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A general purpose two-dimensional mesh generator[†]

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INTRODUCTION

Composing and checking input for finite element programs is very labour intensive; this is particularly true for the division of the area to be studied into elements. In the past many programs, called mesh generators, have been developed in order to automate this job. A survey of these programs is given by Buell and Bush¹. This paper deals with a mesh generator for two-dimensional areas; the principal characteristics of this mesh generator, named TRIQUAMESH, are: (1) a user-oriented input language with debugging aid is provided; the user will only have to supply simple composable input data; (2) both single and multiple coherent two-dimensional areas with complex geometry can be divided into triangular and/or quadrilateral elements; (3) easy specification of the magnitude of the elements; (4) substructuring facilities have been incorporated; (5) the shape of the generated elements is optimized; (6) the mesh generator has some possibilities to reduce the bandwidth of the assembled structural matrices; (7) the output of the mesh generator can be used directly as a part of the input for three finite element programs, including ASKA and MARC.

After introduction of some basic concepts, the method used in TRIQUAMESH will be dealt with. Afterwards the use and possibilities of TRIQUAMESH will be illustrated by means of some examples.

BASIC CONCEPTS

The area to be divided G can be divided into n_s subareas $G_1, \dots, G_{n_s}$ in order to specify element and material properties and to define the substructuring of the area G . It is demanded that subareas are simple coherent; n -fold coherent areas can be made simple coherent by making at least $n-1$ cuts. The contour of an area G is C , subarea G_s has C_s as contour and the overall contour of all subareas is C^* (Fig. 1).

In G a number of so-called basis points is fixed, by means of numbers and coordinates. These points will supply a basis for the geometrical description of contour

Figure 1. An arbitrary two-dimensional area (a) and a division thereof in simple coherent subareas (b).

[†] To be presented at the forthcoming Engineering Software conference to be held at Southampton, 4-6 September 1979.

C^* and for the determination of other user wishes, for example the desired element size. With two or more basis points oriented elementary curves can be defined, for example straight lines, arcs etc. (Fig. 2a). A contour part (identified by a number) is a non-branched coupling of elementary curves and has an orientation. The geometric description of the contour part consists of the coupling of descriptions of the constituent curves (Fig. 2b).

Subcontour C_s consists of the coupling in a closed curve of one or more contour parts. For reasons of univocality, C_s is described by denoting the numbers of the anti-clockwise-sequence of joining contour parts. If a contour part is met in a direction opposite to its orientation, the number of this contour part is denoted negatively (Fig. 2c). A substructure is defined by one or more subareas. All substructures together form one structure: the area to be divided G . The concepts mentioned are hierarchically ordered. The basis points define elementary curves. These define contourparts etc.:

Basispoints $\rightarrow$ elementary curves $\rightarrow$ contourparts $\rightarrow$
subareas $\rightarrow$ substructures $\rightarrow$ structure.

During the division of area G into elements, use is made of the roughness function $g_2(x, y)$. It is postulated that in G , g_2 is directly proportional to the desired magnitude of the element sides; the length l of any element side between the points (x_1, y_1) and (x_2, y_2) therefore shall have to be 'the best fit possible':

$$l = \frac{1}{2} \{g_2(x_1, y_1) + g_2(x_2, y_2)\} \cdot RI \quad (1)$$

The proportional constant RI has the dimension of length and will be called the standard element side. Equation (1) is the definition of the roughness concept.

DESCRIPTION OF THE METHOD

Globally TRIQUAMESH has been developed as follows: (1) checking and manipulation of input data; (2) generation of nodal points on the total contour C^* ; (3) division

Figure 2. Description of elementary curves (a), a contour part (b) and subcontours (c).

Figure 3. Harmonic roughness function on an elementary curve.

Figure 4. Nodal points and element sides on the curve PQ.

of subareas into elements; (4) post-processing, such as: optimization of element shape, bandwidth reduction, transformation to elements with more nodal points and output. The aspects mentioned above will be described subsequently.

Checking and manipulation of input data

A user-oriented input language has been developed so that the input can be interpreted simply and elaborate tests for errors in the input data are possible. The program expects the input to be delivered by means of punchcards or by means of a file to be found on a disc-unit. From this input, arrays are determined which will serve as input parameters for the next steps. During processing, the input is also checked for syntax and semantic errors. Possible error messages are for instance:

```
1 TRIAX3 2, 5)
* >>>> LEFT PARENTHESIS EXPECTED
* >>>> UNKNOWN SUBAREA
```

Generation of contour points

The user will have to supply values for the roughness function $g_2(x, y)$ for each of the basis points in area G . Doing so, using a chosen roughness behaviour along the contour, the roughness on the total contour C^* is fixed. Starting from this, and together with the user given standard element side RI , the nodal points to be generated on C^* will be determined. Because of the assembly of a subcontour out of contour parts, which in turn consist of elementary curves, it will only be necessary to explain the generation of nodal points on an elementary curve. Consider an elementary curve K with length l and curvilinear coordinate s , $0 \leq s \leq l$; for reasons of simplicity a non-essential simplification to a straight line is made. On curve K a number of basis points are denoted, numbered locally with j , $j=0, 1, \dots, m$, which divide the

curve in m pieces, and, by means of the user given values gk_j , a piecewise harmonic roughness function is defined (Fig. 3):

$$gk(s) = \frac{gk_j + gk_{j+1}}{2} + \frac{gk_j - gk_{j+1}}{2} \times \cos \frac{s - s_j}{s_{j+1} - s_j} \pi, \quad j=0, \dots, m \quad (2)$$

Starting from this function $gk(s)$ nodal points are generated on curve K , this is done in two steps: first, the determination of the number of nodal points and subsequently the computation of the correct location. Suppose that $n-1$ nodal points will have to be generated (and therefore n element sides) on the elementary curve (Fig. 4). From equation (1) for element side i the following can be derived:

$$\frac{s_i - s_{i-1}}{gk_i + gk_{i-1}} = \frac{s_{i+1} - s_i}{gk_{i+1} - gk_i} \quad (i=1, 2, \dots, n-1) \quad (3)$$

This can be met 'as good as possible' for all element sides, by computing n as follows:

$$n = \frac{1}{RI} \cdot \int_0^l \frac{l}{gk(s)} ds \quad (4)$$

After which n is rounded off to an integer in a suitable way. As soon as the curvilinear coordinates $s_1 \dots s_{n-1}$ are known, one can easily determine the coordinates in the overall two-dimensional system.

Generating of elements in a subarea

A contour is defined by sequentially connected nodal points on this contour. The connection is made by straight lines (the element sides). Every subarea will have to be divided either in triangles or quadrilaterals, depending upon the user given element type. The nodal points on contour C_s of subarea G_s are numbered locally 1 ... n_{cp} (Fig. 5). Subarea G_s is concave whenever one of the angles enclosed by the contour G_s is concave whenever one of the angles enclosed by the contour $\alpha_i > \pi$. Let i and j , where $i \neq j$, be nodal points on C_s such that the interconnecting line between i and j lies completely within G_s . Whilst dividing G_s into elements such lines are frequently used, and an instantaneous check will have to be made to see whether this line is actually within G_s ; for instance the connection between points 8 and 17 in Fig. 5 is not

Figure 5. Local numbering of contour points on C_s .

Figure 6. Determination of form P visible points.

acceptable. These checks for concave areas are quite complicated and therefore a concave subarea is split into two or more convex partial areas, after which these areas are divided into elements.

Splitting a concave subarea into convex partial areas. A nodal point is called concave if $\alpha_1 > \pi$ (Fig. 5). The splitting is done by the following steps: (1) take a concave nodal point on the contour; call this point P. If no such point exists, the area will be convex; (2) determine the accumulation V_1 of nodal points on the contour which are visible from P; (3) determine out of V_1 that nodal point Q in such a way that, based on given criteria, PQ is the best splitting line; (4) determine the accumulation V_2 of nodal points on the contour which are visible from Q; (5) define on PQ a roughness function, based on the roughness values of the nodal points in V_1 and V_2 ; (6) generate, using that roughness function, nodal points on PQ; (7) define two new areas separated by line PQ; (8) continue with step (1) for both areas.

Explanation. Step (1). Point P is chosen to be the most concave point on the contour or to be the middle point of a series of almost equally concave points. Steps (2) and (4). The determination of visible points is illustrated with an example (Fig. 6a). Consider a continuous contour; β_k is defined as being the angle between line P-K and the tangent of the contour in P; Fig. 6b shows β_k as functions of curvilinear coordinate s. Points of interest in the determination of visible points are those concave points L of the contour where β_L is a local extremum; not visible will be: (1) points K with $s_K > s_L$ and $\beta_K < \beta_L$ if β_L is a local maximum; for example points between K_1 and K_2 ; (2) points K with $s_K < s_L$ and $\beta_K > \beta_L$ if β_L is a local minimum; for example points between K_3 and K_4 . Step (3). Consider Fig. 7: the points P^{-1} and P^{+1} are the neighbouring points of P. The lines PP^{-1} and PP^{+1} divide the visible area in sectors I, II and III. At first the most suitable point Q in sector I is searched for; if such a suitable point in that sector exists, the concave angle in P can be eliminated. However, if line PQ does not meet some minimum demands, the searching-area is extended with sectors II and III.

Possible splitting lines are selected with respect to three criteria: (1) the angles $\gamma_1 \dots \gamma_4$ will have to fit as good as possible multiples of 60° or 90° (respectively for triangles and quadrilaterals); this results in a difference $\Delta\gamma$; (2) the length of the splitting line, which should be as short as possible, yields a difference Δl ; (3) the number of nodal points on a splitting line is rounded off, which gives a Δn . From all possible splitting lines that line is chosen for which $\Delta = C_1 \cdot \Delta\gamma + C_2 \cdot \Delta l + C_3 \cdot \Delta n$ reaches a minimum.

The weighting factor C_1 , C_2 and C_3 have been empirically determined.

Step (5). A two-dimensional roughness polynomial $g2$ is determined by means of a weighted residual method; let gk_k be the roughness in contourpoint k, so its weighted factor will be $1/gk_k$. The result can be reduced to a one-dimensional polynomial gl on the splitting line.

$$g2(x, y) = \sum_{i=0}^n \sum_{j=0}^i a_{i-j,j} x^{i-j} y^j;$$

$$gl(v) = \sum_{i=0}^n b_i v^i \quad (5)$$

Dividing a convex area into triangles. The contour of a convex partial area is determined by n nodal points. For any of these points, locally numbered i, $i = 1 \dots n$, the coordinates, the contour-enclosed angle α_i and the roughness gk_i are known. An angle α_i is called sharp whenever $\alpha_i < 80^\circ$, because cutting and splitting of $\alpha_i = 80^\circ$ yields trivial. Step (3). The aspects with respect to which possible $-\alpha_i/2$. The actual dividing of a convex area can be described as follows: (1) if $n = 3$, one triangle is formed and the area is exhausted; (2) if a sharp angle is found, subsequent layers are cut from the area as long as this is in accordance with given criteria. These layers are divided into triangles; (3) split the remaining area into two partial areas; (4) define two contours for those partial areas and continue for both areas with step (1).

Explanation. Step (2). A cutting line $i+j \rightarrow i-j$ is acceptable if it satisfies two conditions (Fig. 8): (1) the angles $\gamma_1, \gamma_2, \gamma_3$ and γ_4 should be $> 40^\circ$, and (2) if g_{i+j} and g_{i-j} are the roughness values in the end points of the cutting line then roughness g_l on that line should satisfy:

$$\frac{2}{3} \min \{gk_{i+j}, gk_{i-j}\} < gl < \frac{3}{2} \max \{gk_{i+j}, gk_{i-j}\} \quad (6)$$

Figure 7. Search for a suitable splitting-line from point P.

Figure 8. Cutting layers starting at sharp angle i.

Figure 9. Forming triangles within a layer.

Figure 10. Dividing a hexagon into 2, 3, 4 or 5 quadrilaterals.

This criterion reduces possible configurations as shown in Fig. 8b, which causes bad shaped elements. The nodal points on an accepted cutting line are generated based upon the roughness of all contour points in the original convex partial area.

Figure 9 illustrates the dividing of a layer into elements; inside a layer no nodal points will be created. If $LM < KN$ (Fig. 9a) then LKM will be the next triangle to be formed; otherwise KMN will be created. Figure 9b shows a possible end situation and Fig. 9c a similar situation for the first layer; the triangulation of such remaining areas is trivial. Step (3) The aspects with respect to which possible splitting lines are selected (i.e. angles, length and number of nodal points to be generated) are the same as for the splitting of concave areas.

Dividing a convex area into quadrilaterals. The method is mainly analogous to the method for triangles. An angle α_i is now said to be sharp if $\alpha_i < 120^\circ$. The actual division of a convex area into quadrilaterals is shown in the following steps: (1) if $n = 4$; one quadrilateral is formed, and the area is exhausted; (2) if $n = 6$; 2, 3, 4 or 5 quadrilaterals will be formed, and the area is exhausted; (3) find a combination of two contour points i and j for which both α_i and α_j are 'sharp' and the between i and j situated angles $\alpha_k = 180^\circ$, which means that i and j are connected with a straight line. Cut off layers, as long as this is possible with respect to given criteria, and divide these layers into quadrilaterals; (4) split the remaining area into two partial areas; (5) continue with step (1).

Explanation. Step (2) is necessary because of the geometrically and topologically difficult element form. Typical geometries which will cause 2, 3, 4 or 5 quadrilaterals are shown below (Fig. 10).

Step (3). Apart from the demands for the angles with the contour and the minimum and maximum roughness on the cutting-line as in the case of triangles, it will be a demand that the number of nodal points on the cutting-line is equal to the number on the previous cutting-line (or the original line between i and j). In that case, the division of the layer is trivial.

Post processing

Shape improvement of the elements. It has been found that the form of the elements joining at an internal nodal

point k can be improved in an efficient way by giving point k new coordinates as the average of the coordinates of his nk neighbouring points (i.e. nodal points which are connected to k with an element side):

$$x_k = \frac{1}{nk} \sum_{i=1}^{nk} x_{ki} \quad \text{and} \quad y_k = \frac{1}{nk} \sum_{i=1}^{nk} y_{ki} \quad (7)$$

The iteration process is stopped as soon as none of the internal nodal point changes significantly.

Bandwidth reduction. The method which TRIQUAMESH uses for the dividing of an area G into elements causes that the coefficient-matrices of substructures defined in G , will have a bad or no band structure. Because many finite element programs only work efficiently by means of an acceptable band structure, some possibilities have been provided in order to reduce bandwidth by renumbering the nodal points in a suitable way. The user can, for each substructure, define more renumberings; finally that one is chosen which gives the best band structure. The available renumbering methods will be described subsequently.

CMK-renumbering. This method of renumbering was developed by Cuthill and McKee² and starts from the topology of the element division of the substructure. Let $e_1, e_2, \dots, e_n$ ($n \geq 1$) be the elements which contain nodal point k . The $m+1$ nodal points $k, k_1 \dots k_m$ describe the elements $e_1 \dots e_n$ so that nodal point k has degree m and $k_1 \dots k_m$ are surrounding points of k (Fig. 11). CMK-renumbering proceeds as follows: (1) choose a starting point k_1 and give this number 1. The accumulation V of renumbered points now consists of the nodal point k_1 : $V = \{k_1\}$; (2) number sequentially the not yet renumbered surrounding points of each nodal point in V , according to an increasing degree, with their new numbers; (3) define $V = \{\text{nodal points to which new numbers were given in step (2)}\}$; (4) if V is empty the process stops otherwise repeat step (2). Because of the fact that the CMK-renumbering uses no geometrical data, this method will be especially useful in the case of substructures with an irregular geometry.

Line-, distance- and angle-renumbering. These methods are based on the geometry of the substructure (coordinates of nodal points), and are realised by means of a user-given, new Cartesian coordinate system $\bar{x}, \bar{y}$. In order to achieve this two basispoints P and Q will have to be specified. The origin of the new system will be P ; $\bar{y}$ is measured along the line PQ and $\bar{x}$ along a line orthogonal to PQ . In this system a nodal point k can be characterized in two ways: the pair $(\bar{x}_k, \bar{y}_k)$ and the pair $(\bar{r}_k, \bar{\varphi}_k)$ (Fig. 12).

By means of the nodal points k_1 with coordinates $(\bar{x}_1, \bar{y}_1)$ or $(\bar{r}_1, \bar{\varphi}_1)$ and k_2 with coordinates $(\bar{x}_2, \bar{y}_2)$ or $(\bar{r}_2, \bar{\varphi}_2)$.

Figure 11. Degree of a nodal point k .

Figure 15. Element divisions for a crane-hook.

\$GRADING: contains information about the desired magnitude of the elements.

\$RENUMBER: control data for bandwidth-reduction.

\$OUTPUT: defines the way output will have to be presented.

Figure 14b shows the generated contour points and Fig. 15a the division into elements. The divisions according to Figs. 15b, 15c and 15d are created by making some slight alterations in the input above. Figure 15b: the complete division less coarse by a factor 2. ($RI = 7.5$ at line 230). Figure 15c: locally refinement in the basis points 10, 11, 12, 13 and 16 by a factor 2 (add GV 0.4 (10, 11, 12, 13, 16) after line 230). Figure 15d: as Fig. 15c with quadrilaterals (QUAM4 instead of TRIM3 at line 260).

The next example deals with a piston of a diesel engine which is simplified to an axi-symmetric construction (Fig. 16a). The moulded aluminium piston is oil-cooled by means of a moulded-in steel tube which serves as a coolant spiral; in a radial plane one can see 4 cross-sections of the tube: the subareas 2, 3, 4 and 5. At the location of the upper two piston rings, a considerably higher wear resistance material is used: a moulded-in cast iron ring, the cross-section of which is given by subarea 7. In order to be able to make an optimal design of the form of the cast iron ring, it is put into a separate substructure together with its surroundings (subarea 6). Figure 16b shows the mesh generated for the piston.

Figure 16. Generated contourpoints (a) and element division (b).

DISCUSSION AND FUTURE DEVELOPMENTS

The meshgenerator TRIQUAMESH has proved to be quite a useful tool, the most revealing advantages of which are: user orientation; one tends to work with it quickly; freedom in choice of geometry and number of subareas; simple control of element size; possibility of quadrilateral elements. Whilst using the program, a number of useful extensions have become clear which shall be made in the near future; this will be done in the frame work of a pre-processing system, with which one can generate the complete input (mesh, kinematic and dynamic boundary conditions etc.) for two-dimensional problems.

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