) d\psi\right)^2$$

We notice that $g$ is determined completely by $\psi_0$, $\psi_a$ and $n$. Let the $n$th feed of a contoured beam antenna radiate a power $P_{\text{in}}$ and let the diameter of the aperture be $D_1$. We now illuminate the region by a conventional reflector antenna with one feed in the focus. The wavelength $\lambda$, $\psi_0$, $\psi_a$ and the radiation pattern of the feed are equal to those of the contoured beam antenna. The reflector diameter $D_2$ of the conventional antenna should be such that the -3 dB contour of the main beam comprises the entire region. Let further the feed of the conventional antenna radiate a power $P_2$. We use the same normalization as we used for the contoured beam antenna. The ratio $P_{\text{in}}$ to $P_2$ is now

$$\frac{P_{\text{in}}}{P_2} = \left(\frac{D_2}{D_1}\right)^2 |\text{COEF}_n|^2$$
The ratio between the total power $P_1$ supplied by all $n$ feeds to the reflector and $P_2$ now is

$$\frac{P_1}{P_2} = \left(\frac{D_2}{D_1}\right)^2 \frac{2n}{\nu} |\text{COEF}_n|^2 \quad (5.4)$$

Some results are found in the table below:

<table>
<thead>
<tr>
<th>Figure</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$\frac{P_1}{P_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.7</td>
<td>2.5</td>
<td>0.77</td>
<td>0.24</td>
</tr>
<tr>
<td>4.8</td>
<td>4.5</td>
<td>0.77</td>
<td>0.18</td>
</tr>
<tr>
<td>4.9</td>
<td>10</td>
<td>1.67</td>
<td>0.35</td>
</tr>
<tr>
<td>4.22</td>
<td>4.5</td>
<td>0.78</td>
<td>0.15</td>
</tr>
</tbody>
</table>

5.3 The WARC specifications

To check whether the calculated power distribution meets the WARC specifications (see Appendix A), the sidelobes of the contoured beam antenna pattern have to be known. Fig. 5.1 shows the main beam and the sidelobes of the contoured beam antenna radiation pattern for Great Britain - Ireland ($D = 4.5 \text{ m}$) described in Section 4.2.5.

In fact, the WARC specifications are drawn up for antennas with circular and elliptical beams. When applying these specifications to the radiation pattern represented in Fig. 5.1, too strong requirements are imposed on the directions $285^\circ < \phi < 310^\circ$.

In these directions $\theta_{3\text{dB}}/2$ is very small, (see Fig. 5.2). To get over this problem, the WARC specifications can be applied to an ellipse enclosing the $-3 \text{ dB}$ contour of the contoured beam antenna pattern. Fig. 5.3 shows that our power distribution meets these specifications. According to international norms every country has its own ellipse. Fig. 5.4 shows the combined ellipses for Great Britain and Ireland. We notice that our power distribution also meets the WARC specifications applied to these ellipses.

5.4 Frequency dependence of the radiation pattern

In Figure 5.1 the normalized power distribution over Great Britain - Ireland is shown using a symmetrical parabolic reflector with 14 feeds ($D = 4.5 \text{ m}$). The frequency applied is $20 \text{ GHz}$. Generally, one will not transmit at only one frequency : a frequency band is used. That is the reason why it is very important to know the frequency dependence of the radiation pattern. Figs. 5.5 and 5.6 contain the normalized power distributions at 18 and 22 GHz. All computed powers are normed to the power
The latter is related to the power radiated in the $\theta = 0$ direction by the same antenna with one focused feed at 20 GHz the excitation coefficient being equal to 1. We notice that in the case of such a great relative frequency change (10%) the deformation of the original power pattern is small.

5.5 Aperture distribution

The scalar far field of a defocused parabolic reflector antenna is given by (Eq. 4.35, 4.26)

$$\mathbf{E}(R, \theta, \phi) = \frac{j \omega}{2\pi R} e^{-jkR} \left( \frac{\xi}{\hbar} \right)^\frac{P}{2\pi} e^{-2jkF} g(\theta, \phi)$$  \hspace{1cm} (5.5)

with

$$g(\theta, \phi) = \int_0^{2\pi} \int_0^\infty \psi_a \left[ G_{||}^1(\psi, \xi) \right]^\frac{1}{\rho} \left[ e^{jk(\rho' - \rho \alpha_F - 2F)} \right] \rho^2 \sin \psi d\psi d\xi$$  \hspace{1cm} (5.6)

If we use a contoured beam antenna, this formula becomes

$$g(\theta, \phi) = \int_0^{2\pi} \int_0^\infty \psi_a \left[ \sum_{i} \text{COEF}_i \left[ G_{||}^{i'}(\psi, \xi) \right]^\frac{1}{\rho'} \left[ e^{jk(\rho' - \rho \alpha_F - 2F)} \right] \right] \rho^2 \sin \psi d\psi d\xi$$  \hspace{1cm} (5.7)

The plane $z = 0$ is the aperture plane. The approximations $\frac{\xi}{\rho} < < 1$ and $\cos \theta \approx 1$ are now introduced. For the aperture distribution we find

$$F(\psi, \xi) = \sum_i \text{COEF}_i \left[ G_{||}^{i'}(\psi, \xi) \right]^\frac{1}{\rho'} \left[ e^{jk\rho \cdot \xi / \rho} \right]$$  \hspace{1cm} (5.8)

with

$$G_{||}^{i'}(\psi, \xi) = 2(n+1) \left[ \frac{1}{\rho_i} \left( \rho \cos \psi + \xi z_i \right) \right]^n$$  \hspace{1cm} (5.9)

In section 4.3.7 we calculated the power distribution over Great Britain - Ireland using an offset parabolic reflector antenna with $D = 4.5$ m. The normalized power and phase of the aperture field of this antenna is shown in Fig. 5.7. We notice that if $F(r, \phi') = 0$, the phase arg $F(r, \phi')$ is not defined. The coordinates $r$ and $\phi'$ are the polar coordinates in the aperture plane. The latter is no longer an equiphase plane. So it will be
very difficult to produce this aperture distribution with one feed.

5.6 Spillover

5.6.1 Spillover of a defocused parabolic reflector antenna

For the geometry of a defocused reflector antenna see Section 4.3.1, Fig. 4.10. The unit vector in the direction of the incident ray is \( \vec{a}_p \), and the distance from feed to reflector \( \rho' \). The spillover may be written as [12].

\[
\text{spillover} = 1 - \frac{1}{4\pi} \int_0^{2\pi} \int_0^\infty \frac{G'_F(\psi, \xi)}{\rho'^2} \frac{\vec{a}_n \cdot \vec{a}_p}{\vec{a}_n \cdot \vec{a}_p} \frac{2\sin \psi \psi d\xi}{\vec{a}_n \cdot \vec{a}_p} (5.10)
\]

\( G'_F(\psi, \xi) \) is given by Eq. 4.37.

The calculation of \( \vec{a}_n \cdot \vec{a}_p \), and \( \vec{a}_n \cdot \vec{a}_p \) will be performed in the \( x, y, z \) coordinate system. The equation of the paraboloidal surface in these coordinates is

\[
g(x, y, z) = x^2 + y^2 - 4F(z + F) = 0 \quad (5.11)
\]

The unit vector normal to the reflector is

\[
\vec{a}_n = \frac{\vec{v}_g}{|\vec{v}_g|} = \begin{bmatrix} \rho_x \\ \rho_y \\ -2F \end{bmatrix} \frac{1}{\sqrt{\rho_x^2 + \rho_y^2 + 4F^2}}. \quad (5.12)
\]

The ratio of the two scalar products becomes

\[
\frac{\vec{a}_n \cdot \vec{a}_p}{\vec{a}_n \cdot \vec{a}_p} = \frac{\rho \vec{a}_n \cdot \vec{a}_p}{\rho' \vec{a}_n \cdot \vec{a}_p} = \frac{\rho \vec{a}_n \cdot \vec{a}_p + \vec{a}_n \cdot \vec{a}_p}{\rho' \vec{a}_n \cdot \vec{a}_p} = \frac{\rho}{\rho'} \left(1 + \frac{\vec{v}_g \cdot \vec{e}}{\vec{v}_g \cdot \vec{p}}\right). \quad (5.13)
\]

With Eq. 5.10 the spillover yields

\[
\text{spillover} = 1 - \frac{1}{4\pi} \int_0^{2\pi} \int_0^\infty \frac{1}{2(n+1)} \left(\rho \cos \psi + \epsilon \xi\right) \left|\frac{\vec{v}_g}{\vec{v}_g \cdot \vec{p}}\right|^3 \sin \psi \psi d\xi. \quad (5.14)
\]

The spillover has been calculated for feed displacements along the \( x'' \) and \( y'' \) axes. The displacements of the feed have been so chosen that the main
beam is scanned \( k \) beamwidths. These feed displacements are given by

\[
\varepsilon = k \, \text{const} \frac{F}{D} \frac{\lambda}{BDF}.
\]

Fig. 5.8 shows the spillover of a defocused symmetrical parabolic reflector antenna as a function of \( k \) for feed displacements in the \( x^" \) \( y^" \) plane, \( D = 4.5 \) m (Space Shuttle) and \( \lambda = 1.5 \) cm.

Fig. 5.9 shows the same but now for \( D = 2.5 \) m (Ariane rocket) and \( \lambda = 1.5 \) cm.

Fig. 5.10 represents the spillover of a defocused offset parabolic reflector antenna as a function of \( k \) for feed displacements along the \( x^" \) and \( y^" \) axis, \( D = 4.5 \) m and \( \lambda = 1.5 \) cm and Fig. 5.11 for \( D = 2.5 \) m and \( \lambda = 1.5 \) cm.

We can also determine the spillover as a function of \( \varepsilon_D \) by a given \( \psi_0 \) \( \psi_a \) combination.

Fig. 5.12 illustrates the spillover of a defocused symmetrical parabolic reflector antenna as a function of \( \varepsilon_t / D \), and Fig. 5.13 that of a defocused offset parabolic reflector antenna as a function of \( \varepsilon_x / D \) and \( \varepsilon_y / D \).

### 5.6.2 Spillover of a contoured beam antenna

It is easy to see that the spillover of a contoured beam antenna is given by

\[
\text{spillover c.b.a.} = \frac{\sum_n |\text{COEF}_n|^2 \text{spillover}_n}{\sum_n |\text{COEF}_n|^2}\]

with

- Spillover c.b.a. : Spillover of the contoured beam antenna,
- \( \text{spillover}_n \) : Spillover of the \( n^{th} \) feed,
- \( |\text{COEF}_n|^2 \) : The complex excitation coefficient of the \( n^{th} \) feed.

Using Eq. 5.16, the spillover of the offset contoured beam antenna for Great Britain and Ireland is 8.132%.

### 5.7 Beam efficiency

We define the beam efficiency as the ratio of the utilized power to the total power radiated. The utilized power equals the solid angle bounding the region to be illuminated multiplied by the minimum power density within that solid angle. Usually this minimum power density is half the maximum power density (-3dB requirements). For the beam efficiency of a reflector antenna we find
The relation between $P_m$ and $P_T$ is given by

$$P_m = \frac{P_T^{2 \pi \theta_c(\phi)}}{4 \pi \left(\frac{\pi D}{\lambda}\right)^2}.$$  \hspace{1cm} (5.18)

Equations 5.17 and 5.18 yield

$$\eta_B = \frac{g \left(\frac{\pi D}{\lambda}\right)^2}{8 \pi \left(\frac{\pi D}{\lambda}\right)^2} \int_0^{2 \pi} \int_0^{\theta_c(\phi)} \sin \theta d\theta d\phi.$$  \hspace{1cm} (5.19)

The maximum beam efficiency ($\theta_c(\phi) = \frac{\pi}{2} \theta_{3dB}$) is

$$\eta_{B_{max}} = \frac{g}{2} \left(\frac{\pi D}{\lambda}\right)^2 \sin^2 \frac{\pi}{2} \theta_{3dB}.$$  \hspace{1cm} (5.20)

with $\theta_{3dB}$ the half-power beamwidth.

For small $\theta_{3dB}$ Eq. 5.20 becomes

$$\eta_{B_{max}} = g \frac{\pi^2}{32} \text{const}^2.$$  \hspace{1cm} (5.21)

The maximum beam efficiency is completely determined by $\psi_0$ and $\psi_a$, Fig. 5.15 shows $\eta_{B_{max}}$ as a function of $\psi_a$, $\psi_0$ being a parameter. It appears that $\eta_{B_{max}} = 33\%$ if we use a feed with a cosine illumination function. The beam efficiency of the conventional antenna ($D = 0.78 \text{ m}$) to illuminate Great Britain and Ireland is 8.4%. This is considerably less than $\eta_{B_{max}}$. We can also use Eq. 5.17 to compute the beam efficiency of a contoured beam antenna. $P_m$ is then the desired power distribution within the contour, $P_T$ is the total power radiated by the feeds. The beam efficiency of the contoured beam antenna for Great Britain and Ireland described in Section 4.3.7 is $\eta_B = 55\%$. This is larger than $\eta_B$ and $\eta_{B_{max}}$ of the comparable conventional antenna.
5.8 Gain

For the gain of a contoured beam antenna we find the expression

\[ G(\theta, \phi) = \frac{4\pi \, P(\theta, \phi)}{P_1}, \]  

(5.22)

with \( P_1 \) the power radiated by the feeds of the antenna.

We have normed \( P(\theta, \phi) \) to the power \( P_0(0,0) \) of a similar antenna with one focused feed, excitation coefficient being 1. The power radiated by the feed of this antenna is \( P_2 \) and its maximum gain is given by

\[ g\left(\frac{\pi D^2}{\lambda}\right) = \frac{4\pi \, P_0(0,0)}{P_2}, \]  

(5.23)

g being the gain factor.

The ratio of the power \( P_2 \) radiated by the feed of this antenna and the power \( P_1 \) radiated by the feeds of the contoured beam antenna is

\[ \frac{P_2}{P_1} = \frac{1}{\sum_{n} |\text{COEF}_n|^2}. \]  

(5.24)

With Eq. 5.22, 5.23 and 5.24 the gain of the contoured beam antenna is found to be

\[ G(\theta, \phi) = \frac{P(\theta, \phi)}{P_0(0,0)} \frac{4\pi \, P_0(0,0)}{P_2} \frac{P_2}{P_1}\]  

\[ \frac{P(\theta, \phi)}{P_0(0,0)} g\left(\frac{\pi D^2}{\lambda}\right) \frac{1}{\sum_{n} |\text{COEF}_n|^2}. \]  

(5.25)

As the distances from the satellite to the points inside the contour to be illuminated vary only little it is possible to obtain a rather accurate impression of the gain from the normalized power distribution. To this we must add

\[ 10 \log \left[ g\left(\frac{\pi D^2}{\lambda}\right) \frac{1}{\sum_{n} |\text{COEF}_n|^2}\right] \]

to find the gain \( G(\theta, \phi) \). This yields, for example in Fig. 4.22,

\[ 10 \log \left[ g\left(\frac{\pi D^2}{\lambda}\right) \frac{1}{\sum_{n} |\text{COEF}_n|^2}\right] = 51.6 \text{ dB} \]  

(5.26)

We could have illuminated Great Britain and Ireland by a conventional antenna with one focused feed. At 20 GHz an aperture diameter \( D = 0.78 \text{ m} \) would have
been sufficient. The gain $G(0,0)$ belonging to this antenna is 43.4 dB. We notice that the gain of the contoured beam antenna is larger than that of the conventional antenna.

6. Conclusion

Well-designed contoured beam antennas using reflectors offer the possibility of distributing large parts of the transmitted power uniformly over arbitrary regions. The gain and the beam efficiency of the contoured beam antenna are considerably larger than those of a conventional antenna. Therefore, the power to be delivered to the contoured beam antenna is much smaller than that to be delivered to the conventional antenna to obtain the same power density in the illuminated region.

Our calculated radiation pattern of a contoured beam antenna for Great Britain - Ireland ($D = 4.5$ m) meets the 'modified' WARC specifications. For a relatively large frequency change (10%) the deformation of this radiation pattern seems to be small. Because of the relatively small feed displacements (large $D/\lambda$) the spillover of the contoured beam antenna is not much larger than that of the conventional focused antenna.

The cross polarisation properties of the contoured beam antenna and the design of the array feed have to be investigated before deciding to employ this antenna.
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APPENDIX A

Fig. A.1.1 shows the reference pattern for co-polar and cross-polar components according to the WARC specifications for a satellite transmitting antenna.

Curve A: co-polar component.

\[-12 \left( \frac{\theta}{\theta_{3dB}} \right)^2 \quad \text{for } 0 \leq \theta \leq 1.58 \theta_{3dB}\]

\[-30 \quad \text{for } 1.58 \theta_{3dB} < \theta \leq 3.16 \theta_{3dB}\]

\[-17.5 + 25 \log_{10} \left( \frac{\theta}{\theta_{3dB}} \right) \quad \text{for } 3.16 \theta_{3dB} < \theta\]

Curve B: cross-polar component.

\[-(40 + 40 \log_{10} \left| \frac{\theta}{\theta_{3dB}} \right| -1) \quad \text{for } 0 \leq \theta < 0.33 \theta_{3dB}\]

\[-33 \quad \text{for } 0.33 \theta_{3dB} < \theta < 1.67 \theta_{3dB}\]

\[-(40 + 40 \log_{10} \left| \frac{\theta}{\theta_{3dB}} \right| -1) \quad \text{for } 1.67 \theta_{3dB} < \theta\]
Fig. 2.1: Noncircular area $A_1$ on earth; circular area $A_2$ on a sphere around the satellite $S$.

Fig. 2.2: The coordinates $\theta$, $\phi$ of a spherical coordinate system around the satellite.
Fig. 2.3:  
--- The transformed contour $C'$

----- The -3 dB contour of a spotbeam.

Fig. 2.4: The latitude $\alpha$ - longitude $\beta$ system of the earth and a $l, \theta', \phi'$ satellite coordinate system.
Fig. 2.5: The $x'_a$, $y'_a$ coordinate system.

Fig. 2.6: The new $x$, $y$, $z$ satellite coordinate system.
Fig. 2.7: The transformed contour of the Benelux.

Fig. 2.8: The transformed contour of Great Britain - Ireland.
Fig. 4.1: The geometry of a defocused symmetrical parabolic reflector antenna.

Fig. 4.2: The radiation pattern of a defocused symmetrical parabolic reflector antenna, $\xi = k \times 0.75 \lambda$ (k: number of beamwidths scanned), $f(x) = 1$, $D/\lambda = 510$, $F/D = 0.67$. 
Fig. 4.2 (continued)
Fig. 4.2 (continued)
Fig. 4.2 (continued)

Fig. 4.3: Const as a function of p, q being a parameter.
Fig. 4.4: The BDF as a function of $F/D$, $p$ and $q$ being parameters.
Fig. 4.5: \((F/D)_{\text{min}}\) as a function of \(d/\lambda\), \(p\) and \(q\) being parameters.
Fig. 4.5: \((F/D)_{\text{min}}\) as a function of \(d/\lambda\), \(p\) and \(q\) being parameters.

Fig. 4.6a: Radiation pattern of a symmetrical parabolic reflector antenna with 19 equally excited feeds, arranged hexagonally.

---

3dB contour of one spotbeam
lines of constant power.
Fig. 4.6b: Radiation pattern of a symmetrical parabolic reflector antenna with 19 feeds arranged hexagonally with improved excitation coefficients.

Fig. 4.7a: The normalized power distribution over Great Britain - Ireland using a symmetrical parabolic reflector antenna, $D = 2.5 \text{ m}, p = 1, q = 10^{-0.5}, F/D = 0.6$, freq. = 20 GHz.
Fig. 4.7b: The normalized power distribution over Great Britain - Ireland using a symmetrical parabolic reflector antenna, 
D = 2.5 m, p = 1, q = 10^{-0.5}, F/D = 0.6, freq. = 20 GHz.

--- -3dB contour of one spotbeam.
• optimization point.
— transformed contour of Great Britain - Ireland.

Fig. 4.8a: The normalized power distribution over Great Britain - Ireland using a symmetrical parabolic reflector antenna, 
D = 4.5 m, p = 1, q = 10^{-0.5}, F/D = 0.6, freq. = 20 GHz.

--- transformed contour of Great Britain - Ireland.
-- lines of constant power.
Fig. 4.8b: The normalized power distribution over Great Britain - Ireland using a symmetrical parabolic reflector antenna, 
D = 4.5 m, p = 1, q = 10^-0.5, F/D = 0.6, freq. = 20 GHz.

Fig. 4.9a: The normalized power distribution over the Benelux using a symmetrical parabolic reflector antenna, 
D = 10 m, f(r) = 1, F/D = 0.67, freq. = 20.4 GHz.
Fig. 4.9b: The normalized power distribution over the Benelux using a symmetrical parabolic reflector antenna, $D = 10 \text{ m}$, $f(r) = 1$, $F/D = 0.67$, freq. = 20.4 GHz.

Fig. 4.10: The geometry of a defocused offset fed parabolic reflector antenna.
Fig. 4.11: The shifted $\psi'$, $\xi'$ coordinate system.

Fig. 4.12: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function, $n = 16.448$, $\psi = 35^\circ$, $\psi' = 30^\circ$, $D/\lambda = 300$.

\[ a : \epsilon_x = 1.13 \lambda ; \epsilon_y = \epsilon_z = 0. \]  

\[ b-h : \epsilon_x'' = k * 1.13 \lambda \ (k : \text{number of beamwidths scanned}), \epsilon_y'' = \epsilon_z'' = 0. \]  

\[ i-l : \epsilon_y'' = k * 1.13 \lambda \ (k : \text{number of beamwidths scanned}), \epsilon_x'' = \epsilon_z'' = 0. \]
Fig. 4.12 b

\[ 20 \log \left\{ \log (Q, \theta) \right\} - \theta \]

\[ k = -3 \]

Fig. 4.12 c

\[ 20 \log \left\{ \log (Q, \theta) \right\} - \theta \]

\[ k = -2 \]
Fig. 4.12 d

\[ 20 \log \{ \log (\theta, \alpha) \} - P_e \]

\[ \theta (\text{Rad}) \]

\[ k = -1 \]

Fig. 4.12 e

\[ 20 \log \{ \log (\theta, \alpha) \} - P_e \]

\[ \theta (\text{Rad}) \]

\[ k = 0 \]
Fig. 4.12 f

$20 \log \{ \arg g(\Theta, 0) \} - \Phi_0$

$k = 1$

Fig. 4.12 g

$20 \log \{ \arg g(\Theta, 0) \} - \Phi_0$

$k = 2$
Fig. 4.12 h

Fig. 4.12 i
Fig. 4.12 j

Fig. 4.12 k
Fig. 4.12

\[ 20 \log \left\{ \mid g(\theta, \phi) \mid \right\} - P_0 \]

\[ k = -3 \]

\[ \theta (\text{Rad}) \]

\[ 200 \quad 200 \quad 100 \]

\[ -200 \quad -300 \]

\[ \theta (\text{Rad}) \]

\[ 1.0 \times 10^{-3} \]
Fig. 4.13: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function.

\[ \theta_{\text{rad}} = \theta_0, \quad n = 16.448, \quad \varphi_0 = 35^\circ, \quad \varphi_a = 30^\circ, \quad \delta d/A = 300. \]
Fig. 4.13b: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function, $n = 16.448$, $\psi_0 = 35^\circ$, $\psi_a = 30^\circ$, $\phi = 300^\circ$.  

\[ \arg(g(\theta, \phi)) \]

\[ e_x = e_y = e_z = 0 \]
Fig. 4.13c: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function, $n = 16.418$, $\psi_0 = 35^\circ$, $\psi_a = 30^\circ$, $D/\lambda = 300$. 
Fig. 4.13d: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function, \( n = 16.448, \psi_0 = 35^\circ, \psi_a = 30^\circ, D/\lambda = 300. \)
Fig. 4.13e: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function, 

\[ n = 16.448, \psi_0 = 35^\circ, \psi_a = 30^\circ, D/\lambda = 300. \]
Fig. 4.13c: The radiation pattern of a defocused offset parabolic reflector antenna illuminated by a feed with a cosine illumination function, for $n = 16.448$, $\vartheta = 35^\circ$, $\varphi = 30^\circ$, $\lambda = 300$.

\[ \text{arg} \{g(\theta, \phi)\}, \]

\[ 130 \quad \kappa = 1.13 \lambda; \quad k = -2, \]

\[ \epsilon_x''' = \epsilon_y''' = 0. \]
Fig. 4.14: $20 \log (|g(\theta, 0)|)_{\text{max}} - P_0$ and $20 \log (|g(\theta, \frac{\psi}{2})|)_{\text{max}} - P_0'$ as a function of $k$, $\psi_0$ and $\psi_a$, being parameters.
**Fig. 4.15**: The maximum gain factor $g_{\text{max}}$ as a function of $\psi_a$, $\psi_0$ being a parameter.

**Fig. 4.16**: The $n$ of the cosine illumination function belonging to $g_{\text{max}}$ as a function of $\psi_a$. 
Fig. 4.17: Const as a function of $\psi_a$, $\psi_0$ being a parameter.

Fig. 4.18: The BDF as a function of $\psi_a$, $\psi_0$ being a parameter.

Fig. 4.19: The BDF$_{sym}$ as a function of $\psi_a$. 
Fig. 4.20: $\psi_{\text{max}}$ as a function of $d/\lambda$.

Fig. 4.21: The -3 dB contours of two adjacent spotbeams.
Fig. 4.22a: Transformed contour of Great Britain - Ireland.
--- lines of constant power.

Fig. 4.22b: -3 dB contour of one spotbeam.
- optimization point.
--- transformed contour of Great Britain - Ireland.

Fig. 4.22 a,b: The normalized power distribution over Great Britain - Ireland using an offset parabolic reflector antenna,
D = 4.5 m, $\psi_0 = 35^\circ$, $\psi_a = 30^\circ$, $n = 16.448$, freq. = 20 GHz.
Fig. 5.1: The normalized power distribution over Great Britain - Ireland using a symmetrical parabolic reflector antenna, $D = 4.5\, \text{m}$, $p = 1$, $q = 10^{-0.5}$, $F/D = 0.6$, freq. = 20 GHz.
Fig. 5.3: The WARC specifications applied to an ellipse enclosing the -3 dB contour of the contoured beam antenna pattern.
Fig. 5.4 The WARC specifications applied to the combined ellipses for Great Britain and Ireland.
Fig. 5.5: The normalized power distribution over Great Britain - Ireland using a symmetrical parabolic reflector antenna, $D = 4.5 \text{ m}$, $p = 1$, $q = 10^{-0.5}$, $F/D = 0.6$, freq. = 18 GHz.
Diagram 5.6: The normalized power distribution over Great Britain – Ireland

Legend:
- Red = 22 kV
- Green = 11 kV
- Blue = 33 kV

Symbols:
- P = 0.5
- Q = 10
- R = 0.5
- E = 0.6

Using a symmetrical periodic rectifier arrangement.
Fig. 5.7b: The normalized phase (degrees) of the aperture field.
Fig. 5.8: The spillover of the defocused symmetrical parabolic reflector antenna as a function of $k$ for feed displacements in the $x''$, $y''$ plane, $D = 4.5$ m, $\lambda = 1.5$ cm.

Fig. 5.9: The spillover of a defocused symmetrical parabolic reflector antenna as a function of $k$ for feed displacements in the $x''$, $y''$ plane, $D = 2.5$ m, $\lambda = 1.5$ cm.
Fig. 5.10: The spillover of a defocused offset parabolic reflector antenna as a function of k for feed displacements along the x' and y' axes, D = 4.5 m, λ = 1.5 cm.
Fig. 5.11: The spillover of a defocused offset parabolic reflector antenna as a function of $k$ for feed displacements along the $x''$ and $y''$ axes, $D = 2.5$ m, $\lambda = 1.5$ cm.
The spillover of a defocused symm. parabolic reflector antenna as a function of $\varepsilon_t/D$.

Fig. 5.12: The spillover of a defocused offset parabolic reflector antenna as a function of $\varepsilon_{x''}/D$ and $\varepsilon_{y''}/D$. 

Fig. 5.13: The spillover of a defocused offset parabolic reflector antenna as a function of $\varepsilon_{x''}/D$ and $\varepsilon_{y''}/D$. 

$\psi_0 = 15^\circ$, $\psi_a = 10^\circ$
$\psi_0 = 25^\circ$, $\psi_a = 20^\circ$
$\psi_0 = 30^\circ$, $\psi_a = 30^\circ$
$\psi_0 = 40^\circ$, $\psi_a = 40^\circ$
$\psi_0 = 45^\circ$, $\psi_a = 45^\circ$
Fig. 5.14: The angle $\theta_0(\phi)$ at which, at a given $\phi$, the contour of the illuminated region is seen by the satellite.

Fig. 5.15: The maximum beam efficiency $\eta_{B,\text{max}}$ as a function of $\psi_0$, $\phi_0$ being a parameter.
Fig. A1.1: The reference pattern for co-polar and cross-polar components according to the WARC specifications for a satellite transmitting antenna.
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