Sound radiation simulation of a resilient train wheel

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Sound radiation simulation of a resilient train wheel

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2 April 1997

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Summary

Railway rolling noise can be reduced by the use of low noise wheels, such as resilient wheels. Noise measurements on trains with resilient wheels have shown significant reductions. Resilient wheels consist of two steel parts: a wheel body and a tyre. Both parts are structurally isolated by means of resilient rubber elements, which reduce the transmission of vibration and increase the damping of the wheel structure. Therefore the sound radiation is reduced by a smaller strong vibrating - and thus radiating - surface and by higher decay rates.

To investigate the sound radiation of resilient wheels, an analysis, based on simulation models of an existing resilient wheel model, was performed. The stiffness coefficients of the resilient elements are obtained experimentally using ISO standards measurement techniques, and are used to build a finite element model in ANSYS, to describe the vibration behaviour of the wheel. Modal analyses are performed, providing the mode shapes and resonance frequencies. These results are used to model the sound radiation of the resilient wheel using the simulation program TWINS.

The results of the modal analysis show the vibration behaviour of the resilient wheel to be more complex than that of conventional train wheels. Wheel body and tyre have independent modes which are only slightly coupled. An experimental modal analysis supplies the results for the validation of the finite element model. Seventy-five percent out of 34 resonance frequencies are within 4% accurate. Sound radiation measurements and additional simulations are performed to verify the radiation model in TWINS. Minor adjustments to the TWINS radiation module are needed to fit the radiation characteristics of the resilient wheel.

Although the accuracy of the TWINS model is limited, some trends are clear. Significant reduction in sound radiation can be achieved with resilient wheels. However the amount of reduction depends strongly on the excitation spectrum. Future research can result in acoustical optimal resilient wheels with resilient elements having optimised dynamic stiffness characteristics.
Samenvatting


Aan de hand van simulatiemodellen is de geluidafstraling van een veel gebruikt type geveerd wiel onderzocht. Dynamische stijfheiden en dempingsfactoren van de rubberveer-elementen zijn experimenteel vastgesteld op basis van internationale ISO normen. De dynamische stijfheiden zijn gebruikt in een eindige elementen model in ANSYS, waarmee het trillingsgedrag van het treinwiel werd onderzocht. Modaalanalyses leverden trilvormen en resonantiefrequenties. Uitgaande van het eindige elementen model, is de geluidafstraling van het geveerde wiel gemodelleerd. Hierbij is gebruik gemaakt van de simulatieprogramma's TWINS en SYSNOISE.

Uit de resultaten van de modaalanalyse blijkt dat het trilgedrag van een geveerd treinwiel complexer is dan dat van een conventioneel treinwiel. Wielband en wieliijf hebben trilvormen die slechts in beperkte mate gekoppeld zijn. Het eindige elementen model is gevalideerd door middel van een experimentele modaal analyse. Vijfentwintig procent van de eigenfrequenties zijn binnen 4% nauwkeurig.

De simulatiemodellen voor de geluidafstraling zijn ook gevalideerd door experimentele resultaten. Modale geluidvermogens en afstraalfactoren zijn gemeten door middel van geluidintensiteitsmetingen.

Geringe aanpassingen in TWINS zijn nodig om de geluidafstraling van een geveerd wiel juist te beschrijven. Hoewel de nauwkeurigheid van TWINS beperkt blijft, zijn duidelijke trends waarneembaar. Geveerde wielen kunnen een significante geluidreductie leveren, het resultaat is echter wel afhankelijk van het excitatiespectrum. Ververgonderzoek zal kunnen leiden tot een akoestisch optimaal wiel, waarbij de rubberveer-elementen de optimale dynamische stijfheidseigenschappen bezitten.
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1 Introduction

Noise and vibration are major sources of environmental impact due to railway operations. In a section of the Netherlands law "Wet Geluidhinder," titled "Besluit Geluidhinder Spoorwegen," more strict limits for noise levels are announced for the year 2000. Therefore new ways of reducing the noise emission of trains will have to be investigated.

A potential noise reduction in railway traffic is the use of low noise wheels. In the past several tests with different wheel designs have been performed, showing a possible noise reduction up to 10 dB. To achieve an acoustically optimal wheel design it is essential to be able to investigate the acoustical performance of the wheel design in an early design stage. Therefore simulation models are developed, which are very helpful in designing low noise wheels. Numerical models will also enable designers to compare the acoustical performance of different wheel designs.

This report describes a numerical model for a resilient train wheel. This type of wheel, in which rubber elements are used to isolate vibrations, is believed to have a high potential in reducing railway noise. The numerical model represents the vibration as well as the sound radiation behaviour of the wheel. At this moment very little information is available on numerical models for low noise wheel designs.

1.1 Typical noise sources of normal speed train traffic

The primary source of noise at low train speeds is the wheel/rail noise. Wheel/rail noise can be categorised into three types of noise: squeal, impact and rolling noise. Squeal noise appears when a train wagon rounds a curve of small radius. Due to the tangential displacement of the wheel on the rail, a stick-slip effect occurs in the contact point. This results in an intense narrow band noise which is related to the resonance frequencies of the wheel.

Impact noise is the "banging" noise that occurs when a wheel crosses a rail joint or another discontinuity in the rail. On the other hand, wheel flats can have the same result. Sudden changes in vertical velocity of the wheel, result in large forces between wheel and rail. These dynamic forces excite the wheel and rail into vibration and causes them to radiate sound.

Rolling noise dominates on straight track, in the absence of rail joints and wheel flats. On a smaller scale, wheel and rail are not ideally round and straight. These so called surface irregularities are normally referred to as roughness or waviness, and will result in small deflections of the wheel rail contact. Again these changes in vertical velocity result in dynamic forces and thus radiation of noise of wheel and rail.

1.2 Low noise wheel designs

In the design of low noise wheels the main issue is the reduction of rolling noise, however in most cases the impact and squeal noise will also be reduced. World wide several wheel types have been designed to reduce wheel/rail noise. Reviews of different low noise wheel designs were published by Reybardy and Zach [Reybardy et al., 1989] and Kurzweil and Wittig [Kurzweil et al., 1981]. The different solutions can be categorised as described in the next sections.
1.2.1 Resilient wheels

Resilient wheels are wheels in which the metal tyre is structurally isolated from the wheel body by a resilient (rubber-like) material (figure 1.1). The resiliency is achieved by elastomeric material, which supplies the wheel with a reduced stiffness. Due to the resilient material, the vibrations of the wheel tyre are isolated from the wheel body, which generally results in reduced dynamic forces. Hence the vibration excitation of both wheel and rail are reduced. The resilient material not only isolates vibrations, it also provides damping for vibrations with relative displacements of tyre and hub. Resilient wheels were invented 1899 and are nowadays widely used for tramway systems. For standard (heavy) railways only a small number of applications are known.

A large number of resilient wheel designs are available, for example the VSG Bochum54 (Germany) and the SAB resilient wheel (Sweden). Recently new resilient wheels are introduced by Valdunes (France) and GHH Radsatz (Germany). Thrane [Thrane, 1988] has reported a significant reduction on a Danish IC prototype of about 5 dB(A). According to the reviews by Reybardy and Zach, and Kurzweil and Wittig the reduction obtained by resilient wheels varies from 0 to 10 dB(A). In case of squeal noise, reductions up to 13 dB(A) have been found.

![Resilient VSG wheel (Bochum54)](image1.png)

Figure 1.1 Resilient VSG wheel (Bochum54).

1.2.2 Damped wheels

The vibrational response of train wheels to the wheel/rail interaction forces can be reduced by applying damping layers to the wheels or by increasing the internal damping of a wheel. Damping will convert the vibration energy into heat by internal friction processes, hence the vibration levels will be reduced. Interaction of wheel and rail also reduces the vibration levels of the wheel. The rail distributes the vibration energy along the track where, again, damping effects occur and energy is lost. It is believed that a considerable amount of wheel damping will be needed before the damping due to the wheel/rail contact can be exceeded. Only then damping will be effective for reduction for the vibration and noise levels.
Three types of damped wheels applied with success, are ring damped wheels, tuned absorber wheels and web damped wheels. For ring damped wheels, a steel ring is mounted in a notch within the rim. Friction between ring and rim provides the necessary damping. The damping is optimised by adjusting the pressure between ring and rim. The damping treatment of a tuned absorber wheel consists of elements of layered damping material mounted on the wheel rim. The strips enable the wheel vibration energy transported to the "resonance absorbers" to be dissipated as heat. By appropriately tuning the dampers, either the axial or the radial wheel modes can be damped. In this way optimal noise reduction for squeal, respectively rolling noise, can be achieved. Figure 1.2 shows an example of tuned absorber damped wheel.

Web damped wheels have a layer treatment applied to the web. Unlike the tuned absorber dampers, a single web damping layer supplies broad band damping. Two layer configurations are used: an unconstrained layer or a constrained (multi-) layer design (figure 2.1).

![Diagram of tuned absorber wheel and constrained layer web damped wheel](image)

A: Mass fitted to the ring  
B: Absorption device  
A: Synthetic material  
B: Covering steel plate

Figure 1.2 A tuned absorber wheel and a constrained layer web damped wheel.

### 1.2.3 Vibration damped and shielded wheels

Another solution to reduce the noise level of wheels is used for the MAN dampers. More noise reducing principles are used in this damped wheel design (figure 1.3). Wheel covers are supplied on both sides of the wheel web. These covers consist of different plate segments separated by a rubber material. The vibration energy of the wheel is partly transported to the cover plates that will also start to vibrate. The rubber material damps the relative displacement of the plates. On the other hand the covers also shield the radiated sound by the wheel web. Modal tests and calculations with a numerical model (TWINS) have shown that reduction of 5 to 8 dB(A) in the wheel noise can be achieved [Thompson et al., 1993]. The TWINS simulation program is discussed in the next section.
Other solutions to reduce the noise produced by wheels are spoked wheels, resiliently treaded wheels and special material tyred wheels. These solutions are rarely used and beyond the scope of this report.

1.3 TWINS: Numerical model for sound radiation of wheel and rail

For the European Railway Research Institute (ERRI) a theoretical model has been developed to predict wheel/rail rolling noise. This numerical model consists of a number of program modules, which are linked in the computer package TWINS (figure 1.4). The program is based on a first model proposed by Remington [Remington, 1987] and subsequently developed by Thompson [Thompson,1994].

The TWINS program models the interaction of the wheel/rail contact in the frequency domain. The input modules for the program contains measured values of wheel and rail roughness. The combined roughness spectrum is used to calculate the contact forces on wheel and rail. A matrix of elastic springs describes the contact area of wheel and rail, taking into account the roughness as well as the geometry. The wheel/rail interaction is coupled for three up to six degrees of freedom in the contact point. However, the vertical and lateral (axial) directions are most important. The lateral responses of wheel and rail depend strongly on their cross receptances, the ratio of lateral deflections and vertical forces. Cross receptances are more difficult to predict than the point receptances, that is the ratio of forces and displacements in the same direction.

Within TWINS the train wheel is modelled by receptances. These receptances can be predicted from a modal basis, the modal parameters. Modal parameters are the eigenfrequencies (system poles), the mode shapes of vibration (modal vectors) and the modal masses. Finite element models can supply these modal parameters. These models are validated by experimental modal analysis tests. Additionally these tests provide the necessary damping parameters (modal damping values). The calculation of the wheel response uses the same modal basis, and is performed at discrete frequencies. Around the eigenfrequencies of
the wheel the calculations are performed using small frequency steps. TWINS also takes into account the effect of wheel rotation [Thompson, 1993].

For the track dynamics three complementary models are available. The first is a Timoshenko beam, mounted on a two layer continuous support. The second is the same beam on a periodic support, with significant effects at the pinned/pinned frequencies related to the sleeper separation. The third track model uses a continuum model based finite element model, and includes deformation of the rail cross section. But this model again assumes a continuously supported track. For each model, the response to the wheel/rail interaction is calculated separately for each near field and propagating wave. The attenuation of the propagating wave types is an important parameter for prediction of the vibration and radiated noise.

![Diagram of the TWINS model](image)

Figure 1.4 Schematic view of the TWINS model.

The noise radiated by the wheel is calculated as sound power using simple equations for radiation efficiencies for different types of modes. Components for axial and radial radiated noise are calculated separately and summed as uncorrelated sources. This simplified theoretical model, proposed by Thompson [Thompson, 1991], is based on a flat vibrating surface approximation, having constant (average) axial displacements defined for up to six rings.

The radiation of the track is based on a model using distributed fictitious monopole and dipole sources [Heckl and Petit, 1991]. The source strengths are determined by a least squares fit to the acoustic potential of the vibrating surface of the rail. For each wave the
radiated noise is calculated separately and the total sound power is determined by summation. For the sound radiation of the sleepers the same source model is used. The radiated powers are calculated in third octave band frequency spectra. Subsequently the generated noise levels of wheel, rail and sleeper are summed. The last part of the program calculates the sound pressure at any given receiver position.

1.4 Problem definition

Very little is known about the sound radiation of - more complex - low noise train wheels. For resilient wheels no numerical models are known. Test results have shown resilient wheels to be very effective in reducing rolling noise, but the large number of parameters make it hard to compare experimental results.

This research project is aimed at the achievement of a numerical model that describes the sound radiation of a resilient wheel. The vibration behaviour as well as the sound radiation will have to be included, in this simulation model. One existing type of resilient wheel will be selected.

A validated numerical model for a resilient wheel can be used to gain a better understanding of the vibration and sound radiation behaviour of these wheels. Comparison to other wheels and structural optimisation of this type of resilient wheel will also be possible.

1.5 Approach to be adopted

To determine the modal behaviour of the resilient wheels and obtain a modal basis, a finite element model (FE) will be build. The resilient elements will have to be included in this model. Special attention is required for a good description of these elements. The dynamical behaviour of rubber resilient elements will be complex, and might be frequency dependent or even non-linear. Therefore problems might arise using the (principally linear) modal analysis to determine the dynamical behaviour of the wheel. The finite element model will have to be validated by experimental tests. TWINS will be used to simulate the radiation of this selected wheel. Since TWINS is not validated for resilient wheels, experimental tests will be necessary to verify the numerical results. In these tests the radiated sound powers will have to be measured, with known vibration levels of the wheel.

The research program can be divided in a structural vibration and a sound radiation part. Both parts will contain numerical as well as experimental tests. Figure 1.5 shows a schematic view of the adopted approach. A resilient VSG wheel (figure 1.1) is the subject of this research program. In this report, this wheel is referred to as the VSG wheel. Due to an agreement on confidentiality with the manufacturer, detailed information on the specific wheel type will not be presented in this report.

The vibration analysis will start with a numerical analysis based on a finite element (FE) model, using the software package ANSYS. This FE model will describe the solid steel wheel body and tyre like a solid structure. These structures will be modelled using the same elements and solution method, as used for FE models of conventional train wheels. These models have shown that eigenfrequencies of conventional train wheels can be estimated within 5% [van Haaren, 1997]. How the resilient elements will be represented will have to be determined. Anyway the properties of the resilient elements will have to be established. Most appropriate is an experimental test, by which only the (dynamical) properties of interest are measured. These measured dynamical properties of the resilient element will then be used in
the FE model. Modal analysis of the FE model will supply eigenfrequencies and mode shapes. These modal parameters will be validated by an experimental modal analysis. The experimental analysis will also supply modal damping values.

The experimental modal analysis will be performed for a single wheel on an axle. Effects of axle load and rotation will not be included in these measurements. For conventional train wheels this approach has been validated. Whether this holds for resilient wheels, can be verified in a successive analysis applying variations in the rotationally symmetric model. Theoretical and experimental modal parameters supply a modal basis, describing the dynamical behaviour of the resilient wheel. This basis forms the input for TWINS.

Chapter 2 will describe the measurement method to determine the dynamical properties of the resilient elements, as well as the measurement results. Descriptions of both the FE-model and the experimental modal analysis will be reported in Chapter 3.

For the radiation simulation, TWINS will be used. Input of the modal basis for the wheel and a standard rail model and roughness values, results in a wheel response. This response consists of vibration levels which form the input for the radiation calculation. Where necessary additional radiation calculations are performed, using boundary element calculations or modified TWINS radiation equations.

Again the numerical models are validated by experiments. In a laboratory experiment, a single wheel will be excited by a shaker and the radiated noise levels will be measured. Results in narrow band spectra will supply enough information to determine sound power level per mode shape. Although TWINS supplies results in third octave bands, internally in TWINS, sound radiation factors are calculated per mode. These internal TWINS results will be used to compare the theoretical model and the experimental results.

Chapter 4 describes the theoretical model for the radiation as well as the measurement method for the sound power levels. In chapter 5, results of both vibration and radiation parts are evaluated and discussed. This chapter includes results of TWINS simulations for a resilient wheel as well as a conventional train wheel. Finally, in chapter 6, conclusions are drawn and recommendations for further research are conceived.
2 Dynamical properties of resilient elements

2.1 Introduction

Resilient elements of the VSG wheel consist of elastomeric material. In literature these materials are also referred to as visco-elastic materials. Due to the elastomeric material characteristics, these elements result in a more complex dynamical behaviour of the wheel. A material is visco-elastic if both time and temperature have a strong effect on the material representation and the material has an elastic (recoverable) part as well as a viscous (non-recoverable) part.

Visco-elastic damping can be considered to be associated with hysteresis loop effects [Nashif et al, 1985]. Contrary to velocity dependent viscous damping, hysteretic damping is displacement dependent. In this approach $E$, the modulus of elasticity, is replaced by a complex modulus $E^* = E(1+i\eta)$, with loss factor $\eta$ representing the damping. This leads to a differential equation of the following type:

$$m \frac{\partial^2 u}{\partial t^2} + k(1+i\eta)u = F(t)$$

(2.1)

The term $k^* = k(1+i\eta)$ represents the complex stiffness coefficient. Standard viscously damped systems show the resonance frequency, under forced vibration, to be dependent on the damping coefficient. While the resonance displacement amplitude depends on all equation parameters, hysteretic damped systems have resonance frequencies independent of the damping coefficient and the resonance displacement amplitude is independent of mass. In equation (2.1) the damping is linearly related to the displacement amplitude. Under the assumption of a harmonic solution, the equation becomes a velocity term with a non-constant coefficient, due to the complex nature of the stiffness parameter. The resulting equation is still linear.

Dynamical stiffness coefficients of the resilient elements are determined by experiments. Values for these properties depend on preload, frequency and temperature. Therefore measurements are performed under the mean preload, at a number of different frequencies. Temperature effects are beyond the scope of this report. All tests are carried out at about 20°C. Tests are based on the measurement method described in the (draft) international standard ISO 10846 [ISO, 1996]. Figure 2.1 shows one resilient element, the isolator, under test. By exciting the mass on top at a certain frequency and measuring the dynamical forces and displacements at both sides, the complex dynamical transmission coefficients are measured. Verheij extensively describes this test method [Verheij, 1980; 1982].

![Figure 2.1 Schematic diagram of an isolation element under test.](image-url)
In case of a vibration isolator, observed in only one direction, the relations between forces and displacements in the contact points can be expressed as:

\[
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} =
\begin{bmatrix}
k_{1,1} & k_{1,2} \\
k_{2,1} & k_{2,2}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

\(k_{ij}\) is a complex stiffness; the real part represents the stiffness and inertia effects and the complex part represents the damping. \(k_{1,1}\) and \(k_{2,2}\) are the blocked driving point stiffnesses, for output displacement \(u_2=0\) and input displacement \(u_1=0\). \(k_{1,2}\) and \(k_{2,1}\) are blocked transfer stiffnesses, the ratio of the force on the blocked side and the displacement on the opposite side and vice versa. Symmetrically shaped vibration isolators have equal \(k_{1,1}\) and \(k_{2,2}\) coefficients. For a passive isolator such as a resilient element, reciprocity holds. Hence, the off-diagonal transfer stiffness coefficients have the same value. At low frequencies only elastic and dissipative forces act, and all \(k_{ij}\) will be equal. However at high frequencies, inertia effects within the isolator result in different values for the diagonal and off-diagonal terms. Forces and displacements are measured in a experimental set-up, based on ISO standard 10846.

### 2.2 Measurement methods

Three different methods are used to measure the frequency dependent dynamic transfer stiffness \(k_{1,2}\). The three methods are complementary and therefore they are all described in ISO 10846. The Direct Method requires the measurement of the input acceleration \(a_1\) and the blocking output force \(F_2\). Typically this method is used for frequencies up to 500 Hz. For very low frequencies where the driving point stiffness \(k_{1,1}\) and the transfer stiffness \(k_{1,2}\) are equal, the forces and displacements can be measured at the same (driven) side of the vibration isolator, by blocking the output side of the isolator. This method is called the Driving Point Method.

Using the Indirect Method, the vibration transmissibility is measured. In order to measure the blocking output force, the isolator is terminated with a mass element which provides a large dynamic stiffness. The Indirect Method can be used over a wide frequency range, where the upper limit is determined by limitations in the rigid body behaviour of the termination mass.

For the resilient elements within the scope of this report, the Indirect Method is most appropriate. Figure 2.1 shows a diagram for the measurement set-up with the Indirect Method. Vibration levels at both sides of the element under test are measured using the accelerometers \(a_1\) and \(a_2\). For the dynamic transfer stiffness the ratios of force on one side over the displacement at the other side will have to be determined. The force \(F_2\) can be derived by using Newton’s second law:

\[
F_2 = m_2 \ddot{a}_2 = -\omega^2 m_2 u_2
\]

The tilde indicates the primarily measured variable.

In the indirect method \(u_2 \ll u_1\), therefore \(F_2\) is approximated by:

\[
F_2 \approx k_{2,1} u_1
\]
This assumption forms the background of the Indirect Measurement Method. Measurement of the blocked transfer stiffness, for a resilient element under static preload is easier than measurement of the complete stiffness matrix. In fact this is a simple way to determine the representative characteristics for a vibration isolator.

The acceleration transmissibility $T(\omega)$, the ratio of $a_2$ over $a_1$, can be measured by taking the signals of the accelerometers at the input and output of the isolator. Hence, by using (2.3) and (2.4), the dynamic transfer stiffness can be calculated as a function of frequency:

$$k_{2,1}(\omega) = -m_2 \omega^2 \frac{\ddot{a}_2}{\ddot{a}_1} = -m_2 \omega^2 T(\omega)$$  \hspace{1cm} (2.5)

In the frequency range where inertia forces can be neglected, the loss factor can be estimated from the real and imaginary part of the acceleration transmissibility:

$$\eta = \tan(\delta(\omega)) = \frac{\text{Im}(T(\omega))}{\text{Re}(T(\omega))}$$  \hspace{1cm} (2.6)

For this measurement method it is assumed that the isolator under test is the only transfer path between source and receiver. In practice there will be a parallel transmission through the frame of the test set-up, so called flanking transmission. This flanking will have to be minimised, the vibration level of the test frame should be at least 20 dB less, than the level measured at the output mass.

Gaul et al. presented a description for the dynamic transfer behaviour of elastomer mounts in multibody systems [Gaul et al., 1993]. The distributed inertia, restoring and material damping forces of the elastomeric material under static and superimposed dynamic loads are described by a continuum approach. The transfer behaviour is expressed by a force displacement relation based on boundary integral equations representing the field equations of the viscoelastic elastomer domain and boundary conditions. Analogue to equation (2.2), the degrees of freedom for boundary faces I and II are linked by the dynamic stiffness matrix relation:

$$\begin{bmatrix} \mathbf{F}_M^I \\ \mathbf{F}_M^II \end{bmatrix} = \begin{bmatrix} \mathbf{K}_M^{II,II} & \mathbf{K}_M^{II,I} \\ \mathbf{K}_M^{I,II} & \mathbf{K}_M^{II,II} \end{bmatrix} \begin{bmatrix} \mathbf{u}_M^I \\ \mathbf{u}_M^II \end{bmatrix}$$  \hspace{1cm} (2.7)

With the forces and moments partitioned for interface I and II on the left hand side in vector $\mathbf{F}_M$. On the right hand side the corresponding displacements and rotations are grouped in vector $\mathbf{u}_M$ which is multiplied by the stiffness matrix $\mathbf{K}_M$. Making use of reciprocity results in the transfer stiffness matrix $\mathbf{K}_M$ being equal to the transpose of the transfer stiffness matrix $\mathbf{K}_M$. $\mathbf{K}_M^{II}$ and $\mathbf{K}_M^{II}$ are the driven point stiffness matrices at interface I and II.

Equation (2.7) is in fact a generalisation of (2.2), relating all six degrees of freedom and loads in six directions at both sides to each other. The symmetric complex dynamic stiffness matrix $\mathbf{K}_M (12 \times 12)$ can be written as:

$$\mathbf{K}_M = \mathbf{K}_{(\text{Re})} + i \mathbf{K}_{(\text{Im})} = C(\omega) + i \omega . D(\omega)$$  \hspace{1cm} (2.8)
C and D contain frequency dependent elements which represent the lumped parameters of the elastomer mount: elements of spring coefficient matrix C describes the influence of restoring and inertial forces; elements of damping coefficient matrix D describe the energy dissipation. These lumped parameters not only depend on the properties of the viscoelastic material, they also depend on the static preload and the geometry of the resilient element. The entries $k_{i,i}(\omega) = c_{i,i}(\omega) + i \cdot \omega \cdot d_{i,i}(\omega)$ of the stiffness matrix can directly be evaluated from measured data, when, for example, longitudinal motion is excited.

By using the indirect measuring method for longitudinal motion, and including an load sensor measuring the driving force, the measurement results can be used to derive the complex stiffness coefficients in this direction by rewriting (2.2) as:

$$\begin{bmatrix} F_1 \\ -m_2 \bar{a}_2 \end{bmatrix} = \frac{1}{\omega^2} \begin{bmatrix} k_{1,1} & k_{1,2} \\ k_{2,1} & k_{2,2} \end{bmatrix} \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}$$  \hspace{1cm} (2.9)

Where parameters with a tilde represent the measured variables. By using $k_{1,1} = k_{2,2}$ and $k_{1,2} = k_{2,1}$, the complex stiffness coefficients can be derived from the measured variables by:

$$k_{1,1}(\omega) = \frac{m_2 \omega^2}{1 - \left( \frac{\bar{a}_2}{\bar{a}_1} \right)^2 \left( \frac{\bar{a}_2}{\bar{a}_1} \right)^2 + \frac{\bar{F}_1}{m_2 \bar{a}_1}}$$  \hspace{1cm} (2.10)

$$k_{1,2}(\omega) = \frac{-m_2 \omega^2}{1 - \left( \frac{\bar{a}_2}{\bar{a}_1} \right)^2 \left( \frac{\bar{a}_2}{\bar{a}_1} \right)^2 + \frac{\bar{F}_1}{m_2 \bar{a}_1}}$$  \hspace{1cm} (2.11)

In order to determine the complex stiffness coefficients for other directions similar measurements in these directions will be necessary. The measured variables for dynamical stiffness $k$ can be used to relate forces on and displacements of the elastomer element boundaries in all vibration directions: 3 displacements and 3 rotations in both the source and receiver side. Hence the full matrix relates the 12 elements in the force vector to the 12 elements in the displacement vector. Therefore, a stiffness matrix of size $12 \times 12$ will have to be determined. However, due to symmetry of the elements a large number of matrix coefficients are zero [Verheij, 1982].

![Figure 2.2 The resilient element with the defined directions of excitation.](image-url)
The resilient elements in the VSG wheel is a V-shaped body (Figure 2.2), having symmetry planes in two orthogonal directions, normal to the axial and the tangential direction. Therefore the driving point stiffness matrix $K_M^{I}$ contains only 10 non-zero elements:

$$F_M^I = K_M^{I} \cdot U_M^I$$

$$(2.12)$$

$$\begin{bmatrix}
F_x^I \\
F_y^I \\
F_z^I \\
M_x^I \\
M_y^I \\
M_z^I
\end{bmatrix} =
\begin{bmatrix}
k_{1,1} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{2,2} & 0 & 0 & k_{2,6} & u_x^I \\
0 & 0 & k_{3,3} & 0 & k_{3,5} & u_y^I \\
0 & 0 & 0 & k_{4,4} & 0 & u_z^I \\
0 & 0 & k_{5,3} & 0 & k_{5,5} & \gamma_x^I \\
0 & 0 & k_{6,2} & 0 & 0 & k_{6,6} & \gamma_z^I
\end{bmatrix}
\begin{bmatrix}
u_x^I \\
u_y^I \\
u_z^I \\
\gamma_x^I \\
\gamma_y^I \\
\gamma_z^I
\end{bmatrix}$$

$$(2.13)$$

Where $x$, $y$ and $z$ refer to the radial, tangential and axial direction. In radial direction, due the V-shape of the resilient element, the normal plane is not a symmetry plane. Therefore an additional test is required to verify whether the symmetry assumption of the isolator element, and hence equal $K_M^{II}$ and $K_M^{RR}$, is valid.

For the transfer stiffness matrix a similar explanation holds:

$$F_m^I = K_m^{I} \cdot U_M^{II}$$

$$(2.14)$$

$$\begin{bmatrix}
F_x^I \\
F_y^I \\
F_z^I \\
M_x^I \\
M_y^I \\
M_z^I
\end{bmatrix} =
\begin{bmatrix}
k_{1,7} & 0 & 0 & 0 & 0 & 0 \\
0 & k_{3,8} & 0 & 0 & k_{2,12} & u_y^I \\
0 & 0 & k_{3,9} & 0 & k_{3,11} & u_z^I \\
0 & 0 & 0 & k_{4,10} & 0 & \gamma_x^I \\
0 & 0 & k_{5,9} & 0 & k_{5,11} & \gamma_y^I \\
0 & 0 & k_{6,8} & 0 & 0 & k_{6,12} & \gamma_z^I
\end{bmatrix}
\begin{bmatrix}
u_x^I \\
u_y^I \\
u_z^I \\
\gamma_x^I \\
\gamma_y^I \\
\gamma_z^I
\end{bmatrix}$$

$$(2.15)$$

Translational stiffness coefficients will result from the dynamical measurements. The rotational coefficients can be derived from the translational coefficients, by using the dimensions of the resilient elements and assuming linear stiffness characteristics. Integration of compression forces provides the bending moments $M_y$ and $M_z$, hence stiffness coefficients $k_{5,5}$ and $k_{6,6}$ can be determined. In a similar way the torsional moment $M_t$ and $k_{4,4}$ can be derived from integration of shear forces.

Off-diagonal terms in the partitioned transfer stiffness matrix, represent the cross-stiffnesses which relate translation and rotational components. These cross-terms can be deduced from the translational coefficients in a similar way as the rotational coefficients.

### 2.3 Experimental set-up

In order to measure the dynamical stiffness coefficients, an experimental test set-up has been constructed, using an existing MTS dynamic test bench as the basis. This test set-up is based
on the indirect ISO measurement method. An additional load sensor is used to measure driving point stiffnesses as well.

In figure 2.3 a schematic view of this test set-up is shown. In this test set-up the isolator can be tested under preload, which consists of a gravitational part by mass $m_1$ and an additional load by a hydraulic cylinder. The dynamic load of mass $m_2$ upon the test element is measured by a strain gauge load cell. Both loading masses are designed to be dynamically free, by the use of three rather soft vibration isolators at each side.

Figure 2.3 Schematic test set-up for measuring dynamic stiffness in normal direction, with an additional load cell.

Figure 2.4 Detailed test set-up for tests in radial direction.
The resilient element under test is clamped between two plates, which supply a similar closure as within the VSG wheel. This closure will affect the measured stiffness strongly. The components used for this set-up are listed in table 2.1. In figure 2.4 the realised set-up is displayed. The test set-up in figure 2.3 is aimed at measuring the normal dynamic stiffness. The input acceleration $a_i$ is obtained by exciting mass $m_1$ with a vibration exciter system. Accelerometer $a_i$ measures the acceleration above the load cell. Therefore the combined stiffness of load cell and element under test is measured. The stiffness of the load cell has been measured and is used to correct the measured stiffness values. The measured accelerance $a_i$ is used to estimate the accelerance on top of the resilient element.

$$a_{1,\text{cor}}(\omega) = \tilde{a}_i(\omega) + \frac{\omega^2 \bar{F}_1(\omega)}{k_\text{le}} \tag{2.16}$$

While the measured force $F_1$, is corrected with the dynamic force due the mass between load cell and element under test:

$$F_{1,\text{cor}}(\omega) = \bar{F}_1(\omega) - a_1(\omega) m_{\text{cor}} \tag{2.17}$$

Figure 2.4 Test set-up for tests in transverse direction.

For transverse stiffnesses a similar set-up can be used, where the element under test is excited in the transverse direction. A special adapter has been used for this purpose (figure
However, if resonance frequencies in the isolator element occur at high frequencies, extrapolating the measured stiffness will be valid. According to characteristic shear moduli presented by Snowdon [Snowdon, 1968], dynamic stiffness of filled rubbers increase by about 25% over 0.5-5 kHz. Assuming constant stiffness coefficients will supply a good estimate for the actual stiffness, in the frequency range up to half the first internal resonance frequency. The first resonance frequency in an isolator can be roughly be estimated by [Heckl, 1977]:

\[ f = \frac{1}{2} \sqrt{\frac{K_{d_m}}{m_s}} \]  

(2.18)

The measured dynamic stiffness of the resilient element is approximately 70e6 N/m. Taking the effective mass \( m_0 \) of the element 190 g, the first resonance frequency can be expected around 9.5 kHz. Up to half of this frequency, the driving point and the transfer stiffnesses will be equal, and only slight changes in dynamic stiffness are expected. Therefore, the measured stiffness values are expected to apply over the full frequency range of interest.

### 2.4 Measured dynamical stiffness values

Although frequencies up to 5 kHz are of interest, dynamic tests have been carried out up to 2.2 kHz. At higher frequencies vibration levels were not within the required ranges, therefore no tests have been performed above this frequency.

![Figure 2.6 Measured vibration and force levels, radial test.](image)

The shaker is used at discrete frequencies, typically in steps of 100 Hz. Broad band excitation was not possible, due to limited energy supply of the shaker. Only excitation at discrete frequencies results in significant vibration levels. However, above 2.2 kHz, no vibration levels can be measured that are well above the noise level. As can be expected, the acceleration \( a_2 \) of the blocking mass \( m_2 \) is limited. For all recorded levels, the level of signal \( a_2 \)
However, if resonance frequencies in the isolator element occur at high frequencies, extrapolating the measured stiffness will be valid. According to characteristic shear moduli presented by Snowdon [Snowdon, 1968], dynamic stiffness of filled rubbers increase by about 25% over 0.5-5 kHz. Assuming constant stiffness coefficients will supply a good estimate for the actual stiffness, in the frequency range up to half the first internal resonance frequency. The first resonance frequency in an isolator can be roughly be estimated by [Heckl, 1977]:

\[
f = \frac{1}{2} \sqrt{\frac{K_{\text{dyn}}}{m_s}}
\]

(2.18)

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![Graph showing measured vibration and force levels](image)

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The shaker is used at discrete frequencies, typically in steps of 100 Hz. Broad band excitation was not possible, due to limited energy supply of the shaker. Only excitation at discrete frequencies results in significant vibration levels. However, above 2.2 kHz, no vibration levels can be measured that are well above the noise level. As can be expected, the acceleration \( a_2 \) of the blocking mass \( m_2 \) is limited. For all recorded levels, the level of signal \( a_2 \)
is 20 dB above the vibration level of the MTS-frame, therefore the effect of flanking can be neglected. Figure 2.6 shows the recorded levels for a test in radial direction. These results are recorded at a preload of 23 kN, the resilient element is excited at the outer radius (tyre) side. The vibration level \(a_2\) decreases with frequency, due to the isolation behaviour of the resilient element.

![Figure 2.7 Isolation factor in radial direction.](image)

The difference in vibration levels \(a_1\) and \(a_2\) is an important characteristic of a vibration isolator. It determines the amount of isolation as function of frequency. Figure 2.7 displays the difference in acceleration level: \(\Delta a\), or isolation factor.

Around the resonance frequencies of the set-up, that is below 250 Hz, a negative isolation factor is found. This has nothing to do with the resilient element. Above 300 Hz, the isolation factor increases, indicating a stronger decoupling effect at higher frequencies. At 800 Hz the isolation factor is already 25 dB. At 1017 Hz the isolation factor might be affected by the resonance frequency in the load cell, however only a small ripple can be noticed. An increase of 12 dB per octave indicates a constant transfer stiffness. From the measured levels in figure 2.6, the dynamic stiffness is calculated as described in the former section.

In figure 2.8 the dynamic stiffnesses and loss factors are plotted as function of frequency. These stiffness values are determined according to equations (2.10) and (2.11). These stiffness coefficients and loss factors are measured in radial direction under a medium load of 23 kN. Around 750 Hz, magnitudes of \(k_{11}\) and \(k_{12}\) are equal: about 75 MN/m. Around this frequency, the loss factors change of sign. From a positive value of about 0.15, they become about -0.20. The negative value for \(k_{11}\) is maximal at 800 Hz, and tends to zero above 1 kHz. \(k_{12}\) decreases up to the maximal measured frequency. The jumps in the stiffness values are caused by changes in sign in the very small real part of this stiffness coefficient.

According to figure 2.8, the analysis results in different stiffness and loss ratio values for \(k_{11}\) and \(k_{12}\). Moreover the \(k_{11}\) shows to be strongly frequency dependent. These results are in conflict with the expectations. As stated in section 2.3, resonance frequencies in the resilient elements are expected around 9.5 kHz. Therefore, up to 2 kHz, no difference between \(k_{11}\)
and \( k_{1,2} \) is expected and stiffness values should be only slightly frequency dependent. The measurement results, including the load cell, should have confirmed this.

![Graphs showing stiffness and loss factor vs. frequency](image)

Figure 2.8 Measured stiffness magnitude and loss factor in radial direction. (driving point stiffness \( k_{1,1} \) in red, transfer stiffness \( k_{1,2} \) in blue)

In order to find an explanation of these results, the test set-up has been analysed. A dynamical model of all stiffness and mass elements in the set-up has been built. No explanation could be found until the load cell was taking into account. By separating this element in separate masses and taking into account its finite stiffness (table 2.2), the analysis leads to an additional resonance frequency at 1017 Hz. Hence, the force signal \( F_1 \) can not be used around or above this frequency. A second test without load cell could not easily be performed. Therefore, the measured signal for \( a_1 \) in the recorded data is corrected according to equation 2.16. Unfortunately, this limits the frequency range to maximal 800 Hz. In the succeeding part of this report only the dynamic stiffness values will be determined on the accelerometer signals only. The stiffness values are determined according to equation (2.5). This implies the assumption of \( k_{1,1} \) and \( k_{1,2} \) being equal is used.

A large number of tests has been performed. In the frequency range 300-800 Hz stiffness coefficients are determined for different loads, and in different directions: radial, axial and tangential. Test results are summarised in table 2.3. Stiffness coefficients are averaged over the frequency range of 300-800 Hz, using only acceleration signals. In this frequency range no significant frequency dependence of the stiffness coefficients could be traced.
Table 2.3 Frequency and sample averaged dynamic stiffness coefficients (MN/m).

<table>
<thead>
<tr>
<th>Direction</th>
<th>Load [kN]</th>
<th>Real part</th>
<th>Imag.part</th>
<th>Magnitude</th>
<th>Loss factor [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>radial</td>
<td>18</td>
<td>58.8</td>
<td>8.12</td>
<td>69.36</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>66.6</td>
<td>11.3</td>
<td>76.55</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>98.2</td>
<td>12.9</td>
<td>101.04</td>
<td>0.13</td>
</tr>
<tr>
<td>symmetry</td>
<td>20</td>
<td>62.8</td>
<td>8.76</td>
<td>71.41</td>
<td>0.14</td>
</tr>
<tr>
<td>axial</td>
<td>18</td>
<td>25.8</td>
<td>1.91</td>
<td>25.87</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>20.7</td>
<td>4.12</td>
<td>24.11</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>23.3</td>
<td>3.41</td>
<td>23.55</td>
<td>0.15</td>
</tr>
<tr>
<td>tangential</td>
<td>18</td>
<td>16.9</td>
<td>3.79</td>
<td>17.32</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>14.4</td>
<td>4.81</td>
<td>15.18</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>12.1</td>
<td>3.26</td>
<td>12.53</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Averages are determined from 8 to 10 measured values. Under some conditions the variation in measured stiffness values at a single preload is large. In case of the axial and tangential, 18 kN conditions, the standard deviation is about 80%. For higher loads this reduces to 25%. In radial direction the maximum standard deviation is 13%. These large differences are probably caused by variations in actual preload. Due to relaxation of the element under test, load and deflection combinations could not easily be reproduced. Therefore the actual loads might not be correct. On the other hand, relaxation causes some shift in deflection for tests over the frequency range.

Radial dynamic stiffness is strongly preload dependent. 10 kN (56%) rise in load results in 39.4 N/m (67%) increase in stiffness. Increasing the preload by 10 kN, results in a deflection of 1.0 mm. For vibration levels of the wheel, which are in the order of 1e-6 m, assuming the stiffness coefficients to be linear, will be valid.

The test under 20 kN preload, is a test on the symmetry condition assumption: $k_{1,1}$ and $k_{2,2}$ being equal. The resilient element is excited at the inner radius (web) side. The dynamical stiffness value for this test confirms the trend of 18 to 23 kN for the original radial test results. So the symmetry assumption for the resilient element is validated.

In axial direction the stiffness is not significantly dependent on the preload, while in tangential direction the stiffness seems to be inversely dependent on preload. On average the imaginary parts in these transverse directions is larger than in radial (compression) direction. Loss factors for the radial and axial direction are about the same, although the variation in axial direction is larger. Radial loss factors vary from 0.14 to 0.17, while axially values lie between 0.07 and 0.20. In tangential direction the maximum loss factors occur: 0.22 up to 0.33.

### 2.5 Discussion

Dynamic stiffness coefficients of the resilient elements are measured in three directions, in the frequency range up to 2 kHz. Only in the range of 300 up to 800 Hz, the results can be used. Dynamic analysis of the test set-up has shown that inclusion of the load cell results in a resonance frequency at 1017 Hz. Results based on data including the load cell signal, show to be distorted by resonances over a wide frequency range. The dynamic stiffness coefficients are calculated using only the acceleration signals. However, the upper frequency limit is still 800 Hz, because the acceleration $a_1$ is measured above the load cell. If the accelerometer would have been placed under the load cell, the acceleration signals could have been used up to 2 kHz.

In the calculations of the stiffness coefficients, corrections are made for the stiffness of the load cell. Measured stiffness coefficients show large variations. Due to relaxation of the resilient elements load deflection combination could not easily be reproduced. The nominal preload of the resilient elements is about 25 kN. In practice the preload follows from the...
nominal deflection. Actual preload depends on the material stiffness, which also varies per batch. Within the numerical model, stiffness values based on interpolation can be used (table 2.4).

<table>
<thead>
<tr>
<th>Direction</th>
<th>Magnitude</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial direction</td>
<td>70 MN/m</td>
<td>0.15</td>
</tr>
<tr>
<td>Axial direction</td>
<td>20 MN/m</td>
<td>0.18</td>
</tr>
<tr>
<td>Tangential direction</td>
<td>15 MN/m</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 2.4 Dynamic stiffness values to be used for numerical model (preload 25 kN)
3 Modal analysis of a resilient wheel

3.1 Introduction

Vibration characteristics of the resilient VSG wheel are analysed using a numerical model as well as an experimental modal analysis. For the numerical model, a finite element (FE) model has been built with ANSYS (revision 5.2). Special attention is required for the description of the resilient elements. For the selection of the element type, the description of the dynamical behaviour of the resilient elements plays a major role. Other visco-elastic material characteristics, like relaxation and temperature effects, will be out of the scope of this research. Only stiffness characteristics of the resilient elements will be included in the model.

Numerical modal analysis of the FE model will supply the modal parameters, such as mode shapes and eigenfrequencies. The model will be validated by an experimental modal analysis. This experimental analysis will also supply the modal damping factors.

3.2 Finite element model

ANSYS provides a matrix element (MATRIX27) relating two nodes, each having six degrees of freedom [ANSYS, 1995]. The matrix represents an arbitrary element whose dynamic response can be specified by stiffness, damping or mass coefficients. These elements are used in a solid model to describe the resilient elements. Stiffness matrices are used to specify the absolute values of the stiffness coefficient. Therefore each resilient element around the circumference (34 elements) is represented by one matrix element. Experimental stiffness coefficients are used directly for the definition of the matrix coefficients.

Solid brick elements (SOLID45) are employed to model the wheel tyre and body. These elements contain 8 nodes having only translation degrees of freedom. The use of extra (higher order) shape functions makes these elements well suited to the analysis of structural bending [ANSYS, 1995]. For conventional train wheels, the use of these elements has proven to provide results of eigenfrequencies, that are within 5% accurate over a wide frequency range. For high frequencies, the wavelengths decrease, resulting in too high frequency values. The number of elements over one wavelength should not be less than six.

Symmetry conditions in the wheel geometry allow the finite element model to be confined to only half a wheel. In the symmetry plane, symmetrical or anti-symmetrical boundary conditions can be specified. Additional boundary conditions are defined for the inner radius of the wheel body. Applying fixed degrees of freedom at this radius, excludes elasticity effects of the axle. Hence the modal analysis will provide modes of vibration for a rigid axle.

In the frequency range up to 5 kHz, the stiffness coefficients of the resilient elements are slightly frequency dependent. The matrix coefficients however represent constant stiffness values. Therefore frequency averaged mean stiffness values are used in the solution routines. Modal analysis is performed using the subspace method [ANSYS, 1995], using only real dynamic stiffness coefficients for the resilient elements. Modal damping ratios are determined experimentally.

The parameters of the MATRIX27 are defined, according to the matrix description in chapter 2. As a result of the measurement results in section 2.5, all sub matrices in the partitioned matrix in equation (2.7) are taken to be equal in magnitude. Due to different definition of degrees of freedom within ANSYS, the off diagonal terms have a negative sign. Therefore sub-matrices are defined as $K_{mm}^{uu} = -K_{mm}^{lu} = -K_{mm}^{lu} = K_{mm}^{uu}$. Only the diagonal stiffness coefficients in partitioned matrices are defined, however some computations are performed...
with the parameters including the off-diagonal terms. By using the selected SOLID45 elements, rotational stiffness coefficients are not active. Only translational degrees of freedom are coupled by these elements, rotational degrees of freedom are not described. In additional calculations, off-diagonal stiffness coefficients and rotational stiffness coefficients are included as well. For these calculations additional elements, with rotational degrees of freedom, are used to couple the rotational degrees of freedom of the MATRIX elements with the solid elements.

Stiffness coefficients are linearized around a preload of 25 kN (table 2.4). All resilient elements are assumed to have the same preload. Effects of axle loads and different preloads around the circumference are disregarded. The modal model will be validated by modal analysis of an unloaded wheel.

In figure 3.1 the finite element model is displayed. In case of an undamped situation, this model consists of 1088 SOLID45 elements and 17 MATRIX27 elements, having a total of 2275 nodes. The total mass for the model of half wheel is 195.14 kg. The mass of the resilient elements is not included due the massless representation of these elements.

Figure 3.1 Finite element model for the resilient VSG wheel.

Solving the undamped model results in 44 modes up to 5 kHz. One solution with symmetric boundary conditions for the symmetry plane provides all essential modes. The anti-symmetric boundary condition provides some additional, circumferential modes. Extraction of 44 frequencies and expanding the corresponding mode shapes requires a CPU
time of 1115 seconds (HP9000/712). The results of these solution routines are presented in section 3.4.

3.3 Experimental modal analysis

An experimental modal analysis of a single VSG wheel has been performed in a lab situation. The wheel, clamped on the axle, was excited by a an impact hammer, while the response of the wheel was measured by 5 accelerometers. The frequency response functions were acquired according to the moving hammer method. All instruments used for this test, are listed in table 3.1.

![Experimental modal analysis](image)

Figure 3.2 Placing of accelerometers and locations of excitation.

<table>
<thead>
<tr>
<th>Instruments:</th>
<th>type, specifications:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometers</td>
<td>B&amp;K 4395; sensitivity: 0.988 mV/m/s²</td>
</tr>
<tr>
<td>Acquisition system and analyser</td>
<td>CADA-PC (LMS): 8 channels front end</td>
</tr>
<tr>
<td>Excitation hammer</td>
<td>B&amp;K 8202; sensitivity: 0.96 pC/N</td>
</tr>
</tbody>
</table>

Table 3.1 Instruments used for the experimental modal analysis.
The excitation locations are chosen over a quarter of the wheel. Due to symmetry in the modal behaviour of the wheel, modes of vibration can be determined by the analysis of a quarter wheel only. Figure 3.2 shows the distribution of excitation locations and the positions of the accelerometers. The grid of 102 excitation locations is also visible in figure 3.3. This grid is used in the analysis software to analyse the recorded frequency response functions. Excitations are chosen on four lines along the tyre, and seven lines around the web. Two accelerometers on the web, measure the radial and axial vibrations components. On the tyre, radial, axial and torsional vibration components are measured by three accelerometers.

![Figure 3.3 Grid of excitation nodes on tyre and web.](image)

The frequency response functions are recorded and analysed by running the CADA-PC software. The modal parameters are estimated using a multi degree of freedom technique. This technique uses a time domain algorithm, called the Least Squares Complex Exponential method to identify the poles (frequency and damping values). In a second phase, the residues (mode shapes) are determined with the Least Squares Frequency Domain method.

The analysis is performed with a single reference method. Results of several analyses are evaluated. Each analysis is carried out using another accelerometer as a reference. Summed frequency response and mode indicator functions have been calculated to provide the necessary information to select the poles. For selected frequency bands, solution routines are performed and stable poles are selected. Subsequently the complex residues are extracted.

The results of the experimental modal analysis are presented and compared to the numerical results in section 3.4.

### 3.4 Results of the modal analysis

This type of resilient wheel appears to have two classes of modes. A distinction can be made between dominant tyre and dominant web modes. As will be seen, the resilient elements decouple both wheel and web vibration components, allowing each component to have its
own mode shapes. At lower frequencies some amount of coupling still exists; but at frequencies above 1500 Hz the effect of coupling can almost be neglected.

Additional solution routines are run to analyse the effect of the rotational stiffnesses and the off-diagonal terms in the stiffness matrices. No significant effect of including these stiffness coefficients could be found. All eigenfrequencies and mode shapes were identical. Moreover, the solution was not very sensitive to changes in the rotational stiffness coefficients. Increasing these coefficients by even a factor of 100, does not affect the solutions in eigenfrequencies significantly. Therefore it can be concluded that the resilient elements can be represented by taking only the translational stiffness coefficients $k_{1,1}$, $k_{2,2}$ and $k_{3,3}$.

Modes are characterised as being dominant tyre or dominant web mode. Additionally the modes are identified by the number of nodal diameters (nd) and nodal circles (nc). The number of nodal diameters in the modeshape is the number of sections around the circumference, having zero displacements. The vibration amplitude takes the form $A \cos(n \theta)$ or $A \sin(n \theta)$, with angular coordinate $\theta$. So the number of nodal diameters equals the number of waves around the circumference. For axial modes the modeshape shows circular section of zero displacement, referred to as the number of nodal circles. In figure 3.4, two typical modeshapes are displayed. The sign indicates the directions of vibration.

![Figure 3.4 Two typical mode shapes, having one nodal circle (left) and two nodal diameters.](image)

<table>
<thead>
<tr>
<th>Table 3.2 Numerical eigenfrequencies VSG wheel (ANSYS).</th>
<th>Numeral</th>
<th>Dominant tyre modes</th>
<th>Dominant web modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodal diameters (nd)</td>
<td>tyre radial</td>
<td>tyre circ.fer.</td>
<td>tyre axial</td>
</tr>
<tr>
<td>nd=0</td>
<td>1988</td>
<td>225.1</td>
<td>202.7</td>
</tr>
<tr>
<td>nd=1</td>
<td>434.3</td>
<td>2710</td>
<td>148.9</td>
</tr>
<tr>
<td>nd=2</td>
<td>563.8</td>
<td>4229</td>
<td>351.6</td>
</tr>
<tr>
<td>nd=3</td>
<td>832.7</td>
<td>977.6</td>
<td>3140</td>
</tr>
<tr>
<td>nd=4</td>
<td>1335</td>
<td>1771</td>
<td>3993</td>
</tr>
<tr>
<td>nd=5</td>
<td>2007</td>
<td>2881</td>
<td>4937</td>
</tr>
<tr>
<td>nd=6</td>
<td>2818</td>
<td>3661</td>
<td>4110</td>
</tr>
<tr>
<td>nd=7</td>
<td>3747</td>
<td>4686</td>
<td></td>
</tr>
<tr>
<td>nd=8</td>
<td>4778</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sound radiation simulation of a resilient train wheel/A - 29/58
Dominant tyre and web modes were identified in the experimental modal analysis. Table 3.3 lists the frequencies of 36 mode shapes found determined in the experimental modal analysis. Some modes having 0 or 1 nodal diameters could not be found, presumably due to the mounting of the wheel or high damping. Additional analysis of the measurement data did not supply any additional poles that could be linked to these modes. These poles are of special interest, because these modes might indicate whether the right stiffness coefficients are used. The resilient elements are expected to play a major role in these modes. To evaluate the analyses, numerical and experimental eigenfrequencies are compared.

Differences in simulation and experimental results are listed in table 3.4. Seventy-five percent of 34 eigenfrequencies are within 4% accurate. The finite element model approximates the eigenfrequencies of the tyre modes very well. Except for the radial tyre mode, having 2 nodal diameters, a difference of 11.3% is found. The higher resonance frequency for the numerical model might indicate that the resilient elements are modelled to stiff.

Larger differences are found for the web modes. The finite element model provides higher eigenfrequencies. The web being modelled as one solid body does not accurately describe its vibration behaviour. In fact the web consists of two parts, bolted together. An accurate description of such an interface, which includes friction, is beyond the scope of this report. Due to the solid body description, the finite element model overestimates eigenfrequencies of the web modes.

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Dominant tyre modes</th>
<th>Dominant web modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodal diameters (nd)</td>
<td>tyre radial</td>
<td>tyre cinc,fur.</td>
</tr>
<tr>
<td>nd=0</td>
<td>1970</td>
<td>1693</td>
</tr>
<tr>
<td>nd=1</td>
<td>2693</td>
<td>1890</td>
</tr>
<tr>
<td>nd=2</td>
<td>506.6</td>
<td>340.4</td>
</tr>
<tr>
<td>nd=3</td>
<td>823.4</td>
<td>963.3</td>
</tr>
<tr>
<td>nd=4</td>
<td>1340</td>
<td>1741</td>
</tr>
<tr>
<td>nd=5</td>
<td>2005</td>
<td>2653</td>
</tr>
<tr>
<td>nd=6</td>
<td>2777</td>
<td>3595</td>
</tr>
<tr>
<td>nd=7</td>
<td>3649</td>
<td>4566</td>
</tr>
<tr>
<td>nd=8</td>
<td>4591</td>
<td>5547</td>
</tr>
</tbody>
</table>

Table 3.3 Measured eigenfrequencies VSG wheel (Bochum, 11/10/1996).

<table>
<thead>
<tr>
<th>Difference</th>
<th>Dominant tyre modes</th>
<th>Dominant web modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodal diameters (nd)</td>
<td>tyre radial</td>
<td>tyre cinc,fur.</td>
</tr>
<tr>
<td>nd=0</td>
<td>0.91%</td>
<td>0.47%</td>
</tr>
<tr>
<td>nd=1</td>
<td>0.63%</td>
<td>0.05%</td>
</tr>
<tr>
<td>nd=2</td>
<td>11.29%</td>
<td>3.29%</td>
</tr>
<tr>
<td>nd=3</td>
<td>1.13%</td>
<td>1.48%</td>
</tr>
<tr>
<td>nd=4</td>
<td>-0.37%</td>
<td>1.27%</td>
</tr>
<tr>
<td>nd=5</td>
<td>0.10%</td>
<td>1.06%</td>
</tr>
<tr>
<td>nd=6</td>
<td>1.48%</td>
<td>1.84%</td>
</tr>
<tr>
<td>nd=7</td>
<td>2.69%</td>
<td>2.63%</td>
</tr>
<tr>
<td>nd=8</td>
<td>4.07%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4 Differences in numerical and measured frequencies.
Figure 3.5 Web mode in axial direction, having 2 nodal diameters (f.e. modal)

Figure 3.6 Web mode in axial direction, having 2 nodal diameters (experimental).
Figure 3.7 Tyre mode in radial direction, having 3 nodal diameters (f.e. modal)

Figure 3.8 Tyre mode in radial direction, having 3 nodal diameters (experimental).
Modal and response analysis of a resilient VSG wheel.

Figure 3.9 Tyre mode in axial direction, having 3 nodal diameters (f.e. modal)

Modal and response analysis of a resilient VSG wheel.

Figure 3.10 Tyre mode in axial direction, having 3 nodal diameters (experimental)
Environmental Issues

Figure 3.11 Tyre, rotational mode, having 4 nodal diameters (f.e. modal)

Figure 3.12 Tyre, rotational mode, having 4 nodal diameters (experimental)
In figure 3.5 to 3.12 mode shapes provided by numerical and experimental analysis methods are shown. In both cases mode shapes can be identified easily. These results show that the observed mode shapes indeed can be classified as dominant tyre or dominant web modes.

At higher frequencies there is less coupling of tyre and web, hence the modes of vibration at higher frequencies are not sensitive to variation in the stiffness of the resilient elements. The stiffness of the resilient elements can be neglected compared to the stiffness of tyre and web. To verify whether the resilient elements are simulated correctly, some special attention is required for lower frequency modes. The amount of coupling of tyre and wheel depends only on the resilient elements. In table 3.5, the amount of coupling is compared for two tyre modes, in the radial and axial direction.

<table>
<thead>
<tr>
<th>Mode shape</th>
<th>Type of analysis</th>
<th>Tyre amplitude [m]</th>
<th>Web amplitude [m]</th>
<th>Tyre/web ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial tyre mode</td>
<td>ANSYS model</td>
<td>0.1288</td>
<td>0.4918e-2</td>
<td>28.3</td>
</tr>
<tr>
<td>823.4 Hz, nd=3</td>
<td>Exp. modal anal.</td>
<td>0.2340</td>
<td>0.0280</td>
<td>8.42</td>
</tr>
<tr>
<td></td>
<td>Vibr. level meas.</td>
<td>6.717e-8</td>
<td>1.054e-8</td>
<td>6.38</td>
</tr>
<tr>
<td>Axial tyre mode</td>
<td>ANSYS model</td>
<td>0.1203</td>
<td>0.0259</td>
<td>4.60</td>
</tr>
<tr>
<td>963.3 Hz, nd=3</td>
<td>Exp. modal anal.</td>
<td>0.2090</td>
<td>0.1229</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>Vibr. level meas.</td>
<td>4.457e-8</td>
<td>1.321e-8</td>
<td>3.37</td>
</tr>
</tbody>
</table>

Table 3.5 Coupling of tyre and web vibrations for two modes.

Results of the finite element model are compared to experimental results: the experimental modal analysis as well as vibration measurements under excitation by a shaker. Absolute displacement amplitudes do not correspond due to different normalisation methods. ANSYS results are mass matrix normalised, while experimental modal analysis results are scaled to unity length. Only the vibration measurement results are absolute values. For comparison the dimensionless amplitudes ratios are used. Differences between the ANSYS model and the experimental and measured levels are 10 dB for the radial mode and 3 dB for the axial mode. These values indicate the numerical model to have too little coupling. In other words, this might indicate that the resilient element might be modelled too compliant.

Modal damping values are estimated by the pole estimation algorithm. Modal damping ratios are within the range of 0.1 to 3.5 %. The highest damping values occur at lower frequencies. The radial tyre mode with 2 nodal diameters has the highest damping. At higher frequencies damping ratios decrease. The amount of damping depends on the amount of deflection of the resilient elements, and thus on the modeshape. The damping ratios are significantly higher than damping ratios measured for conventional train wheels. For example the NS-ICM wheel shows damping ratios of 0.002 up to 0.2% [van Haaren, 1997].

3.5 Discussion.

Results of this modal analysis indicate that the resilient VSG wheel can be simulated as a linear dynamical system. Its vibrational behaviour can be decomposed into modes of vibration. Hence the dynamical behaviour can be represented by a modal basis.
However, care is required in comparing numerical and experimental modal analysis results. Although the resilient elements play an essential role in the transmission of vibration, their effect on the overall mode shapes is limited. Comparison of vibration levels show that large differences in the transmission of vibration are found. According to these figures, the numerical model contains underestimated stiffnesses for the resilient elements.

Comparison of the eigenfrequencies show the radial tyre mode with 2 nodal diameters, to be overestimated by the numerical model. This might indicate that the stiffness for the resilient elements in the numerical model is too high. The relation between these contradictory indications is not yet understood. The dynamic stiffness coefficients are measured according to the international standard. There is no reason to question the validity of the measured stiffness coefficients. Possibly that inclusion of the damping of the resilient elements in the numerical model, will improve the model. Damping will reduce the numerical resonance frequency of the radial tyre mode, while the local existence of the damping might reduce the difference of vibration levels of tyre and web.

In the finite element model only translational stiffness coefficients are defined. Additional solution routines, including rotational and off-diagonal stiffness coefficients, showed the effect of these coefficients to be negligible. Therefore it is concluded that the resilient elements can be represented by the translational stiffness coefficients only.

The results of these modal analyses is used to compose a modal basis, which can be used as an input for the TWINS program. This modal basis contains 42 modes up to 4927 Hz. In case no experimental modes could be identified, numerical eigenfrequencies are used. Corresponding damping values are given a default damping ratio of 1%.
4 Sound radiation of a resilient wheel

4.1 Introduction.

Modal parameters of the VSG wheel can be used as an input for the TWINS program (revision 2.3). However the wheel radiation module in TWINS is expected not to describe the radiation of a resilient wheel properly. Therefore TWINS calculation results will have to be validated. Boundary element simulations are performed to analyse the noise radiation and to supply a reference to the TWINS results. The boundary element model is based on the modal analysis results of the finite element model. Additionally experimental sound radiation tests are performed to research the radiation characteristics of the resilient wheel and validate both radiation models.

TWINS' sound radiation module and the boundary element analysis are described in section 4.2. In section 4.3, the sound intensity measurement method for the resilient wheel is described. Results of these measurements are used to determine the radiation characteristics of the resilient wheel. Sound radiation calculations for the resilient wheel as well as results of the intensity measurements are presented in section 4.4. In section 4.5, the TWINS simulation results for a resilient wheel are compared to those of a standard intercity wheel. The discussion in section 4.6 finishes this sound radiation chapter.

4.2 Calculation of the wheel sound radiation.

4.2.1 TWINS calculation module

Within TWINS, the normal velocity for six locations along a radial line over the wheel, as well as the contact point are calculated. In the radiation calculation, these velocities and additional information on the mode shapes are used to predict the sound power levels in third octave spectra. The calculation of the sound radiation by the train wheel is based on semi-empirical formulae. These formulae result in estimates for the sound power that are accurate to within 2 dB for average values (Appendix A).

Due to the more complex vibrational behaviour of the resilient wheel, these empirical formulae probably will not yield accurate results for the VSG wheel. Therefore the sound radiation module of the TWINS program will have to be validated for this case. Results of TWINS simulations consist of sound power levels in third octave bands. To compare radiated powers per mode shape, modal radiation efficiencies and modal radiated powers will be evaluated. How radiation efficiencies are determined, according to the TWINS module, is also described in Appendix A.

In order to evaluate the wheel radiation module of TWINS, a simulation model of the VSG wheel has been put together. The modal basis consists of 42 modes up to 5 kHz, including 6 rigid body modes. Measured eigenfrequencies and modal damping values of experimental determined modes are used, where possible. For modes that were not found in the experimental modal analysis (table 3.3), eigenfrequency results of the finite element solution are used (table 3.2). For these 'numerical' modes, a default damping ratio of 1% is used.

The dimensional parameters input for the TWINS calculation is at some points more or less arbitrary. Normally, these parameters refer to conventional train wheels. For the inner and outer diameter of the tyre, the dimensions of the structurally isolated tyre are used. This is not straightforward, because typically these dimensions refer to the total non web outer part of
the wheel. Because the VSG wheel has a rim, one might argue that the inner rim diameter should be taken instead. The width of web and tyre are simply taken as they are. However, again this is questionable, but is believed to be the best guess. Used dimensions are listed in Appendix B.

According to the TWINS calculation method, modal radiation efficiencies and radiated powers are determined. These calculations are based on the modal basis and the parametrical input according to Appendix C. They are carried out outside of TWINS. Results are compared with boundary element model results and experimental values in section 4.5.

### 4.2.2 Boundary element model

The sound radiation of the vibrating resilient wheel is calculated with SYSNOISE. This model is based on the finite element model as used for the numerical modal analysis in chapter 3. The geometry as well as the modal analysis results are used to calculate the radiated power for each modeshape [Lier, 1996].

The volume mesh from the finite element model is converted into a surface mesh for the boundary element model. The finite element faces of the wheel surface are taken as boundary elements. The radiation simulation is performed using the Direct Boundary Element Method [SYSNOISE, 1996]. For each mode shape, the modal displacements of the finite element result file are taken as nodal velocities at the boundary elements. The boundary element model consists of 2255 nodes and 1858 elements. Again symmetry conditions are defined at the symmetry plane of the wheel.

Due to the narrow slit between the tyre and the web, the exterior surface is not closed. In the boundary element model, the slit is filled up with additional elements. Velocities for the additional areas are interpolated from tyre and web velocities. Local effects of the slit are therefore not included. Resonances in the interior volumes between tyre and web, might affect the overall sound radiation. These effects will not be included in the model. Geerts has performed additional radiation calculations including the slit in the SYSNOISE model as well as by using an alternative Fourier based radiation program BARD [Geerts, 1997].

For all 42 eigenfrequencies of the finite element solution, radiation calculations are carried out. The calculation routine consists of producing the boundary element mesh, definition of radiation parameters, generation of velocity boundary conditions and performing the matrix inversion. Postprocessing supplies the necessary results, as radiated sound power, radiation efficiencies and radiation directivity. In this analysis only modal sound power and radiation efficiencies are considered. Results are presented in section 4.4.

### 4.3 Measurement method of wheel sound radiation

Radiated sound power levels of the VSG wheel are measured in a laboratory experiment. The wheel is excited by a shaker, while the radiated sound is measured with an intensity probe. The experiment is performed according to the ISO standard for sound intensity measurements [ISO 9614/1, 1995]. In table 4.1, the instruments for this test are listed.

By using white noise input for the shaker, vibration levels of the VSG wheel are assumed to be stationary. During the test, radial and axial vibration levels for tyre and web are recorded. To verify whether the vibration levels are stationary, levels are recorded at the end of every measurement cycle.
Sound intensity is measured on half a sphere around the wheel. The sphere has a radius of 0.8 m. With the outer radius of the wheel of 0.455 m, the minimum distance between radiating surface and measurement probe is 0.345 m. The experimental set-up is shown in figure 4.1.

<table>
<thead>
<tr>
<th>Instruments:</th>
<th>type, specifications:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity sensor</td>
<td>B&amp;K 3547 with B&amp;K 4181 microphones</td>
</tr>
<tr>
<td>Acquisition system and front end</td>
<td>HP 35650/35652A/35651B</td>
</tr>
<tr>
<td>Acoustic analyser</td>
<td>HP382 workstation, CADA-X acoustic intensity module</td>
</tr>
<tr>
<td>Accelerometers</td>
<td>B&amp;K 4395; sensitivity: 0.988 mV/m/s²</td>
</tr>
<tr>
<td>Acquisition system and analyser</td>
<td>CADA-PC: 8 channels front end</td>
</tr>
<tr>
<td>Shaker</td>
<td>B&amp;K 4802/4817</td>
</tr>
</tbody>
</table>

Table 4.1 Instruments used for measuring the acoustic intensity.

The intensity levels are measured at a total of 48 measurement points at the sphere. The measurement points are chosen at every 30° around the circumference in X-Y plane and along four circles, at intermediate angles of 22.5°.

Figure 4.1 Side view of test set-up for sound intensity measurement.

Acoustic intensity measures the amount of sound energy through a unit area, in normal direction. The acoustic intensity is the time averaged product of the instantaneous acoustic pressure and the instantaneous particle velocity. Radiated powers are calculated by taking the sum of the products of measured intensity [W/m²] and the corresponding areas [m²]. This measurement method has the advantage that radiated sound powers can be measured in situ.

The accuracy of this acoustic measurement depends on the type of microphones, the spacing and the measured sound levels. With the used microphones and a spacer of 12 mm, the precision of the measurement is within 0.5 dB. That is, if the ISO indicators (Appendix C) are within specified values [ISO 9614/1, 1995]. The actual values, as well as the ISO limits, for these indicators are listed in table 4.2. These indicators are a result of the analysis with the CADA-X system. Indicator F1, which is related to whether the results are stationary, is not available. Table 4.2 includes indicators over the entire range as well as indicators around two modes of vibration at 823.4 Hz and 963.4 Hz.

Indicator F2 is used to verify whether the sound intensity level is high enough relative to the sound pressure level and is above the residual intensity. In this calculation the residual...
intensity index of the measurement probe: 16.8 dB, is used. The recorded sound intensity levels appears to be well above the required levels.

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>Field indicators</th>
<th>ISO9614 limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F2</td>
<td>F3</td>
</tr>
<tr>
<td>200-5000 Hz</td>
<td>2.807</td>
<td>2.807</td>
</tr>
<tr>
<td>750-900 Hz</td>
<td>1.503</td>
<td>1.512</td>
</tr>
<tr>
<td>930-1000 Hz</td>
<td>0.996</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Table 4.2 Field indicators of the intensity measurement.

Differences of F3-F2 above 3 dB indicate negative intensities, which occurs when noise enters the sphere. This is not the case for this measurement. The last check is related to the number of measurement points. If the number of points is smaller than C*F4^2, variations between points can be reduced by increasing the number of measurement points. Strong directivity of the source might affect the calculated sound power. For the second mode, this value is slightly higher than the 48 measurement points. However, this effect is expected to be small. Details upon the ISO indicators can be found in Appendix C.

![Measuring surface diagram](image)

Figure 4.2 Top view of test set-up with orientation of co-ordinate system in the measurement surface.

Figure 4.2 shows the orientation of the co-ordinate system in the measurement surface, which is 15 degrees rotated in the X-Y plane. Results are presented in section 4.4. Plots of the directivity of the sound radiation will be provided, having the same co-ordinate system as in figure 4.2.

Vibration levels of the tyre and web are measured in both radial and axial direction. Results of these measured vibration levels will be used to determine the radiation efficiencies of the VSG wheel. By taking small bands out of the measured sound power levels, around eigenfrequencies, modal radiation efficiencies will be determined.

### 4.4 Validation of the sound radiation module in TWINS

In order to validate the sound radiation calculation in TWINS, TWINS results are compared with results of the boundary element analysis and the intensity measurements as described in section 4.2. Figure 4.3 shows the sound power level, for the wheel excited in radial direction. It is clear that the sound radiation is dominated by modal radiation. Peaks in this power spectrum correspond to the eigenfrequencies as described in chapter 3.
To evaluate the sound radiation, radiation efficiencies are determined. For a limited number of strong modal peaks, modal radiation efficiencies are determined. Radiated sound power and vibration levels are calculated over the frequency bands, as indicated in Table 4.3. These frequency bands at least include the -3 dB range. In this table results are compared with numerical calculated efficiencies according to TWINS and SYSNOISE.

<table>
<thead>
<tr>
<th>Measured SWL [dB]</th>
<th>Resonance frequency</th>
<th>Radiation efficiency [-]</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>freq. range</td>
<td>SWL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500-550</td>
<td>77.20</td>
<td>506.6</td>
<td>0.28 0.02 0.53</td>
</tr>
<tr>
<td>600-680</td>
<td>72.60</td>
<td>631.7</td>
<td>0.86 0.99 1.11</td>
</tr>
<tr>
<td>750-900</td>
<td>76.00</td>
<td>823.4</td>
<td>2.12 0.03 0.63</td>
</tr>
<tr>
<td>930-1000</td>
<td>74.20</td>
<td>963.3</td>
<td>2.51 0.54 0.51</td>
</tr>
<tr>
<td>1320-1370</td>
<td>68.50</td>
<td>1340</td>
<td>1.75 0.01 1.27</td>
</tr>
<tr>
<td>1950-2050</td>
<td>77.20</td>
<td>2005</td>
<td>1.08 0.02 0.68</td>
</tr>
<tr>
<td>2400-2450</td>
<td>60.60</td>
<td>2426</td>
<td>0.53 0.31 0.69</td>
</tr>
<tr>
<td>2750-2800</td>
<td>64.20</td>
<td>2777</td>
<td>0.48 0.38 0.45</td>
</tr>
<tr>
<td>3100-3200</td>
<td>66.00</td>
<td>3148</td>
<td>0.63 0.30 0.78</td>
</tr>
<tr>
<td>3550-3625</td>
<td>54.40</td>
<td>3595</td>
<td>0.56 0.68 0.87</td>
</tr>
</tbody>
</table>

Table 4.3 Comparison of measured radiation efficiencies with calculated values by TWINS and SYSNOISE.

Experimental radiation efficiencies are based on the method described in section 4.3. The measurement method according to the ISO standard, ensures the reliability of the measured powers. However, some uncertainty may be caused by the measurement of the vibration levels. Accelerations are only measured at a number of discrete points, and are only sampled at discrete intervals.

The measured vibration levels lie at one radial line in line with the point of excitation. For mode shapes with a certain number of model diameters, the maximum levels are measured. In the calculation of the radiation efficiency the effective vibration levels over the wheel...
surface are used. More detailed information, like the number of nodal circles is not used in this calculation.

Large differences are found between the measured radiation efficiencies and the efficiencies calculated by TWINS. For three modes: 823, 963 and 1340 Hz, the measured values are high. This might indicate the existence of acoustic resonances.

The TWINS values for radial tyre modes are very low, but this is due to limitation of TWINS to model a uncoupled wheel tyre. For conventional wheels radial tyre modes include strong web vibration levels, for the resilient wheel this is not the case. For two axial modes, the TWINS method overestimates the radiation efficiency.

The SYSNOISE values correlate somewhat better with the measured values, but still large differences are found. Especially the modes at 823 and 963 Hz, SYSNOISE shows too low values. For the latter, the difference is 7 dB. However, for 7 out of these 10 modes, SYSNOISE overpredicts the radiation efficiency.

In order to find an estimate for the difference in estimated sound power levels between TWINS and SYSNOISE, actual sound power levels are determined. For the modes in table 4.3, actual vibration levels of the sound radiation tests are taken. Modal radiated powers are scaled with the square of the vibration ratio, and summed for these 10 modes. The total radiated power is 101.6 dB according to TWINS, and 105.5 dB according to SYSNOISE. Although the large number of radial modes might suggest that TWINS would largely underestimate the radiated power, the difference is only 3.9 dB. Not the largest difference, but the difference in the dominant modes determines the off-set.

![Figure 4.4 Sound intensity in the frequency range 200-5000 Hz, radial excitation. Top view of the measurement surface as seen in figure 4.2.](image)

Figure 4.4 shows the sound intensity plot at the measurement surface of the intensity measurement. This is the total intensitiy for the frequency range of 200-5000 Hz. The total

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noise is radiated uniformly in all directions. Directivity of the sound source is less than 5 dB. No clear structure can be found in the radiation pattern. Around some resonance frequencies some radiation patterns are identified. However strong directivities, for example dominant radial or axial radiation, do not occur. In these modal radiation patterns, maximum observed variations do not exceed 10 dB. Radiation directivities are not further analysed within this report.

### 4.5 TWINS simulation of the resilient wheel

Finally TWINS simulations of the wheel/rail noise are performed. The sound radiation of the resilient wheel on a standard track is evaluated, and compared with a standard intercity wheel.

Standard 'low' and 'high' roughness spectra are used as input for the model. The track for this simulation is a standard UIC54 rail on concrete monobloc sleepers. More details of the model input can be found in Appendix B.

TWINS results are presented in sound power levels. Table 4.4 shows sound power levels for the VSG wheel, for standard high and low roughness spectra. A standard NS-ICM wheel, under the same circumstance, is used as a reference. The NS-ICM wheel is a solid wheel, having a slightly curved web in radial direction, which is used for Dutch intercity trains [van Haaren, 1997].

<table>
<thead>
<tr>
<th>SWL [dB(A)]</th>
<th>low roughness</th>
<th>high roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICM</td>
<td>VSG</td>
<td>change</td>
</tr>
<tr>
<td>Total</td>
<td>103.1</td>
<td>99.8</td>
</tr>
<tr>
<td>Wheel</td>
<td>100.5</td>
<td>93.2</td>
</tr>
<tr>
<td>Rail</td>
<td>97.5</td>
<td>98.5</td>
</tr>
<tr>
<td>Sleeper</td>
<td>95.6</td>
<td>87.8</td>
</tr>
</tbody>
</table>

Table 4.4 Sound power levels of the VSG wheel compared to an NS-ICM wheel.

Low roughness input gives the VSG wheel a reduction in total power of 3.3 dB(A). At high roughness, however, an increase in total power of 1.1 dB(A) occurs. For low roughness, the change in total wheel noise is -7.3 dB(A), while the rail noise is slightly higher: 1.0 dB(A). At high roughness, the change in wheel noise is -3.2 dB(A) and the rail noise significantly increases by 2.7 dB(A). Sleeper noise is lower with the VSG wheel at both low and high roughness input, 7.8 respectively 3.0 dB(A).

In figures 4.5 and 4.6 the TWINS calculated sound power spectra for the VSG wheel are presented in dB. Figure 4.5 shows the total and the partial spectra for at low roughness, while 4.6 shows the spectra at high roughness input. Total sound power levels in dB are indicated by the level lines at the right. Similar spectra for the reference NS-ICM wheel are presented in figures 4.7 and 4.8.

As can be seen in Appendix B, both roughness input spectra have different spectra, which also affects the noise spectra with both wheels. This explains the differences in noise levels at low and high roughness. The spectrum of the high roughness shows an increase of 20 dB in frequency range of 500-1500 Hz. The VSG wheel has a relatively high response in this range. While, on the contrary, the response of the NS-ICM wheel shows a dip in this range. Therefore the NS-ICM is not sensitive for high roughness in this frequency range.
Figure 4.5 TWINS calculated sound power levels of the VSG wheel (low roughness).

Figure 4.6 TWINS calculated sound power levels of the VSG wheel (high roughness).
Figure 4.7 TWINS calculated sound power levels of the NS ICM wheel (low roughness).

Figure 4.8 TWINS calculated sound power levels of the NS-ICM wheel (high roughness).
In case of the NS-ICM wheel, the low frequency noise is dominated by sleeper radiated noise. Above 500 Hz up to 2 kHz the rail is the dominant source, while above 2 kHz the wheel radiates most noise. This is the case for low as well as high roughness excitation. In case of the VSG wheel, wheel noise is never dominant. Above 3-4 kHz, rail and wheel have equal sound power levels. The switch in dominant source from sleeper to rail again takes place at about 500 Hz. In case of the VSG wheel the rail becomes the dominant noise source. Vibration levels of the rail are high over a wide frequency range. The wheel resonances are coupled to the rail. Therefore the rail also radiates noise at the resonance frequencies of the wheel. Lateral propagating waves dominate in the rail radiation up to 1 kHz. Above this frequency, vertical propagating waves are dominant.

The dynamic model of the NS-ICM wheel includes axle modes of vibration, while they are not included in the VSG model. Including the axle modes in the VSG model, should increase the low frequency noise up to about 500 Hz. However no significant increase in the total noise levels is expected.

### 4.6 Discussion

To validate the TWINS wheel radiation module for simulation of the resilient VSG wheel, TWINS radiation calculation results are compared with findings of experimental radiation tests and boundary element simulations. Field indicators according to the ISO standard indicate that the measured sound powers are within 0.5 dB. Although the experimental sound power levels are accurate, some uncertainty in the calculation of the radiation efficiencies is expected. Vibration levels of the wheel are measured only at five locations. Knowledge of the mode shapes is used to calculate average vibration levels.

The measured sound power level shows the radiation of the VSG wheel to be dominated by modal radiation. Therefore it should be possible to use the modal approach to describe the radiation of this wheel.

Comparison of radiation efficiencies calculated by TWINS and the experimental and boundary element model values show that, for this type of wheel, the modal efficiencies are not very well estimated within TWINS. The results of the radial modes are strongly underestimated by TWINS. For most modes, the efficiencies based on the boundary element analysis are higher than the experimental values. Comparison of radiated powers under broad band radial excitation, show that the wheel power according to TWINS, is 3.9 dB less than the power based on the boundary element model. This quantifies the underestimation of the TWINS calculation for the VSG wheel. The semi-empirical formulas used in TWINS to calculate the radiation efficiency and critical frequency for radial modes need to be adapted to fit the measured efficiencies.

No strong radiation directivity is found in the broad band intensity measurements. The total level shows variations of maximum 5 dB around half a sphere. Directivity is not investigated in detail is this report.

Compared to the NS-ICM wheel, TWINS results show the total noise level from the VSG wheel to be 3.3 dB(A) lower for low roughness and 1.1 dB(A) higher for high roughness. If the TWINS' underestimation of 4 dB in wheel noise is taken into account, the changes in total noise become 3 dB reduction and a 2 dB increase respectively.
5 Evaluation and discussion

Measurement of the dynamic stiffnesses of the resilient elements are performed up to 2 kHz. Results up to 800 Hz are used to determine the translational stiffnesses in three directions. The strain gauge load cell can not be used above roughly 1 kHz. The exact limit depends on the effective mass at the load cell. For radial test in this the test set-up the additional mass was 1.9 kg. Higher frequency tests can be performed by using piezo load cells. Additionally, the input accelerometer should be positioned at the mass on top of the element under test. On top of the load cell the measured dynamic stiffness includes the stiffness of the load cell, for which corrected are made within this report.

The simulation strategy is believed to incorporate all significant characteristics in the vibrational and radiational behaviour of the resilient wheel. Eigenfrequencies of the finite element model are within acceptable limits. However, large differences are found in the modeshapes. The vibration ratio of tyre and web, that is the transmission of vibration from tyre to web, is underestimated by about 10 dB for radial modes. The influence of this mismatch in the model on the simulation of the vibration behaviour and radiation is hard to quantify. Underestimation of the vibration transmission indicates that the stiffness of the resilient elements in the finite element model is too low.

The eigenfrequency of the radial modeshape at 823 Hz, is overpredicted by the f.e.-model, which indicates that the resilient elements are modelled too stiff. On the other hand the web is modelled too stiff, which might also decrease the transmission. More detailed modelling of the web is needed to come to a more precise model. Constant stiffness values are defined in the f.e.-model. In radial direction, it is shown that the first resonance frequency in the element will occur at frequencies out of the range of interest for this model. For the axial and tangential directions, however, resonances can occur at lower frequencies. Nevertheless, it is the transmission in radial direction that is underestimated most strongly.

According to equation (2.5), the transmission of vibration is proportional to the transfer stiffness coefficient. An underestimation of the transmission of 10 dB, therefore corresponds to an underestimation in the radial stiffness by a factor 10. It is highly unlikely that the stiffness coefficients are wrong by a factor 10. It is therefore expected that a combination of factors result in underestimating the transmission of vibration. Possible causes are the too stiff modelled web, differences between measured resilient elements and elements applied in the wheel or other effects that are not included in the f.e.-model. Taking this limitation into account, the model has been used to shows the characteristics of the resilient wheel in the radiation of wheel/rail noise.
6 Conclusions and recommendations

6.1 Conclusions

Very little was known on the sound radiation of resilient train wheels. Only several noise measurements results were available. To investigate the potential of resilient wheels, in relation to the need for reducing railway rolling noise, a typical resilient wheels (VSG) has been investigated. Numerical simulation models have been built to describe the vibration and radiation characteristics of the resilient VSG wheel. The numerical models are validated by an experimental modal analysis and sound radiation measurements. Results of the vibration and radiation analyses show that the wheel can be modelled as a linear system. The sound radiation of the wheel indeed can be related to the modes of vibration.

Results of numerical and experimental modal analyses of the resilient VSG wheel, show that the dynamic behaviour of the wheel can be described by modes of vibration. Tyre and web have separate modes of vibration which are only slightly coupled. Therefore the number of modes is increased, and the modes shapes are more complex, compared to a conventional solid train wheel. Eigenfrequencies of the finite element model are within acceptable limits. However, care is required in defining the coupling between tyre and web. The finite element model underpredicts the transmission of vibration between tyre and web. This might suggest that the resilient elements are modelled too compliant, however other effects might also play a role, such as the web that is modelled too stiff.

Results of the finite element model and experimentally determined modal damping values are used to provide a simulation model within TWINS. Under the assumption that the transmission of vibration can be improved, TWINS should be able to simulate the sound radiation of a resilient wheel. However small adjustments are required to calculate the sound radiation of the wheel accurately. Especially for tyre modes in radial direction, radiation efficiencies are strongly underestimated by TWINS, while they are overestimated by the boundary element model. Corrections within TWINS are required for the definition of parameters for the resilient wheel, as well as for the calculation of the radiation efficiency for the radial tyre modes.

TWINS simulation results show that by using this resilient wheel, wheel/rail noise can be reduced. However reductions are not as high as indicated in literature. Indeed the area which is vibrating strongly, is reduced by isolation of the tyre vibrations. However the level of vibration of the tyre modes increases significantly. Although radiation of the tyre is not always very efficient, the tyre radiates significantly. On the other hand, tyre vibrations are transmitted to the rail which becomes the strongest radiating source. Therefore, reductions in noise are limited when using the VSG-wheel. Under certain conditions the radiated noise can even increase. Comparisons are for the VSG wheel and a conventional NS-ICM intercity wheel. A standard high roughness, with high roughness levels at 1000 Hz, did result in an increase of about 2 dB. For standard low roughness, a reduction of 3 dB can be achieved. The VSG wheel shows a high response in the frequency range 500-1500 Hz. Therefore the wheel is particularly sensitive to roughness in this frequency range.
6.2 Recommendations

Additional simulations with various stiffness values for the resilient elements, will supply valuable information on the trends of vibration transmission and sound radiation of the resilient wheel. Modal analysis simulations with different stiffness values will show how resonance frequencies and transmission of vibration vary with these stiffness coefficients. Results of these simulations then can be used as an input for TWINS simulations, which will indicate how the sound radiation changes with the stiffness of the elements. This sensitivity analysis will also supply basic rules to optimise the acoustical performance of the resilient wheel.

Within this project some response analyses have been performed with the finite element model. These finite element calculations did not include damping. By using the measured damping, frequency response analyses supply frequency response functions, which can be compared to the experimentally determined response functions. This will provide an additional way of comparing vibration transmission of the numerical model and the experiments.

The finite element model might be improved by improving the description of the wheel web. Different web components can be modelled as separate volumes. However, this requires also a proper handling of the friction in the metal to metal contact between these components. Therefore, a more detailed model for the web will require extensive effort.

Similar to the definition of the stiffness matrices, damping matrices can be defined within the finite element model. Measured loss factors can be used to define constant displacement amplitude damping coefficients, or $1/\alpha$ proportional viscous damping. These models should then supply complex eigenfrequencies and mode shapes. The numerical damping factors can be compared to measured modal damping values. Including damping in the numerical model, will give an additional tool to optimise the acoustical performance of the wheel. However, the definition of the constant damping or loss factor for single elements is not straightforward in ANSYS. Perhaps other simulation software is needed to perform such simulations.

To verify high measured radiation efficiencies of the VSG wheel, more detailed radiation measurements are needed. By performing measurements close to the wheel surface (about 10 cm), local sources can be identified. Possible local sources can be expected near the slit between tyre and web.

It would be very interesting to validate the calculated wheel/rail noise. For that purpose pass-by measurements would have to be performed. By measuring wheel, rail and sleeper vibrations, partial noise sources can be identified. This will supply information which will validate the overall performance of the TWINS simulation of this wheel. Additionally, comparison to other types of low noise train wheels is of interest.
References

Acknowledgement

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At last but not least I would like to thank Herrn Weber at VSG Verkehrstechnik for his kind help and for providing a resilient wheel for experimental testing.
Appendix A  Sound radiation calculation within TWINS

Within TWINS, the sound radiation of the wheel is calculated by the module 'wxrad' [Thompson, 1994]. This model assumes the mode shapes to be axi-symmetric. The calculations use the number of waves around the circumference, that is the number of nodal diameters \( n_d \), as characteristic parameter of the radiation. In the model, the sound radiated by axial, radial and (tyre) torsional vibrations are treated as independent components. For the axial component, the wheel is divided into a number of concentric rings (figure A.1). Up to 6 concentric rings can be used to describe axial vibration amplitudes. For each ring \( j \) between \( r_i \) and \( r_{i+1} \), the vibrating surface \( S_j \) and the mean square normal velocity \( v_{an}^2 \) are calculated. The total radiated sound power \( W \) is calculated with:

\[
W = \rho \cdot c \cdot \sum_{i}^{n_d} \left( \sigma_{axi}(n) \sum_{j}^{n_d} S_j \left( \bar{v}_{an}^2 \right) + \sigma_{rad}(n) S_j \bar{v}_{rad}^2 + \sigma_{tor}(n) S_j \bar{v}_{tor}^2 \right)
\]  

(A.1)

where \( \sigma \) denotes the radiation efficiency, the index relates to the vibration component. \( \rho \) and \( c \) the mass density and the speed of sound of air. Total radiated power is summed over \( n_f \) modes of vibration. The first term in (A.1) denotes the axial radiated power, with the axial web velocities. Followed by the radial power using \( v_r \) and the torsional radiated power with \( v_t \) respectively.

Figure A.1 Geometry parameters used in the wheel radiation model.

Radiation efficiencies are determined for each vibration component, depending on the number of nodal diameters and frequency. The radiation efficiency \( \sigma \) is the ratio of radiated power and normal velocity squared (spatially averaged rms value), of the vibrating surface \( S \):

\[
\sigma \equiv \frac{W}{\rho \cdot c \cdot S \cdot v_n^2}
\]  

(A.2)

Radiation efficiencies usually lie between 0 (in-efficient) and 1 (efficient radiation).
Radiation efficiencies of elementary sources

Elementary sources are pulsating and oscillating spheres. The radiation efficiency of a pulsating sphere, of which the whole surface vibrates radially in phase, can be approximated by two asymptotes:

\[ \sigma = \left( \frac{f}{f_c} \right)^2 \text{ for } f < f_c \]
\[ \sigma = 1 \text{ for } f > f_c \]

where the critical frequency can be found by:

\[ f_c = \frac{c}{\sqrt{\pi S}} \] (A.3)

Sound radiation is inefficient (increase of 6 dB/octave) for large wavelengths, but becomes very efficient for wavelengths, which are small compared to the sphere radius. The sphere radiates equally strong in all directions. This so-called monopole model is a good approximation for a vibrating surface of which a part moves in phase.

The radiation efficiency of an oscillating sphere, which vibrates as a rigid body in one direction, can be approximated by two asymptotes:

\[ \sigma = \left( \frac{f}{f_c} \right)^4 \text{ for } f < f_c \]
\[ \sigma = 1 \text{ for } f > f_c \]

where the critical frequency can be found by:

\[ f_c = 1.41 \frac{c}{\sqrt{\pi S}} \] (A.4)

For frequencies below \( f_c \), the oscillating sphere radiates sound less efficiently than the pulsating sphere (increase of 12 dB/octave). In a similar way, the quadrupole model can be described by two spheres oscillating in the same direction but with opposite phase. Higher-order elementary sound sources can be described as well.

In case of a vibrating plate the radiation efficiency is somewhat more difficult. Adjacent maximum amplitudes of the bending waves act as local sound sources, out of phase. So some pressure cancellation can occur. The radiation model of bending plates takes this phenomenon into account. The critical frequency is given by

\[ f_c \approx \frac{c^2}{18 c_l t} \]

with \( t \) the thickness of the plate, \( c_l \) the speed of sound inside the plate. For short wavelengths in air, compared to the wavelengths in the plate (\( f > f_s \)), \( \sigma \approx 1 \). Larger wavelengths of the sound waves result in decreasing radiation efficiency. For frequencies in the order of \( f_c \), the efficiency can be larger than 1. This is due to the coincidence effect: the wavelength of the bending vibration in the plate coincides with the wavelength of the sound wave in air for the
same frequency, in certain directions constructive interference of pressure waves occurs, and the over-all sound radiation can be even more efficient than a plane wave.

Radiation efficiencies calculation within TWINS

The formulas for the radiation efficiency used in TWINS have been derived with the help of extensive calculations with the Boundary Element Method. At low frequencies, the sound radiation from the wheel depends strongly on the number of nodal diameters \( n \) in a given mode shape. The formulas are:

- **Axial motion:**
  \[
  \sigma_a(n) = \frac{1}{1 + \left( \frac{f_{ca}(n)}{f} \right)^{2n+4}} \quad \text{with} \quad f_{ca}(n) = \frac{c}{2\pi r} \sqrt{(n+1)(n+4)}
  \]  
  (A.5)

- **Radial motion:**
  \[
  \sigma_r(n) = \frac{1}{1 + \left( \frac{f_{cr}(n)}{f} \right)^{2n+2}} \quad \text{with} \quad f_{cr}(n) = \frac{c}{2\pi W} \sqrt{(n+1)(n+4)}
  \]  
  (A.6)

- **Torsional motion:**
  \[
  \sigma_t(n) = \frac{1}{1 + \left( \frac{f_{ct}(n)}{f} \right)^{2n+4}} \quad \text{with} \quad f_{ct}(n) = \frac{c}{\pi W} \sqrt{(n+1)(n+4)}
  \]  
  (A.7)

for frequencies higher than the corresponding critical frequency: \( \sigma = 1 \).

These formulas are based on basic physics, similar to the equations of the elementary sources. For low frequencies equation (A.5) becomes:

\[
\sigma_a(n) \approx \left( \frac{f}{f_{ca}(n)} \right)^{2n+4}
\]  
(A.8)

In case of 0 nodal diameters, that is the whole wheel vibrates axially in phase, the exponent in (A.8) is 4. This is like a dipole radiation: both sides of the wheel vibrate and radiate sound in opposite phase, partly cancel each other out, and thus lower the radiation efficiency. The characteristic length of the radiating surface is the wheel radius \( r \), which can be seen in the expression for \( f_{ca} \). For 1 nodal diameter the exponent is 6, this is the same as is the case for quadrupole radiation. Similar arguments hold for higher numbers of nodes and for radial and torsional motion.

The calculation of the radiation factor above, is based on vibration components. TWINS sums the radiated power in third octave bands, over the components of the modes within that band. Modal radiation efficiencies are not determined, but can by derived from (A.1). The summed component radiation factors weighted by the areas and amplitudes, divided by the sum of the products of areas and square modal (velocity) amplitudes, supplies the modal radiation efficiency.
## Appendix B  
Model input for the TWINS simulation

<table>
<thead>
<tr>
<th>Rail type: UIC 54</th>
<th>Ballast</th>
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<tr>
<td>Bending stiffness vertical $E_l$ [Nm$^2$]</td>
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<tr>
<td>Shear coefficient vertical $K_s$ [-]</td>
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<tr>
<td>Loss factor vertical $\eta_v$ [-]</td>
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<tr>
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<tr>
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<tr>
<td>Mass per unit length $m_A$ [kg/m]</td>
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</tr>
<tr>
<td>Cross-admittance ampl. $X_{dB}$ [dB]</td>
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<tr>
<td>Cross-admittance phase $X_{sign}$ [-]</td>
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<td>Rail pads</td>
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<tr>
<td>Length $l$ [m]</td>
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<tr>
<td>Width $w$ [m]</td>
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<tr>
<td>Intermediate distance $d$ [m]</td>
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<tr>
<td>Train speed $V_t$ [km/h]</td>
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<tr>
<td>Standard roughness input</td>
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</tr>
<tr>
<td>&quot;standard&quot; third octave spectra</td>
<td>low/hig</td>
</tr>
</tbody>
</table>

### Response calculations
- Integration length $L$ [m]: 26.0

### Wheel parameters
- Web position 1 $R_{w1}$ [m]: 0.130
- Web position 2 $R_{w2}$ [m]: 0.210
- Web position 3 $R_{w3}$ [m]: 0.280
- Web position 4 $R_{w4}$ [m]: 0.325
- Web position 5 $R_{w5}$ [m]: 0.390
- Inner tyre radius $R_{il}$ [m]: 0.400
- Outer tyre radius $R_{lo}$ [m]: 0.455
- Tyre width $W_t$ [m]: 0.135
- Web width $W_w$ [m]: 0.020

### Contact parameters
- Wheel radius $R_{11}$ [m]: 0.455
- Wheel radius $R_{12}$ [m]: $\infty$
- Rail profile radius $R_{22}$ [m]: 0.3
- Static load $F$ [kN]: 50

| Table B.1 Input parameters for TWINS simulation. |
Figure B.1 Combined roughness spectra: high and low standard roughness.
Appendix C  Calculation of field indicators

According to ISO 9614, field indicators are calculated in the following ways.

**F2: Surface pressure - intensity indicator**

\[ F_2 = \bar{L}_p - \bar{I}_{1a} \]

with:

\[ \bar{L}_p = 10 \log \left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{P_i}{P_0} \right)^2 \right), \]

sound pressure level averaged over N measurement points

\[ \bar{I}_{1a} = 10 \log \left( \frac{1}{N} \sum_{i=1}^{N} I_{1a} \right) \]

normal absolute intensity level averaged over N measurement points

Requirement:

\[ F_2 < I_{sd} \]

where:

\[ I_{sd} = \delta_{p0} - K \]

F2 should be smaller than the dynamical capability index, which is the Residual Pressure Intensity Index, minus a constant. The ISO standard specifies a value of 10 dB for K. This value is related to an accuracy of 0.5 dB. The Residual Pressure Intensity Index (RPII) is determined by measuring a sound wave perpendicular to the probe. This is done by using a intensity calibrator, with the microphones at exact the same distance from the source. The RPII is a specified quantity for each probe.

**F3: Negative partial power indicator**

\[ F_3 = \bar{L}_p - \bar{I}_n \]

with this time the non-absolute intensity level averaged over N measurement point. F3 - F2 should be smaller than 3 dB.

**F4: Non-uniformity indicator**

\[ F_4 = \frac{1}{\bar{I}_n} \sqrt{ \frac{1}{N-1} \sum_{k=1}^{N} (I_{nk} - I_n)^2 } \]

with:
Indicator $F_4$ is used to evaluate the measurement mesh accuracy. The requirement is: $N > C \cdot F_4^2$. 
$N$ is the number of measurement points, $C$ is constant dependent on the frequency range and accuracy limit. For this report a value for $C$ of 29 has been used.