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A note on the Forming Limit Curve.

Forming Limit Curves, theory and application.

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Abstract

A new theory to estimate the Forming Limit Curve (FLC) is presented in this paper. This theory is verified by practical results. Some materials show a disagreement to the presented theory. Several items like inhomogeneity, ductility, strainrate sensitivity and logarithmic strain hardening are discussed in this paper to give an explanation for this disagreement.

Introduction

In sheet metal working, FLC's, serve as one of the guidelines to estimate the forming characteristics of sheet metal (Keeler\textsuperscript{1} Goodwin\textsuperscript{2}). The FLC is the combination of loci of the strains with necking initiation. This can be presented in an $\varepsilon_1$-$\varepsilon_2$ or $\varepsilon_1$ - $\varepsilon_2$ diagram (see fig. 1).

Usually, the FLC is measured experimentally, but because of the importance of the FLC, many attempts have been made in order to describe the FLC mathematically. First theories, Hill\textsuperscript{4}, and Swift\textsuperscript{5} have been based on the mathematical estimation of necking in sheet metal working. Other authors, like Marciniak\textsuperscript{6,7}, need to assume an inhomogeneity in the sheet metal, which is introduced as a smaller thickness.

The above-mentioned theories are complex and do not satisfy available data in the first quadrant of the FLD, see fig. 1.

The theory presented in this paper professes to contribute to the understanding of the phenomenon and to supply a relatively simple theory for the FLC, without claiming completeness.
Basics of anisotropic material behaviour

For the formulation of an instability criterion, certain material behaviour has to be assumed. Here, a simplified formulation, based on Hill, Backofen and Hosford, is used. Only the essential equations are presented here.

\[
\begin{align*}
\varepsilon_1 &= \ln \left( \frac{a}{a_0} \right) \\
\varepsilon_2 &= \ln \left( \frac{b}{b_0} \right) \\
\varepsilon_3 &= \ln \left( \frac{c}{c_0} \right)
\end{align*}
\]

with: \( \varepsilon_1 \geq \varepsilon_2 \) and \( \varepsilon_1 > 0 \).

Starting points of the analysis are:
- Shear strains are negligible.
- \( \sigma_3 = 0 \) The stress perpendicular to the sheet is negligible:

\[
\frac{\sigma_2}{\sigma_1} = l, \; l \; (<1) = \text{constant.}
\]

This implies together with (1) and (2) that a straight stress path also exists:

\[
\frac{\sigma_2}{\sigma_1} = l, \; l \; (<1) = \text{constant.}
\]

- The material exhibits normal anisotropy, i.e. the planar anisotropy \( \Delta R \) is zero. Anisotropy then can simply and solely be described with the \( R \) factor as measured in the tensile test:

\[
R = \frac{\varepsilon_2}{\varepsilon_3 (l=0)}
\]
Taking the above assumptions into account, the following equations can be obtained:

- The effective strain:
  \[ \bar{\varepsilon} = J_R \varepsilon_1 \]  
  \[ J_R = \sqrt{\frac{R+1}{2R+1}} (1+R+2jR+(R+1)j^2) \]  
  \[ R > 1 \]  

- The equivalent or Von Mises stress:
  \[ \bar{\sigma} = I_R \sigma_1 \]  
  \[ I_R = \sqrt{\frac{1-j^2}{R+1} + j^2} \]  

- The stress strain relations according to Levy-Von Mises then become:
  \[ \varepsilon_1 = \frac{\bar{\varepsilon}}{\sigma_1} \left( \frac{\sigma_1 - R}{R+1} \sigma_2 \right) \]  
  \[ \varepsilon_2 = \frac{\bar{\varepsilon}}{\sigma_1} \left( \frac{\sigma_2 - R}{R+1} \sigma_1 \right) \]  
  \[ \varepsilon_3 = -\frac{\bar{\varepsilon}}{\sigma} \frac{1}{R+1} (\sigma_1 + \sigma_2) \]  

- The material shows exponential strain hardening behaviour:
  \[ \sigma_f = C(\bar{\varepsilon} + \varepsilon_0)^n \]  
  - \( \sigma_f \) the flow stress.
  - \( C \) characteristic deformation resistance.
  - \( n \) strain hardening exponent.
  - \( \varepsilon_0 \) pre-strain.

- The yield criterion remains:
  \[ \bar{\sigma} = \sigma_f \]  

By means of (2), (4) and (8), \( i \) and \( j \) can be expressed in each other:

\[ j = \frac{(R+1)i-R}{R+1-iR} \quad \text{and} \quad \frac{(R+1)i+R}{R+1+jR} \]  

See figure 3. This means that a straight stress path (i), implies an \( R \) dependent, straight strain path and vice versa. Since a straight strain path implies a minimum of plastic work, the assumption of a straight strain path is likely to be correct in most sheet metal forming processes. We are dealing with processes like bending and deep drawing.
The local necking in the tensile test

In a tensile test necking is related to the maximum of the force in the force-path diagram:

\[ dF = 0 \]  \hspace{1cm} (12)

![Figure 3 Relation between strain path \( j \) and stress path \( i \) with the normal anisotropy \( R \) as a parameter.](image)

![Figure 4 The initiation of necking caused by the weakest section.](image)

The hypothesis that necking occurs when the force reaches its maximum is plausible, with the condition that there is always a weakest cross-section in the test specimen. As is shown in fig. 4 a weakest spot implies, in the case of a decreasing force, that there will be a strain concentration, called necking, at the weakest section. By the decrease in force, the stronger cross-sections will lose part of their elastic deformation (spring-back).

From figure 2 it follows:

\[ F_1 = \sigma_1 b c \]

and with (1), (6), (7), (9) and (10) it yields:

\[ F_1 = \frac{C}{I_R} (J_F \varepsilon_1 + \varepsilon_0)^n e^{-\varepsilon_1} b_0 s_0 \]  \hspace{1cm} (13)

With the instability criterion:

\[ dF_1 = \frac{\partial F_1}{\partial \varepsilon_1} d\varepsilon_1 = 0 \]

It then yields:
\[ \varepsilon_{1cr} = n \frac{\varepsilon_0}{J_R} \]  

This is the well known critical strain in a tensile test, but actually only for \( \varepsilon_2 = 0 \) correct with respect to the experimentally obtained FLCs.

**The local necking in the formed sheet**

Experimentally it is found that at the moment of instability the strain \( \varepsilon_2 \) perpendicular to the major strain \( \varepsilon_1 \) remains stationary (Veerman\(^9\)). This can be explained by the fact that the rapid local increase of the strain in 2-direction \( \varepsilon_2 \) as observed in the tensile test will be suppressed by the neighbouring material, which is relatively stable compared to the weakest spot in a relatively large sheet. Figure 5 shows the strain path as has been found experimentally.

From this, a second condition for instability can be obtained, in the case of a relatively large sheet:

\[ d\varepsilon_2 = \frac{\partial \varepsilon_2}{\partial \varepsilon_1} d\varepsilon_1 = 0 \]  

(15)

So, as an instability criterion it is proposed that:

\[ dF_1 = 0 \text{ and } d\varepsilon_2 = 0 \]  

(16)

Relation (13) can be rewritten in several ways as a function of the strains. Using the instability criterion (16), three different formulations for the critical strain \( \varepsilon_{1cr} \) are obtained.

**First formulation**

\[ F_1 = \frac{C_b \varepsilon_0}{|J_R|} (-J_R(\varepsilon_2 + \varepsilon_3) + \varepsilon_0)^n \, e^{\varepsilon_2^2 + \varepsilon_3} \]  

(17)

and:

\[ dF_1 = \frac{\partial F_1}{\partial \varepsilon_2} d\varepsilon_2 + \frac{\partial F_1}{\partial \varepsilon_3} d\varepsilon_3 \]
Then the instability criterion becomes:

\[
\frac{\partial F_1}{\partial \epsilon_3} = 0
\]

and the critical major strain becomes:

\[
\epsilon_{1\sigma} = n \frac{\epsilon_0}{J_R}
\]  

which conforms to the former result.

**Second Formulation**

\[
F_1 = \frac{C b_0 s_0}{l_R} \left( -J_R (\epsilon_2 + \epsilon_3) + \epsilon_0 \right)^n e^{\frac{\epsilon_3}{1+j}}
\]  

(19)

and:

\[
dF_1 = \frac{\partial F_1}{\partial \epsilon_2} d\epsilon_2 + \frac{\partial F_1}{\partial \epsilon_3} d\epsilon_3
\]

Then the instability criterion becomes:

\[
\frac{\partial F_1}{\partial \epsilon_3} = 0
\]

and the critical major strain becomes:

\[
\epsilon_{1\sigma} = (1+j) n \frac{\epsilon_0}{J_R}
\]  

(20)

**Third Formulation**

\[
F_1 = \frac{C b_0 s_0}{l_R} \left( -\frac{J_R (\epsilon_3 + \epsilon_0)}{1+j} \right)^n e^{\epsilon_2 + \epsilon_3}
\]  

(21)

and:

\[
dF_1 = \frac{\partial F_1}{\partial \epsilon_2} d\epsilon_2 + \frac{\partial F_1}{\partial \epsilon_3} d\epsilon_3
\]
Then the instability criterion becomes:

\[
\frac{\partial F_1}{\partial \varepsilon_3} = 0
\]

and the critical major strain becomes:

\[
\varepsilon_{1c} = \frac{n}{1+j} - \frac{\varepsilon_0}{J_R}
\]  

(22)

The Forming Limit Curve

With the three previously described formulations (18), (20) and (22), the FLC can be drawn (see figure 6).

In sheet metal forming it is obvious that the highest values of the critical strain are closest to the experiments, in the case of relatively large sheets. However, if the instability occurs near an edge of the sheet, then the second condition \( \varepsilon_2 = 0 \) is not fulfilled (Veerman\textsuperscript{10}), and the critical strain moves downward towards line I.

Figure 6 The FLC in absence of prestrain, with line I according to (18), line II according to (22) and line III according to (20).

Figure 7 The influence of prestrain and anisotropy on the FLC.

Figure 7 shows the influence of the prestrain \( \varepsilon_0 \) and anisotropy \( R \). In most cases the prestrain is so small that it can be neglected.
Experimental results

From Painter, experimental results of several materials are used to verify the theory presented in this paper. The leftside values are obtained by tensile tests; grooved teststrips (to produce different strain ratio) were pulled to failure. The rightside values are obtained using the hydraulic bulging technique. In figure 8, 9, 10 and 11 the theoretical lines (according to (18), (20) and (22)) and experimental values for steel, aluminium and brass are plotted.

Figure 8 CRf-steel tensile test.

Figure 9 FLC for CRf-steel.

Figure 10 FLC for aluminium.

Figure 11 FLC for brass.
Factors influencing the FLC

Quite a number of measurements deviate substantially from the theoretical curves. Four causes will be discussed here:
- Inhomogeneity of the material.
- Ductility.
- Strain rate sensitive material.
- Logarithmic strain hardening.

A : Inhomogeneity of the material

Inhomogeneity of the material causes stress-concentration in the workpiece and consequently early crack initiation, even before the necking point is achieved (Ramaekers\textsuperscript{12}). This fracturing is often interpreted as fracture preceded by necking instability. In fact, the material shows lacking ductility.

B : Ductility

Ductility is defined as the strain until crack initiation in the workpiece. As is shown by Bolt\textsuperscript{13}, and Sillekens\textsuperscript{14}, the strain to fracture depends on the material (homogeneity (A), structure, grain size) and the state of stress, in particular the hydrostatic stress (see fig. 12).

For the hydrostatic stress the following holds:

\[ \frac{\sigma_m}{\sigma} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \]  \hspace{1cm} (23)

it then follows for the current analysis:

\[ \frac{\sigma_m}{\sigma} = \frac{1}{3}(1+i) \left( 1 - i \frac{2R}{R+1} + i^2 \right)^{-\frac{1}{2}} \]  \hspace{1cm} (24)

and so with (11):

\[ \frac{\sigma_m}{\sigma} = \frac{1}{3} \left( 1 + \frac{j+R^*}{1+jR^*} \right) \left( 1 - 2 \frac{j+R^*}{j+1} + \left( \frac{j+R^*}{1+jR^*} \right)^2 \right)^{-\frac{1}{2}} \]  \hspace{1cm} (25)

in this:

\[ R^* = \frac{R}{R+1} \]
As can be seen from fig. 13, early crack initiation will preferably occur at the right part of the FLC (\( j > 0 \)). This is also illustrated by Pearce\(^3\) (see fig. 14).

C : Strain rate sensitivity.

Strain rate sensitive material shows a delay in the occurrence of necking. With a constant velocity \( (v) \) of the moving head of the tensile tester, it yields the following for the strain rate during uniform deformation (see fig. 15):

\[
\dot{\varepsilon}_1 = \frac{V}{L}, \text{L the length of the test piece.}
\]

During necking this becomes:

\[
\dot{\varepsilon}_1 = \frac{V}{L_N}, \text{L}_N \text{ length of the necking zone.}
\]

Thus with \( L_N < < L \), the strain rate will increase substantially in the necking zone, so the strain resistance or yield stress will increase, and consequently a more uniform deformation will appear.

Generally, it is assumed that carbon steel is strain rate sensitive, contrastingly, aluminium is not. Experimental curves of C-steel show some delay in the occurrence of necking, while aluminium mostly does not show this delay. This is in agreement with the statements made.
D : Logarithmic strain hardening.

This behaviour is experimentally found for copper and brass (KMS 63) (Ramaekers\(^{14}\)). Fig. 12 shows the results of the tensile test.

For \( \bar{\varepsilon} \leq \bar{\varepsilon}_s \) the well-known exponential strain hardening behaviour is found and for \( \bar{\varepsilon} \geq \bar{\varepsilon}_s \) the so-called logarithmic strain hardening:

\[
\sigma_f = B + q \ln(\bar{\varepsilon} + \varepsilon_0)
\]

A possible explanation for this behaviour is the appearance of cross-slip after a certain amount of plastic work. With (26), (13) becomes:

\[
F_1 = \frac{Bb_0c_0}{l_R}(1 + q^* \ln(J_R\varepsilon_1 + \varepsilon_0))\exp^{-\varepsilon_1}
\]

From a limited number of experiments on copper and brass (Ramaekers\(^{14}\)), it follows that:

\[
q^* = \frac{q}{B} \approx \frac{n}{2}
\]

Analogue to exponential strain hardening material, the critical strain \( \varepsilon_{1cr} \) is derived. Fig 17 shows the result of the calculations.

The n-value of formula (28) is the real value obtained from the tensile test for \( \bar{\varepsilon} < \bar{\varepsilon}_s \).

Normally this deviant material behaviour is not taken into account, and one will find an average value of the strain hardening exponent, which is smaller than the one above-mentioned for \( \bar{\varepsilon} < \bar{\varepsilon}_s \).

The theoretical FLC is in fairly good agreement with measurements for \( \varepsilon_2 < 0 \) (fig. 17). For \( \varepsilon_2 > 0 \) the ductility (B) will influence the measurements.
Conclusions

A plausible theory for the FLC is derived. For aluminium and carbon steel, theory and experiment correspond very well. For some materials, for example low carbon steels, the strain rate sensitivity is a possible explanation for the delay in the occurrence of necking. Some materials, i.e. brass, show a large deviation, for $\varepsilon_2 > 0$, from the theory, which can be explained by the moderate ductility of the material. In certain conditions the material tends to fracture before localised necking. For brass the different strain hardening behaviour also gives a possible explanation for the deviation.

References