The design of a pulley position measurement system

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The design of a pulley position measurement system
W.D. Versteden (s479891)

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Chapter 1: Introduction

In the section of Vehicle Drive Trains of the faculty Mechanical Engineering, a lot of research is being done on the subject of continuous variable transmissions (CVT's), mainly for automotive applications. The application of these transmissions brings along advantages as the vehicle speed and the speed of the engine can be partly decoupled. Therefore different working points of the motor can be used for different driving conditions. This means that the motor can always be used in its most ideal working point and so performance and overall efficiency of a car can be improved.

The CVT is based on a plain principle. This principle, which is initially developed by Hub van Doorne, contains three main parts. These are two pulleys and one V-belt. The V-belt runs through both pulleys, the primary pulley drives the belt and at the secondary pulley is driven by the belt.

Both these pulleys contain one fixed sheave and one movable sheave. The fixed sheave is a part of the shaft on which the pulley is mounted. As the movable sheave can move inwards, the space between both pulley sheaves is reduced and the belt has to move outwards. This means the running radius of the belt is bigger than before. As the belt has a constant length, the space between the opposite pulley sheaves has to increase, to let the running radius of the belt decrease. With the changing running radii of course the ratio of the transmission is changing. This principle can be seen in figure 1.1. In the upper condition the belt is on a small running radius at the right pulley. What happens when the pulley moves inwards can be seen in the lower figure. The running radius on the right pulley is enlarged and so the ratio of the transmission is changed.

As said, the running radii of the belt on both pulleys determine the theoretical ratio of the transmission. These running radii are both determined by the space between the fixed and the movable pulley sheave. So, if we are able to measure the axial position of the
movable pulley sheaves, we can determine the running radii of the belt, and with that also the ratio of the transmission. This ratio can be very useful for transmission control systems, motor management systems, and also for the determination of slip.

In the Vehicle Drive Trains section a test rig is present at which a so-called belt-box transmission can be tested under several conditions. The transmission is mainly tested on slip between the belt and the pulley. Slip can be determined by comparison of the theoretical and actual ratio of the transmission. The actual ratio of the belt-box transmission can be accurately determined at the test rig by measuring the ingoing and outgoing rotational speeds. At the moment the theoretical ratio can only be exactly determined when the movable pulley sheaves are in a constant prescribed position. As has been explained this ratio can also be determined by measuring the axial positions of the movable pulley sheaves. If such a measurement system is present, the belt-box transmission can then be tested over its complete range, and not only in the prescribed positions.

The goal of this six weeks internship is to develop such a measurement system for the transmission that is easy to apply and reliable. Furthermore we have to be able to measure slip with an accuracy of 0.1%. This will be done in four different steps.

First the different types of measuring will be discussed in Chapter 2. Two main types of measuring, contactless measuring and measuring with contact are investigated and furthermore different sensing methods are investigated. Next some measurement systems have been designed. Two systems seemed feasible, both systems are worked out and their (dis) advantages are discussed. The chosen system is worked out in Chapter 3; the other system is explained in Appendix B. In Chapter 4 the chosen system is completely analysed. The forces in the system are calculated, the tribological effects in the contact are discussed and the total system stiffness is calculated. Finally in Chapter 5 conclusions are drawn and some recommendations are made.
Chapter 2: How to measure

For the measuring of one translational degree of freedom different sensors are available. All these sensors can be divided in two main groups, namely sensors that make contact with the object of which the movement should be measured and contactless sensors. Both classes of sensors have been researched and a choice has been made.

§2.1 Contactless measuring

Contactless measuring will in most cases, if applicable, less difficult than measurement with contact. This is because of the fact that the contact between sensor system and pulley has, mainly tribological, complications. When contact is made, one should take into account difficulties as wear, lubrication films and vibrations of the sensor system.

The great disadvantage of a contactless measuring is that it is not easy to be applied in the belt-box transmission. There are some problems to overcome, like the high temperature in the transmission (up to 130° C), the combination of air and oil droplets and the small space the sensor should fit into. Also the measuring range of 16mm. is very large for the general contactless sensors.

Research has shown that contactless measuring is indeed very difficult. The main problem is the mixture of oil and air in the transmission. Almost every sensor will react if oil will fly between the sensor and the pulley. Sensors that react on the oil are not applicable in the system, which means optical sensors and capacitive sensors are not applicable on forehand.

One type of sensor that could work is the inductive sensor. It may not respond to the oil and is able to measure over large measuring ranges. The main problem of these sensors is the size. Sensors that are able to measure the complete range of the pulley are generally large. Figure 2.1.1 shows a sensor of Pepperl & Fuchs and has dimensions of L x B x H = 80 x 80 x 40 mm.

Figure 2.1.1: Pepperl & Fuchs inductive sensor
§2.2 Measuring with contact

Although there are a lot of advantages of contactless measuring, also measuring with contact has advantages. If contact is made between the sensor system and the pulley, the real measuring can be done outside the transmission. With this kind of design a lot of problems will be overcome. The media inside the transmission does not influence the measurement, the high temperatures do not affect the actual sensor and outside the transmission is much more space to place the sensor.

One of the most broadly used transducers is the Linear Variable Differential Transformer (LVDT). The working principle is very easy and robust. The main advantages of the LVDT are the low friction while moving the shaft of the sensor, the high resolution of the sensor and the relative low cost.

§2.3 Sensor choice

Because of the many difficulties with the application of contactless measuring, the choice has been made to measure with contact between the pulley and sensor system. This means that a system has to be designed which has a guiding to follow the movement of the moving pulley sheave. Also a pointing tip has to be designed with which the contact is made. The third task is that the system has to deliver a force, to hold the feeler against the pulley sheave. The greatest disadvantage of this choice is that effects of film lubrication, wear and vibrations have to be taken into consideration.

Because of its positive properties the choice has been made to use an LVDT sensor. For more details of the chosen LVDT see appendix A. A second option could be an incremental sensor. The specifications of this sensor are also given in Appendix A. This sensor is more accurate but therefore also much more expensive.
Chapter 3: Measuring system

As said a system has to be designed which guides the shaft in the direction of movement and delivers a force in this direction to keep the feeler against the pulley.

First the placement of the system has been considered. After the placement of the system is defined, different systems have been designed. A choice has been made and the design of the system and its parts is worked out in §3.2. In chapter 4 calculations for this system are made.

§3.1 System placement

The placement of the sensor system has to fulfil to these demands:

- There has to be enough space for the system
- The movement of the pulley has to be transferred through the transmission housing
- The system has to be easily mountable and removable
- The transmission must be easy to work on, the sensor system may not interfere with the accessibility of the transmission

After consideration the easiest place for the system is on top of the bearing blocks of the shaft of the transmission. There it is easily mountable and can directly move out of the transmission. The system only has to have small height to reach the pulley. To guide the shaft out of the transmission, some material has to be removed from the cover of the transmission. Also the cover has to be easily removable from the transmission. With the movement of the cover it is very important not to touch the shaft, because it can be bend in that way. Third problem is that the sealing of oil has to be ensured.

§3.2 System design

In the beginning of this chapter some demands of the system are formulated. With these tasks in mind several designs have been made. After consideration two applicable designs were left. One was a system with linear bearings, the other a system with a cylinder. The choice has been made to go for the design with the linear bearings. This was mainly because of the simplicity of this system. The cylinder system and the choice for the bearing system will be explained in Appendix B, the bearing system is shown in figure 3.2.1.
§3.2.1 Linear bearings

In this design the shaft is guided by linear guiding bearings. These bearings can be bought completely with housing at several companies. The main advantage of these bearings is the small shaft diameter and the low friction. With a small shaft diameter also the moving mass will be small. Smallest diameters that can be bought are 3 mm. An example of these bearings can be seen in figure 3.2.1, the dimensions are in table 3.2.1. Reliance Gear Company Limited manufactures these bearings.

The placement of these bearings cannot be directly on top of the bearing blocks. A base plate is placed on top of the blocks and the guiding system is placed on this block.

Table 3.2.1: Dimensions of the bearing

<table>
<thead>
<tr>
<th>Shaft diameter</th>
<th>Height</th>
<th>Centre height</th>
<th>Width</th>
<th>Centre distance</th>
<th>Overall length</th>
<th>Hole distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>d 3</td>
<td>H 14</td>
<td>h 9</td>
<td>t 3</td>
<td>W 10</td>
<td>T 59</td>
<td>L 14</td>
</tr>
</tbody>
</table>
§3.2.2 Base plate

The base plate of the system has some advantages. It makes sure the system is easily mounted and removed. The system can be assembled outside the transmission and completely be installed onto the transmission with two bolts. Figure 3.2.2 shows the base plate. Complete drawings of the base plate can be found in Appendix C.

![Base plate](image)

Figure 3.2.2: Base plate

With the bigger M6 holes the base plate can be fixed onto the bearing block. The left side of the base plate in the figure reaches towards the pulley sheave. This is in order to get some length between the two linear bearings, to improve the bending stiffness of the shaft.

The right side of the base plate in the figure, the triangular part, is the part through which the shaft moves out of the transmission. This part rests on the flange of the shaft onto which the pulley is mounted. It is added to the base plate to protect the axis of the system against the cover of the transmission. As this cover is often removed, the shaft has to be protected against the cover touching and bending it. Now the cover falls over this triangle and the shaft is protected. The groove in this part is added to be able to insert a sealing rubber, which avoids the oil coming out of the transmission.

§3.2.3 Spring

The force applied to hold the shaft against the pulley sheave is, in this case, generated by a spring. As the force generated by a spring linearly rises with the compression of it, very low spring stiffness is wanted. Calculations in §4.1 show that the spring has to deliver a force with a minimum of 4 N. Also it has to be able to make the stroke of 16 mm after it has been prestressed to the 4 N.

§3.2.4 Shaft and couplings

Together with the linear bearings a shaft is available at Reliance Gear Company Limited.
This shaft is made from stainless steel and has a diameter of 3mm. An advantage of this shaft is the stainless steel. This steel is easier to treat than the normal bearing steel.

From Appendix A we learn that the LVDT has an extension with M5 thread onto it. This means the shaft and the LVDT can be easily connected to the shaft. As we have a 3mm. shaft at which thread can be cut a simple coupling can be designed. This coupling has to have one side with M5 thread and the other side with M3 thread.

Also on the other end of the shaft M3 thread can be cut. This means a pointing tip can be mounted on this side of the shaft. This feeler will have a pointing tip in the form of a ball, which makes the contact with the pulley. The choice for this ball is lying in the lubrication type, which is pointed out in chapter 4.
Chapter 4: System analysis

Before we can make some calculations to determine some main parameters of the system as stiffness and vibrations, first we have to consider the contact between the feeler and the pulley. It has to be decided what type of lubrication is wanted in the contact. This has been done in Appendix D. Also the theory for the chosen elastohydrodynamic lubrication is explained in this appendix.

When the type of lubrication is defined, different calculations can be made. First we will look at the accelerations in the system and we will determine the spring force that should be applied. With this force and the lubrication theory we can calculate some of the main properties of the system. So in the undergoing sections the spring force and accelerations, the radius of the ball, the rms surface roughness of the pulley, the lubrication film thickness and stiffness and the vibrations in the system are determined.

All these calculations have been done with MATLAB. The m-file for these calculations can be found in Appendix E.

§4.1 Spring force

For the spring force a compromise has to be made. The force should be as small as possible because it affects the lubrication film thickness negatively. The force may not be too low otherwise the contact between the feeler and the pulley sheave may be broken when the shaft is accelerated too much. This will lead to errors.

In purpose of an estimation of the accelerations in the system it is supposed that the pulley sheave has a stroke in the direction of the shaft, so in the only degree of freedom of the system. This stroke is supposed to have the form of a sinus and appears two times per rotation of the pulley. The maximum height of this stroke is varied from 0 mm. to 0.3 mm., so that the maximum allowable height can be specified.

One last thing that should be taken into consideration is the following speed of the LVDT. As can be seen in Appendix A the sensor can only measure up to 195 m/s². The incremental sensor is able to measure much higher accelerations. Therefore all calculations are based on the LVDT sensor, as we want be able to apply both sensors.

As the highest forces occur at the highest rotational speeds the maximum force will be calculated at the highest rotational speed of the belt box transmission. At the moment this is 5500 rpm, or 575 rad/s. This speed can even be higher when a reduction is applied between the secondary pulley and the secondary electro motor. As the sinus is supposed to appear two times per rotation of the pulley it has a rotational speed \( \omega_{\text{stroke}} \) of 1150 rad/s in the present configuration. When the stroke is known as a sinus, the accelerations can be derived in the following way:

\[
\text{Stroke: } \quad x = h \sin(\omega_{\text{stroke}} \times t) \quad (4.1)
\]
Speed: \[ v = x' = h\omega_{stroke} \cos(\omega_{stroke} \times t) \] (4.2)

Acceleration: \[ a = v' = -h\omega_{stroke}^2 \sin(\omega_{stroke} \times t) \] (4.3)

In (4.3) can be seen that the maximum absolute acceleration occurs when \( \sin(\omega_{stroke} \times t) \) is 1. The maximum absolute acceleration is then \( |h\omega_{stroke}^2| \). A plot is made for these accelerations against the height of the stroke h.

![Figure 4.1.1: Accelerations of the LVDT](image)

With the dotted line in figure 4.1.1 the maximum acceleration of the LVDT is plotted. In the figure can be seen the maximum height of the stroke may only be 0.15 mm. If the pulley has a stroke of greater height it has to be reduced. First measurements have shown that the stroke of the unadapted pulley surface will be very much smaller than this 0.15 mm.

The maximum height of the stroke is now specified so the maximum forces can also be calculated. The mass of the moving part of the sensor system is mainly determined by the shaft of the system and the mass of the moving part of the LVDT. As these masses are known the total of the moving mass is estimated on 20 gram. Now the accelerations and mass are known the force can be calculated with \( F = m \times a \). In figure 4.1.2 the force is plotted against the stroke height (solid line). The dotted line indicates the maximum height of the stroke, as was determined in figure 4.1.1.
In the figure can be seen that the maximum force required to keep the feeler against the pulley is only 4 N. This force is needed when the stroke is 0.15 mm and the rotational speed of the pulley is 5500 rpm.

### §4.2 Lubrication

From Hamrock (3) we learn that a dimensionless film parameter exists that gives us an indication of the lubrication range. This parameter characterises the degree of separation of the surfaces by the lubrication film and is given by:

\[
\Lambda = \frac{h_{\text{min}}}{\sqrt{\sigma_1 + \sigma_2}} \quad (4.4)
\]

With: \( \Lambda \) = dimensionless film parameter  
\( h_{\text{min}} \) = minimal film thickness  
\( \sigma_1 \) = rms surface roughness of surface 1  
\( \sigma_2 \) = rms surface roughness of surface 2

In general it is taken that if \( \Lambda \geq 2.5 \), complete film lubrication is present. If \( 1 \leq \Lambda \leq 2.5 \) mixed lubrication is present and if \( \Lambda \leq 1 \) only boundary lubrication is present.

In the lubrication theory of Appendix D an equation is derived which relates the different lubrication conditions to the minimal film thickness \( h_{\text{min}} \). This equation (D.9) is used to examine the influence of the different parameters. With this theory the radius of the
touching ball can be specified, as well as the wanted rms surface roughness of the pulley sheave. In figure 4.2.3 the influence of the ball radius on the dimensionless film thickness is examined.

Clearly can be seen the lubrication is not sufficient for total film lubrication which begins at approximately $\Lambda=2.5$. At a ball radius of 100 mm the value of $\Lambda$ is still only 1.4. This means that, for full film lubrication, the rms surface roughness of the pulley should be reduced. If the surface roughness is lowered also the ball radius can be in proportion with the rest of the system.

To decide which rms surface roughness for the pulley and which ball radius are qualified we have to examine the effect of both on the value of $\Lambda$. In figure 4.2.4 the values of $\Lambda$ are plotted against the rms surface roughness. This is done for different ball radii from 3 to 9 mm.
The figure points out that if the pulley roughness is lowered the value of \( \Lambda \) increases exponentially. With this result the choice has been made for a pulley roughness of 0.1 \( \mu \text{m} \). This is because this roughness is one of the lowest achievable roughnesses. This roughness can be achieved with polishing, following the polytechnisch zakboekje (5). The ball radius is chosen at 5 mm. This ball radius is applicable in the sensor system and delivers a good lubrication.

Now the system is fully determined we can look at the lubrication film thickness. The film thickness is depending on the rotational speed of the pulley. After calculations it pointed out that the film thickness varies from 0 to 0.3 \( \mu \text{m} \) within the speed range. This is below the desired precision, which means the variation of the film thickness does not have a large influence on the precision of the measurement.

Other calculations point out that the rotational speed of the pulley should not be too long beneath 1000 rpm. At the lower speeds of course the film thickness will be smaller, sometimes even so small that no sufficient lubrication is present. If this happens problems like wear become relevant. Of course the transmission has to run beneath 1000 rpm when starting up, but one should try to keep this period as short as possible.

It should be taken into consideration that the lubrication increases when the ball will be worn a bit. The surface at which a lubrication film is formed increases and with this a higher force can be generated.
Third problem that should be taken care of is the temperature of the contact. One has to make sure that sufficient cooled oil has to reach the contact to avoid the problem of the temperature rising over the melting temperature of the ball.

§4.3 System vibrations

Under the pressure of the lubrication film the surface of the ball will flatten. Because of the high pressure a deforming contact is generated and following the theory of Herz one can calculate the flattening of the surface. The theory of Herz can also be found in Hamrock (3). From this theory we learn that the flattened surface of the ball will become a circle with a diameter of:

\[ D = 2 \left( \frac{6\Sigma FR}{\pi E'} \right)^{1/3} \]

or

\[ D = 0.13 \times 10^{-3} \]  

With:

- \( \Sigma \) = \( \pi/2 \) for a ball
- \( F \) = Force on the ball, 4 [N]
- \( R \) = Ball radius, \( 5.10^{-3} \) [m]
- \( E' \) = Effective elastic modulus, \( 2.2527 \times 10^{11} \) [Pa]

With the data of the problem the diameter of the surface can be calculated. This is also done with the m-file that is presented in Appendix D. The results point out that the diameter of the surface is 0.13 mm.

One has to make sure the fluid film is able to follow the vibration (see §4.1) on the pulley sheave. If the fluid film is not able to follow the stroke the measurement will be much less reliable because the system does not exactly follow the pulley. To be sure the system follows the pulley one should compare the passing time of the vibration with the passing time of an oil particle. The passing time of the oil should be much lower than the passing time of the vibration otherwise the fluid film is not able to follow the vibration. Otherwise situations can occur that the ball will float that much on the oil that the vibrations in the pulley will not be measured.

The mean surface speed indicates the oil speed and together with the diameter of the flattened ball one can calculate the passing time of the oil \( t_{\text{trans}} \):

\[ t_{\text{trans}} = 2 \left( \frac{D}{L} \right) \left( \frac{u_{1} + u_{2}}{2} \right) = \frac{2 \times 0.13 \times 10^{-3}}{38.5} = 6.7 \times 10^{-6} \text{ s} \]

As we assumed in §4.1 there are two vibrations per rotation and the test rig has a rotational speed of 5500 rpm, there are 11000 vibrations per minute. So the transition time of one vibration is:

\[ t_{\text{vibration}} = \frac{60}{11000} = 5.45 \times 10^{-3} \text{ s} \]

This means the system can easily follow the pulley and its vibrations.
§4.4 System stiffness

For the total stiffness of the system also the stiffness of the lubrication film and the hertz contact should both be determined. The stiffness of the lubrication film can be examined with the minimum film thickness equation. Stiffness of the film is defined as:

\[ S_{\text{film}} = -\frac{\partial F}{\partial h_{\text{min}}} \]  \hspace{1cm} (4.6)

The spring force \( F \) appears in (C.9) in the dimensionless load parameter \( W \). With these facts one can easily derive the film thickness \( S_{\text{film}} \) as:

\[ S_{\text{film}} = \frac{1}{0.073 h_{\text{min}} F^{-1}} \]  \hspace{1cm} (4.7)

The same sort of derivation can be done for the equations of Herz, especially the equation for the flattening of the ball. With this derivation the stiffness of the Hertz contact can be defined as:

\[ S_{\text{hertz}} = \frac{1}{2.5 \delta F^{-1}} \]  \hspace{1cm} (4.8)

With \( \delta \) = compression length of the ball

These two stiffnesses define the total stiffness of the contact. This stiffness should be much higher than the spring stiffness. If this is not the case the pulley is able to move by compressing the film or ball. This means no movement will be detected in the system although the pulley sheave moves. The stiffnesses are calculated in MATLAB and in figure 4.2.5 the resulting stiffnesses can be seen as a function of the pulley speed.
From this figure can be seen that the Hertz stiffness almost completely determines the total contact stiffness. This can also be seen in the magnified part of figure 4.2.5. We see that the stiffness is lying in the order of $10^7$ N/m, and so it is much higher than the spring stiffness of 150 N/m. From this we can conclude that the spring stiffness determines the total system stiffness and so the Herz compression of the ball and the lubrication film compression will not have any effect on the measurement.

§4.5 System accuracy

Calculations have shown the system is able to follow the polished surface and that the lubrication film stiffness large enough in comparison to the spring stiffness, so it doesn’t influence the total system stiffness significantly. This means the total accuracy of the system is completely determined by the sensor and the influences of the belt-box.
transmission. The sensor already has a maximum error in its linearity of 0.5\% of the complete range (See appendix A). For the stroke of 16mm, this means a maximum error of 0.08mm. Vibrations will also be a problem in the sensor system. As the transmission is actuated it will always vibrate and the sensor will of course measure these vibrations. These vibrations can be partly removed from the signal by some data acquisition, for example filtering the signal. Other errors may occur by uncertainties of the transmission, for example the play in the bearings of the shafts. On the other hand, extensive calibrations, and also application of a more accurate sensor can improve the accuracy of the measurements.

To determine the exact accuracy of the measurement system would take a serious study of all the errors induced by the transmission and the sensor. This will not be done in this report due to the size of such a research. If we take into account errors and uncertainties as bearing play, sensor errors and vibrations a first estimation tells us that the measurement system can be used for the slip measurements. Especially with the very accurate incremental sensor the axial pulley position and with that the theoretical ratio can be determined accurately to determine slip with an accuracy of 0.1\%.
Chapter 5: Conclusions and recommendations

In the foregoing text a measuring system has been designed with a plain background. The designed system only has one major component that should be custom made; the other components can be bought with the given specifications.

Because of the design with a base plate, the system is easily mountable in the transmission. Also the placing of the system, on top of the bearing blocks makes the system very easy to reach. It can be assembled outside of the transmission and then the complete system can be placed in the transmission afterwards.

Measuring with contact between the measurement system and the pulley sheave brings along some specific advantages. The system is more reliable and easier to apply than measuring without contact. Errors may occur due to moving metal parts and oil droplets when using contactless sensors.

Measuring with contact has some disadvantages. The system is more sensitive for vibrations in the transmission and especially some tribological problems occur in the contact. The polishing of the pulley sheave mainly solves these problems. With this treatment a sufficient film thickness can be formed for full film lubrication at higher speeds. Also small vibrations at the surface can be removed. With this fact also the spring force can be lowered, which has also has a positive effect on the lubrication. An effect that also has a positive influence is a little bit of wear of the ball. As the ball wears a little, the lubrication surface enlarges a little and the lubrication is improved.

Calculations have shown the system is able to follow the polished surface and that the lubrication film stiffness is large enough in comparison to the spring stiffness, so it doesn’t influence the total system stiffness significantly. This means the total accuracy of the system is completely determined by the sensor and the influences of the transmission. With this conclusion in mind the system should be able to determine the ratio accurate enough to determine slip with an accuracy of 0.1%.

When working with the system one should make sure that the system is not working to long beneath a rotational speed of 1000 rpm. Otherwise the wear of the ball can become significant. Also enough cooled oil has to be supplied to the contact to avoid the temperatures to increase to a temperature that is higher than the melting temperature of the ball. If the lubrication in practice seems not to be sufficient also another form of lubrication type can be used, namely hydrodynamic lubrication. A wedge is used instead of a ball and the lubrication film will be much thicker. The problem with this type of lubrication is that a specific construction should be designed to guarantee the wedge to be perpendicular to the tangential speed of the pulley. The hydrodynamic lubrication theory can be found in Appendix D.1.
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   Bernard J. Hamrock, Bo Jacobson, Steven R. Schmid.  
   ISBN 0-07-228933-3

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   H.J. van Leeuwen  
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   Koninklijke PBNA BV.  
   ISBN 90-6228-266-0
Appendix A: LVDT and Heidenhain sensor

<table>
<thead>
<tr>
<th>Resolution</th>
<th>nearly infinite, depending on following circuit (ripple) and measurement range.</th>
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<tbody>
<tr>
<td>Range</td>
<td>±0,50/ ±2,50/ ±5,00/ ±10,00/ ±12,50/ ±15,00/ ±25,00/ ±50,00/ ±75,00/ ±100,00/</td>
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<tr>
<td></td>
<td>±125,00/ ±150,00/ ±175,00/ ±200,00/ ±250,00/ ±300,00/ ±400,00/ ±500,00/ ±550,00 mm</td>
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<tr>
<td>Linearity</td>
<td>≤ ±4,5% range, others on request.</td>
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<tr>
<td>Output AC</td>
<td>Excitation 5 Vrms at 3 kHz</td>
</tr>
<tr>
<td>(external electronics)</td>
<td>Temperature: -30...+85°C (Standard) optional -30...+150°C</td>
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<tr>
<td></td>
<td>Frequency Response: 3 dB at 180 Hz</td>
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<tr>
<td>External electronics</td>
<td>Using the external electronic device 8100 it is possible to adapt LVDT’s with</td>
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<tr>
<td></td>
<td>smaller housing and an extended temperature range up to 150 °C. For critical</td>
</tr>
<tr>
<td></td>
<td>applications in harsh environments we recommend LVDT’s with external</td>
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<tr>
<td></td>
<td>electronics.</td>
</tr>
<tr>
<td></td>
<td>Please see technical details in the data sheets 8100 and 6000 (19&quot; rack, modular</td>
</tr>
<tr>
<td></td>
<td>system)</td>
</tr>
<tr>
<td>Output:</td>
<td>0...5 VDC, 0...10 VDC, 4...20 mA</td>
</tr>
<tr>
<td>Internal electronics</td>
<td>Built in electronics. Following outputs are available (others on request).</td>
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<tr>
<td></td>
<td>±2,5 VDC</td>
</tr>
<tr>
<td></td>
<td>Supply: 10...30 VDC (to be specified)</td>
</tr>
<tr>
<td></td>
<td>Current consumption: 35 mA (at 12 VDC)</td>
</tr>
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<td></td>
<td>Ripple: max. 30 mV</td>
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<tr>
<td></td>
<td>Output Bandwidth: 300 Hz</td>
</tr>
<tr>
<td></td>
<td>Zero Temperature Coefficient: 0,01% FS/°C</td>
</tr>
<tr>
<td></td>
<td>Span Temperature Coefficient: 0,03% FS/°C</td>
</tr>
<tr>
<td></td>
<td>Working Range: -50...+85°C.</td>
</tr>
<tr>
<td>0...10 VDC</td>
<td>Supply: 15...30 VDC (to be specified)</td>
</tr>
<tr>
<td></td>
<td>Current consumption: 35 mA (at 12 VDC)</td>
</tr>
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<td></td>
<td>Ripple: max. 30 mV</td>
</tr>
<tr>
<td></td>
<td>Output Bandwidth: 300 Hz</td>
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<tr>
<td></td>
<td>Zero Temperature Coefficient: 0,01% FS/°C</td>
</tr>
<tr>
<td></td>
<td>Span Temperature Coefficient: 0,03% FS/°C</td>
</tr>
<tr>
<td></td>
<td>Working Range: -50...+85°C.</td>
</tr>
<tr>
<td>4...20 mA</td>
<td>Supply: 14...24 VDC</td>
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<td>Ripple: max. 0,1% at 20 mA</td>
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<td></td>
<td>Null: 12 mA at zero set within 0,5%</td>
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<td>Working Range: -20...+95°C.</td>
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<tr>
<td>Type</td>
<td>Plain core with extension front end guided, core with extension and rod end</td>
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<td>bearings, sprung loaded core with extension front end guided max. stroke ± 75</td>
</tr>
<tr>
<td></td>
<td>mm extension rod wiper.</td>
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<td>Housing</td>
<td>Stainless steel</td>
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<td>Connection</td>
<td>2000 mm cable output (others on request)</td>
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<td>Protection</td>
<td>IP54</td>
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<td>Shock</td>
<td>1000g (10 ms)</td>
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<tr>
<td>Vibration</td>
<td>20 g (2 kHz)</td>
</tr>
</tbody>
</table>
Internal electronics

±2.5 VDC
supply: 10...30 VDC
current cons.: 35 mA at 12 VDC
ripple: max. 30 mV
output bandwidth: 300 Hz
zero temp. coefficient: 0.01% /°C
span temp. coefficient: 0.03% /°C
temperature range: -50...+55°C

0..10 VDC
supply: 15...30 VDC
current cons: 35 mA at 15 VDC
ripple: max. 30 mV
output bandwidth: 360 Hz
zero temp. coefficient: 0.01% /°C
span temp. coefficient: 0.03% /°C
temperature range: -50...+85°C

4...20 mA
supply: 14...24 VDC
ripple: max. 0.1% at 20 mA
null: 12 mA ± 0.5%
temperature range: -20...+95°C

External electronics

Please see technical details in the data sheets §100 and 6000 (19" rack, modular system). Outputs available:
0...5 VDC, 0...10 VDC, 4...20mA
HEIDENHAIN-SPECTO
Length Gauges with ±1 µm Accuracy

- Very compact dimensions
- Splash-proof

Thanks to their quite small dimensions, the HEIDENHAIN-SPECTO length gauges are the product of choice for multipoint inspection apparatus and testing equipment.

Plunger actuation
The length gauges of the ST 12x8 and ST 30x8 series feature a spring-loaded plunger that is extended at rest.

In the pneumatic length gauges ST 12x7 and ST 30x7, the plunger is retracted to its rest position by the integral spring and is extended to the measuring position by the application of compressed air.

Mounting
The HEIDENHAIN-SPECTO length gauges are fastened by their 8H standard clamping shank.

Output signals
The HEIDENHAIN-SPECTO length gauges are available with three different output signals. The ST 120x and ST 300x versions supply sinusoidal current signals with 11 µA peak levels for HEIDENHAIN subsequent electronics.

The ST 128x and ST 308x length gauges provide sinusoidal voltage signals with 1 V peak levels, which permit high interpolation.

The ST 127x and ST 307x feature integrated digitizing, edge detection and interpolation electronics with 5-bit and 10-bit interpolation (as ordered) and square-wave signals in TTL levels.

Lateral cable exit
Alternative possible with
ST 12x8
ST 30x8

ST 1200

ST 3000

Mechanical data
- Plunger actuation: Plunger position at rest
- Measuring standard
- System accuracy
- Recommended measuring step
- Reference mark
- Measuring range
- Gauging force with retracting plunger:
  Vertically downward
  Vertically upward
  Horizontal
- Permissible radial force
- Operating attitude
- Vibration (55 to 2000 Hz)
- Shock (11 ms)
- Protection (IEC 60529)
- Operating temperature
- Fastening
- Weight (without cable)

Electrical data
- for length gauges
- Output signals:
  Signal period
- Measuring velocity
- Electrical connection
  Connecting elements
  Cable length to subsequent electronics
- Power supply

mm
DIN ISO 2015
ISO 2768 - m H
Ø = Reference mark position
Ø = Beginning of measuring length
<table>
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<td>~11 μA</td>
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<td>HEIDENHAIN connector (male) 15-pin</td>
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<td>D-sub connector (male) 15-pin^*</td>
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<td>HEIDENHAIN connector (male) 15-pin</td>
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<td>30 m max. (98.4 ft)</td>
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<tr>
<td>5 V ±5% &lt; 50 mA</td>
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<td>5 V ±10% &lt; 120 mA</td>
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<td>5 V ±5% &lt; 50 mA</td>
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</tbody>
</table>

^ Interface electronics integrated in connector

- **DIACUR grating on silica glass; grating period 20 µm**
- ±1 µm
- 1 µm ± 0.5 µm
- Approx. 5 mm below upper stop
- 12 mm (0.47 in.)
- 30 mm (1.2 in.)
- 12 mm (0.47 in.)
- 30 mm (1.2 in.)
- 0.6 to 1.8 N
- 0.4 to 1.6 N
- 0.5 to 1.7 N
- 0.7 to 1.4 N
- 0.3 to 1.0 N
- 0.5 to 1.2 N
- 0.4 to 3.4 N depending on pressure and position
- 0.4 to 2.4 N depending on pressure and position
- ± 0.8 N
- Any
- ± 100 mNcm (IEC 80068-2-6)
- ± 1000 mNcm (IEC 80068-2-27)
- IP 64
- 10 to 40 °C (50 to 104 °F); reference temperature 20 °C (68 °F)
- Clamping slants Ø 30 x
- 40 g
- 50 g
- 40 g
- 50 g

24
Appendix B: Cylinder system

In chapter 3 the design of the bearing system has been explained. In the design period also a design of a cylinder system has been made. This system is explained in this appendix.

In this system also the fixing onto the bearing blocks can be applied. The rest of the system is completely different. A cylinder does both the guiding of the shaft and the force on the shaft. The complete system can be seen in figure B.1.

For the guiding of the shaft in the cylinder there are two bronze guiding bushes (black) in at the ends of the cylinder. These bushes are fixed with the circlips. Especially bronze is used for its low friction coefficient. The cylinder is fixed onto the base plate by the two clamps (grey).

Inside the cylinder is a small piston (yellow) that fits exactly into it. This piston is also shrunken onto the shaft. This piston divides the two parts of the cylinder of each other. If now air pressure is brought onto one side of the piston a force is generated on the shaft. As air pressure is available at the test rig from 0 bar to 5 bar overpressure, the forces on the shaft can easily be adjusted by increasing or decreasing the pressure. The side of the cylinder at which there is no air pressure, the right side of the cylinder in figure B.1, contains some holes at the end. This is done to guarantee that there is only atmospheric air pressure in this part of the cylinder.

As a first estimation we want a force of approximately 10 N. Air pressure is available from 0 bar to 5 bar. The cylinder is an inner diameter of 10mm and the shaft has a diameter of 2mm. So the surface of the piston is:
As 1 bar is approximately \(1 \times 10^5 \text{ N/m}^2\) and the required force (\(F\)) 10 N, the pressure that should be supplied is:

\[
P = \frac{F}{A} = \frac{10}{75.4 \times 10^{-6}} = 1.3 \times 10^5 \text{ N/m}^2 = 1.3 \text{ bar}
\]

So the force can be increased to almost 4 times the required force.

This design has some advantages above the design with the bearings. First the alignment of the guiding bushes is always guaranteed, if the cylinder is correctly manufactured. In the second place the force on the shaft is not a function of the movement of the pulley. As the pressure on the piston remains constant, also the force on the piston will not change.

The major disadvantage of the cylinder system is the complexity. As almost none of the parts are standard products, the entire system has to be manufactured only for this application. Because of this the costs will rise and the delivery time of the system will be longer. Also there has to be made an air supply into the transmission, which is a difficult problem.

Both the cylinder and the bearing system have there own advantages and disadvantages. The cylinder system seems to be the best solution for accurate measuring. The force on the shaft stays constant and the alignment of the guiding bushes seems easier than aligning the linear bearings.

Although the cylinder system seems a bit better, the choice has been made to go for the design with the linear bearings. This is mainly because of the simplicity of the system. Except for the base plate and some small things, all of the parts can be bought at different companies. The system is, by it’s simplicity, very reliable and easy to implement.
Appendix C: Base plate drawings

In this appendix first the complete assembly will be shown. Next the base plate will be shown, both to remind how the system works and how the base plate functions in the system. On the next page the base plate dimensions will be given.

Figure C.1: Assembly of the system

Figure C.2: Base plate in TFR-ISO view
Appendix D: Lubrication

For the wear of a feeler along which the pulley slides, calculations can be made with the Archard wear theory in Hamrock (3). Results of these calculations show that, under certain circumstances the feeler could have wear in the order of 1mm/hour. These calculations point out film lubrication definitely has to be generated to keep the wear into proportions.

Different subtypes of full film lubrication can be applied in this case. Three main lubrication types are hydrodynamic lubrication (HL), elastohydrodynamic lubrication (EHL) and hydrostatic lubrication (HSL).

At HSL the fluid film is generated by a pressure source from the outside of the lubricated contact. Because this means that a more difficult construction is needed, pressure has to be guided from a pump to the contact point, HSL will not be applied.

The two types of lubrication left, HL and EHL, will be further examined in the following paragraphs and a choice for the lubrication type will be made. All of the formulas derived can be found in Hamrock (3) and are derived from the well-known Reynolds equation. This equation is also derived in Hamrock (3).

§D.1 Hydrodynamic lubrication

At hydrodynamic lubrication (HL) the film is generated by the relative movement of the surfaces and the geometry of the contact. The geometry of the feeler is formed as a wedge. Figure D.1.1 shows an example of such a wedge.

![Diagram of Hydrodynamic lubrication](image)

The oil is dragged into the wedge and a positive pressure is generated by the so called 'wedge effect'. The magnitude of the pressure developed (usually less than 5 Mpa) is not generally large enough to cause significant deformation of the surfaces.

In HL the film thickness normally is in the order of micrometers.
In Van Leeuwen (4) equations for a wedge with infinite width are derived from the Reynolds equation. If we make a few assumptions for the geometry of the wedge a first estimation of the lubrication film thickness can be made with these equations. With an estimated spring force of 10 N, given oil viscosity and a rotational speed of 5500 rpm, the lubrication film thickness is estimated at 19 μm.

From Hamrock (3) we learn that a film parameter (see §4.2) exists that gives us an indication of the lubrication range. This parameter characterises the degree of separation of the surfaces by the lubrication film and is given by:

\[ \Lambda = \frac{h_{\text{min}}}{\sqrt{\sigma_1 + \sigma_2}} \]  

With:
- \( h_{\text{min}} \) = minimal film thickness
- \( \sigma_1 \) = rms surface roughness of surface 1
- \( \sigma_2 \) = rms surface roughness of surface 2

In general it is taken that if \( \Lambda \geq 3 \), complete film lubrication is present. If \( 1 \leq \Lambda \leq 3 \) mixed lubrication is present and if \( \Lambda \leq 1 \) only boundary lubrication is present.

As the pulleys are grinded at the moment, the roughness is estimated with the polytechnisch zakboekje (5) at 0.8 μm. The roughness of the wedge is also estimated but at 1.6 μm. With the known \( h_{\text{min}} \) of 19 μm the film parameter becomes:

\[ \Lambda = \frac{19 \times 10^{-6}}{\sqrt{(0.8 \times 10^{-6})^2 + (1.6 \times 10^{-6})^2}} = 20.6 \]

This means that good film lubrication is guaranteed.

A very large disadvantage of hydrodynamic lubrication is the characteristic that the wedge has to be completely square on the pulley movement. If the wedge is not positioned in the right way the lubrication film will decrease in height very fast and may lose its stability. Therefore a special construction should be designed to fix the wedge to the shaft. If HL is applied to this problem one should design an attachment of the wedge to the shaft that guarantees the wedge to be completely square to the pulley movement.

### §D.2 Elastohydrodynamic lubrication

Elastohydrodynamic lubrication (EHL) is a form of hydrodynamic lubrication where elastic deformation of the lubricated surfaces becomes significant. As the pressure in the contact rises because of another geometry of the feeler, it is typically between 0.5 and 4 Gpa. Under these circumstances the surfaces of contact will elastically deform and have a significant influence to the lubrication film thickness.

The main advantage of EHL is that, in this case it can be realised with a feeler in the form of a bearing ball. A ball is symmetric and so because of this it does not need a specific
type of positioning with respect to the pulley. This fact makes the solution to the contact problem much easier and stable. Figure D.1.2 shows the EHL solution with a ball.

![Diagram of EHL solution with a ball](image)

**Figure D.1.2: Elastohydrodynamic lubrication**

The main disadvantage of EHL in comparison to HL is the smaller minimal film thickness, \( h_{\text{min}} \). With Hamrock (3) also for EHL a first estimation of the lubrication can be made. As the feeler is implemented as a ball with a radius of 5 mm and the speed of the pulley is again 5500 rpm, the film thickness can be approximated. With given parameters of the oil, the modules of elasticity of the materials etcetera, the film thickness of the EHL becomes approximately 0.4 \( \mu \text{m} \). The film parameter of equation (D.4) in this case reduces to:

\[
\Lambda = \frac{0.4 \times 10^{-6}}{\sqrt{(0.8 \times 10^{-6})^2 + (0.01 \times 10^{-6})^2}} = 0.5
\]

The rms surface roughness of the ball is estimated at 0.01 \( \mu \text{m} \). This means boundary lubrication is present and also wear becomes a significant parameter in the problem.

As both lubrication types are considered a choice between them can be made. The choice has been made to apply elastohydrodynamic lubrication. This is because of the simplicity and stability of this lubrication type. As a ball can be used as a feeler and in this case the does not have to be especially positioned to the pulley this seems the easiest solution. The only problem is the dimensionless film parameter. As this parameter is only 0.5 and it has to be 3 or higher, the roughness of the pulley surface has to be reduced. This means some extra work but if the pulley is adapted, also vibrations in the system can be reduced which means an output signal of the LVDT with less noise. Hydrodynamic lubrication can also be applied, but a smart attachment of the wedge to the shaft should be designed in that case.

§D.3 EHL theory

As has been said, all equations of the elastohydrodynamic lubrication can be derived from the Reynolds equation. These derived equations can all be found in Hamrock (3).
From the contact in the sensor system different parameters can be found or derived. These parameters and their units will now be given.

- $E' = \text{effective elastic modulus, [Pa]}$  
  
  $E' = \frac{2}{\left(\frac{1-v_a^2}{E_a}\right) + \left(\frac{1-v_b^2}{E_b}\right)}$  
  
  With:  
  $\nu_a = \text{poisson ratio of material a}$  
  $\nu_b = \text{poisson ratio of material b}$  
  $E_a = \text{modulus of elasticity of material a}$  
  $E_b = \text{modulus of elasticity of material b}$

- $u = \text{mean surface velocity in motion direction, [m/s]}$  
  
  $u = \frac{u_a + u_b}{2}$  
  
  With:  
  $u_a = \text{velocity of surface a}$  
  $u_b = \text{velocity of surface b}$

- $F = \text{spring force, [N]}$
- $h = \text{film thickness, [m]}$
- $R_x = \text{effective ball radius in motion direction, [m]}$
- $R_y = \text{effective ball radius in transverse direction, [m]}$
- $\xi = \text{pressure-viscosity coefficient of the oil, [m}^2/\text{N}]$
- $\eta_0 = \text{atmospheric viscosity, [N-s/m}^2]$ 

From these parameters the following five dimensionless groupings can be established.

- **Dimensionless film thickness:**  
  $H = \frac{h}{R_x}$  
  
  (D.4)

- **Ellipticity parameter**  
  $k_e = \left(\frac{R_x}{R_y}\right)^{2/\Gamma}$  
  
  (D.5)

- **Dimensionless load parameter**  
  $W = \frac{F}{E'R_x^2}$  
  
  (D.6)

- **Dimensionless speed parameter**  
  $U = \frac{\eta_0 u}{E'R_x}$  
  
  (D.7)

- **Dimensionless materials parameter**  
  $G = \xi E'$  
  
  (D.8)
The dimensionless minimum film thickness can thus be written as a function of the other four parameters. This is equation is known as the minimum film thickness equation and is empirically determined by Hamrock. The minimum film thickness equation is given by equation (D.9)

\[ H_{\text{min}} = \frac{h_{\text{min}}}{R_x} = 3.63U^{0.68}G^{0.40}W^{-0.072} \left(1 - e^{-0.68k_x}\right) \]  

(D.9)
Appendix E: Matlab m-file

% M-file for calculations on the sensor system
% All equations can be found in: 'Fundamentals of machine elements'
% of Bernard Hamrock, Bo Jacobson and Steven R. Schmid.

clear all
close all

% First the calculations for the created accelerations
% and the spring force that is required. See section 4.2.1

m = 20e-3; % Moving mass of the sensor system
number = 2; % Number of vibrations per rotation
omega = 575; % rad/s Maximum transmission speed

H = [ ];
A = [ ];
P = [ ];

for h = 0:1e-5:0.3e-3
    % Height change of the vibration
    H = [H, h];
    a = h * (number * omega)^2;
    A = [A, a];
    P = [P, m*a];
end

figure(1)
plot(H(1,:), A(1,:), 'k', H(1,:), 195, 'k-.'), % 195 m/s^2 is sensor maximum
xlabel('stroke height [m]'),
ylabel('accelerations [m/s^2]'),
title('acceleration')
legend('acceleration of the sensor system', 'maximum LVDT')

figure(2)
plot(H(1,:), P(1,:), 'k', 0.15e-3, P(1,:), 'k-'),
xlabel('stroke height [m]'),
ylabel('needed spring force [N]'),
title('spring force calculations')
legend('springforce', 'maximum stroke height')

% Now we are going to look at the effects of the ball radius
% on the tribological effects. See section 4.2.2

clear all

ua = 0; % Speed of the ball
ub = 36.5; % Maximum speed of the test rig
ugem = (ua+ub)/2; % Mean surface velocity
eta = 7e-3; % Oil viscosity
ksi = 2e-6; % Pressure/viscosity ratio of the oil
Ea = 205e9; % E-modulus of the pulley
Eb = 205e9; % E-modulus of the ball
nu_p = 0.3; % nu of the pulley
nu_b = 0.3; % nu of the ball
Etot = 2/((1-nu_p^2)/Ea + (1-nu_b^2)/Eb); % Effective modulus of elasticity
\begin{verbatim}
R = 0.0001:0.001:0.1
r = [r, R];
W = F/(E*t^2);
U = (eta/ugem)/(E*t^2);

Hmin = 3.63*(1^0.68)*G^0.49 ...

hmin = Hmin*R;
sigma = sqrt(sigma^2 + sigma^2);
lambda = hmin/sigma;
L = [L, lambda];
end

figure(3)
plot(r(1,:),L(1,:), 'k', r(1,:), 2.5, 'k-.', r(1,:), 1, 'k-.')
xlabel('ball radius [m]')
ylabel('lambda')
legend('lambda', 'full film lubrication', 'partial film lubrication')
title('film parameter as a function of ball radius')
axis([0 0.1 0 3])

clear all

ua = 0;
ub = 38.5;
ugem = (ua+ub)/2;
eta = 7e-3;
ksi = 2e-8;
Ea = 205e9;
Eb = 205e9;
nuA = 0.3;
nuB = 0.3;

Etot = (1-nuA^2)/Ea + (1-nuB^2)/Eb; % Effective modulus of elasticity

sigma = 0.01e-6;
F = 4;
k = 1;

Sigmanu = [ ];
L = [ ];

for R = 3e-3:1e-3:9e-3
i = for sigmanu = 0:1e-9:1e-6
Sigmanu = [Sigmanu, sigmanu];
W = F/(E*t^2);
G = ksi*Etot;
\end{verbatim}
\[ U = \frac{(\eta \cdot u_{gem})}{(E_{tot} \cdot R)}; \]
\[ H_{min} = 3.63 \cdot U^{0.68} \cdot G^{0.49}; \]
\[ * (W^{(-0.073)}) \cdot (1 - \exp(-0.68 \cdot k)); \]
\[ \text{Minimum film thickness} \]
\[ \text{Dimensionless speed} \]
\[ \text{Formula 13.72 HAMROCK} \]
\[ \text{Formula 8.25/8.26} \]
\[ \text{Formula 8.27} \]
\[ \text{Contact stiffness} \]

\[ \sigma = \sqrt{(\sigma_a^2 + \sigma_{nu}^2)}; \]
\[ \lambda = \frac{H_{min}}{\sigma}; \]
\[ \text{Roughness coefficient} \]
\[ \text{Lambda value} \]

\[ \text{Figure 4} \]
\[ \text{Plot} \]
\[ \text{Xlabel} \]
\[ \text{Ylabel} \]
\[ \text{Legend} \]
\[ \text{Title} \]
\[ \text{Hold on} \]
\[ \text{Axis} \]

\[ \text{Now the effect of the pulley speed on the lubrication film thickness is being examined.} \]

\[ \text{Clear all} \]
\[ \text{Ksi} = 2e-8; \]
\[ \text{Ea} = 205e9; \]
\[ \text{Eb} = 205e9; \]
\[ \text{Nu}_a = 0.3; \]
\[ \text{Nu}_b = 0.3; \]
\[ \text{Etot} = 2/(1 - \text{Nu}_a^2); \]
\[ \text{Etot} = 2/(1 - \text{Nu}_b^2); \]
\[ \text{Effective modulus of elasticity} \]
\[ \text{Formula 8.25} \]

\[ \text{Delta} = \frac{1}{2} \cdot \left( \frac{9 \cdot (2 \cdot E_{tot})}{(1 + k^2)} \right)^{1/3} \]
\[ \text{Ball deformation height [m]} \]
\[ \text{Formula 8.27} \]

\[ \text{Sh} = \frac{1}{2} \cdot \text{Delta} \cdot \left( \frac{1}{F} \right); \]
\[ \text{Contact stiffness} \]
for ub=0.001:0.5:38.501;
    %Different pulley velocities
    U1=[ub;]
    Lm=[U1,ub];
    RPM=[RPM, rpm];
    ua=0;
    %Ball speed
    ugme=(ua+ub)/2;
    W=F/(Etot*E^2);
    G=ksi*Re;
    U=(eta*ugme)/(Etot*E);
    Hmin3.63*(U^0.68)*(G^0.49)...%Formula 13.72 HANROCK
    hmin=Hmin*K;
    H=[H, hmin];
    s=-1/(0.073*Hmin*E-1); %Lubrication film stiffness
    S=[s,s];
    stot=1/((1/Sh)+(1/s));%Dimensionless force
    Stot=[Stot, stot];
    sigma=sqrt(sigma1^2+sigmanu^2); %Dimensionless filmparameter
    lambda=hmin/sigma;
    L=[L, lambda];
end

figure(5)
plot(U1(1,:),L(1,:), 'k',U1(1,:),2.5, 'k',U1(1,:),1,'k')
xlabel('Pulley speed [m/s]')
ylabel('lambda')
legend('lambda', 'Full film lubrication'...
    , 'boundary lubrication')
%title('Lambda as a function of the pulley speed')

figure(6)
plot(U1(1,:),H(1,:), 'k')
xlabel('Pulley speed [m/s]')
ylabel('Film thickness [m]')
%title('Lubrication film thickness as a function of
% pulley speed')

figure(7)
plot(RPM(1,:),L(1,:), 'k',RPM(1,:),2.5, 'k',RPM(1,:),1,'k')
xlabel('rpm [1/min]')
ylabel('lambda')
legend('lambda waarde', 'volledige smering', 'grenssmering')
%title('lambda as a function of the rpm')

figure(8)
semilogy(RPM(1,:),S(1,:), 'k',RPM(1,:),Sh, 'k-',RPM(1,:),Stot(1,:),'k')
xlabel('rpm [1/min]')
ylabel('stiffness [N/m]')
legend('film stiffness', 'Hertz stiffness', 'total contact stiffness')
%title('stiffnesses as a function of the rpm')