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POTENTIAL MEASUREMENTS

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1. Introduction.

The goal of this study is the transformation of torso-skin potentials into an instructive form, which provides direct insight into the depolarisation wave through the ventricle walls. Especially an answer is sought to the question, what is attainable without apriori restrictions with respect to the information about the normal, expected pathway of the activation front.

As the septum activity is largely short circuited by the well conducting intracavitary blood mass, this septum activity will be hardly observable beyond the heart. The positions of the sources in the outer walls with respect to the intracavitary blood mass are very critical for the consequent epicardial and skin potentials. Without apriori knowledge about the depolarisation wave, it is impossible to distinguish the endocardial sources from the epicardial sources. Already in 1853 Helmholtz stated, that an equivalent double layer enclosing arbitrary electromotive forces can produce the same currents and potentials for an observer outside the double layer as the original forces do. Consequently we restrict ourselves preliminary to the determination of the epicardial potentials, as a reflection of mainly the joint activities of endocardial through epicardial sources in the outer walls of the ventricles. Also are implicitly incorporated the secondary sources, that represent the inhomogeneity effects inside the epicardial surface.
We allow ourselves to define extreme simplifications. If there will be no perspective at all even with these oversimplifications and especially in model-to-model adjustments, it will make little sense to continue the study. If, on the other hand, some promising results appear, this output will define an upper bound to what can be expected for more detailed and thus more realistic models.

The simplifications chosen are:

1. The epicardial surface is not modelled exactly, but we confine ourselves to a sphere, that tightly encloses the heart.

2. The medium between the epicardial sphere and the torso skin is supposed to be linear, isotropic and homogeneous.

3. As far as the calculations are concerned, the body extremities are ignored.

4. The truncated torso is approximated by 200 plane triangles.

For a human subject, the torso geometry is measured as well as the torso skin potentials. On the basis of these data and using the simplifications as defined above the epicardial potentials are estimated.

In a forward simulation the potentials on that same torso surface are evaluated for a simplified, mathematical model of the electrical heart activity. These simulated skin potentials are used to estimate inversive-ly the epicardial potentials and the results are compared with the known, simulated sources. From the torso skin potentials the epicardial potentials are evaluated by means of an equivalent set of current multipoles somewhere in the center of the heart.

The motivation for this approach is explained in section 2.

Section 3 is devoted to remarks about the gathering of data, to the projection of the closed, epicardial and torso surfaces onto a flat plane and to the model, which simulates the heart activity. Finally section 4 presents the results and contains a discussion; at the end some conclusions will be drawn.
2. Why current multipoles?

Several theoretical methods are available for obtaining epicardial potentials from skin potentials. They all have been developed from Green's theorem for a twofold bounded medium as shown in figure 1.

![Diagram](image)

**Figure 1** Homogeneous medium \( \tau \) with conductivity \( \sigma \), bounded by the skin interface \( S_s \) and surrounded by air \( (\sigma = 0) \). The inside boundary consists of an epicardial surface \( S_e \).

\( \mathbf{n}_e \) and \( \mathbf{n}_s \) are normals on \( S_e \) and \( S_s \).

Green's theorem is stated here as:

\[
\iint_{S_e} (\phi \frac{\partial \psi}{\partial n_e} - \psi \frac{\partial \phi_e}{\partial n_e}) \, dS_e = \iint_{S_s} \phi \frac{\partial \psi}{\partial n_s} \, dS_s \quad \ldots \ldots \quad (1)
\]
where: \( \phi_e \) is the real potential on the epicardial surface \\
\( \phi_s \) is the real potential on the skin \\
\( \partial / \partial n \) denotes the normal derivative \\
\( \psi_i \) is any potential function caused by a source outside the 
open area \( \Omega \). For this field the conductivity outside 
may take arbitrary values.

The term \( \partial \phi / \partial n \) on the right hand side of equation (1) is missing, be-
cause no current can leave the body. Like in most similar studies, 
the fields are supposed to be quasi-stationary; consequently equation 
(1) holds for any time \( t \).

In practice the bounding surfaces have to be approximated by a set of 
flat triangles; consequently the integrals degenerate into summations 
of finite length.

Each different choice for \( \psi_i \) may lead to a new independent equation.

If the \( \psi_i \) are chosen in such a way, that \( \psi_i = 0 \) on the epicardial 
surface, equation (1) can be written as a vector equation:

\[
P \phi_e = x = Q \phi_s \quad \text{............... (2)}
\]

where \( \phi_e \) is a vector, containing the potentials attached to the smal-
lest divisions of the epicardial surface.

\( \phi_s \) analogously for the torso surface 

\( P \) and \( Q \) are matrices, depending upon the choice for \( \psi_i \)

\( x \) is generally a meaningless vector, which can be given a phy-
sical interpretation by suitable choices of \( \psi_i \).

The number of elements of the vector \( x \), denoted by \( [x] \), can be chosen 
freely, as it equals the number of different functions \( \psi_i \).

Nevertheless \( [x] \) may not exceed \( [\phi_e] \), as the \( [\phi_e] \) equals the number 
of divisions of the torso surface and this division limits the reso-
lution for the different \( \psi_i \)-fields. Furthermore \( [\phi_e] \) will be smaller 
than \( [\phi_s] \) and \( [x] \), as we cannot require the epicardial potential dis-
tribution to be more detailed than the one measured on the skin.

Consequently:

\[
[\phi_e] \leq [x] \leq [\phi_s] \quad \text{............... (3)}
\]
A systematic survey of the different methods can now be discussed on the basis of figure 2.

**Figure 2**: Different methods for obtaining $\Phi_e$ from $\Phi_s$.

See text for further explanation.

The general approach (equation 2) is indicated sub. I.

A disadvantage of this direct approach is, that the found $\Phi_e$ reflects not only the primary and secondary sources inside $S_e$, also the effects of the torso boundary are incorporated. We would like to eliminate these additional influences and in all subsequent methods the potential $\Phi_e$ represents the epicardial potentials for an unbounded medium. This is not completely true, however, as the secondary intracardial sources depend upon the torso boundary too.

II-Martin and Pilkington (lit. 1) have split up the problem into a subsystem without $S_e$ and a subsystem without $S_s$. For each subsystem they have chosen another set of $\Phi_i$ and the result can be written as:

$$D\Phi_e = \Phi_\infty = B\Phi_s \text{............ (4)}$$

where $\Phi_\infty$ denotes the potentials on the skin at the electrode positions, if the medium outside the body is unbounded and has the same conductivity $\sigma$ as inside.
IV-Another choice of $\psi_i$ implies the determination of multipole coefficients (lit. 2):

$$\psi_i = (2-\delta_m^0) (n-m)! \frac{(n-m)!}{(n+m)!} r^n Y_{nm}^{\text{eu}} \quad \ldots$$  \hspace{1cm} (5)

where: $\delta_m^0 = \begin{cases} 1 & \text{if } m = 0 \\ 0 & \text{if } m \neq 0 \end{cases}$

$n = 1, 2, 3 \ldots \ldots N$

$N = \text{maximum order of the set of multipoles}$

$m = 0, 1, \ldots \ldots n$

$$Y_{nm}^{\text{eu}} = p_m^n (\cos \theta). (\cos m\phi \sin m\phi) \ldots$$  \hspace{1cm} (6)

$v = \text{"or"}$

$p_m^n$ are associated Legendre functions of the first and second kind.

$r, \theta, \phi$ are spherical coordinates according to the usual definition.

In that case the vector $\mathbf{x}$ is called $\mathbf{a}$ and represents the multipole field coefficients, generally abbreviated as M.P.C.:

$$a_i = \alpha_{nm}, \beta_{nm}$$

The potential in an unbounded, homogeneous medium is described by:

$$\phi_\infty (r, \theta, \phi) = \frac{1}{4\pi \sigma} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} p_m^n (\cos \theta) (\alpha_{nm} \cos m\phi + \beta_{nm} \sin m\phi) \frac{1}{r^{n+1}}$$

$$= \frac{1}{4\pi \sigma} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (\alpha_{nm} Y_{nm}^{e} + \beta_{nm} Y_{nm}^{o}) \frac{1}{r^{n+1}} \ldots$$  \hspace{1cm} (7)

Especially this represents the epicardial potentials, when for $r, \theta, \phi$ the epicardial coordinates are substituted. In vector notation this yields:

$$\mathbf{\Phi}_e = \mathbf{H}_e \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 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\ldots 

Furthermore this special set of $\psi_i$ transforms the right hand side of equation (2) into:
\[ a = A \hat{\varphi}_s \].............................. (9)

For obtaining the matrix \( A \) by means of discretising the right hand side of equation (2) the integrand is assumed to be constant over the smallest divisions \( \Delta S_s \). This assumption implies quite a big error. In order to diminish this imperfection another approach is used, where the M.P.C. are estimated on the basis of \( \hat{\varphi}_o \), which follows from equation (7) when the torso surface coordinates are substituted:

\[ \hat{\varphi}_o = F a \].............................. (10)

Combination with formulas (4) and (8) leads to the method III. Method III offers an extra advantage. If the M.P.C. are directly derived from \( \hat{\varphi}_s \), the integration has to be performed over the complete, closed surface \( S_s \). Not all potentials are available, however, because of the truncation of the extremities. In method III we can account for this omission, as we have:

\[ F a = \hat{\varphi}_o = B \hat{\varphi}_s \].............................. (11)

With the aid of the well-known deflation technique, \( B \) may be inverted:

\[ \hat{\varphi}_s = B^{-1} F a \].............................. (12)

Omitting the \( \hat{\varphi}_s \)-elements, which are not measured, and eliminating the corresponding rows of \( B^{-1} F \) leads to:

\[ (\hat{\varphi}_s)_r = (B^{-1} F)_r \hat{a} \].............................. (13)

where the index \( r \) stands for 'reduced'.

A simple pseudo-inverse of \((B^{-1} F)_r\) will result in an estimation \( \hat{a} \) of \( a \) according to a least squares criterion:

\[ \hat{a} = (B^{-1} F)^+ (\hat{\varphi}_s)_r \].............................. (14)

Of course this does not eliminate the error, due to the geometrical truncation, but we do not use implicitly the interpolated elements of \( \hat{\varphi}_s \) in the truncated areas.
Why did we select method III?

If the number of multipoles and the number of divisions of the surfaces increase and tend to infinity, methods II, III and IV should be equivalent. E.g. the equivalence of methods II and III can be proven by showing, that \( F = H D \). Yet, if one tries to study the equivalence of III and IV, one is confronted with the problem, that \( B \neq FA \), "see appendix". It also follows, that method IV is very sensitive to small measurement errors, which convinced us to reject this method.

The choice of method III has been dictated by the preference of multipole series truncation above the discretisation of the geometry and potentials of the epicardial surface. The truncation of the infinite multipole series seems to be a profit rather than a loss. Diminishing the number of multipoles provides us indeed with a tool to restrict the epicardial potentials not on the basis of the expected depolarisation wave, but based on the expected influence of the inhomogeneous medium between the heart and the skin. This influence can be regarded as noise superimposed on the real multipole fields due to the electrical heart sources. This noise will be broad-banded in the sense of space frequencies. Generally speaking the medium between the heart and the skin will act as a low pass filter on the multipole fields for the space frequencies. Because of the real noise and the unknown inhomogeneities we cannot expect to be able to derive multipoles of higher order reliably from the skin potentials. Efforts in this direction, which in fact are always made by an inverse low pass filter, will enlarge the high space frequency noise up to unrealistic amounts, if the band of the inverse filter is too wide.

The influence of the inhomogeneities will be especially high frequent, as can be expected from the irregular, small dimensions of ribs, sternum, spinum, arteries etc. Only the lungs may act upon the multipoles of lower order, so that in the future we really have to correct for them. The high space frequencies may be filtered out by the truncation of the multipole series.

Furthermore we are not much interested in extremely high-ordered multipoles, as they would only provide unnecessary details. Of course we want more details than a simple dipole can offer, but few multipoles may be sufficient and attainable.

The practical implementation will teach us how many multipoles can reliably be estimated above the noise level and whether these provide a correct plot of the epicardial potentials. This practical implementation is now briefly commented upon.
3. Practical Realisation.

Two types of data have to be gathered: measurements of the torso geometry and the skin potentials. Both measurements require a definition of a set of points on the torso surface. The distribution of these points has to be chosen in such a way, that the density increases according as the distance to the heart decreases. Close to the heart the potentials are more pronounced, more reliable and have higher spacefrequencies than far away from the heart. Such a desired electrode distribution is accomplished in the following way:

Choose the origin of a rectangular coordinate system in the center of the heart. Define a regular octahedron, the center of which coincides with the origin and whose vertices are positioned on the axes. The triangles in each octant can regularly be divided into 25 congruent, smaller triangles. The octahedron can be expanded until it is transformed into a sphere. This expansion is performed in such a way, that the edges of the octahedron are transformed into orthogonal greater circles of the sphere, while the vertices of the smaller triangles are projected equally spaced on parallel circles. The axis of the parallels is defined as z-axis and they intersect the meridians at equal distances, measured over the sphere. In that way a rather equally spaced set of 200 points on a sphere is defined. The 102 vertices of the 200 spherical triangles can be connected with the origin. The connecting lines are predefined in their direction by the corresponding spherical coordinates $\theta$ and $\phi$. The radii $r$, where the lines intersect the torso surface, can be measured by a special apparatus as shown in figure 3.

About 10 radii cannot be measured because of the extremities. The remaining 92 points are approximated by a set of spherical harmonics up to the order 5, that defines the torso surface by 35 parameters in a least squares sense. The points in the truncated areas are interpolated and a resulting torso is shown in figure 4.

The points on the torso defined in this way are marked using the apparatus of fig. 3 and the electrical potentials are measured as a function of time. The E.C.G.-amplifiers possess a built-in 6 dB/oct. bandfilter ($Q_1 = 500$ Hz). The E.C.G.-signals are supposed to be completely repetitive during the period of measuring.
Fig. 3
Apparatus to measure the torso geometry. Provisions are made to position the heart center in the origin of the coordinate system.

Fig. 4
The truncated torso approximated by 200 triangles.

H = Head Side
RA = Right Arm Side
LA = Left Arm Side
A = Abdomen
Therefore the data processing can be performed on a quasi-simultaneous basis, although the measurements were made in a sequential way. Groups of six signals are recorded simultaneously on an analog recorder together with one reference signal for the time alignment. After analog-to-digital conversion (accuracy: 10 bit, sample time: 1 ms) the signals are corrected for baseline drift on the basis of the silent periods between the complexes. The time alignment is obtained by way of cross correlation techniques and 10 successive complexes are averaged in order to reduce the noise.

Because plots of equipotential lines are desirable, some method has to be defined for projecting the closed surfaces of the torso and of the epicardial sphere.

Therefore the four posterior faces of the original octahedron can be opened until they are positioned in the same planes as their neighbouring anterior faces. The vertices of the smaller triangles are projected perpendicularly on a vertical frontal plane. In that way the faces of the octahedron are ultimately projected into a quadrate. Each equilateral triangle is transformed into a isosceles right triangle. The epicardial and torso surface points are uniquely projected via the vertices of the octahedron triangles into a quadrate. Characteristic epicardial and torso surface areas are illustrated in figure 5.

The combined data of torso geometry and potentials enable us to evaluate the infinite medium potentials and from these a set of multipole coefficients. The z-axis for the multipole series has been chosen to point from the right side to the left side of the subject. This seems profitable, since determination of the coefficients of the Legendre functions for \( m = 0 \) requires particular information in the form of potentials in the poles (\( x = y = 0 \)). If the z-axis were chosen to be vertical, the information in the poles would be lacking, because of the torso truncation. In practice we followed this reasoning, though experiments for verification have not yet been performed.

In a forward evaluation the potentials in the truncated areas can be calculated from the estimated M.P.C. \( \tilde{A} \). This provides us with a first tool to decide, whether too many multipoles are estimated. If the interpolated potentials are highly overshooting their neighbouring measured potentials, \(*\) The projection method is related to the one, that C.S. Peirce suggested in lit. 3, but this relation could not yet be verified completely by the author.
Figure 5: Projection of torso surface and epicardial surface.

B = backside  
R = right side  
L = left side  
CS = coronary sulcus  
IS = interventricular sulcus  
RA = right atrium  
LA = left atrium  
RV = right ventricle  
LV = left ventricle

the upper limit on the dimension of \( a \) will be exceeded.

As a further test of the reliability of the found epicardial potentials, a model-to-model adjustment was suggested. The generating model of the heart action is a very simple one, which is not claimed to represent the depolarisation wave meticulously. It generates, however, a field which shares the features with the real situation considering the accepted simplifications and restrictions of this study. An extensive description of the generating model, called "string model", can be found in lit. 4. We confines ourselves here to an example shown in figure 6.
4. Results and discussion.

For real measurements as well as simulations estimations have been made of sets of multipoles varying from the order 1 through 6. This implies that an equivalent heart generator is used consisting of respectively 3, 8, 15, 24, 35 and 48 independent time signals. The equipotential lines found on an epicardial sphere of 6 cm radius are shown in figure 6 for the real measurements and in figure 7 for the simulated potentials. The higher the order of the multipole set, the more detailed the plots become, until the estimations become unreliable because of the noise. For higher orders the tendency exists to enlarge extrema at the posterior areas. In the real situation this may well be attributed to two phenomena:

1) the epicardial sphere is badly positioned and intersects the posterior heart walls;
2) the lower conductivity of the lungs is not accounted for.

Note, however, that the same tendency exists in the simulation, where none of these phenomena can affect the result.

In the simulation an equivalent generating source is handled, that would produce an epicardial potential of a constant, positive level in the shaded areas of figure 7 and a constant negative level in the remaining areas.

The field is decomposed in Tesseral harmonics and the torso surface potentials are generated by truncating the series after the order 6. A gaussian, white noise is added with a variance $\sigma^2$, where $\sigma$ equals 1/60 of the maximum peak-to-peak value of the simulated skin potentials. This noise level is higher, than what can be expected in the real situation. Also the noise is not filtered, as it has to account for the additive instrumental disturbances as well as for the electrode displacements, body movements, inhomogeneity influences etc.

From the results of the model-to-model adjustments it is clear, that the details become more pronounced if more multipoles are identified, until the influence of the noise reaches a level, where no reliable estimations can be made. The optimum seems to lay somewhere between the order 3 and 4.
Figure 6: Equipotential lines on an epicardial sphere radius 6 cm, where an equivalent set of multipoles is used of respectively the order 1 through 6. The middle between the maximum and the minimum potentials is defined as zero level and all positive valued areas are shaded in grey.

In each plot nine equipotential lines have been drawn, that are linearly distributed between the maximum and minimum potentials.
At the left the simulated homogeneous double layers are shown; they are shaded in grey.

Figure 7: Equipotentials lines, analogously to figure 6, for the simulated case.

Besides the noise two other imperfections have to be put into the picture, viz. the truncation of the torso and the finite length of the multipole series in the simulation. As far as these two influences are concerned the following experiments have been performed.

When the equivalent multipole series up to the order six were estimated on the basis of a simulation of the same order with no noise and only 92 electrodes, the results were perfect. When, on the other hand, under identical circumstances, the order of the simulation was 10, the estimated M.P.C. above the order 4 were extremely overvalued, while the multipoles below the order 4 were badly adjusted. This effect may be explained by
formulas:
Without noise and torso truncation effects, the simulation up to the order 10 can be written as:

\[ \mathbf{s}_s = \mathbf{B}^{-1} \mathbf{F} \mathbf{a} + \mathbf{B}^{-1} \mathbf{G} \mathbf{b} \]  

(15)

where \( \mathbf{b} \) contains the real M.P.C. for the order \( n > 6 \).
The estimated coefficients \( \mathbf{\hat{a}} \) will be given by:

\[ \mathbf{\hat{a}} = \mathbf{F}^+ \mathbf{F} \mathbf{a} + \mathbf{F}^+ \mathbf{G} \mathbf{b} = \mathbf{a} + \mathbf{F}^+ \mathbf{G} \mathbf{b} \]  

(16)

The last term represents the unwanted interaction with higher ordered multipole terms. It may not be expected (also not for the real situation) that \( \mathbf{G} \mathbf{b} \) is small compared to \( \mathbf{F} \mathbf{a} \). Neither will the rows of \( \mathbf{F}^+ \) and the columns of \( \mathbf{G} \) be perpendicular. Only in case the torso surface were a concentric sphere, the columns of \( \mathbf{F} \) and \( \mathbf{G} \) would be orthogonal. In that situation the rows of \( \mathbf{F}^+ \) would equal the columns of \( \mathbf{F} \) multiplied by a constant and consequently \( \mathbf{F}^+ \mathbf{G} \) would be zero. Because the torso surface is far from a concentric sphere the interaction between \( \mathbf{a} \) and \( \mathbf{b} \) cannot be neglected. The interaction effect may also be visualised by drawing the equipotential lines for the different columns of \( \mathbf{F} \) and \( \mathbf{G} \), i.e. the equipotential lines on the torso surface for a special multipole element. These plots show rather indistinguishable patterns for several elements of \( \mathbf{a} \) and \( \mathbf{b} \).

Now we are confronted with the dilemma, that the multipoles of higher order are not neglectable, but their influence is only prominent in the precordial area on the torso, where they are not distinguishable from multipoles of lower order. If we still identify higher ordered multipoles, the anterior, epicardial potentials will be correct, but the posterior, epicardial potentials will be completely out of range.

If we restrict ourselves to a lower number of multipoles, the anterior parts will be less detailed, but the posterior potentials will be more reliable.

A choice for the maximum order of the multipole series is based upon a criterion, that is illustrated in figure 8.
Figure 8: Root Mean Square error (in time and space) between the measured torso skin potentials and the cumulative contributions of estimated multipole generators. The ultimate order of the estimated sets of multipoles ranges from 1 through 6. The incorporated multipole order is denoted by \( n \) along the horizontal axis.

A special number of multipoles is estimated simultaneously. Then we can make a diagram of the least squares error between the real potentials and the cumulative contributions of the respective multipoles up to the ultimate, which is estimated.

If only low ordered multipoles are estimated, the error will decrease monotoneously if the number of contributive multipoles increases, just as we expect. But as soon as too many multipoles are estimated, the error will suddenly increase extremely, which at the end only leads to
a relative small improvement, when all estimated multipoles are incorporated. This effect is due to the interaction between the multipole contributions on the torso surface. Small improvements for the skin potential adjustment can only be achieved by almost compensating, but nevertheless enormous contributions of the different multipoles. And especially the fields of these multipoles will not compensate each other at the posterior parts of the epicardial surface.

From figure 8 we can conclude, that for the real situation the order 4 will be the upper limit.

In this context it is worthwhile to remind that Barr c.s. (lit. 5) also found a maximum of 24 independent heart generators above the noise level, which is consistent with the order 4 of a multipole series. It would be worthwhile to test whether the time signal space of the M.P.C. covers largely the time signal space of the intrinsic components, obtained by factor analysis of the measured torso skin potentials.

The various tests leads to an upper limit for the order of 4. Even with this order we have to be very careful at the interpretation of the results. We continuously find ourselves in the paradoxial mood, where optimism takes turns with scepticism. Conceding to the optimism, we plotted for the complete electrical heart action (P, Q R S and T) the equipotential lines on the epicardial sphere at several characteristic moments (see fig. 9). At least these plots do not display an image of the depolarisation wave, which is contradictory to what Durrer has measured under open thorax conditions (lit. 6). On the contrary some patterns can be recognized, which are not so evident in the torso surface fields, as figure 10 shows. In figure 9 the positive values may be correlated roughly with the depolarisation fronts. During the P-wave one can then recognize the activity of the atria. During the QRS-complex the initial apical activity spreads out over the ventricle wall in the direction of the basis, and a clear separation between the right and left ventricle may be observed. The last area which depolarises is positioned in the basical right ventrical wall. The T-wave reveals the same polarity as the depolarisation, which points to an opposite path way of the repolarisation.
Fig. 9: Epicardial equipotential lines
Black fields are positive. Time in ms.
P-wave: ~ 15 - 90 ms
QRS-complex: ~ 130 - 240 ms
T-wave: ~ 300 - 500 ms
Fig. 10: Torso skin equipotential lines. Black fields are positive. Time in ms. The dots mark the places, where no measurements could be made.
5. Conclusions.

Several rather intuitive and not yet mathematically defined criteria are used to obtain an upper limit for the order of an equivalent set of multipoles, which is observable from skin potentials:

- On the basis of the estimated set of M.P.C., the potentials in the truncated areas of the torso may be interpolated. These interpolated potentials have to form a smooth function with their neighbouring measured potentials.

- Unrealistically high values for the posterior epicardial potentials together with an upstroke of the high time frequencies in that areas indicate the exceeding of a limit.

- Unrealistically high values for the M.P.C. of higher order, that almost compensate each others' influence in the torso fields, indicate the exceeding of a limit.

An upper limit of the order 4 seems attainable.

In the applied method the estimated set of M.P.C. are corrupted by the neglected M.P.C. of higher order. Because of insufficient information at the posterior parts of the torso the lower orders cannot be separated from the higher orders. The reliability of the found epicardial potentials is therefore still under study. An estimated set of M.P.C. of the order 4 produces epicardial fields, that are at least not contradictory to real myocardial measurements.
References


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Appendix

a. Equivalence of method II and III:

\[ [F]_{sj} = \frac{1}{r_s^{n+1}} Y_j(\theta_s, \phi_s) \]
\[ [H]_{ej} = \frac{1}{r_e^{n+1}} Y_j(\theta_e, \phi_e) \]
\[ [D]_{se} = \frac{r_s^2 - r_e^2}{4\pi r_e |r_s - r_e|^3} \Delta S_e \]

\[ [DH]_{sj} = \sum \frac{r_s^2 - r_e^2}{4\pi r_e |r_s - r_e|^3} \Delta S_e \frac{1}{r_e^{n+1}} Y_j(\theta_e, \phi_e) \]

if \( \Delta S_e \) tends to zero, this may be transformed into:

\[ [DH]_{sj} = \frac{r_s^2 - r_e^2}{4\pi r_e n+2} \int_{\Delta S_e} Y_j(\theta_e, \phi_e) \frac{1}{|r_s - r_e|^3} dS_e \]
as \( r_e \) is constant.

We will use the following equalities:

\[ \frac{r_s^2 - r_e^2}{|r_s - r_e|^3} = \nabla_e \left( \frac{1}{r_s - r_e} \right) \cdot (r_s + r_e) \]
\[ \nabla_e \left( \frac{1}{|r_s - r_e|^3} \right) = -\nabla_s \left( \frac{1}{|r_s - r_e|^3} \right) \]

which results in:

\[ \frac{r_s^2 - r_e^2}{|r_s - r_e|^3} = \nabla_e \left( \frac{1}{|r_s - r_e|^3} \right) \cdot r_e - \nabla_s \left( \frac{1}{|r_s - r_e|^3} \right) \cdot r_s \]

\[ = \frac{\partial}{\partial r_e} \left( \frac{1}{|r_s - r_e|^3} \right) r_e - \frac{\partial}{\partial r_s} \left( \frac{1}{|r_s - r_e|^3} \right) r_s \]
Using the series:
\[
\frac{1}{|r_s - r_e|} = \sum_{q=0}^{\infty} \frac{r_e^n}{r_s^{n+1}} \frac{(n-m)!}{(n+m)!} (2 - \delta_{m}^{q}) Y_{q}(\Theta_e, \phi_e) Y_{q}(\Theta_i, \phi_i)
\]
(where \( q \) is a combination of \( n \) and \( m \))

Substitution in the integral form leads to:
\[
[DH]_{s_j} = \frac{1}{4\pi} \frac{2\pi}{r_s^{n+1}} \int \sum_{q=0}^{\infty} \frac{r_e^n}{r_s^{n+1}} \frac{(n-m)!}{(n+m)!} (2 - \delta_{m}^{q}) Y_{q}(\Theta_e, \phi_e) Y_{q}(\Theta_s, \phi_s) Y_{j}(\Theta_e, \phi_e) \sin \Theta_e \, d\Theta_e \, d\phi_e
\]

\( y_{q}(\Theta_s, \phi_s) \) is constant on the epicardial sphere.

Because of the property of orthogonality for the spherical harmonics at an integration over a spherical surface, this integral equals:
\[
[DH]_{s_j} = \frac{1}{r_s^{n+1}} Y_{j}(\Theta_s, \phi_s) = [F]_{s_j} \quad \text{QED}
\]

b. Discrepancy between method III and IV.
\[
[F]_{s_j} = \frac{1}{r_s^{n+1}} Y_{j}(\Theta_s, \phi_s)
\]
\[
[A]_{j_t} = \frac{\partial \psi_j(r_t)}{\partial n_t} \Delta S_t
\]
\[
[B]_{s_t} = \frac{1}{\partial n_t} \frac{\partial (|r_s - r_t|)}{\partial r_t} \Delta S_t
\]
\[
[F_A]_{s_t} = \sum_{j} \frac{1}{r_s^{n+1}} Y_{j}(\Theta_s, \phi_s) \nabla_t \psi_j(r_t) \cdot n_t \Delta S_t =
\]
\[
= \nabla_t \left[ \sum_{j} \frac{r_t^n}{r_s^{n+1}} \frac{(n-m)!}{(n+m)!} (2 - \delta_{m}^{q}) Y_{j}(\Theta_s, \phi_s) Y_{j}(\Theta_t, \phi_t) \right] n_t \Delta S_t
\]
The expression between the rectangular brackets equals $\frac{1}{|r_s - r_t|}$ if and only if $r_s > r_t$, but this is only true for half of the set $(r_s, r_t)$. If $r_s < r_t$ then the series diverges. So:

$$FA \neq B$$