Comparative study of tyre models for the simulation of landing gears on impact

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Comparative study of tyre models for the simulation of landing gears on impact

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COMPARATIVE STUDY OF TYRE MODELS
FOR THE SIMULATION OF LANDING GEARS ON IMPACT

Thierry Wiertz
electro-mechanical engineering
Aerospace tendency

Graduation project
The areas of contact between tyres and road ... are the very front line trenches in the furious battle between space and time

Evans, 1935
thanks

Many people have contributed to this work and I would need more than a few lines to be exhaustive in expressing my gratitude. Yet, I want to address very special thanks to Mr Geradin, the promoter of this work for his enthusiasm and his trust. I am also very grateful to Mr A. De Kraker and G. Verbeek and to all the people at Eindhoven University for the enlightened advice and for the favourable atmosphere in which I have been able to work there. I also thank my parents for translating this study in English, for re-reading it and for all the rest; Ph. Jetteur, I. Besselink and the research workers of Samtech and Fokker for their help and well informed strategical and practical advice.
COMPARATIVE STUDY OF TYRE MODELS FOR THE SIMULATION OF LANDING GEARS ON IMPACT

by Thierry Wiertz

Electro-mechanical engineering, aerospace tendency

SUMMARY

The landing phase of an aircraft is and will always remain a delicate negotiation. It is of utmost importance for safety reasons to attain an accurate conception of the landing gear. Presently, this conception is affected by computer simulation of the behaviour. Thanks to its finite elements formalism the Mecano module of the Samcef software appears to be a performing tool for this type of problem. The modelling of the forces generated by the tyre is extremely important for the accuracy of these simulations.

Pacejka has recently proposed a new method of computing the forces at the tyre-road interface. The proposed model named magic formula has great advantages due to the physical meaning of the coefficients it incorporates.

The present study consists in implementing and testing this tyre model on a new wheel element for Samcef Mecano. For the building of this element a new kinematic interface was created, which takes the camber angle of the wheel into account. This interface is designed to receive eventually other tyre models. The tests performed with the new wheel element show that it works correctly. However, in the present state of things, it is not yet possible to get results that are quantitatively significant.

The possible developments of the model are the implementation of combined longitudinal and lateral slip cases, the adaptation of the magic formula to bias ply tyres used in aviation and the modelling of the transient behaviour.

In a remoter future, it will be possible to consider extending the kinematic interface to non flat roads.
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APPENDIX
INTRODUCTION
I.1 The landing gear characteristics

For centuries man has been dreaming of flying. Once this dream true, it was yet important to come back to firm ground safely.

Many aviation pioneers paid with their limbs or lives for having failed to design an adequate landing gear. From that heroic time to the present day, much progress has been made. The landing of an airplane however remains a critical phase of the flight.

A modern civil aircraft, the mass of which amounts to a hundred tons, rests on a landing gear consisting of about ten wheels. This landing gear is designed to resist high vertical loads occurring on impact up to 5 times the nominal load. It should withstand 90,000 flights in complete safety. This involves more than 900,000 kilometres, that is to say more than a standard car would run.

fig 1.1 forward landing gear of Fokker 100
The present day landing gears are complex systems including rigid members and oleo pneumatic shock absorbers. All these systems are hinged in order to be retracted into the aircraft body. The tyres which are replaced over about 200 to 500 flights, must be able to work in total safety, sometimes on fully drenched runways and at very high speeds.

The safety requirements in force in modern aviation and the extraordinary working conditions are such that the design of landing gears is nowadays an engineering problem claiming the most careful attention. Any progress made is the result of varied types of research and necessitates more and more sophisticated technologies. The development of computer simulation represents without any doubt a great leap forward in the knowledge to be acquired about the landing processes.
1.2 The simulations of vehicle behaviour

The invention of the wheel goes back to a time when man could hardly have considered flying and yet, it can be stated that the pneumatic wheel has only made great strides since the beginning of this century. Up to that time man had acquired an empirical knowledge about the behaviour of wheel vehicles he designed and did not ask himself many questions about the justification of that behaviour.

The end of the 19th and the beginning of the 20th Century saw the boom of a certain number of kinematic models aiming at explaining the vehicle behaviour of the first cars. The twenties had to be reached before the first steps were taken towards a dynamic model. Then appeared the notion of cornering force and its relation to lateral slip. The first paper attempting at modelling forces that a tyre can generate in sliding was published by Evans in 1935. The complexity of observable phenomena at the tyre-road interface is described in a vivid way. The development of industry and car racing generated a better knowledge of vehicle behaviour and the ways to control it.

Nowadays, the computer has allowed much liberty and can take over tasks that could not have been imagined even ten years ago. A car manufacturer rarely markets a new model before it was "road tested" on computer. To allow these simulations, it is necessary to develop models describing the forces generated by the tyre. The door is now open for the use of these models in other fields.
1.3 The purpose of this study

The necessity of effecting accurate simulations of landing gear behaviour is justified by the important role played by these simulations in the process of design. Most of these simulations are effected on multibody simulation software. The Mecano module of the Samcef software is a very adequate response to the problems of the flexibility of the members and joints which play a great role in this type of problem. The accuracy of the force model for tyre-road interface representation is the key to the quality of the simulation. The present study aims at equipping Samcef Mecano with a wheel element allowing the most accurate representation of the characteristics of the tyre-road interface. A maximal flexibility as well as a generality of use will be sought and the computation cost will be taken into account for the selection of an adequate tyre model. As a basis to this wheel element, a kinematic interface will be developed, liable to receive other tyre models and representing an extensive range of kinematic situations.
II

THE TYRE AND THE TYRE-ROAD INTERFACE
II.1 Function of the tyre

The tyre being the only entity that links the vehicle to the ground its role is to insure the transfer of all the forces of the vehicle towards the road and vice versa. Any tyre has to support the radial loads, to insure the transmission of the acceleration torque with a minimum of slip, contribute to the lead and to a sound road behaviour of the vehicle and to participate in junction with the suspension system to the comfort of the passengers on rough roads. The requirements of maintenance require of course a definite resistance to wear. On the other hand, an ideal tyre should provide a rolling resistance as low as possible in order to reduce consumption.

Given the conditions under which it is brought to operate, the plane tyre is probably one of the most stressed; indeed it must produce a far greater resistance to vertical stress and must be able to behave correctly at very high slip and rolling speeds.
II.2 Conception and materials

All those requirements make that the conception of a tyre constitutes an unequalled technological challenge. The merciless competition and the secrecy between the main producers are proof of it. That competition tends also to prove that much progress is still possible in the field of tyres.

One can yet say that roughly all the tyres follow more or less the same principles of elaboration. They consist of a rubber carcass reinforced by layers of textile and metal cables. Within that scheme one can although distinguish two types of manufacturing that are different: radial tyres and bias ply tyres.

II.2a Radial-ply tyres

These are almost of general use in automobile use. The fibres inside the plies lay perpendicular to the normal translation direction of the wheel. The carcass is reinforced by a braided belt under the tread, this belt insures a low distortion liability of the part of the tyre that is in contact with the road. Their two important qualities are a low heating and a low resistance to rolling.

II.2b Bias-ply tyres

The reinforcement layers are now laid obliquely towards the circumferential direction by alternative cord directions in order to obtain more or less homogenous properties. The advantage of the bias-ply tyre is its good behaviour by camber. This makes it an ideal tyre for two wheeled vehicles. Its drawbacks are an important heating due to internal friction between the plies as well as rapid wear. It is much used in aviation as it has a great radial rigidity. Not rarely are bias-ply tyres reinforced by a belt at the tread in order to make this one more stable like in the radial-ply tyres.

Materials used are natural or synthetic fibre (cotton, rayon, polyester) for the reinforcement layers, a braided steel wire for the belt, a steel cable for the leads as well as a mixture of natural and artificial rubber added with sulphur, carbon black and other additives of which each manufacturer keeps the secret. This mixture insures the cohesion of the
whole and is determinative of the friction properties of the interface tyre-ground.

Its present conception makes the tyre a composite material of which the general properties are very difficult to fix: the more that they greatly vary from one specimen to the other, even within the same model and that they are not invariable throughout the life of the tyre. This fact reduces greatly the acuteness of the measures that can be made on the properties of tyres, because the factors to be taken into account are so numerous that the measures rarely are reproducible.
II.3 Kinematics of the wheel

We shall restrict this description to a rather simple description of the kinematics of the wheel, a more extensive description will follow in chapter IV. Let us first consider the case of a free rolling wheel in a straight line on an even plane. The rotation speed of the axle is \( \omega \). Three radiuses are defined for the wheel: the non-distorted radius \( R_0 \) measured by the distance between the axis and the upper limit of the wheel, the distorted radius \( R \) which corresponds to the distance between the ground and the axis of the wheel and to finish with the real radius \( R_e \) which corresponds to the quotient of the translation speed of the wheel by its spinning speed.

Some authors define this radius in the same way in the case of slip, so for them the real radius is defined by wheel lock and zero by skidding on the spot. For an easier computing of the slip speed it will be more advisable to link the definition of the real radius with the case of free rolling wheel. In that case, the point \( S \) located at a distance \( R_e \) on the vertical that joins the centre of the wheel with the ground is called the instantaneous rotation centre of the wheel, while the point \( C \) located on the same line at a distance \( R \) is the contact point (in fact the centre of the trace in simple cases).

fig II.1 kinematics of the tyre
Once the wheel starts skidding point $S$ acquires a non-zero speed which is simply the slip speed of the wheel. You can define slip quantities. Two slip quantities are commonly used: one named theoretical is defined by the quotient of slip speed by free wheeling translation speed; the other one, called practical is given by the quotient of the same slip speed by the true translation speed.

**theoretical slip**
$$\sigma_x = \frac{V_{sx}}{V_{cx} - V_{sx}}$$
$$\sigma_x = \frac{V_{sx}}{\omega \times R_e}$$

**practical slip**
$$\kappa = \frac{V_{sx}}{V_{cx}}$$

When the wheel does not move in a straight line there is a lateral component to the slip speed. You define then the quantities of side slip in the same way as in the case of longitudinal slip.

When the wheel is not in an upright situation you define the camber angle as the angle between the axis of the wheel and the horizontal. That angle is usually written $\gamma$.

**theoretical slip**
$$\sigma_y = \frac{V_{sy}}{\omega \times R_e}$$

**practical slip**
$$\tan(\omega) = \frac{V_{sy}}{V_{cx}}$$

**drift angle**
$$\alpha = \arctan \left( \frac{V_{sy}}{V_{cx}} \right)$$

All the cinematic relationships formerly stated remain valid for wheels with a non-zero camber as far as the yaw angle of the wheel remains constant (the wheel does not swivel around the axis that joins the contact point to the centre). In those cases where the product is not zero, the kinematic problem becomes more complex and it is necessary to define a new slip ratio called turnslip, but this does not concern the present problem.
II.4 Way in which the contact forces appear

At the level of the wheel-road interface four forces appear: a vertical force, a longitudinal braking or acceleration force, a lateral force (also named side force) and a self aligning torque. You can clearly distinguish between the first one, which is a reaction caused by internal pressure of the tyre and by its structure and the other three, which find their origin in the friction phenomena that appear at the footprint.

The vertical force works along the perpendicular (normal) to the ground, the longitudinal force is positive in the forward direction along the intersection of the wheel plane with the ground plane; the side force works along the ground plane perpendicular to the longitudinal force and finally the self aligning torque works along the same axis as the vertical force.

fig II.2 conventions defining the directions of forces
II.4.a The vertical force

The internal pressure of the tyre and its structural resistance make that once passed a certain degree of crush the weight working on the wheel is compensated. The role of the internal pressure is not so much to produce a vertical resistance but well to span the outer cover of the tyre so as to impart it an important radial stiffness. Past a certain degree of crush this stiffness becomes higher and higher up to the point where it reaches in extreme cases the thrust value constituted by the rim stiffness.

II.4.b Friction forces

The mechanism that leads to the advent of the three longitudinal, side and self aligning forces is one sole phenomenon which occurs at the level of the footprint.

When the wheel is in motion, the footprint constitutes for any element of the tread a deformation zone that will be crossed progressively as the wheel spins. When the rubber-road contact occurs the lead distorts under the effect of the contact force. Once the structural resistance of the tread exceeds the contact force, the slip phenomenon appears; that is why the footprint is divided in two distinct regions, an adhesion region and a sliding region. To determine the total force working at the level of the footprint it will be enough to integrate the contact or friction forces working on each of the infinitesimal elements. In order to define that elementary force it is necessary to know the trajectories of all the elements constituting the footprint, there of course lies the whole difficulty of the step.
II.4.c Longitudinal force

In simple cases (zero camber angle, flat ground, no side slip) the longitudinal slip is rather easily fixed because you can simplify the tyre model by taking a unidimensional model for the print. The important quantities to be fixed are the velocity field of the print elements, the pressure diagram at the level of the print, the stiffness of the tyre and the friction law of the material.

![Diagram showing longitudinal force, slip distance, pressure, and friction](image)

**Figure II.3** Way of appearing of the longitudinal force
II.4.d The side force

In very simplified cases one can resort to a unidimensional model similar to the one used for the longitudinal force; a number of tyre models are indeed based on that type of simplification (see ch.III) In addition to the longitudinal case, the lateral stiffness of the material is necessary to be able to calculate the side force. The problem of the strain at the level of the print is rather more complicated because it is bi-dimensional.

fig II.4 unidimensional model giving the lateral force as a function of a single foundation stiffness

II.4.e The friction forces in combined cases of longitudinal and side slip

For the detailed computation of the phenomena that appear here it is necessary to define the trajectories of the whole of the print. This implies the solution of a complex problem of strain of the tread. Indeed now the three types of strain of the carcass combine: longitudinal, lateral and torsional. In addition, as already mentioned, the mechanical properties of the tyre are generally badly known.

The summit of complexity is reached when, in addition, you consider a non-zero camber angle. In that case the strain of the carcass becomes rather important and it is not easy to represent it by a simple model.
II.4.f Vibration phenomena

The print is not the only distorted part of the tyre under load; actually the strain on the print propagates on both sides of it. The speed with which this distortion propagates is of course finite and depends on the structural characteristics of the carcass and of the inflation pressure. For a definite rotation speed of the wheel the elements of the tread directly upstream of the print get hearer at a speed equal to the propagation speed of the distortion. They get then in a zone of forbidden signals (like the air in front of a plane flying over Mach 1) and in the same way as for the sound barrier, there occurs a shock wave in front of the print. That phenomenon should absolutely be ruled out as it causes an important loss of energy (hence an increased rolling resistance and overheating) and this explains why there is a maximum speed of use for each tyre. This also shows that the rotation speed of the wheel has an influence on the behaviour of the print (the effect of the speed intervenes also over the centrifugal force).

This type of phenomenon also plays a certain role in the transient behaviour of the tyre and particularly in the impact problems where the shaping of the print goes together with the in spin up of the wheel.

II.4.g Conclusion

Except for the largely simplified cases which occur only very rarely in practice, it is extremely difficult to compute analytically the forces that appear between the tyre and the road. The multiplicity of influences and the lack of constancy in the measures realised practically make it more difficult to validate an analytical model used for theoretical calculation. Luckily many applications do not need an extreme accuracy of computation of the contact forces. For those applications have been developed a whole series of more or less sophisticated models which are presented in the next chapter.
The simulation of landing gear impacts is probably a problem that requires one of the most accurate tyre models. It is however inconceivable to use a plain description of all the phenomena that occur at the tyre-road interface. It fits consequently to find a simplified model which is able to represent with accuracy the most important characteristics of the tyre-road interface without involving a too high computational cost.
As seen in the preceding chapter, the problem of the tyre-road contact is very complex to be treated in detail. Since the first study of forces appearing between the tyre and road by Evans in 1935, a large number of models have been developed. They offer a wide range of possibilities as well on the way of approaching the problem as on the level of accuracy reached in the determination on forces. A complete and exhaustive classification is not possible and we shall have to limit ourselves in this chapter to giving the great tendencies which define the different types of models and to showing the principle of functioning of each type.

III.1 Two approaches to simplification

In the way of simplifying the problem of determination of contact forces, two approaches can be singled out, which lead to totally different models. The first approach consists in simplifying the structural model of tyre and to perform the calculation of forces in a classical way by using this physically simplified model. The brush, stretched string and beam models and their combinations fall into this category. These models involve a certain computation load to determine the forces but require only a few measures of the structural characteristics of the tyre. The physical phenomena represented are well known for they are at the basis of the model and the degree of complexity can be extended rather far. In the latter case, these models appear to be good tools for the tyre manufacturers in order to determine the performances of a tyre theoretically. In their simplest forms, on the other hand, they prove to be sufficiently accurate to carry out the computing of vehicle performances.
In the second approach, all computing of forces is suppressed by representing them in the form of charts or tables, with respect to kinematic variables. It is thus necessary to make numerous measures of forces for all the kinematic situations encountered. In cases in which the values are taken up in formulas, these will generally appear in polynomial form and the coefficients of the polynomials will be determined by fitting. This type of model is very practical for simulation problems as the computing load is then very low and the accuracy can be great, depending on the quality of the measures. The physical phenomena included in the model are not explicitly known, which could make the interpretation of some results hazardous.
III.2 The tyre models used at present time: Survey

III.2.a Models based on a simplified physical model

Most of these models use a unidimensional representation of the print. Starting from the slip ratio, the strain of the centre line of the print is determined, taking into account that beyond a certain strain, slip is reached. The slip limit depends on the vertical load applied to the tyre. It will consequently be necessary to determine the vertical pressure diagram at the print. Many authors adopt a parabolic diagram as a matter of facility; other seek a representation nearer to reality and get intermediate diagrams between parabola and ellipse.

fig III.1 simplified model of lateral force with one single stiffness [11]
For the simplest models in this category, one single stiffness called foundation stiffness is adopted. In others, only the stiffness of the thread rubber is taken into account. The results obtained in this way are already close to reality. If a greater accuracy is required, an additional stiffness will be adopted for the carcass. The deflexion of the thread is then divided in two distinct parts, one due to the flexibility of the carcass, the other one due to the flexibility of thread rubber. In this case, the computing is more difficult but the results can justify such an effort.

This type of model permits the computation of longitudinal, lateral and self aligning forces in cases of pure slip. It also allows the determination of the relaxation length of the tyre, which plays an important role in transient phenomena (see chap VI).

Two models can be adopted to represent the carcass deformation, either a stretched string model or a beam model. The beam model is a little more difficult to implement for it is of the fourth degree whereas the string model is of degree two only. Experience has shown that the string model is more suitable for bias-ply tyres whereas the beam model gives better results for radial tyres.
An additional refinement of this type of model consists in considering a variable friction coefficient of rubber depending on the slip speed.

This type of model generally gives a bad estimate of the variation of friction forces with vertical load, especially in case the flexibility of the carcass is not taken into account. It does not account for the saturations of longitudinal and lateral forces observed in reality.

III.2. Models based on empirical formulas

There is rather little to tell about these models, if only on the model of Pacejka which falls into this category and will be presented in greater detail at the end of this chapter. All the models of the category require extensive measures of the contact forces in a wide range of kinematic situations. In relatively simple cases, the only entry variables for a force are the vertical load and a slip ratio; the force can then be represented by a Bezier patch or by another interpolation of the same type.

A rather interesting species of this kind of model consists in building a model based on a certain number of basic curves (generally one for each force) and to make fittings of the coefficients describing these curves in order to cover the complete field of the entry variables.
III.2.c The finite elements models

Since the birth of powerful computers, the finite elements computing techniques have known a tremendous development. They now allow to simulate the emergence of forces in the tyre-road contact. The large number of degrees of freedom necessary for this kind of simulation does not (yet?) allow however, for reasons of computing capacity, to include these finite elements models in the simulation model for dynamic behaviour. Yet, these models can prove to be very useful for they will allow once rescaled to save a great deal of the necessary measures for the setting up of other models.

The present day research uses thick shell and a non symmetrical topology to represent the tyre carcass. The elements used are of anisotropic type and their properties are computed by using the same techniques as for laminate composite materials. The model is finally refined by using the whole set of rescaling methods.

fig III.3 bidimensional shell elements and tridimensional volume elements models of tyre [10]
III.2.d Models including dynamic phenomena

A certain number of models allow to include directly the computation of forces in a system of differential equations describing the dynamic behaviour of the carcass. These models are especially used to simulate shimmy phenomena. In this case, there is a coupling between the lateral force and the self aligning torque. These models are rather difficult to implement for their rescaling goes through a series of measures on the transient behaviour of the tyre.

\[
\begin{bmatrix}
F \\
M
\end{bmatrix} = A_0 \begin{bmatrix} y \\ \psi \end{bmatrix} + A_1 \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix}
\]

\[cF_t = \psi_t + C_1 \dot{\psi}_t,\]

In the preceding formula, \(F\) and \(M\) represent the lateral force and the aligning torque whereas \(\psi\) and \(\phi\) represent the lateral deflexion of the tyre and its lateral slip angle. The matrices \(A_0\) and \(A_1\) must be experimentally determined, which is not without raising some difficulties. The second equation expresses the coupling relation which exists between the lateral force and the self aligning torque in shimmy cases.
III.3 The model of Pacejka

As could well be noticed, the model used in the simulation applications are generally extremely empirical. The coefficients that describe them are determined by fittings and have no physical meaning. Because of that, no extrapolation of measured values can be made, no adaptation of the model either, to fit new situations.

This led some authors, among them Pacejka, to look for a type of mathematic function liable to represent the longitudinal lateral and self aligning forces in cases of pure slip and in steady state conditions. At the end of his research, Pacejka [1] proposes the following description for the tree forces:

\[ Y = D \sin(C \arctan(B \phi)) + \Delta S_v \]

with

\[ \phi = (1-E)(x + \Delta S_h) + \frac{E}{B}(\arctan(B(x + \Delta S_h))) \]

where \( x \) is the entry variable (longitudinal or lateral slip ratio) and \( Y \) the force.

This formula, quickly dubbed "magic formula" by the first who used it, proves very practical and can be adapted to all situations and almost to all types of tyres. Besides, the six coefficients which describe it have a physical meaning and/or a well-known influence on the curves obtained.

The influences of the different coefficients are the following ones. Coefficient \( D \) determines the maximal value reached by the curve. Product \( BCD \) represents the slope at the origin; coefficient \( C \) determines the general aspect of the curve and \( E \) permits to control the tension of the curve in its inflected part. Both coefficients \( \Delta S_v \) and \( \Delta S_h \) are but vertical and horizontal shifts allowing the best possible adjustment of the curve to the measures taken.
Under the preceding terms, the coefficients are named as such:

- **B**: slope coefficient
- **C**: form coefficient
- **D**: amplitude coefficient
- **E**: curvature coefficient
- **ΔS_v**: vertical shift
- **ΔS_h**: horizontal shift

Starting from this formula, it is possible to adjust accurately the curve to the measures taken, by using non-linear fitting techniques that are rather simple.

Then we get coefficient tables depending on the vertical load and on the camber angle. At this stage, the formula does not yet offer a prominent advantage on its competitors but progress can be made in the simplification by performing fittings of the coefficients themselves. At first, the influence of the load only is taken into account. The coefficients can then be described by the following formulas:

\[ C = 1.30 \]
\[ D = a_1F_x^2 + a_2F_z \]
\[ BCD = a_3\sin(a_4\arctan(a_5F_z)) \]
\[ E = a_6F_x^2 + a_7F_z + a_8 \]

If the camber angle is also taken into account, the formulas will be altered as follows and the supplementary formulas defining the shifts will be adopted:

\[ ΔS_h = a_9γ \]
\[ ΔS_v = (a_{10}F_x^2 + a_{11}F_z)γ \]
\[ BCD = a_3\sin(a_4\arctan(a_5F_z))(1 - a_{12}|γ|) \]
This set of formulas represents the first proposal made by Pacejka to express the relations between the coefficients describing the curve, the vertical load and the camber angle. Later, other more complex versions were proposed. They can contain up to 60 coefficients and become very difficult to handle. In the future, for easier implementing the formulas above will be adopted in the wheel model. In these formulas, the constancy of the form factor C can be observed, which is rather normal since it is the factor describing the type of force represented. Also to be noticed is the particular form of the BCD coefficient which will be studied in more detail in chapter V.

The great advantage of this model over its competitors lies in the easy control that can be exerted on the form of the force curves produced. So, any physical phenomena can be rather easily translated in terms of influence on the coefficients describing the curves. That is expressed by a sort of "variable geometry" of the model, which, by keeping an identical frame can offer a high degree of accuracy in the representation of contact forces.

With this model, post-treatment techniques have also been developed to compute the forces in combined slip cases and transient phenomena. These techniques will be approached in chapter VI.
IV

CREATION OF A WHEEL MODEL FOR
SAMCEF MECANO
IV.1 Samcef Mecano

IV.1.a Functioning and developments

The Mecano module of the Samcef software aims at the simulation of flexible multibody systems. It is based on a finite elements formalism, which offers numerous advantages for this type of model, especially concerning its taking flexibility into account and the topological representation of the mechanism. The library of finite elements available in this module now allows to solve a large number of problems of mechanical simulations.

The use of superelements permits the creation of links between Mecano and other modules of linear or non-linear dynamical analysis. In this way, you can include structures with large dimensions in a multibody model. These structures are then represented by their vibratory behaviour. It is also possible by the inverse technique to make the modal analysis of a flexible mechanism. Many elements such as Hinges springs, dampers can be defined with non-linear force laws. Finally, the user can rather easily define his own elements with a view to completing the library at his disposal.

The principal developments of the program concern the adjunction of shell and gear elements as well as the improvement of the wheel element. But still many more improvements can be imagined to be brought on the module in order to extend its capacities to simulate complete systems or its possibilities in optimisation.
IV.1.b Principles of modelling

The principal problems that arise in the matter of simulation of mechanisms are the taking into account of non-linearities due to displacement of the members, the sometimes complex topological description and the kinematic constraints. The finite elements formalism, associated with the use of cartesian coordinates are a good response to the problem of topology. The choice concerning the representation of finite rotations has fallen upon the rotation vector, which presents advantages concerning the its translation in terms of rotation matrix. As for the adoption of cartesian coordinates, it is very helpful in the expression of constraints. The constrained problem will be treated by the method of Lagrange multipliers, which allows a direct access to reactions in constraints.

Each member is represented by a certain number of nodes that define its dimensions and its characteristics. To each node are generally associated six degrees of freedom corresponding to the three possible translations and the three rotations. For each element described in this way, we can determine:

- the strain energy $\mathcal{W}$
- The potential of external forces $P$
- the kinematic energy $\mathcal{K}$
- the constraints $\Phi$

From these quantities we can write the Lagrange functions to be minimised to determine the kinematic, static or dynamic response. We have:

For the kinematic problem
$$\min_{q, \lambda} (\mathcal{W} + \lambda \Phi)$$

For the static problem
$$\min_{q, \lambda} (\mathcal{W} + P + \lambda \Phi)$$

For the dynamic problem
$$\min_{q, \lambda} (\mathcal{W} + P + K + \lambda \Phi)$$
In the case of the dynamic response, the primary variations of the variational principle lead to the following equations:

\[
\begin{align*}
\begin{cases}
M\ddot{q} + B\dot{\lambda} &= g(q, \dot{q}, t) \\
\Phi(q, t) &= 0
\end{cases}
\end{align*}
\]

After linearisation:

\[
\begin{align*}
\begin{cases}
M\Delta\ddot{q} + C\Delta\dot{q} + K\Delta q + B\Delta\lambda &= r(q^*, \dot{q}^*, t) \\
B\Delta q &= -\Phi(q^*, t)
\end{cases}
\end{align*}
\]

The mass matrix \( M \), the tangent stiffness \( K^t \), the tangent damping \( C^t \) and the constraints gradients \( B \) can be determined for each element. They will be assembled in so many matrices unique for the whole mechanism, according to its topology. So the equations written above remain valid for the whole mechanism. The superscript * indicates the approximate starting point.

**IV.1.c The rotation vector**

Given two reference frames having a common origin and different orientations, we search for a relation allowing to pass from one to another. The operator universally used for this type of transformation is the rotation matrix. It is given for example by the direction cosine of the vectors of the transformed reference frame.

\[
t_i = R E_i
\]

\[
R = \begin{bmatrix}
(E_1, t_1) & (E_1, t_2) & (E_1, t_3) \\
(E_2, t_1) & (E_2, t_2) & (E_2, t_3) \\
(E_3, t_1) & (E_3, t_2) & (E_3, t_3)
\end{bmatrix}
\]

Axes \( t \) represent axes \( E \) after transformation, we can thus write for a vector \( x \) expressed in axes \( E \)

\[
x' = Rx
\]
Matrix \( R \) contains nine terms but its orthonormal property reduces its degrees of freedom to three. So, we can define any tridimensional rotation with three parameters; for example Euler’s angles or Bryant’s angles, which correspond to series of three plane rotation, Euler’s parameters or Rodrigues’ parameters, which are more linear descriptions of finite rotations.

The technique of the rotation vector consists in defining a unique rotation around a single vector, which sends at once the basic reference frame on the transformed one. We get then four parameters: the components of the unit vector around which the rotation is made and the magnitude of the rotation. By imposing this magnitude as the norm of the rotation vector, the number of parameters is reduced to three.

\[ \psi = n\|\psi\| \]

Here, \( \psi \) represents the rotation vector, \( n \) is the direction around which one turns.

The kinematic relations between the rotation vector, its derivatives and the rotation matrix and the angular velocity and acceleration matrices are rather easy to express. We have for example:

\[ R = \exp(\tilde{\psi}) \]

Where \( \tilde{\psi} \) represents the skew-symmetric matrix corresponding with a vector, which allows among other things to express a cross product in the form of a matrix product.

\[ \Omega = T(\psi_{inc})\tilde{\psi}_{inc} \]

\[ A = T(\dot{\psi}_{inc})\tilde{\psi}_{inc} + \dot{T}(\psi_{inc})\dot{\psi}_{inc} \]
Operator T is an operator depending only on the rotation vector:

$$T(\psi_{inc}) = \frac{\sin\|\psi_{inc}\|}{\|\psi_{inc}\|} \mathbf{I} + \left(1 - \frac{\sin\|\psi_{inc}\|}{\|\psi_{inc}\|}\right)n^n - \frac{1}{2} \left(\frac{\sin\left(\frac{\|\psi_{inc}\|}{2}\right)}{\left(\frac{\|\psi_{inc}\|}{2}\right)}\right)^2 \vec{\psi}$$

The index inc expresses the fact that we take the vector corresponding to the rotation between two consecutive time steps. That is to say the incremental rotation vector.

In the mode of operation of the Samcef Mecano module, the total rotation vector is kept as far as time step n and the incremental rotation vector between positions at time step n and n+1 is considered. We get:

$$R_{ref} = \exp(\psi_{ref})$$

$$R_{inc} = \exp(\psi_{inc})$$

$$R = R_{ref}R_{inc}$$

The rotation vector is thus an easy and economical way to represent tridimensional rotations, it permits an easy expression of the kinematic relationships with the angular velocity and acceleration matrices. It also permits an easy integration of the incremental rotations in the complete rotation.

**IV.1.3 Time integration**

The integration of the algebraic differential equations is performed with the help of an implicit scheme, either Newmark's or Hilbert-Hughues-Taylor's (HHT). In the case of Newmark's scheme the velocity and position at instant n+1 are given by:

$$\dot{q}_{n+1} = \dot{q}_n + (1-\gamma)\dot{q}_{n+1} + \gamma \ddot{q}_{n+1}$$

$$q_{n+1} = q_n + h\dot{q}_n + \frac{1}{2}(1-\beta)h^2\ddot{q}_n + \beta h^2\ddot{q}_{n+1}$$
where \( h \) is the time step and \( \gamma \) and \( \beta \) are parameters to be freely chosen. In order to get a maximal accuracy, the parameters giving a constant acceleration between the two time steps will be chosen, namely:

\[
\gamma = \frac{1}{2} \quad \beta = \frac{1}{4}
\]

Moreover, this choice of parameters ensures the unconditional stability of the integration scheme.

When HHT scheme is used, \( \gamma \) and \( \beta \) are given as a function of one parameter \( \alpha \) only:

\[
\beta = \frac{1}{4} (1 + \alpha)^2 \quad \gamma = \frac{1}{2} (1 + 2\alpha)
\]

In both cases, we finish with a system of equations of the first degree, the variables of which are the degrees of freedom and the Lagrange multipliers. This will be solved for example with a Newton-Raphson's algorithm.

\[
\begin{bmatrix}
S & B^T \\
B & 0
\end{bmatrix}
\begin{bmatrix}
\Delta q \\
\Delta \lambda
\end{bmatrix} =
\begin{bmatrix}
r(q^*) \\
\Phi(q^*, t)
\end{bmatrix}
\]

In the preceding equations, matrix \( S \) is called the iteration matrix. The accuracy with which this matrix is computed has no influence on the accuracy of the result but it is of capital importance to ensure a good convergence of the scheme. As for the accuracy of the result, it is conditioned by the quality of evaluation of the residue \( r \) and thus by the accuracy on the inertia, elastic, and damping forces acting on the element.
IV.2 Strategy for the construction of the wheel element

A wheel model already exists in the Mecano software or actually, two models exist. One is a classical wheel model in which only a vertical load and a horizontal friction force are computed. The vertical force is given by a polynomial of the 4th degree, whereas the horizontal force is given by a single friction coefficient depending on the slip speed of the wheel. The other wheel model is aimed at the simulation of shimmy. The forces are there given by a set of differential equations determining the self-aligning torque and the side force. The tyre model used for this element is the one due to Black and Moreland.

Rather than by a modification of one of these models, the new wheel model will be introduced by using the user element possibilities of Samcef Mecano. To build this user element, it is necessary to specify the forces acting at the wheel centre and the iteration matrix. The entry variables for this element are the position, velocity and acceleration of the degrees of freedom of the nodes composing the element. It is moreover possible to introduce a certain number of general parameters which allow to specify some properties of the element. These will help in this particular case to specify the parameters to be introduced in the tyre model used. It is also possible to keep some variables defining the state at the preceding time step but this characteristic will not be used in the present work.

The diversity of problems integrating wheel elements is such that it is utopic to hope to be able to solve all of them by using one and the same tyre model. In the construction of the wheel element, some care has been taken concerning the kinematic interface. Besides, the construction of the element is such that it is liable to function with different tyre models. The element functions as follows:
Acquisition of general data defining the wheel
Determination of the contact point
Computation of the contact point velocity
Tyre deflection and slip quantities
Call to the tyre subroutine
  Tyre subroutine
    Acquisition of the tyre model parameters
    Computation of forces
    Computation of the derivatives of forces with respect to the entry variables
Computation of forces acting at the wheel centre
Computation of the iteration matrix

By using such a resolution scheme for this problem, it is possible to change the tyre model without having to alter the whole procedure. Later, it will only be necessary to alter the procedure for the user to have a direct choice between different tyre models.
IV.3 Kinematic description

The whole modelling of the wheel is based of the contact point theory, that is to say that in the future, it will be considered that the tyre-road interface can be summed up to one and only one contact point. Besides, in the determination of this point, only the radial deformation of the wheel will be taken into account. The latter hypothesis is such that the tyre models in which the forces are given by a set of differential equations containing a lateral and a torsional deformation will not possibly be used with this kinematic description.

In order to build a wheel model valid for numerous applications, it is necessary that the kinematic interface of the wheel should be as accurate as possible. At first, it will be attempted to describe a wheel with non zero camber angle and on non flat road. However, a restriction will at once be imposed on the shape of the ground, which will be represented by a one dimension function $Y=f(x)$ extruded along axis $Z$. In this way a ground of corrugated iron type will be obtained.

In the solution of the problem 4 reference frames interfere, which are important for the comprehension of what is next :

- The spatial or Galilean reference frame (G)
- The material reference frame, fixed to the wheel. It follows the movements of spin, camber and yaw of the wheel (M)
- The road reference frame, associated to the origin of the road, it allows to define the directions $X$, $Y$ and $Z$ of the road profile with respect to the Galilean reference frame (RRF)
- The contact point reference frame, associated to the tangent plane to the road containing the contact point, and the wheel plane. It permits the definition of the directions along which the forces act (CPF).
The entry variables available in the Samcef formalism are the following ones:

- $x_a$: position of the wheel centre (G)
- $\psi_a$: incremental rotation vector (M)
- $\psi$: total rotation vector (G)
- $\dot{x}_a$: velocity of the wheel centre (G)
- $\dot{\psi}_a$: rotation vector of the angular velocities (M)

**fig IV.1 the four reference frames**
The data attached to the element include besides:

- \( x_p \) origin of the road
- \( x \) longitudinal direction of the road
- \( y \) normal to the ground reference plane
- \( \omega_a \) original wheel axle
- \( R_0 \) nominal radius of the wheel
- \( y(x) \) function giving the road profile

The first stage of the kinematic description goes through the determination of the contact point. An interesting property of this point is that it is such that the normal to the road passing through the contact point meets the wheel axle.

![Diagram of the contact point](image-url)

Fig IV.2 property of the contact point
When the road is not flat, the contact point cannot generally be determined analytically, except if the road profile is a simple function or if the camber angle of the wheel is zero. An algorithm of research of the contact point can be used, which is based on the minimisation of the distance between the wheel axle and the normal to the road at the contact point. Such an algorithm, based on Newton-Raphson's method, already exists in Mecano's wheel element. It can thus be considered that the contact point and the normal to the road passing by this point are known in all cases, and continue the description on this basis.

In case the road is flat, it is not necessary to determine the position of the contact point; the deformed radius and the camber angle can be determined directly.

Let be $x_p$ a point of the road plane, $x_a$ the wheel centre, $n_p$ the normal to the road plane, $v$ the wheel axle. All these quantities are known at the beginning of the procedure or can be determined by executing the correct rotations on initial quantities. To begin with, the longitudinal vector of the contact point reference frame is determined.

$$x_{lon} = v \times n_p$$

Then, we get the projection of the normal vector in the wheel plane:

$$n_{pp} = x_{lon} \times v$$

The camber angle and the deformed radius are easily determined

$$\gamma = \arccos(n_p \cdot n_{pp})$$

$$R = n_p \cdot (x_a - x_p)$$

We can possibly determine the position of the contact point

$$x_c = x_a - R n_{pp}$$
The velocity of the contact point can now be determined by differentiation of its position vector. We have:

\[ \dot{x}_c = \dot{x}_a - \dot{R} \mathbf{n}_{pp} - R \omega_a \times \mathbf{n}_{pp} \]

where the angular velocities \( \omega_a \) can be deduced from the derivative of the rotation vector by a simple relation.

The velocity computed here is the velocity of the contact point as belonging to the wheel. Thus, in the preceding formula, \( \omega_a \times \mathbf{n}_{pp} \) is not the actual derivative of vector \( \mathbf{n}_{pp} \), since this vector normally does not spin with the wheel.

In order to determine the derivative of \( \mathbf{R} \), we express it in an artificial way:

\[
\dot{R} = \frac{\mathbf{n}_p \cdot (\dot{x}_a - x_c)}{\mathbf{n}_p \cdot \mathbf{n}_{pp}}
\]

So its derivative is given by:

\[
\dot{R} = \frac{(\mathbf{n}_p \cdot \mathbf{n}_{pp})(\dot{n}_p \cdot \dot{x}_a + \dot{n}_p \cdot (x_a - x_c)) - \mathbf{n}_p \cdot (x_a - x_c)(\mathbf{n}_p \cdot \omega_a \times \mathbf{n}_{pp} + \dot{n}_p \cdot \mathbf{n}_{pp})}{(\mathbf{n}_p \cdot \mathbf{n}_{pp})^2}
\]

\[
= \frac{\mathbf{n}_p \cdot \dot{x}_a - \dot{R} \mathbf{n}_{pp} \omega_a \times \mathbf{n}_{pp}}{\mathbf{n}_p \cdot \mathbf{n}_{pp}}
\]

One can notice in this formula that the derivative of \( \mathbf{n}_p \) has no influence on the result, it is interesting for this derivative is not computable on non-flat roads. It is also to be noted that the wheel spin has no influence either. Indeed if we consider the wheel spin only, the vectors \( \mathbf{n}_p \), \( \mathbf{n}_{pp} \) and \( \omega_a \) are in a same plane and so their mixed product is zero. This product is kept however to take the effects of camber and yaw on the deformed radius into account.
In the formula giving the contact point velocity, the deformed radius appears. It will be replaced by an approximation of the effective radius, which allows a better approach of the kinematic reality of the wheel.

\[ R_e = \frac{2R_h + R}{3} \]

\[ \dot{x}_c = \dot{x}_e - \frac{n_p \times n_p \times \omega_s \times n_{pp} + R_s n_{pp} \times \omega_s}{n_p, n_{pp}} \]

It is now possible to compute the slip ratios necessary for the computation of forces. Therefore, the contact point reference frame vectors must first be determined:

\[ \mathbf{e}_1 = \frac{x_{ion}}{\|x_{ion}\|} = \frac{\mathbf{v} \times \mathbf{n}_p}{\|\mathbf{v} \times \mathbf{n}_p\|} \]

\[ \mathbf{e}_2 = \mathbf{n}_p \times \mathbf{e}_1 \]

\[ \mathbf{e}_3 = \mathbf{n}_p \]

Here, a major problem arises in the determination of the slip ratios. Indeed, for the computing of the slip quantities, it is necessary to know the slip speed of the contact point (velocity of the contact point as belonging to the wheel) but also the translation speed of this point (velocity of the contact point as belonging to the ground). On flat road, it does not raise any problem, for the translation speed of the contact point is the same as the translation speed of the wheel centre. On non flat road, on the other hand, there can be a great difference between these two speeds, due to the ground curvature.
fig IV.3 "ground effect on the velocities at the wheel centre and the contact point

This simplified case of a wheel rolling without slipping on a curved surface shows quite well the possible difference between the at the wheel centre and the speed at the contact point. This phenomenon is rather unimportant if the curvature radius of the road is large but raises serious problems if the road is defined by a piecewise function.
fig IV.4 Acceleration due to the passing of the wheel over an apex

The problem appearing here does not only lie in the existence of this difference but in the impossibility to appraise it analytically (since it depends on the derivative of $np$, which is not known) except on very simple road profiles.

In the future, only the simplified case of flat roads will be considered, given the problem raised by the computation of slip ratios. However, non flat roads will be mentioned as well as methods to be explored to solve the deriving problems in chapter VI.

In cases of flat roads, we get for the slip ratios:

$$\tan(\alpha) = \frac{\dot{x}_c \cdot e_2}{\dot{x}_a \cdot e_1} \quad \kappa = \frac{-\dot{x}_p \cdot e_1}{\dot{x}_a \cdot e_1}$$
IV.4 Tyre model

After the kinematic description given above, we get the slip ratios and the contact point frame vectors. These allow us to use several tyre models. Among these, the model of Pacejka has been chosen for it seems to be the most supple to be used and parameters describing the tyres (for automobiles at least) are available with this model.

The version of the model of Pacejka used is the simplest among those incorporating the effects of the vertical load and of the camber angle. The vertical load model is a polynomial of the 4th degree, which seems to be accepted as the most adequate one by many authors. The forces are thus given by:

Vertical force
\[ F_z = a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4 \]
\[ w = R - R_0 \]

Longitudinal force
\[ F_x = D \sin(C \arctan(B \phi)) \]
\[ \phi = (1 - E) \kappa + \left( \frac{E}{B} \right) \arctan(B \kappa) \]
\[ C = 1.65 \]
\[ D = a_1 F_z^2 + a_2 F_z \]
\[ BCD = \frac{a_3 F_z^2 + a_4 F_z^3}{\exp(a_5 F_z)} \]
\[ E = a_6 F_z^2 + a_7 F_z + a_8 \]
Lateral force

\[ F_y = D \sin(C \arctan(B \phi)) + \Delta S_v \]

\[ \phi = (1 - E)(\alpha + \Delta S_h) + \left( \frac{E}{B} \right)(\arctan(B(\alpha + \Delta S_h))) \]

\[ C = 1.30 \]

\[ D = a_1 F_z^2 + a_2 F_z \]

\[ BCD = a_3 \sin(a_4 \arctan(a_5 F_z))(1 - a_1 |\gamma|) \]

\[ E = a_6 F_z^2 + a_7 F_z + a_8 \]

\[ \Delta S_h = a_9 \gamma \]

\[ \Delta S_v = (a_{10} F_z^2 + a_{11} F_z) \gamma \]

Self aligning torque

\[ M_z = D \sin(C \arctan(B \phi)) + \Delta S_v \]

\[ \phi = (1 - E)(\alpha + \Delta S_h) + \left( \frac{E}{B} \right)(\arctan(B(\alpha + \Delta S_h))) \]

\[ C = 2.40 \]

\[ D = a_1 F_z^2 + a_2 F_z \]

\[ BCD = a_3 F_z^2 + a_4 F_z \exp(a_5 F_z)(1 - a_{12} |\gamma|) \]

\[ E = (a_6 F_z^2 + a_7 F_z + a_8) / (1 - a_{13} |\gamma|) \]

\[ \Delta S_h = a_9 \gamma \]

\[ \Delta S_v = (a_{10} F_z^2 + a_{11} F_z) \gamma \]
The slip ratio $\kappa$ will be given in % whereas the slip angle will be given in degrees. The derivatives of the forces with respect to their entry variables must be computed together with the forces in the tyre model subroutine. These values will be introduced in the iteration matrix. In the computational construction of the model, the computing of forces is made in three stages. The first stage consists in computing coefficients $B$, $C$, $D$, $E$, $\Delta Sv$ and $\Delta Sh$ and their derivatives with respect to the vertical force and the camber angle. A subroutine is then called upon, which, according to these coefficients and to the slip quantity, computes the forces and their derivatives with respect to $B$, $D$, $E$, and the slip ratio. The formulas being rather complicated, the derivatives can be computed using a special software of symbolic derivation (e.g. Maple). We get:

\[ Y = D \sin(C \arctan(B \phi)) + \Delta S_v \]

\[ \phi = (1 - E)(x + \Delta S_h) + \left(\frac{E}{B}\right)(\arctan(B(x + \Delta S_h))) \]

Preliminary derivatives are computed:

\[ \frac{dY}{d(B \phi)} = \frac{CD}{1 + (B \phi)^2} \cos(C \arctan(B \phi)) \]

\[ \frac{d\phi}{dB} = \frac{E}{B^2} \arctan(B(x + \Delta S_h)) + \frac{E}{B} \frac{x + \Delta S_h}{1 + (B(x + \Delta S_h))^2} \]

\[ \frac{d\phi}{dE} = -(x + \Delta S_h) + \frac{1}{B} \arctan(B(x + \Delta S_h)) \]

\[ \frac{d\phi}{dx} = 1 - E \left(1 - \frac{1}{1 + (B(x + \Delta S_h))^2}\right) \]
We get next:

\[
\frac{dY}{dx} = B \frac{dY}{d(B\phi)} \frac{d\phi}{dx}
\]

\[
\frac{dY}{dB} = \frac{dY}{d(B\phi)} \left( B \frac{d\phi}{dB} + \phi \right)
\]

\[
\frac{dY}{dD} = \sin(C\arctan(B\phi))
\]

\[
\frac{dY}{dE} = B \frac{dY}{d(B\phi)} \frac{d\phi}{dE}
\]

In the third stage, the assembly of derivatives is performed in order to build the complete derivatives of the forces with respect to \(F_z\), \(\gamma\), \(\alpha\), or \(\kappa\). We get for example:

\[
\frac{dF_z}{dF_z} = \frac{dF_z}{dB} \frac{d\beta}{dF_z} + \frac{dF_z}{dD} \frac{d\gamma}{dF_z} + \frac{dF_z}{dE} \frac{d\alpha}{dF_z} + \frac{dF_z}{dS} \frac{d\kappa}{dF_z}
\]

\[
\frac{dM_z}{d\gamma} = \frac{dM_z}{dB} \frac{d\beta}{d\gamma} + \frac{dM_z}{dE} \frac{d\gamma}{d\gamma} + \frac{dM_z}{dS} \frac{d\gamma}{d\gamma} + \frac{dM_z}{dS} \frac{d\kappa}{d\gamma}
\]

All the other derivatives of the same kind are built in a similar way.

In order to render the first implementation of the element easier, the case of combined slip will not be treated here. It will be partly done in chapter VI.
IV.5 Nodal forces and iteration matrix

IV.5.a Nodal forces

The contact forces, such as given by the tyre model are not directly applicable to the degrees of freedom of the element. It is necessary to make them undergo a transformation in order to determine the nodal forces.

In the finite elements formalism, the forces are considered as internal forces of the element. They are non conservative and so, cannot derive from a potential. Thus, we use the virtual work principle to determine the nodal forces corresponding to the admissible displacements of the degrees of freedom. We have:

$$\delta w F + \delta x \cdot \left( F_x e_1 + F_y e_2 \right) + \delta \varphi . M_x e_3 = \delta q . C$$

To be able to determine $G_{int}$ in this formula, it is necessary to express admissible virtual displacements $\delta w$, $\delta x$, and $\delta \varphi$ in terms of variation of the degrees of freedom. We get:

$$\delta w = - \delta R = \frac{\delta x \cdot n_p - R n_p \cdot \delta \varphi \times n_p}{\cos(\gamma)}$$

$$= \frac{\delta x \cdot n_p + R \delta \varphi \cdot R^T n_p \cdot n_p}{\cos(\gamma)}$$

For the preceding formula and those that follow, the following notations are used:

- dot product of two vectors
- matrix product
- cross product
- skew-symmetric matrix corresponding to a vector
- $a \ast b$ equivalent form of the cross product
- $\delta \varphi$ variation of the spatial angular position
- $\delta \theta$ variation of the material angular position
- $T$ transpose matrix
\[ \delta \mathbf{c}^T = \delta \mathbf{a}^T - \delta \mathbf{R} \mathbf{n}_{pp}^T + \mathbf{R} (\mathbf{n}_{pp} \times \delta \mathbf{a})^T \]

\[ = \delta \mathbf{a}^T - \delta \mathbf{R} \mathbf{n}_{pp}^T - \mathbf{R} (\delta \mathbf{a}^T \mathbf{R} \mathbf{n}_{pp}^T \mathbf{n}_{pp}) \]

By replacing \( \delta \mathbf{R} \) with its expression and by bringing the terms together, we finally get for \( \delta \mathbf{c}^T \):

\[ \delta \mathbf{c}^T = (\delta \mathbf{a}^T - \mathbf{R} \delta \mathbf{a}^T \mathbf{R} \mathbf{n}_{pp}^T \mathbf{n}_{pp}) \left( I - \frac{\mathbf{n}_{pp} \mathbf{n}_{pp}^T}{\cos(\gamma)} \right) \]

Then we can write the vector of nodal forces

\[
G_{\text{int}} = \begin{bmatrix}
\frac{\mathbf{n}_{pp} \cdot F_c}{\cos(\gamma)} & \left( I - \frac{\mathbf{n}_{pp} \mathbf{n}_{pp}^T}{\cos(\gamma)} \right) (F_x c_1 + F_y c_2) \\
\frac{\mathbf{R}^T \mathbf{n}_{pp} \cdot R}{\cos(\gamma)} & \frac{- \mathbf{R}^T \mathbf{n}_{pp} \cdot R}{\cos(\gamma)} \left( F_x c_1 + F_y c_2 \right) + \mathbf{R}^T \mathbf{n}_{pp} \cdot M_z
\end{bmatrix}
\]

A certain number of remarks are to be made concerning the transformation that the forces undergo in their application to the degrees of freedom. In the beginning, the usually named vertical force is radial, that is to say acts along \( \mathbf{n}_{pp} \). We see that the transformation renders it vertical at once but divides it by \( \cos \gamma \). By way of simplification, this division will be suppressed, which makes us consider the force not a radial but a vertical one from the start. This simplification can be justified by the fact that the physical model which is at the basis of the computation of these nodal forces - a wheel that is only deformed radially - is not an accurate description of the real behaviour of the wheel. The appearing of the division by \( \cos \gamma \) is due to the rigidity of the wheel model outside the wheel plane, rigidity that does not exist reality.

For the same reason, the nodal forces conjugated to both horizontal forces are also altered. Indeed, when examining the way in which the side force is transformed, we see that it gives rise to a vertical component. This state of things is all the more disturbing that it prevents from computing the side force explicitly for this depends on the vertical force.
The parry consists once more in suppressing the effects of stiffness outside the wheel plane by suppressing the factor \((1 - n_p n_{pp}^T)\) of the nodal forces vector.

We finally get for the nodal forces:

\[
G_{\text{sd}} = \begin{bmatrix}
F_x c_1 + F_y c_2 \\
- R_s^T n_p n_{pp} R F_x - R_s^T \tilde{n}_{pp} R (F_x c_1 + F_y c_2) + R_s^T n_p M_z
\end{bmatrix}
\]

IV.5.b Iteration matrix

Whatever the integration scheme retained (Newmark or HHT), the iteration matrix has the following form:

\[
S = aK^t + bC^t + cM
\]

In this formula, the mass matrix \(M\) is determined by the kernel of the program and the coefficients \(a, b\) and \(c\) depend on the integration scheme. So the user limits himself to specifying the tangent stiffness matrix \(K^t\) and the tangent damping matrix \(C^t\) of the element. Besides, it is not necessary to compute a constraint gradient matrix since the element contains no kinematic constraint.

Just like the nodal forces, the iteration matrix can be computed from a variational principle.

\[
\delta G = K^t \delta q + C^t \delta q
\]

A matrix form of the vector of nodal forces is adopted to make the operations easier

\[
G = A^* F_x c_3 + B^* (F_x c_1 + F_y c_2) + C M_z c_3
\]

\[
A = \begin{bmatrix} I \\ R_s^T \tilde{n}_{pp} \end{bmatrix} \quad \quad \quad B = \begin{bmatrix} I \\ - R_s^T \tilde{n}_{pp} \end{bmatrix} \quad \quad \quad C = \begin{bmatrix} I \\ R_s^T \end{bmatrix}
\]

Then we get for the variation of \(G\) vector

\[
\delta G = \delta A^* F_x + \delta B^* (F_x c_1 + F_y c_2) + \delta C M_z c_3 + A \delta F_x + B (\delta F_x c_1 + F_x \delta c_1 + \delta F_y c_1 + F_y \delta c_2) + C (\delta M_z c_3 + M_z \delta c_3)
\]
It is known that the accuracy of the evaluation of the iteration matrix is not essential to the achievement of the integration. A rather important number of simplifications can thus be made in order to reduce the computation load even if it necessitates a larger number of iterations to converge. So, in the preceding expression, factors $\delta A$, $\delta B$, $\delta C$, $\delta e_1$, $\delta e_2$, and $\delta e_3$ will be considered null, which means neglecting the geometrical stiffness of the element. We shall limit ourselves to computing the force variations of the tyre model.

$$\delta F_z = \frac{dF_z}{dw} \delta w$$

$$\delta F_x = \frac{dF_x}{d\kappa} \delta \kappa + \frac{dF_x}{df_x} \delta f_x + \frac{dF_x}{dy} \delta y$$

$$\delta F_y = \frac{dF_y}{d\alpha} \delta \alpha + \frac{dF_y}{df_y} \delta f_y + \frac{dF_y}{dy} \delta y$$

$$\delta M_z = \frac{dM_z}{d\alpha} \delta \alpha + \frac{dM_z}{df_z} \delta f_z + \frac{dM_z}{dy} \delta y$$

All the derivatives of the forces with respect to variables $\alpha$, $\kappa$, $F_z$, and $\gamma$ have already been computed in the tyre model subroutine. There remains to evaluate the variations of $\delta q$ and $\delta q$.

The determination of $dw$ and $dg$ is direct surpriseless

$$\delta w = -\frac{n_p^T \delta \kappa_a + R n_p^T \tilde{n}_{pp}^T R_a \delta \theta_a}{\cos(\gamma)}$$

$$= \begin{bmatrix} \frac{n_p^T}{\cos(\gamma)} & -R n_p^T \tilde{n}_{pp} \frac{R_a}{\cos(\gamma)} \end{bmatrix} \begin{bmatrix} \delta \kappa_a \\ \delta \theta_a \end{bmatrix}$$

$$= U \delta q$$

$$\delta y = \begin{bmatrix} 0 & \frac{n_p^T \tilde{n}_{pp} \frac{R_a}{\sin(\gamma)}}{\sin(\gamma)} \end{bmatrix} \begin{bmatrix} \delta \kappa_a \\ \delta \theta_a \end{bmatrix}$$

$$= V \delta q$$

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In order to compute $\delta \alpha$ and $\delta \chi$ it is necessary to determine $\delta \chi_c$ (which is done in a straightforward way)

$$\delta \chi_c = \left[ I \ 0 \right] \left[ \begin{array}{c} \delta \chi_a \\ \delta \theta_a \end{array} \right]$$

$$\delta \chi_c = P \left[ \begin{array}{c} \delta \chi_a \\ \delta \theta_a \end{array} \right] + Q \left[ \begin{array}{c} \delta \chi_a \\ \delta \theta_a \end{array} \right]$$

$\chi_c$ is given by the expression

$$\dot{\chi}_c = \left( I - \frac{n_{pp}^T n_p}{\cos(\gamma)} \right) \left( \dot{\chi}_a + \tilde{n}_{pp} \times R \omega_a \right)$$

Two remarks are to be made concerning this equation. On one hand, the spatial angular velocity $\omega_a$ was computed beforehand and can be kept as such in the equations; on the other hand, the first factor of the expression giving $\dot{\chi}_c$ is constant for the two contributions of $n_{pp}$ neutralise each other in the expression of the derivative. Consequently we get for $\dot{\chi}_c$:

$$\delta \chi_c = \left( I - \frac{n_{pp}^T n_p}{\cos(\gamma)} \right) \left( \dot{\chi}_a + \tilde{n}_{pp} \times R \omega_a \right)$$

$$P = \left[ \begin{array}{cc} I - \frac{n_{pp}^T n_p}{\cos(\gamma)} & \left( I - \frac{n_{pp}^T n_p}{\cos(\gamma)} \right) \tilde{n}_{pp} \times R \right]$$

$$Q = \left[ \begin{array}{cc} I - \frac{n_{pp}^T n_p}{\cos(\gamma)} & \tilde{n}_{pp} \times \omega_a \times n_p \tau \\ \tilde{n}_{pp} \times \omega_a \times n_p \tau & I - \frac{n_{pp}^T n_p}{\cos(\gamma)} \end{array} \right]$$

$$= \left( R \omega_a \times \tilde{n}_{pp} \times R_{pp}^{\tau} + \tilde{n}_{pp} \times \omega_a \times R \tilde{n}_{pp} \frac{n_{pp}^T n_p}{\cos(\gamma)} \right)$$

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Starting from matrices $P$ and $Q$ it is rather easy to deduce the expressions of $\delta \alpha$ and $\delta \kappa$. Let us not forget, when effecting the derivatives of $\alpha$ and $\kappa$ that the contact point frame vectors are considered constant. So we get:

$$\delta \alpha = \frac{(e_1^T \cdot \dot{x}_a)(e_2^T \cdot \dot{x}_a) - (e_2^T \cdot \dot{x}_c)(e_1^T \cdot \dot{x}_c)}{(e_1^T \cdot \dot{x}_a)^2}$$

$$\delta \kappa = \frac{(e_1^T \cdot \dot{x}_a)(e_1^T \cdot \dot{x}_c) - (e_1^T \cdot \dot{x}_c)(e_1^T \cdot \dot{x}_a)}{(e_1^T \cdot \dot{x}_a)^2}$$

A matrix form is used as above

$$\delta \alpha = W \cdot \delta \dot{q} + X \cdot \delta q$$

$$\delta \kappa = Y \cdot \delta \dot{q} + Z \cdot \delta q$$

Matrices $W$, $X$, $Y$ and $Z$ are deduced

$$W = \frac{c_2^T \cdot P}{c_1^T \cdot \dot{x}_a} \cdot \frac{(c_2^T \cdot \dot{x}_c) c_1^T \cdot T}{(e_1^T \cdot \dot{x}_c)^2}$$

$$X = \frac{c_2^T \cdot Q}{c_1^T \cdot \dot{x}_a}$$

$$Y = \frac{c_2^T \cdot P}{c_1^T \cdot \dot{x}_a} \cdot \frac{(c_1^T \cdot \dot{x}_c) c_1^T \cdot T}{(e_1^T \cdot \dot{x}_c)^2}$$

$$Z = \frac{c_1^T \cdot Q}{c_1^T \cdot \dot{x}_a}$$

All the elements necessary to the construction of the iteration matrix are now available. So we can easily find for the tangent stiffness and damping matrices:

$$\delta G = A \delta F_e + B (\delta F_e c_1 + \delta F_\nu c_2) + C \delta M_e c_3$$
\[ \mathbf{K}^t = \left( A \mathbf{e}_1 + \frac{dF_y}{d\kappa} \mathbf{B} \mathbf{e}_1 + \frac{dF_y}{d\zeta} \mathbf{B} \mathbf{e}_2 + \frac{dM}{d\zeta} \mathbf{C} \mathbf{e}_3 \right) \frac{dF_z}{d\omega} \mathbf{U} + \left( \frac{dF_x}{dy} \mathbf{B} \mathbf{e}_1 + \frac{dF_y}{dy} \mathbf{B} \mathbf{e}_2 + \frac{dM_z}{dy} \mathbf{C} \mathbf{e}_3 \right) \mathbf{V} + \frac{dF_x}{d\kappa} \mathbf{B} \mathbf{e}_4 \mathbf{Z} + \left( \frac{dF_y}{d\alpha} \mathbf{B} \mathbf{e}_2 + \frac{dM_z}{d\alpha} \mathbf{C} \mathbf{e}_3 \right) \mathbf{X} \]

\[ \mathbf{C}^t = \frac{dF_x}{d\kappa} \mathbf{B} \mathbf{e}_1 \mathbf{Y} + \left( \frac{dF_y}{d\alpha} \mathbf{B} \mathbf{e}_2 + \frac{dM_z}{d\alpha} \mathbf{C} \mathbf{e}_3 \right) \mathbf{W} \]
V
IMPLEMENTING THE ELEMENT, TESTS AND RESULTS
V.1 Fitting of the parameters of the model

Before venturing into simulation it is necessary to fix the parameters to be introduced into the tyre model in order to describe it. Therefore a number of measurements are necessary. The most practical way in the present case is to resort to force measurements. Ideally it would be best to make measurements on the spot in the real functioning conditions but with regard to the prohibitive costs of such measurements most of the tests will be performed in laboratories. That type of measurements is generally done by rolling the tyre on an even cylinder. By controlling the relative orientations of the tyre and the cylinder as well as their relative rotations one can simulate the slip factors and the camber angle. The measurements will also be done for different vertical loads. Those measurements should first be corrected in order to suppress the influence of the curvature of the cylinder used and to take into account the difference between the quality of the cylinder surface and that of the road. The manufacturers who apply those tests usually know rather well the kind of corrections to be applied and we obtain valuable measurements at this stage. One can complete those force measurements by vertical, longitudinal, lateral and torsional stiffness which will enable to fix rates for definite coefficients.

Once those measurements done, one can work in two different ways to fix the coefficients of the model which will describe the measured forces. Either you adjust the basic coefficients for each force curve and then you determine the way in which those coefficients vary with the vertical load and the camber angle, or you fix a set of formulas to describe the variation of the basic coefficients and you include those formulas in the curve model and you apply only one regression on the whole of the measurements. The latter technique has the advantage of fixing all the necessary coefficients in one step but it is necessary to have predetermined the formulas that define the basic coefficient and the algorithms to be used in this case are more complicated. The former technique has as an advantage to be easier to use and more flexible the more if allows to notice more easily if a problem shows up in the regression. That is the reason why it will be used here.
We have the measurements made on an airplane tyre at our disposal. Those measurements represent the side force and the self aligning torque for drift angles up to 8° and vertical loads up to 100 KN. We fix the coefficients by non-linear regression. The used algorithm is the Nelder-Mead simplex, implemented in the non-linear regression function of the Matlab program. The adjustment of the data to the side force does not present too many problems: at the utmost should you allow the coefficient C to vary somewhat and accept a vertical translation even for a non-zero camber. You can yet already notice that the fact that the range of drift angles is not very wide will prejudice the stability of the parameters. Ideally the range of the drift angles should exceed the maximum of the force curve.

Those problems increase largely when you try to apply the same type of regression on the curves of the self aligning torque. Here appears a more crucial problem: The general trend of the torque curve is absolutely not the same in the figure data as in the model to be adjusted. It is yet possible to work a resetting, but therefore it is necessary to relax the C coefficient and the vertical and horizontal shifts of the curve. You can then wonder if the coefficients keep their physical significance.
fig V.1 data representing the self aligning torque for an airplane tyre

fig V.2 Curves to be fitted to the self aligning torque data above

The reasons of such a difference in behaviour don’t appear very clearly. One can yet note an important number of rather fundamental differences between the two problems at hand.
On one side the measurements have been done for an airplane tyre. This one is intended to work by very high vertical loads and by rather small drift angles. This sort of tyre is the bias type.

The model of Pacejka on the other hand was chiefly contrived for application to automobile tyres behaviour. The modelled tyres are of the radial type. In addition to that, here the working conditions involve low vertical loads and drift angles that can reach important values.

There remains to know whether those differences alone can justify such incompatibility between the model of Pacejka and the measurements made. The answer to that question would require deep studies on airplane tyres and their force curves and lies out of the range of the present work.
V.2 Behaviour of the model by high loads

Due to the problems encountered by the resetting of the data about an airplane tyre, the first trials of the element will be done with a car tyre. The big advantage is that the coefficients describing that tyre are available in the literature. We shall yet keep the problem for some trials in order to be able to examine the general behaviour of the element in that case. It will be necessary to pay attention to a characteristic of the model of Pacejka which is its behaviour under high vertical loads. Actually, the way in which the coefficients B, C, D and E are described in function of the vertical load makes it that there is a limited field of validity for the model. Over a definite vertical load the contact forces will start decreasing highly and will unavoidably lead to erroneous simulations;

fig V.3 Evolution of coefficient BCD of the lateral force with the vertical load

With the available data we shall not take the risk to exceed 40 Kn. So it will be necessary to adapt the characteristics of the landing gear in order not to exceed that load. It is not very difficult to perform. It is sufficient to reduce the mass of the plane and the vertical nearing speed and to adapt the stiffness and the damping of the shock-absorber of the landing gear in order to avoid a rebounding of the plane.
If we wish to cope with the problems where higher vertical loads appear it will be necessary to adapt the coefficients of the functions describing B C D an E to those higher loads. That too is not too difficult to perform but you cannot be sure that the modification of the model will be done correctly. Anyway as the used tyre is not a plane tyre the results will have no quantitative value at all.
V.3 Description of the test cases

For the tests, we use a forward landing gear model. This model is very simple and does not correspond to an existing model. The values for the stiffness and the damping of shock-absorbers have been chosen in order to avoid the problems of excessive crushing and rebounding. As the behaviour of the model is not safe for high loads all the tests have been performed with a reduced load and non excessive speeds but yet higher than the nearing speed of a plane landing under normal conditions.

![Diagram of landing gear model used for tests](image)

fig V.4 landing gear model used for tests
Element 1: rigid body representing the plane resting on the landing gear

\[ m: 400 \text{ Kg} \quad I_x = I_y = I_z = 5 \times 10^6 \text{ Kg.m}^2 \quad \text{length: 80 cm} \]

Element 2: circular steel beam

\[ \text{Rint 2.5 cm Rext 4.5cm} \quad \text{length: 60 cm} \]

Element 3: main shock absorber

\[ K : 1 \times 10^5 \text{ N/m} \quad C : 1 \times 10^4 \text{ N.s/m} \quad \text{length: 50 cm} \]

Element 4: secondary shock absorber

\[ K : 4 \times 10^5 \text{ N/m} \quad C : 4 \times 10^4 \text{ N.s/m} \quad \text{length: 1.05 m} \]

Elements 5, 6, 7: hinges, the main shock absorber not being hinged with the gear supporting beam

Element 8: wheel element, mounted on two hinges, one vertical and one horizontal. The vertical hinge is damped to avoid wobble of the wheel.

\[ K : 1 \times 10^6 \text{ N.m/rad} \quad C : 1 \times 10^5 \text{ N.m.s/rad} \]

The 56 coefficients that characterise the wheel and the road (non distorted radius, initial axis, force model coefficient etc...) are introduced into the data bound to the wheel. You will find their values in appendix 2 and 3.

The role of the secondary shock absorber is mainly to avoid longitudinal vibrations induced by the drag of the spin up of the wheel. So the results are free of any perturbing effect.

As the implemented force model is not foreseen for combined slips, we have tried to near as much as possible the case of pure slip in the tests. The force test consists of a vertical impact with a non-zero translation speed, the wheel being initially without any spin. So we can test its spin up and the drag induced by this as well as the vertical force model. For this test the wheel is blocked in the lateral direction.
The second test is double. The question is to simulate a pure case of lateral slip. We perform it either by giving a transversal speed to the wheel or by offsetting the wheel according to its longitudinal position. To perform this test we try at the start to realise the most possible stable conditions i.e. that we give the wheel the crushing corresponding to the nominal vertical load and the spin corresponding to the translation speed at the distorted radius. The initial vertical speed is of course zero.
V.4 Results of the tests

V.4.a Vertical impact

We simulate the oncoming of the gear described in the preceding paragraph with a vertical speed of 4 m/s and a horizontal speed of 50 m/s. As explained previously the stiffness of the shock absorbers have not been chosen to simulate exactly a landing gear but to avoid rebounding or a too important crushing.

![Graph showing radial deflection of the tyre on impact](image)

The graph of the radial deflection shows that the impact shock is very quickly absorbed. After 0.4 sec. the vertical load already reaches its nominal value. So we can also see that the maximal load, which is enormous if you take into account the fact that the nominal load is already rather high with regard to the size of the tyre.
The effects of the spin up drag are felt for a much longer time than that of the impact. After more than 2 seconds there is still a spin up drag. One will note in the very first hundredths of a second that the spin up drag is very important. This is due to the high vertical load that the wheel undergoes at that moment. This makes that the landing gear also undergoes a longitudinal shock by impact. For the absorption of this shock the secondary shock absorber plays an important role. Without it one would observe important vibrations in the whole of the gear. Those vibrations may reflect at the level of the tyre and pervert the results particularly if you work with very small time steps.

The results observed for this test are appreciably the same if you use the classical wheel element of Mecano or the new wheel element. This is completely understandable as the classical element is apt to represent correctly the vertical and longitudinal forces. If the vertical loads become very high one will note a difference between the two models as the classical model is proportional to the vertical load while the new one leads to saturation of the longitudinal force in regard of the vertical force.
V.4.6 Side slip due to a lateral speed

We perform here two tests aiming at simulating a pure side slip. For each test the obtained results with each type of wheels are compared.

The first test consist in giving the wheel an initial speed of 50 m/s and a side speed of 10 m/s. This corresponds to a slip angle slightly over 10 degrees.

![Graph of side slip]

fig V.7 lateral movement of the drifted wheel for the classical wheel (to the left) and the new wheel (to the right)

The graphs showing the side slip of the tyre are a fair reflection of the trajectory followed by the wheel, the longitudinal speed being practically constant. We note in both cases a tendency of the wheel after a definite time to resume the longitudinal direction. It is the effect of the side drag induced by the side slip of the tyre. One can see that the effect is more important in the case of the new model. The longitudinal direction is resumed more quickly and the total drift is lower.
fig V.8 evolution of the wheel spin velocity for the classical wheel (l) and the new wheel (r)

By observing the spin speed of the wheel one can show one of the advantages of the new model. We can indeed note that the steadied spin speed - and by the way the final translation speed - is not exactly the same from one case to the other. It is lower for the classical wheel. This is due to the fact that with the classical wheel there is no separation between the longitudinal and the lateral forces. So there is a longitudinal component to the friction force which depends on the total force and the slip angle. This component creates a longitudinal drag that works for a rather long time and brakes the wheel.

In the case of the new wheel, the longitudinal and lateral forces are clearly separated. So the lateral force has no influence on the longitudinal behaviour of the wheel. One can see here that the wheel settles more quickly longitudinally and that the total drag is less important.
The present analysis requires yet some commentaries. The new wheel as it is implemented at this level does not include a force model for the combined cases of slip. You cannot fully trust the results obtained about the longitudinal behaviour. We shall nevertheless retain some independence of the forms in combined cases (see ch.VI) especially for the low slips as it is the case here.

![Graph](image)

fig V.9 yaw angles of the wheel for the classical wheel (l) and the new one (r)

We also notice for the new wheel the effect of the self aligning torque on the orientation of the wheel. For reasons that do not clearly appear, the vertical hinge angle at the level of the wheel has a tendency to open up: when the airplane shifts towards the left, the wheel tends to shift towards the right with regard to the airplane. We can see with the new model a definite tendency of the wheel to come back to the direction of the airplane, to the left. This tendency is due to the self aligning torque which as its name shows tends to align the wheel with its speed vector.
The passing of that torque over positive values at the very beginning is due to the fact that for important drift angles the torque shifts sign and is no longer self aligning. The maximum value for this torque is obtained here about 3° of drift angle and the reverse value about 10°.

**V.4.c Side slip due to the deviation of the wheel**

We give now an initial deviation of 0.1 radian to the wheel. The initial speed is 50 m/s and purely longitudinal. The drift angle at the start is also approximately -5.5°. We shall again observe a left hand drift of the gear. This time the lateral force is no longer a drag but a driving power which tends to deviate the wheel from its trajectory.
We can see here that the classical wheel deviates more off the longitudinal trajectory. In both cases you can justify the fact of realignment of the wheel by the stiffness of the vertical hinge mounted on the wheel. The fact that realignment is quicker in the case of the new wheel derives from the self aligning torque which adds with the return torque due to the stiffness of the hinge.
We can notice with this test the efficiency of the camber angle calculation. In the initial situation the landing gear geometry is such as the wheel is located slightly ahead of all the rest, the bearing girder is thus not completely vertical. This means that if you give the wheel a light yaw around the axle of the bearer you must observe a non zero camber angle which is the case. This camber angle fades when the wheel returns to its longitudinal position.

Let us finally note the multiplicity of the available results for the new wheel element: the four entry variables $a$, $K$, $y$ and $w$ and the four forces $F_x$, $F_y$, $F_z$ and $M_z$. This eases the interpretation of the wheel behaviour.
VI
DEVELOPPEMENTS
In this chapter improvement possibilities of the representation of the
tyre in simulations are presented. They have not been performed because
either they would have required a longer time to be implemented or because
some obstacle still bars their application. In the latter case, proposals are
shown on the way in which the solution might be reached.

VI.1 Case of combined slip

In many problems the case of combined slip will have to be
considered. As mentioned in chapter III the problem of fixing the forces
becomes much more complex in the case of combined slip. It is the same
when you use a tyre model, evolved or not. In the presentation of his
model, Pacejka explains how to transform the curves giving the forces in
the pure cases in order to fix the forces in a case of combined slip. In
fact one resorts to basic curves where the forces are expressed in function
of the theoretical slip ratio.

Computation of the forces

As the element is basically foreseen to work with the practical slip
ratios, we shall work the transformations in order to express the forces in
combined cases in function of those practical ratios. The elements of the
iteration matrix should also be altered by an additional factor.

The transformation here described applies only to the longitudinal and
the lateral forces. The self aligning torque will be handled separately and
by another technique. The way to follow will be illustrated by the
 corresponding graphs, drawn for the case of the longitudinal force.

In the cases of combined slips the entry variables for the forces
become the theoretical slip ratios defined by

\[ \sigma_x = \frac{v_x}{v_x - v_{\text{ax}}} \qquad \sigma_y = \frac{v_y}{v_x - v_{\text{ax}}} \]
They can be directly linked with the practical ratios by the following formulas

\[ \sigma_x = \frac{-\kappa}{1+\kappa} \quad \sigma_y = \frac{-\tan(\alpha)}{1+\kappa} \]

We perform a horizontal translation in order to let the curves pass over the origin. The shift to operate depends on the translations \( \Delta S_v \) and \( \Delta S_h \) and of the slope BCD. You obtain the total slip factor.

\[ \delta \kappa = \Delta S_h + \frac{\Delta S_y}{BCD} \quad \delta \alpha = \Delta S_h + \frac{\Delta S_y}{BCD} \]

\[ \sigma_{x*} = \sigma_x - \frac{\delta \kappa}{1-\delta \kappa} \quad \sigma_{y*} = \sigma_y - \frac{\tan(\delta \alpha)}{1+\kappa} \]

We normalise the curve by working out that the maximum should be reached for the value 1 of the variable. Therefore we divide the total slip ratio by its maximum value. We obtain the normalised slip ratio.

\[ \sigma_x^* = \frac{\sigma_{x*}}{\sigma_{x*_{\text{max}}}} \quad \sigma_y^* = \frac{\sigma_{y*}}{\sigma_{y*_{\text{max}}}} \]

\[ \sigma^* = \sqrt{\sigma_x^{*2} + \sigma_y^{*2}} \]

The curve so obtained is called basic curve. It depends on the normalised slip ratio which represents the total amount of the longitudinal and lateral slip.
By combining the preceding steps we obtain the global slip ratio transformation. All the parameters are expressed in function of their values in terms of practical ratios.

\[
\sigma_x^* = \frac{\kappa - \delta \kappa}{1+\kappa_M} \frac{1 - \delta \kappa}{\delta \kappa}
\]

\[
\sigma_y^* = \frac{\tan(\alpha) + \tan(\delta \alpha)}{\tan(\alpha_M) + \tan(\delta \alpha)}
\]

The reverse of this transformation allows to find again the basic force corresponding to a value of the normalised slip ratio.

\[
\kappa' = \frac{\sigma^* \left( \frac{\kappa_M}{1+\kappa_M} - \frac{\delta \kappa}{1 - \delta \kappa} \right) + \frac{\delta \kappa}{1 - \delta \kappa}}{1 + \sigma^* \left( \frac{\kappa_M}{1+\kappa_M} - \frac{\delta \kappa}{1 - \delta \kappa} \right) + \frac{\delta \kappa}{1 - \delta \kappa}}
\]

\[
\tan(\alpha') = -\sigma^* (\tan(\alpha_M) - \tan(\delta \alpha)) - \tan(\delta \alpha)
\]
The two basic curves, longitudinal and lateral have usually not the same value in the area of important slip. It would be more adequate to define in those cases a unique slip force of the direction which would be fixed in function of the slip factors. We so correct the values of the basic curves by realising a progressive interpolation.

\[ F_{x0}^* = F_x(\kappa^*) \quad F_{y0}^* = F_y(\alpha^*) \]

\[ F_{x0}^* = |F_{x0}| - \varepsilon(|F_{x0}| - |F_{y0}|) \left( \frac{\sigma_y^*}{\sigma^*} \right) \]

\[ F_{y0}^* = |F_{y0}| - \varepsilon(|F_{y0}| - |F_{x0}|) \left( \frac{\sigma_x^*}{\sigma^*} \right) \]

\[ \varepsilon = |\sigma^*| \quad \text{for } \sigma^* < 1 \]

\[ \varepsilon = 1 \quad \text{for } \sigma^* > 1 \]

fig VI.2 normalisation of the forces on a same final value for high slip
The angle defining the direction in which the horizontal force works is given by

\[ \lambda = \eta + (\Theta - \eta) \frac{\sin(q_8 \arctan(q_9 \sigma^*)^2)}{\sin \left( \frac{1}{2 \pi q_6} \right)} \]

\[ \eta = \arcsin \left( \frac{\sigma_y^*}{\sigma^*} \right) \]

\[ \Theta = \arcsin \left( \frac{\sigma_{\text{tot}}^*}{\sigma_{\text{tot}}} \right) \]

\[ F_x = -F_{x_0} \cos(\lambda) \text{sign}(\sigma_x^*) \]

\[ F_y = -F_{y_0} \sin(\lambda) \]

So we finally obtain the horizontal force in a case of combined slip. The construction of that force is illustrated by the next drawing.

fig VI.3 illustration of the way of building the combined forces [12]
Concerning the self aligning torque we also define a basic curve relying on the transformation operated on the lateral slip ratio. You do not obtain a normalised curve because the realised transformation was about the lateral force. You subtract the residual torque force (for \(\sigma \gamma^*=0\)) and by dividing that basic torque by the lateral force you obtain the pneumatic trail (lever arm to apply to the lateral force to obtain the self aligning torque).

\[
M_{\alpha r} = \frac{M_{\alpha r 0}}{1 + 5\sigma^* \gamma^*^2}
\]

\[
M_{\alpha 0} = M_{\alpha 0}(\alpha')
\]

\[
t = \frac{-(M_{\alpha 0} - M_{\alpha r})}{F_{\gamma 0}}
\]

\[
M_{z}' = -tF_{\gamma} + M_{\alpha r}
\]

The final torque is determined by adding the residual torque and the eventual contribution to the longitudinal force.

\[
M_z = M_z' + (q_2F_{\gamma} + (q_6 + q_7F_z)\gamma + q_3)F_z
\]

**VI.1.b Altering of the iteration matrix**

The computation of the iteration matrix requires in principle that you fix the derivatives of the forces with respect to variables \(F_z\), \(\alpha\), \(\kappa\), and \(\gamma\), but we also know that the iteration matrix does not need to be extremely precise. In a first step it would be useful to try to realise the first simulations of combined cases while maintaining the iteration matrix used in the pure cases. If we then notice that the element does not converge, it will be necessary to recalculate that iteration matrix for the combined cases.

That computation is rather complicated as it gives birth to new terms, all the forces depending now on \(\alpha\) and \(\kappa\).

The kinematic interface should be altered at the showing up of the new derivatives but the changes that have to be realised are minor and have no influence on the processing of the pure cases.
VI.1.c Conclusion

No apparent obstacle prevents the implementation of the forces in the combined cases. That implementation will require an important alteration of the tyre subroutine and a slight alteration of the kinematic interface. The computing load runs the risk of increasing for a rather large amount; this is why it would be useful to find a simplified form for the iterative matrix.
VI.2 Transient behaviour

Up to now the used tyre models are models determining the forces one can observe in steady state conditions. You can easily understand that the kinematic changes operated on the tyre do not instantly reflect on the contact forces. The distortion of the tyre producing the forces takes a definite time to appear. To modelise the transient behaviour of the tyre accurately it will be absolutely necessary to include in the model a set of differential equations. An approximation of the first degree seems to suffice, it has already been used successfully complementarily to the magic formula [2]; This approximation consists of three uncoupled equations linking the forces under steady state conditions and the transient forces that really occur:

\[ L_y \frac{dF_y}{ds} + F_y = F_{yss} \]

\[ L_x \frac{dM_x}{ds} + M_x = M_{xss} \]

\[ L_m \frac{dF_x}{ds} + F_x = F_{xss} \]

The relaxation lengths can be measured for the different possible situations or neared by using the physical and geometrical characteristics of the tyre. If the time steps used are sufficiently small one can use equations under a discretised form:

\[ L_y \frac{F_y(n+1) - F_y(n)}{\Delta x_{d(n)}} + F_y(n+1) = F_{y(n+1)ss} \]

In the opposite case the numerical approximation of the derivative of the force is no longer representative of that derivative by the usual time step. It will then be necessary to operate an integration of the equations inside each time step.
Apparently it will not be necessary to alter the iteration matrix provided again the chosen time step should not be too large. If this is not the case you will be obliged to resort to particular techniques such as the perturbations method to evaluate the derivatives of the forces with respect to the slip ratios. This would produce a tremendous increase in the computing load of the element and would probably induce the loss of the whole benefit of the use of a long time step.
VI.3 Non-plane grounds

Chapter IV allowed us to raise a definite number of problems that occur when you want to modelise the kinematics of the wheel on non-flat ground. As a reminder it chiefly concerns the determination of the contact point and the computation of the derivative of the normal to the road at the contact point.

The beginning of a solution has already been raised to the determination of the contact point through the algorithm that already exists in the standard wheel element of Samcef Mecano. The latter does not yet solve another problem which rises in the simulations on non-plane grounds and which is the possibility to have several contact points. For grounds with continuous slope this can occur if the curvature radius of the ground becomes smaller than the wheel radius. In the cases of grounds defined by a series of points that phenomenon occurs in the hollows encountered.

![Diagram of double contact](image)

fig VI.4 cases of double contact
In both cases you find two contact points for the wheel; except if the curvature radius of the ground is exactly equal to the radius of the wheel, in which case you observe a finite contact surface. It should be possible to find a reduction technique of those two contact points to one by taking into account the positions of the contact points and the deformed radiuses observed for each of them. There would remain to devise an algorithm capable of determining these two contact points.

fig VI.5 proposal for the reduction of double contact. the forces corresponding to each contact point are computed and added. Using the direction so defined, we can take a third point into account and consider it as the reduced contact point.
About the computation of the time derivative of the vector perpendicular to the ground we should advise again the use of a rather small time step so that the difference between the normal vectors between two time steps should be sufficiently representative of the derivative by the usual time step. So you obtain a good approximation of the derivative of the normal vector along the trajectory of the wheel. This derivative will intervene in the computation of the slip ratios.

The time derivative should in principle intervene again in the computation of the iteration matrix. If the curvature of the ground is not too important you can suppose a locally flat ground at the contact point and maintain the iteration matrix as it would be computed for plane grounds. The more difficult cases will be treated by the method of perturbations. This one will be used to determine the derivatives of the forces with respect to the position and the velocity of the centre of the wheel.

The solution of that type of problems remains very critical and it is not sure that one will be able in near future to operate simulations on any type of road. The roads with a continuous slope and with large curvature radiiuses should not raise problems and would already allow to reach a category of simulations that cannot be realised now.
The detailed treatment of problems of impact simulation concerning landing-gears requires the disposal of a performing tool for the simulation of multibody systems. The non linearities are numerous and the flexibility of members plays a prominent role in the behaviour of the whole. It is also essential to be able to modelise the forces that are the basis of the behaviour of the landing-gear: the forces that appear at the tyre-road interface. As the finality of some research-work in the field of impact problems is the simulation of the whole airplane, it is not possible in the present state of computation technology to afford a too important computation cost to the simple determination of these forces. You feel compelled to find a tyre model at once accurate and not too costly in computation.

The magic formula appears to be an attractive tool for the computation of interface forces. It allows to take the influences of the vertical load and the camber angle into account in a detailed way. Its advantage lies especially in its "variable geometry". Since the parameters characterising the curves used by the model have a well determined physical meaning, it is possible to express clearly the influence of some phenomena on the forces.

The increasing complexity of the formula with the effects it can take into account makes it more and more difficult to use in more than one respect. The cost of computation not only increases but the necessary measures to determine all the coefficients characterising a tyre become so numerous and so expensive that the economical interest of the use of this formula is dramatically reduced. The solution to this problem could be derived from the important advance accomplished by the finite element models of tyres. These will ultimately allow to reduce a great number of today still necessary measures by replacing them by much less costly computer simulation.
The success of a correct representation of the wheel depends not only on an accurate tyre model but also and to begin with, on a kinematical description, as accurate as possible. The contact-point theory is sufficient in most applications. It permits to modelise the wheel correctly with weak camber on a flat road. However, it shows its limits in certain cases of non-flat roads but can remain effective with the help of some adaptations.

The tests performed on the wheel element developed in this work show that it is perfectly applicable in its present state in the problems of vehicle behaviour. The feasible developments in a near future: combined slip cases and transient behaviour will allow to cover a large field of simulation.

Some difficulties still stand in the way for efficient functioning concerning impact problems of landing gears. The magic formula doesn't seem well adapted to tyres used in aircraft technology. It would consequently be useful to undertake more thorough research in the field of bias-ply tyres with a view to adaptation of the magic formula. Other developments on the formula could concern a revision of its behaviour in high vertical loads. The very structure of the formula should allow these adaptations without major problems.
APPENDIX 1

FORTRAN Routines of the Element
SUBROUTINE MC531 (NOEL, TIME, DEPL, VIT, ACC, THE, SS, IFORCE, IMAT, 
IVIT, IENRG, NDIM, NOP, IOP, COEF, CAG, FINT, FINE, 
XDEF, XKIN, NBRT)

IMPLICIT REAL*8 (A-H, O-Z)

DIMENSION DEPL(6), VIT(6), ACC(6), THE(6), FINT(6),
FINE(6), COEF(3), RP(3), CAG(4, *)

DIMENSION VAUX(6), THEO(3), OME0(4), XNP(3), PCON(3), AR(3), TTA(3, 3),
ROTTA(3, 3), V(3), UTR(3), XNPP(3), D(3), VITC(3), ULAT(3)

DIMENSION F(4), DF(4, 4), CNPP(3, 3), TOR(3, 3), EYE3(3, 3), A(6, 3),
B(6, 3), C(6, 3), RTOTA(3, 3), A1(6), B1(6), B2(6), C1(6),
* RNPP(3, 3)

DIMENSION CR(3, 3), CRT(3), R(6), T(6), U(3, 6), FGX(3, 3), FG(3, 3),
P2(3, 3), Q121(3), Q11(3, 3), CVIT(3, 3), Q22(3, 3), Q22(3, 3),
Q22(3, 3), Q1(3, 3), Q2(3, 3), P(3, 6), Q(3, 6), W1(6), WY2(6),
* XI(6), Y1(6), Z1(6), W6(6), X(6), Y(6), Z(6)

DIMENSION A11(6), B11(6), B21(6), C11(6), B12(6), B22(6), C12(6),
* SKA1(6, 6), SKB1(6, 6), SKB2(6, 6), SKC1(6, 6), SCB1(6, 6),
* SCB2(6, 6), SCC1(6, 6), SK(6, 6), SC(6, 6), SS(6, 6)

THEO(1) = 0.0D0
THEO(2) = 0.0D0
THEO(3) = 0.0D0
OMEO(1) = 0.0D0
OMEO(2) = 0.0D0
OMEO(3) = 0.0D0
OMEO(4) = 0.0D0

C C READING THE DATA
C C RADIUS
C RPO = CAG(1, 1)
C C NORMAL TO THE ROLLING SURFACE
C XNP(1) = CAG(1, 2)
XNP(2) = CAG(2, 2)
XNP(3) = CAG(3, 2)
ONE POINT OF THE ROLLING SURFACE

PCON(1) = CAG(1,3)
PCON(2) = CAG(2,3)
PCON(3) = CAG(3,3)

ORIGINAL AXLE OF THE WHEEL
THIS AXLE MUST BE THE POSITIVE AXUS OF THE NORMAL ROTATION OF THE WHEEL

AR(1) = CAG(1,4)
AR(2) = CAG(2,4)
AR(3) = CAG(3,4)

COMPUTATION OF THE ROTATION MATRICES

TTA = SPATIAL ROTATION
ROTTA = TTA * INCREMENTAL ROTATION = TOTAL ROTATION

IT=0
CALL MC285(THE(4),TTA,IT)
CALL MC285(DEPL(4),ROTTA,IT)
CALL MULMA2(TTA,ROTTA,ROTTA,3,3,3,3,3,3,VAUX,0)

ACTUALISATION OF THE WHEEL AXLE

CALL MULMA2(ROTTA,AR,V,3,3,1,3,3,3,VAUX,0)

PROCEDURE LEADING TO THE CONTACT POINT

PROJECTION OF THE NORMAL VECTOR IN THE WHEEL PLANE

CALL MC299(UTR,V,XNP)
CALL MC299(XNPP,UTR,V)

DNPP = 0.DO
DTR = 0.DO

DO 30 I=1,3
   DNPP = DNPP+XNPP(I)*XNPP(I)
   DTR = DTR+UTR(I)*UTR(I)
30 CONTINUE

DNPP = DSQRT(DNPP)
DTR = DSQRT(DTR)

UNIT VECTORS IN THE RADIUS AND TRAVEL DIRECTIONS

UNIT VECTOR FROM THE CONTACT POINT TO THE WHEEL CENTER = XNPP
UNIT VECTOR IN THE TRAVEL DIRECTION = UTR

DO 40 I=1,3
   XNPP(I) = XNPP(I)/DNPP
   UTR(I) = UTR(I)/DTR
40 CONTINUE

UNIT LATERAL VECTOR

CALL MC299(ULAT,XNP,UTR)
C CAMBER ANGLE ; ANGLE=GAMAR(RAD) GAMA(DEC); COSINE=CGAM
C
CGAM = XNP(1)*XNPP(1)+XNP(2)*XNPP(2)+XNP(3)*XNPP(3)
SG = XNPP(1)*ULAT(1)+XNPP(2)*ULAT(2)+XNPP(3)*ULAT(3)
SG = -DSIGN(1,SG)
GAMAR = SG*DACOS(CGAM)
GAMA = (GAMAR*1.8D2)/3.1415927D0
C
C DEFORMED RADIUS ; RADIUS=RRP RADIAL DEFLECTION=HP
C
RRP = 0.D0
DO 50 I=1,3
   RP(I) = DEPL(I)-PCON(I)
   RRP = RRP+RP(I)*XNP(I)
50 CONTINUE
C
HP = RPO-RRP
C
TREATING THE CASE WHERE THERE IS NO CONTACT
C
IF (HP.GT.1.D-4) GOTO 58
ICONT = 0
HAPA = 0.D0
ALFA = 0.D0
F(1) = 0.D0
F(2) = 0.D0
F(3) = 0.D0
F(4) = 0.D0
DO 52 I=1,6
   FINT(I) = 0.D0
DO 52 J=1,6
   SS(I,J) = 0.D0
52 CONTINUE
C
GOTO 101
C
58 CONTINUE
ICONT = 1
C
PROCEDURE LEADING TO THE CONTACT POINT VELOCITY
--------------------------------------------------
C
SPATIAL ANGULAR VELOCITY AND ACCELERATION
C
ANGULAR MATERIAL ACCELERATION = ACC
SPATIAL ANGULAR VELOCITY = VIT(4,5,6)
C
IT=0
CALL MC2881(DEPL(4),VIT(4),ACC,IT,THEO,OME0)
CALL MULMA2(ROTTA,VIT(4),VIT(4),3,3,1,3,3,3,VAUX,0)
C
CALL MC299(D,XNPP,VIT(4))
C
RADIAL VELOCITY ; TIME RATE OF CHANGE OF DEFORMED RADIUS = VITR
C
VITR = 0.D0
DO 60 I=1,3
   VITR = VITR+XNP(I)*VIT(I)+RRP*XNP(I)*D(I)
60 CONTINUE
C
VITR = VITR/CGAM
CONTACT POINT VELOCITY

VELOCITY OF THE CONTACT POINT AS BELONGING TO THE WHEEL = VITC
EFFECTIVE RADIUS COMPENSATING THE DEFORMATION = RE

\[
\text{RE} = \frac{\text{RPO}-\text{HP}}{3.0}
\]

\[
\text{DO 70 } \text{I}=1,3
\]

\[
\text{VITC(I)} = \text{VIT(I)} - \text{VITR} \times \text{XNPP(I)} + \text{RE} \times \text{D(I)}
\]

ENTRY VARIABLES FOR FORCES FORMULAS

PROJECTION OF VELOCITIES IN THE CONTACT POINT FRAME

\[
\text{VSLO} = 0.0
\]

\[
\text{VSLA} = 0.0
\]

\[
\text{VREF} = 0.0
\]

\[
\text{DO 80 } \text{I}=1,3
\]

\[
\text{VSLO} = \text{VSLO} + \text{VITC(I)} \times \text{UTR(I)}
\]

\[
\text{VSLA} = \text{VSLA} + \text{VITC(I)} \times \text{ULAT(I)}
\]

\[
\text{VREF} = \text{VREF} + \text{VIT(I)} \times \text{UTR(I)}
\]

\[
\text{IF (DABS(VREF).LT.1.0D-2) VREF} = \text{DSIGN(1,VREF)} \times 1.0D-2
\]

CONTINUE

SLIP RATIOS

\[
\text{ALFAR} = \frac{\text{VSLA}}{\text{VREF}}
\]

\[
\text{HAPA} = \frac{\text{VSLO}}{\text{VREF}}
\]

\[
\text{HAPA} = 1.02 \times \text{HAPA}
\]

\[
\text{ALFAR} = \text{DATAN(ALFAR1)}
\]

\[
\text{ALFA} = \frac{(\text{ALFAR} \times 1.8D2)}{3.1415927}
\]

COMPUTATION OF FORCES

CALL MCTYRE(HP,GAMA,ALFA,HAPA,UTR,ULAT,XNP,VITC,CAG,VIT,F,DF)

PROCEDURE LEADING TO THE NODAL FORCES

CALL MC289(XNPP,CNPP)

\[
\text{DO 90 } \text{I}=1,3
\]

\[
\text{DO 90 } \text{J}=1,3
\]

\[
\text{RNPP(I,J)} = \text{RRP} \times \text{CNPP(I,J)}
\]

\[
\text{EYE3(I,J)} = 0.0
\]

\[
\text{IF (I.EQ.J) EYE3(I,J)} = 1.0
\]

CONTINUE

CALL ATBREC(ROTTA,RNPP,TOR,3,3,3,3,3,3,VAUX,0)
CALL ATBREC(ROTTA,EYE3,RTOTA,3,3,3,3,3,3,VAUX,0)

\[
\text{DO 95 } \text{I}=1,3
\]

\[
\text{DO 95 } \text{J}=1,3
\]

\[
\text{A(I,J)} = -\text{EYE3(I,J)}
\]

\[
\text{B(I,J)} = \text{EYE3(I,J)}
\]

\[
\text{C(I,J)} = 0.0
\]

\[
\text{A(I+3,J)} = \text{TOR(I,J)}
\]

\[
\text{B(I+3,J)} = -\text{TOR(I,J)}
\]

\[
\text{C(I+3,J)} = \text{RTOTA(I,J)}
\]

CONTINUE
MULTIPLICATION MATRICES

CALL MULMA2(A,XNP,A1,6,3,1,6,3,6,VAUX,0)
CALL MULMA2(B,UTR,B1,6,3,1,6,3,6,VAUX,0)
CALL MULMA2(B,ULAT,B2,6,3,1,6,3,6,VAUX,0)
CALL MULMA2(C,XNP,C1,6,3,1,6,3,6,VAUX,0)

NODAL FORCES VECTOR

DO 100 I=1,6
   FINT(I) = A1(I)*F(3)+B1(I)*F(1)+B2(I)*F(2)+C1(I)*F(4)
100 CONTINUE

IF (IMAT.NE.0).OR.(IFORCE.NE.0) GOTO 102
SS(1,1) = HP
SS(2,1) = HAPA
SS(3,1) = ALFA
SS(4,1) = TIME
SS(5,1) = F(3)
SS(6,1) = F(1)
SS(1,2) = F(2)
SS(2,2) = F(4)
SS(3,2) = GAMA
NBRT = 9

RETURN

PROCEDURE LEADING TO THE ITERATION MATRIX

COMPUTATION OF DELTA(PPP), DELTA(GAMA) AND DELTA(VIT)

CALL MULMA2(CNPP,ROTTA,CR,3,3,3,3,3,3,VAUX,0)
CALL MULMA1(XNP,CR,CRT,1,3,3,3,3,3,VAUX,0)

IF (CGAM.GT.1.D0) CGAM=1.D0
SGAM = SG*DSQRT(1-CGAM**2)

DO 110 I=1,3
   R(I) = -XNP(I)/CGAM
   T(I) = 0.D0
   R(I+3) = -RRP*CRT(I)/CGAM
   T(I+3) = 0.D0
   IF (SGAM.LT.1.D-6) GOTO 105
   T(I+3) = CRT(I)/SGAM
105 CONTINUE

DO 110 J=1,3
   U(I,J) = EYE3(I,J)
   U(I,J+3) = 0.D0
110 CONTINUE
COMPUTATION OF DELTA(VITC) WITH D(VITC) = P*D(vit) + Q*D(depl)

CALL MULMA1(XNPP, XNP, FGX, 3, 1, 3, 3, 3, VAUX, 0)
DO 120 I = 1, 3
DO 120 J = 1, 3
   FG(I, J) = EYE3(I, J) - FGX(I, J)/CGAM
120 CONTINUE

COMPUTATION OF THE P COMPONENTS
CALL MULMA2(FG, CR, P2, 3, 3, 3, 3, VAUX, 0)

COMPUTATION OF THE Q COMPONENTS
CALL MULMA2(CNPP, VIT(4), Q121, 3, 3, 1, 3, 3, VAUX, 0)
CALL MULMA2(Q121, XNP, Q11, 3, 1, 3, 3, VAUX, 0)
CALL MC289(VIT(4), CVIT)
CALL MULMA2(CVIT, CR, Q221, 3, 3, 3, 3, VAUX, 0)
CALL MULMA2(Q221, CRT, Q222, 3, 1, 3, 3, VAUX, 0)
DO 130 I = 1, 3
DO 130 J = 1, 3
   Q22(I, J) = RRP*Q221(I, J) + RRP*Q222(I, J)/CGAM
130 CONTINUE

CALL MULMA2(FG, Q11, Q1, 3, 3, 3, 3, VAUX, 0)
CALL MULMA2(FG, Q22, Q2, 3, 3, 3, 3, VAUX, 0)
P AND Q MATRICES THEMSELVES
DO 140 I = 1, 3
DO 140 J = 1, 3
   P(I, J) = FG(I, J)
   Q(I, J) = Q1(I, J)
   P(I, J+3) = P2(I, J)
   Q(I, J+3) = Q2(I, J)
140 CONTINUE

COMPUTATION OF DELTA(ALFA) AND DELTA(HAPA)
D(ALFA) = W * D(vit) + X * D(depl)
D(HAPA) = Y * D(vit) + Z * D(depl)
CALL MULMA1(ULAT, P, W1, 1, 3, 6, 3, 6, 6, VAUX, 0)
CALL MULMA1(UTR, U, WY2, 1, 3, 6, 3, 6, 6, VAUX, 0)
CALL MULMA1(ULAT, Q, X1, 1, 3, 6, 3, 6, 6, VAUX, 0)
CALL MULMA1(UTR, P, Y1, 1, 3, 6, 3, 6, 6, VAUX, 0)
CALL MULMA1(UTR, Q, Z1, 1, 3, 6, 3, 6, 6, VAUX, 0)
DO 150 I = 1, 6
   W(I) = W1(I)/VREF - VSLO*WY2(I)/VREF**2
   X(I) = X1(I)/VREF
   Y(I) = Y1(I)/VREF - VSLO*WY2(I)/VREF**2
   Z(I) = Z1(I)/VREF
150 CONTINUE

103
C STIFFNESS AND DAMPING MATRICES
C
DO 160 I=1,6
   A11(I) = DF(3,3)*R(I)
   B11(I) = DF(1,2)*Z(I)+DF(1,3)*A11(I)+DF(1,4)*T(I)
   B21(I) = DF(2,1)*X(I)+DF(2,3)*A11(I)+DF(2,4)*T(I)
   C11(I) = DF(4,1)*X(I)+DF(4,3)*A11(I)+DF(4,4)*T(I)
   B12(I) = DF(1,2)*Y(I)
   B22(I) = DF(2,1)*W(I)
   C12(I) = DF(4,1)*W(I)
160 CONTINUE
C TERMS OF THE MATRICES
C
CALL MULMA2(A1,A11,SKA1,6,1,6,6,6,VAUX,0)
CALL MULMA2(B1,B11,SKB1,6,1,6,6,6,VAUX,0)
CALL MULMA2(B2,B21,SKB2,6,1,6,6,6,VAUX,0)
CALL MULMA2(C1,C11,SKC1,6,1,6,6,6,VAUX,0)
CALL MULMA2(B1,B12,SCB1,6,1,6,6,6,VAUX,0)
CALL MULMA2(B2,B22,SCB2,6,1,6,6,6,VAUX,0)
CALL MULMA2(C1,C12,SCC1,6,1,6,6,6,VAUX,0)
C MATRICES THEMSELVES SK=STIFFNESS SC=DAMPING
C
DO 170 I=1,6
   DO 170 J=I,6
      SK(I,J) = SKA1(I,J)+SKB1(I,J)+SKB2(I,J)+SKC1(I,J)
      SC(I,J) = SCB1(I,J)+SCB2(I,J)+SCC1(I,J)
170 CONTINUE
C SYMMETRISATION OF THE STIFFNESS MATRIX
C
DO 180 I=1,6
   DO 180 J=1,6
      SK(I,J) = (SK(I,J)+SK(J,I))/2.DO
      SC(I,J) = (SC(I,J)+SC(J,I))/2.DO
180 CONTINUE
C ITERATION MATRIX
C
IF (IMAT.NE.2) RETURN
C
DO 190 I=1,6
   DO 190 J=1,6
      SS(I,J) = COEF(1)*SK(I,J)+COEF(3)*SC(I,J)
190 CONTINUE
C RETURN
C END
SUBROUTINE MCTYRE (HP, GAMA, ALFA, HAPA, UTR, ULAT, XNP, VITC, CAG, 
  VIT, F, DF)

**Release 1.2 of the MCTYRE subroutine 08/04/93**
**Corresponding to the release 1 of the Pacejka formula**
**Includes FZ and camber influences**
**31/03/93 suppression of the CAG variables**
**08/04/93 reintroducing the CAG**

IMPLICIT REAL*8 (A-H, O-Z)

DIMENSION F(4), DF(4, 4), RA(4), PLO(8), PLA(12), PAT(13), DFM(4), 
  UTR(3), ULAT(3), XNP(3), VITC(3), VIT(6), CAG(4, *)

READING THE DATA

COEFFICIENTS FOR THE RADIAL FORCE

RA(1) = CAG(1, 5)
RA(2) = CAG(2, 5)
RA(3) = CAG(3, 5)
RA(4) = CAG(4, 5)

COEFFICIENTS FOR THE LONGITUDINAL FORCE

PLO(1) = CAG(1, 6)
PLO(2) = CAG(2, 6)
PLO(3) = CAG(3, 6)
PLO(4) = CAG(4, 6)
PLO(5) = CAG(1, 7)
PLO(6) = CAG(2, 7)
PLO(7) = CAG(3, 7)
PLO(8) = CAG(4, 7)

COEFFICIENTS FOR THE LATERAL FORCE

PLA(1) = CAG(1, 8)
PLA(2) = CAG(2, 8)
PLA(3) = CAG(3, 8)
PLA(4) = CAG(4, 8)
PLA(5) = CAG(1, 9)
PLA(6) = CAG(2, 9)
PLA(7) = CAG(3, 9)
PLA(8) = CAG(4, 9)
PLA(9) = CAG(1, 10)
PLA(10) = CAG(2, 10)
PLA(11) = CAG(3, 10)
PLA(12) = CAG(4, 10)

COEFFICIENTS FOR THE ALIGNING MOMENT

PAT(1) = CAG(1, 11)
PAT(2) = CAG(2, 11)
PAT(3) = CAG(3, 11)
PAT(4) = CAG(4, 11)
PAT(5) = CAG(1, 12)
PAT(6) = CAG(2, 12)
PAT(7) = CAG(3, 12)
PAT(8) = CAG(4, 12)
PAT(9) = CAG(1,13)
PAT(10) = CAG(2,13)
PAT(11) = CAG(3,13)
PAT(12) = CAG(4,13)
PAT(13) = CAG(1,14)

C RADIAL FORCE AND ITS DERIVATE

C

F(3) = 0.D0
DF(3,3) = 0.D0
DO 10 I=1,4
   F(3) = F(3)+RA(I)*HP**I
   DF(3,3) = DF(3,3)*I*RA(I)*HP**(I-1)
10 CONTINUE

F1 = F(3)/1000
F2 = F1**2

C LONGITUDINAL FORCE

C

PC = 1.65D0
PD = PLO(1)*F2+PLO(2)*F1
PB = (PLO(3)*F2+PLO(4)*F1)/(DEXP(PLO(5)*F1)*PC*PD)
PE = PLO(6)*F2+PLO(7)*F1+PLO(8)
SV = 0.D0
SH = 0.D0
VAR = HAPA

C BUILDING THE FORCE AND ITS DERIVATES WRT THE MAIN COEFFICIENTS

C

CALL MCMAGIC (PB,PC,PD,PE,SV,SH,VAR,FM,DFM)
F(1) = FM

C DERIVATES OF THE COEFFICIENTS WRT THE VERTICAL FORCE

C

DPD = 2*PLO(1)*F1+PLO(2)
DPE = 2*PLO(6)*F1+PLO(7)
DPB = (2*PLO(3)*F1+PLO(4)-PB*PC*(DPD+PLO(5)*PD))/
     *(PC*PD*DEXP(PLO(5)*F1))

C VECTOR OF DERIVATES

C

DF(1,1) = 0.D0
DF(1,2) = DFM(4)
DF(1,3) = DPB*DFM(1)+DPD*DFM(2)+DPE*DFM(3)
DF(1,4) = 0.D0

C LATERNAL FORCE

C

PC = 1.3D0
PD = PLA(1)*F2+PLA(2)*F1
PB = PLA(3)*DSIN(PLA(4)*DATAN(PLA(5)*F1))*(1-PLA(12)*DABS(GAMA)) *
     /(PC*PD)
PE = PLA(6)*F2+PLA(7)*F1+PLA(8)
SH = PLA(9)*GAMA
SV = (PLA(10)*F2+PLA(11)*F1)*GAMA
VAR = ALFA

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C BUILDING THE FORCE AND ITS DERIVATIVES WRT THE MAIN COEFFICIENTS
C
CALL MCMAGIC (PB,PC,PD,PE,SV,SH,VAR,FM,DFM)
F(2) = FM
C
C DERIVATIVES OF THE COEFFICIENTS WRT THE VERTICAL FORCE
C
DPD = 2*PLA(1)*F1+PLA(2)
DPB = (PLA(3)*PLA(4)+PLA(5)*DCOS(PLA(4)+DATAN(PLA(5)*F1))*/
     (1+(PLA(5)*F1)**2)-PB*PC*DPD)*(1-PLA(12)*DABS(GAMA))
     /(PC*PD)
DPE = PLA(6)*2*F1+PLA(7)
DSV = GAMA*(PLA(10)*2*F1+PLA(11))
C
C DERIVATIVES OF THE COEFFICIENTS WRT THE CAMBER ANGLE
C
GPB = -PLA(12)*(DABS(GAMA)/GAMA)*(PLA(3)*DSIN(PLA(4)*
     DATAN(PLA(5)*F1))/(PC*PD))
GSH = PLA(9)
GSV = PLA(10)*F2+PLA(11)*F1
C
C VECTOR OF DERIVATIVES
C
DF(2,1) = DFM(4)
DF(2,2) = 0.0D0
DF(2,3) = DPB*DFM(1)+DPD*DFM(2)+DPE*DFM(3)+DSV
DF(2,4) = GPB*DFM(1)+GSH*DFM(4)+GSV
C
C ALIGNING TORQUE
C
PC = 2.4D0
PD = PAT(1)*F2+PAT(2)*F1
PB = ((PAT(3)*F2+PAT(4)*F1)/(DEXP(PAT(5)*F1)*PC*PD))*
     (1-DABS(GAMA)*PAT(12))
PE = (PAT(6)*F2+PAT(7)*F1+PAT(8))/(1-DABS(GAMA)*PAT(13))
SV = (PAT(10)*F2+PAT(11)*F1)*GAMA
SH = PAT(9)*GAMA
VAR = ALFA
C
C BUILDING THE FORCE AND ITS DERIVATIVES WRT THE MAIN COEFFICIENTS
C
CALL MCMAGIC (PB,PC,PD,PE,SV,SH,VAR,FM,DFM)
F(4) = FM
C
C DERIVATIVES OF THE COEFFICIENTS WRT THE VERTICAL FORCE
C
DPD = 2*PAT(1)*F1+PAT(2)
DPB = ((2*PAT(3)*F1+PAT(4)-PB*PC*(DPD+PAT(5)*PD))/
     (PC*PD*DEXP(PAT(5)*F1)))*(1-DABS(GAMA)*PAT(12))
DPE = (2*PAT(6)*F1+PAT(7))/(1-DABS(GAMA)*PAT(13))
DSV = (PAT(10)*2*F1+PAT(11))*GAMA
C
C DERIVATIVES OF THE COEFFICIENTS WRT THE CAMBER ANGLE
C
GPB = -PAT(12)*(DABS(GAMA)/GAMA)*((PAT(3)*F2+PAT(4)*F1)/
     (DEXP(PAT(5)*F1)*PC*PD))
GSH = PAT(9)
GSV = PAT(10)*F2+PAT(11)*F1
GPE = PAT(13)*(DABS(GAMA)/GAMA)*(PAT(6)*F2+PAT(7)*F1+PAT(8))/
     (1-PAT(13)*DABS(GAMA))**2
VECTOR OF DERIVATES

DF(4,1) = DFM(4)
DF(4,2) = 0.0
DF(4,3) = DPB*DFM(1) + DPD*DFM(2) + DPE*DFM(3) + DSV
DF(4,4) = GPB*DFM(1) + GPE*DFM(3) + GSH*DFM(4) + GSV

RETURN

END
SUBROUTINE MCMAGIC (PB, PC, PD, PE, SV, SH, VAR, FM, DFM)

** RELEASE 1 OF THE MCMAGIC SUBROUTINE 23/03/93 **

** CLASSICAL MAGIC FORMULA AND ITS DERIVATIVES WRT B, D, E, VAR **

** FOR GENERAL USE WITH MC TYRE SUBROUTINE **

** ALSO VALID FOR THE COMBINED CASES **

** DERIVATES PERFORMED WITH AN HP28-S **

IMPLICIT REAL*8 (A-H, O-Z)

DIMENSION DFM(4)

BUILDING THE FORCE FROM THE MAGIC FORMULA

PHI = (1-PE)*(VAR+SH)+(PE/PB)*DATAN(PB*(VAR+SH))
FM = PD*DSIN(PC*DATAN(PB*PHI))+SV

DERIVATIVE WRT THE MAIN VARIABLE VAR OR THE SHIFT SH

DMP = PC*PD*DCOS(PC*DATAN(PB*PHI))/(1+(PB*PHI)**2)
DFM(4) = (1-PE*(1-(1/(1+(PB*(VAR+SH))**2))))*DMP*PB

DERIVATIVE WRT THE COEFFICIENTS B, D AND E

DFM(1) = DMP*(-PE/PB*DATAN(PB*(VAR+SH))+PE/(1+(PB*(VAR+SH))**2))
         * (VAR+SH)+PHI)
DFM(2) = DSIN(PC*DATAN(PB*PHI))
DFM(3) = (-(VAR+SH)+1/PB*DATAN(PB*(VAR+SH)))*PB*DMP

RETURN

END
APPENDIX 2

BANKFILES OF THE TEST CASES
LAND

TRAIN d'atterissage elabore

.i land

.TRAIN

.tit

noe i 1 x 0 y 0 z 1.4
  i 1
  i 1 r 1
  i 3 x 0 y 0 z 0.8
  i 1
  i 1 r 1
  i 5 x 0 y 0 z 0.3
  i 1
  i 1 r 2
  i 8 x -0.8 y 0 z 1.4
  i 1 r 1
  i 10 x 0 y 0 z 0
  i 1 r 4
  i 15 x 0 y 1 z 0.4

.mce
  i 1 rigi n 1 8
  i 2 hing n 1 2 10
  i 3 beam n 2 3 15
  i 4 hing n 3 4 11
  i 5 pris n 3 5
  i 6 spri n 3 5
  i 7 hing n 5 6 12
  i 9 hing n 6 7 13
  i 10 user n 7
  i 11 pris n 4 9
  i 12 spri n 4 9
  i 13 hing n 8 9 14
  i 14 rigi n 7

.mcc
  i 1 rigi mass 400. mix 5.e6 miy 5.e6 miz 5.e6
  i 2 hing axel 2
  i 6 spri kr 1.e5 c 1.e4 1 0.5
  i 4 hing axel 2
  i 3 beam ri 0.025 re 0.045
  y 2.1e1 n 0.33 m 7800
  i 7 hing axel 3 code 30 1.e6 -1 -1.e6 l
  c 1.e5
  i 9 hing axel 2
  i 10 user 13 6

  code 5001 v 0.3 0. 0. 0.
  code 5002 v 0. 0. 1. 0.
  code 5003 v 0. 0. 0. 0.
  code 5004 v 0. 1. 0. 0.
  code 5005 v 7.e5 7.e6 1.6e7 1.e7
  code 5006 v -21.3 1.144e3 49.6 226.
  code 5007 v .069 -.006 .056 .486
  code 5008 v -22.1 1.01le3 1.078e3 1.82
  code 5009 v .208 0. -3.54 .707
  code 5010 v .028 0. 14.8 .022
  code 5011 v -2.72 -2.28 -1.86 -2.73
  code 5012 v .11 -.07 .643 -4.04
  code 5013 v .015 -.066 .945 .03
  code 5014 v .07 0. 0. 0.
  i 12 spri kr 4.e5 c 4.e4 1 1.05
  i 13 hing axel 2
  i 14 rigi mass 10. mix 5. miy 10. miz 5.

.gel g -9.81

.dei

  i 7 c 3 v 0.3
  i 1 j 9 c 2 v 0.
  i 1 c 1 v 0.
  i 9 c 1 v -0.8
.vii
i 1 j 9 c 1 v 50.
i 1 j 9 c 3 v -4.
! i 1 j 9 c 2 v 2.
dge meca 1 itma 25
dgr prcs 1 e9
cat t1 0 t2 2 npas 2000 ia4 100 ia16 100 ia19 1 impd 100 impe 100
.sad i 2 c 1 3 n 1 2
   i 2 c 1 3 n 3
   i 2 c 5 2 n 1 2
   i 2 c 5 2 n 3
   i 4 c 1 3 n 1 2
   i 4 c 1 3 n 3
   i 4 c 5 2 n 1 2
   i 4 c 5 2 n 3
   i 6 c 1 3 n 1 2
   i 6 c 1 3 n 3
   i 6 c 6 2 n 1 2
   i 6 c 6 2 n 3
   i 7 c 1 3 n 1 2
   i 7 c 1 3 n 3
   i 7 c 5 2 n 1 2
   i 7 c 5 2 n 3
.sae i 6 c 1 2
   i 12 c 1 2
   i 10 c 1 2
   i 10 c 3 4
   i 10 c 5 6
   i 10 c 7 8
   i 10 c 9
.sie
.exit
TRAINS COMPLETS AVEC ROUES CLASSIQUES

 importantes

Train d'aérisage élaboré

\[ \begin{align*}
    \text{.tit Train d'aérisage élaboré} \\
    \text{.noe i 1 x 0 y 0 z 1.4} \\
    \text{i 1 r 1} \\
    \text{i 1 r 1} \\
    \text{i 1 r 2} \\
    \text{i 8 x -0.8 y 0 z 1.4} \\
    \text{i 1 r 1} \\
    \text{i 10 x 0 y 0 z 0} \\
    \text{i 1 r 4} \\
    \text{i 15 x 0 y 1 z 0.4} \\
    \text{i 16 x 0 y 0 z 0} \\
    \text{.mce i 1 rigi n 1 8} \\
    \text{i 2 hing n 1 2 10} \\
    \text{i 3 beam n 2 3 15} \\
    \text{i 4 hing n 3 4 11} \\
    \text{i 5 pris n 3 5} \\
    \text{i 6 spri n 3 5} \\
    \text{i 7 hing n 5 6 12} \\
    \text{i 9 hing n 6 7 13} \\
    \text{i 10 whee n 7 16} \\
    \text{i 11 pris n 4 9} \\
    \text{i 12 spri n 4 9} \\
    \text{i 13 hing n 8 9 14} \\
    \text{i 14 rigi n 7} \\
    \text{.mcc i 1 rigi mass 4000. mix 5.e7 miy 5.e7 miz 5.e7} \\
    \text{i 2 hing axel 2} \\
    \text{i 6 spri kr 1.e6 c 1.e5 l 0.5} \\
    \text{i 4 hing axel 2} \\
    \text{i 3 beam ri 0.05 re 0.1} \\
    \text{y 2.lell n 0.33 m 7800} \\
    \text{i 7 hing axel 3 code 30 1.e6 -1 -1.e6 1} \\
    \text{c 1.e5} \\
    \text{i 9 hing axel 2} \\
    \text{i 10 whee axel 2 axe2 3 r0 0.3 tol 0.01} \\
    \text{code 30 0. 0. 1. 10.} \\
    \text{code 31 0.5 20 0.25 60} \\
    \text{ul 0.01 fl 1.e5 u2 0.15 f2 1.e7} \\
    \text{i 12 spri kr 4.e6 c 4.e5 l 1.05} \\
    \text{i 13 hing axel 2} \\
    \text{i 14 rigi mass 50. mix 25. miy 50. miz 25.} \\
    \text{.gel g -9.81} \\
    \text{.dei i 7 c 3 v 0.3} \\
    \text{i 1 j 9 c 2 v 0.} \\
    \text{i 1 c 1 v 0.} \\
    \text{i 9 c 1 v -0.8} \\
    \text{.vii i 1 j 9 c 1 v 50.} \\
    \text{i 1 j 9 c 3 v -4.} \\
    \text{! i 1 j 9 c 2 v 2.} \\
    \text{.dge meca 1 itma 25} \\
    \text{.dgr prcs 1.e9} \\
    \text{.cat tl 0 t2 2 npas 2000 ia4 100 ia16 100 ia19 1 impd 100 impe 100} \\
\end{align*} \]
APPENDIX 3
USER'S GUIDE FOR THE NEW WHEEL ELEMENT
Using the Pacejka wheel element on Samcef Mecano

Introduction of the data

In the .MCE command, introduce the USER element under its USER name. The element contains only one node which is the wheel centre. Do not forget to add hinges to the wheel element to allow its spin and yaw movements.

.MCE I xx USER N yy

In the .MCC command you must specify the maximum number of DOF per node that is 6. Using the property codes of the user element, introduce the coefficients necessary for the description of the wheel. Each code permits the introduction of 4 variables:

- code 5001 undeformed radius
- code 5002 normal to the road plane
- code 5003 one point of the contact surface
- code 5004 original axle of the wheel
- code 5005 coefficient of the vertical force model (growing powers)
- codes 5006 and 5007 coefficients of the longitudinal force
- code 5008 to 5010 coefficients of the lateral force
- code 5011 to 5014 coefficients for the self aligning torque

.MCC USER L3 6

  code 5001 vRo  0  0  0
  code 5002 v

  ...

  ...

  code 5014 v al3  0  0  0

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Results of the element

With the help of the .SAE command, you can get the data concerning the behaviour of the element during the simulation. There are 8 variables available:

- comp 1: radial deflection
- comp 2: longitudinal slip ratio
- comp 3: lateral slip angle (deg)
- comp 4: time
- comp 5: vertical force N
- comp 6: longitudinal force N
- comp 7: lateral force N
- comp 8: self aligning torque N.m
- comp 9: camber angle deg

.SAE I xx c y z