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19. GLOBAL MECHANICAL PROPERTIES OF TRABECULAR BONE: EXPERIMENTAL DETERMINATION AND PREDICTION FROM A STRUCTURAL MODEL

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1. INTRODUCTION

Trabecular bone is a porous material composed of rods and plates of bone tissue. The orientation of these trabeculae accounts for the anisotropic mechanical behaviour of the material. This behaviour further depends on the location in the bone since, shape, dimensions and orientation of the trabeculae are not constant. In order to gain more insight in the mechanical behaviour of trabecular bone, both a method for the experimental determination of material properties and a structural model have been developed.

2. EXPERIMENTAL INVESTIGATION

The mechanical properties were measured on 27 cubic samples of fresh trabecular bone with marrow in situ. These specimens were taken from the proximal epiphysis of a human tibia. Figure 1 shows the location of the samples. On a Reichert-Jung microtome with milling attachment the surfaces were milled until the sides of the specimen measured 8 mm. This procedure ensures smooth and plane surfaces without damage being done to the trabecular structure.

The specimens were compressed between two parallel plates in a specially designed loading device. The force on the movable plate was measured with a load cell and the displacement of that plate was measured with a displacement transducer (LVDT). Both signals were combined to a load/displacement curve on an x-y recorder. From the slope of this curve and the dimensions of the specimen Young's modulus was determined. Young's moduli in the two other orthogonal directions were found by repeating this procedure two times with other sides of the sample facing the plates.

FIGURE 1: Location of the bone samples with global coordinate directions

During loading, the sideward expansion of the sample in the two directions perpendicular to the loading direction was determined. This was accomplished by measuring the displacement of two small mirrors bonded to opposite sides of the cube with laser interferometers. The sideward expansion in the remaining direction was determined by repeating the compression test with the mirrors placed on the other free sides. The output signals of the interferometers were recorded together with the signals of the displacement transducer and the load cell on a four line x-t recorder. Per sample six Poisson's ratios were calculated from the deformation results.

The ultimate compressive strength of the cancellous bone in proximal-distal direction was determined in a MTS testing machine. The samples were compressed with a constant strain rate of 0.01 per second. After determination of the ultimate strength the marrow was extracted from the specimens in order to perform density measurements like the apparent density and the density of the trabecular material. This was still possible, because in most cases the deformation had not exceeded 5%. The methods used to determine the densities were those described by Carter and Hayes (1976).

3. RESULTS AND DISCUSSION

The results are listed in table 1. The dimensions of the Young's moduli E_i and the strength σ_c are MPa, the densities ρ_a and ρ_b are given in g/cm^3 .

3.1 Young's moduli

The load/displacement curves appeared to be almost linear except for the first part. All samples were loaded twice in each direction. In a few cases an increase in the slope up to 5% occurred at a second loading cycle. Subsequent loading had no effect on the stiffness. The moduli were determined from the slope of the linear part of the curve at the second loading cycle. The highest values for the E-moduli were found near the cortical shell, especially near the medial and lateral sides of the tibial plateau. This spatial variation of the stiffness is similar to that reported by Goldstein et. al. (1983). All specimens showed a larger stiffness in proximal-distal direction than in the two other directions. This anisotropy became stronger with increasing distance from the tibial plateau, while at the same time the proximal-distal stiffness remained almost constant. The mean value for E_3 appeared to be about two times as large as the mean values for E_1 and E_2 .

3.2 Poisson's ratios

The values for the Poisson's ratios ranged from 0.07-0.65. It appeared that ν_{31} and ν_{32} were considerably lower than the four other ratios. (ν_{ij} relates the strain in i-direction with the stress in j-direction). Generally the obtained values were much higher than those reported by Knauss (1981) for trabecular bone from the proximal part of the femur (0.01-0.34).

3.3 Compressive strength

The magnitude of the compressive strength in proximal-distal direction ranged from 3-17 MPa, consistent with previously published data (Goldstein et. al., 1983; Carter and

Hayes, 1977)). There existed a strong correlation between the compressive strength and the elastic modulus in the same direction.

SAMPLE	E ₁	E ₂	E ₃	ν ₁₂	ν ₁₃	ν ₂₁	ν ₂₃	ν ₃₁	ν ₃₂	σ _c	ρ _a	ρ _b
1	918	617	952	0.37	0.24	0.56	-	0.20	0.24	-	-	-
2	952	460	990	0.27	0.33	0.41	0.41	0.18	0.25	8.7	0.39	1.97
3	580	203	741	0.24	0.29	0.40	0.44	0.25	-	6.0	0.33	1.97
4	793	441	1043	0.33	0.35	0.45	0.44	0.18	0.21	14.7	-	-
5	790	774	1667	0.46	0.30	0.45	0.32	0.17	0.21	17.0	0.58	1.96
6	693	817	1134	0.57	0.52	0.38	0.37	0.17	0.20	10.6	0.47	1.95
7	276	357	711	0.51	0.42	0.47	0.47	0.24	0.25	-	0.34	1.85
8	187	317	349	0.45	0.50	0.28	0.42	0.37	0.36	3.4	0.25	1.98
9	234	437	899	0.64	0.55	0.39	0.41	0.12	0.24	-	0.35	1.96
10	989	957	1388	0.22	0.26	0.42	0.27	0.23	0.07	15.2	-	-
11	685	566	1175	0.50	0.38	0.42	0.32	0.19	0.27	11.9	0.50	1.94
12	462	427	1015	0.37	0.25	0.38	0.25	0.13	0.17	7.9	0.39	1.94
13	150	291	504	0.42	0.46	-	0.49	-	-	-	0.27	1.85
14	170	224	569	0.45	0.38	0.43	0.39	0.14	0.16	-	0.31	1.98
15	458	509	1134	0.51	0.36	0.41	0.23	0.16	0.18	-	0.42	1.94
16	625	345	765	0.32	0.27	0.53	0.49	0.18	0.21	6.8	0.37	1.97
17	614	714	838	0.38	0.30	0.27	-	-	-	8.8	0.46	1.95
18	-	-	987	-	-	-	0.28	0.13	-	8.6	0.35	1.99
19	175	310	1140	0.43	0.63	0.30	0.41	0.13	0.15	9.8	0.34	1.96
20	338	327	1449	0.51	0.44	0.44	0.39	0.20	0.12	11.5	0.42	1.97
21	270	425	1273	0.45	0.60	0.34	0.48	-	0.21	10.8	0.38	2.00
22	95	132	748	0.25	0.59	-	0.46	0.18	0.13	5.2	0.26	1.97
23	64	191	293	0.50	0.60	0.26	0.38	-	0.28	3.0	0.22	1.90
24	11	161	766	0.64	0.65	-	0.33	0.16	-	-	0.29	2.01
25	342	338	1452	0.57	0.42	0.50	0.31	0.16	0.12	10.4	0.42	1.99
26	156	319	1102	0.49	0.73	0.34	0.45	0.17	0.23	10.1	0.31	-
27	250	437	1106	0.51	0.47	0.37	0.58	-	0.19	-	-	-
mean	434	427	970	0.44	0.43	0.40	0.39	0.18	0.20	9.5	0.37	1.95

TABLE 1: Technical constants for trabecular bone

Linear regression on the logarithmic data yielded

$$\sigma_c = 0.012 E^{0.97} \quad (r = 0.94) \tag{1}$$

This close relationship between strength and stiffness is practically a linear one, which implies an almost constant strain at failure regardless the wide variation in strength found in trabecular bone.

3.4 Correlation between mechanical properties and density

Linear regression between apparent density and each of the three elastic moduli resulted in

$$E_1 = 63 \cdot 10^2 \rho_a^{3.1} \quad (r = 0.73) \quad E_2 = 19 \cdot 10^2 \rho_a^{1.7} \quad (r = 0.83)$$

$$E_3 = 41 \cdot 10^2 \rho_a^{1.5} \quad (r = 0.82) \tag{2}$$

A power relationship was chosen, although this did not yield better correlation coefficients than a first degree relationship between these parameters. Linear regression between apparent density and strength in proximal distal direction resulted in

$$\sigma_c = 38 \rho_a^{1.5} \quad (r = 0.88)$$

(3)

Also in the case of strength the power relation model did not correlate stronger than the linear model. The results implie that density does not fully account for the wide variation in mechanical properties of trabecular bone.

4. A STRUCTURAL MODEL OF TRABECULAR BONE

A structural model enables the prediction of the global behaviour of the bone, knowing the behaviour of the individual trabeculae, i.e. it provides a relation between the macroscopic and the microscopic mechanical behaviour of the bone structure. An intra-medullary fixed prostheses is cemented for a large part in a plate/rod type of trabecular bone. Therefore this structure has been chosen for the mechanical modelling. The essential premise of the model is the assumption that the loads on the structure are transferred from one plate to another by means of the interconnecting rods. Fig. 2a shows a 2-dimensional cross section of the structural model.

FIGURE 2: Two-dimensional cross section of the structural trabecular model

A simplification of the structure consists of the adoption of various symmetry planes. In this way, the mechanical behaviour of a small part of the model (shown in fig. 2b) characterizes the behaviour of the complete structure. Each interconnecting rod in fig. 2a in fact represents a system of rods as indicated in fig. 2b. The rods have various angles with respect to the parallel plates but are assumed to have some preferred orientation.

FIGURE 3: Characteristic part of the plate/rod model (only one rod is shown)

An interconnecting cylindrical rod has length L , radius R , Young's modulus E and is loaded at its ends by the parallel plates with forces and moments (see fig. 3). Beam theory is used to relate the force on the rod ends and the displacement difference between the rod ends. The plate transmits the resulting force of a finite number of rods and the direction (α, β) of the rods is supposed to be characterized by a bivariate normal distribution G with the mean angles α_0 and β_0 , standard deviations σ_α and σ_β and correlation coefficient ρ .

$$G = \frac{1}{2\pi\sigma_\alpha\sigma_\beta\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\left(\frac{\alpha-\alpha_0}{\sigma_\alpha}\right)^2 - 2\rho\left(\frac{\alpha-\alpha_0}{\sigma_\alpha}\right)\left(\frac{\beta-\beta_0}{\sigma_\beta}\right) + \left(\frac{\beta-\beta_0}{\sigma_\beta}\right)^2\right]\right\} \quad (4)$$

The total force which is exerted on each plate by the rods then may be found by integrating over all possible angles α and β (see for further details Klever (1984)). In this way a relation between the forces exerted on a plate and the displacements of that plate is obtained, which may be transformed in a relation between global stresses and strains by means of the dimensions of the model (B, BB, H, D) shown in fig. 3. Thus all the orthotropic elastic constants are expressed as functions of the geometrical parameters H, D, BB, B and R , the Young's modulus E of the bone rods, the number of bone rods N and as functions of the bone rod angle distribution parameters $\alpha_0, \beta_0, \sigma_\alpha, \sigma_\beta$ and ρ .

FIGURE 4: Dimensionless Young's moduli E_x and E_z and Poisson's ratio ν_{xz} of the model as functions of the mean angle β_0 and the standard deviation σ_β

In this paper results will be presented for the case that all the bone rods lie in the x - z plane, i.e. $\alpha_0 = 0$ and $\sigma_\alpha = 0$. The Young's moduli E_x and E_z and Poisson's ratio ν_{xz} are shown in fig. 4 as functions of the mean angle β_0 and the standard deviation σ_β .

The following dimensionless quantities are used:

$$\frac{E_1}{E_{10}} = \frac{E_x BB H D}{E N \pi R^2 (BB+B)} \quad \frac{E_3}{E_{30}} = \frac{E_z BB (BB+B) D}{E N \pi R^2 H} \quad \frac{\nu_{13}}{\nu_{130}} = \nu_{xz} \frac{BB+B}{H} \quad (5)$$

As a result of the model the shear modulus G_{zx} appears to be strongly related to the Young's modulus E_z and for realistic parameters this relation is shown to be

$$G_{zx} \approx \left\{ \frac{(BB+B)}{H} \right\}^2 E_z \quad (6)$$

The dimensionless modulus E_1/E_{10} appears to be only moderately dependent on the mean angle β_0 and the standard deviation σ_β .

The modulus E_3/E_{30} varies strongly with the deviation, but the mean angle does not play any role.

The Poisson's ratio ν_{13}/ν_{130} is determined mainly by the mean angle (as could be expected), but also to some extent by the deviation. It should be noticed that ν_{zx} is a dependent quantity due to the symmetry requirements of the constitutive matrix and equals $\nu_{xz} E_x/E_z$.

Within a realistic range of its parameters the trabecular bone model presented here describes a great variety of the anisotropic elastic material constants. For realistic model parameters these properties lie within the range of actually measured material constants of trabecular bone.

REFERENCES

- [1] Carter, D.R. and Hayes, W.C. (1976), Bone compressive strength: The influence of density and strain rate. Science, Vol. 194, 1976, pp. 1174-1175.
- [2] Goldstein, S.A., Wilson, D.L., Sonstegaard, D.A. and Matthews, L.S. (1983), The mechanical properties of human tibial trabecular bone as a function of metaphyseal location. J. Biomech., Vol. 16, 1983, pp. 965-969.
- [3] Knauss, P. (1981), Material properties and strength behaviour of spongy bone tissue at the human coxal femur (in German). Biomed. Techn., Vol. 26, 1981, pp. 200-210.
- [4] Carter, D.R. and Hayes, W.C. (1977), The compressive behaviour of bone as a two-phase porous structure. J. Bone Jt. Surg., Vol. 59-A, no. 7, 1977, pp. 954-962.
- [5] Klever, F.J., (1984), On the Mechanics of failure of artificial knee joints. Ph.D. Thesis, Twente University of Technology, Enschede, 1984, The Netherlands.