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Determination of Young's Modulus or Poisson's Ratio Using Eddy Currents

Young's modulus and Poisson's ratio of a small conducting specimen can be determined by a new method using the strains caused by eddy currents in a magnetic field

by Th. Kwaaitaal and A.J.G. Schoofs

ABSTRACT—Combined a-c and d-c magnetic fields cause strains in a conducting sample. The amplitude of this strain is dependent on the value of Young's modulus and Poisson's ratio. The strain amplitude, being in the order of 10 pm, can be measured with a stabilized Michelson interferometer, described elsewhere.^{1,2} An expression is derived, relating the axial strain in cylindrical samples to the magnetic-field quantities, the elastic properties and the electrical resistivity of the sample.

The finite-element method is used to treat more complicated configurations. Samples of aluminum, copper, gold and tin are used for comparing the measured and calculated results. To this end, the elastic properties of the copper samples were also determined from measurement of the ultrasonic-wave velocity. The agreement between both methods is very satisfactory.

List of Symbols

- a, b = inner and outside radii, m
- B_o, B_i = magnetic induction, T
- E = Young's modulus, Pa
- E_θ = field strength, Vm^{-1}
- F = body force, Nm^{-3}
- G = shear modulus, Pa
- H = magnetic-field strength, Am^{-1}
- J = current density, Am^{-2}
- l = sample length, m
- t = time coordinate, s
- u, v, w = displacements in r, θ and z -directions, respectively, m
- v_l, v_t = longitudinal and transverse-wave velocity, ms^{-1}
- γ = shear strain
- ϵ = strain
- μ = magnetic permeability, $VsA^{-1}m^{-1}$
- ν = Poisson's ratio
- ρ = electrical resistivity, Ωm
- ρ_m = mass density, kgm^{-3}
- σ = stress, Pa
- τ = shear stress, Pa
- ω = circular frequency, $rad s^{-1}$
- r, θ, z = as subscripts designate radial, circumferential and longitudinal directions, respectively

Introduction

The method described in this paper arose from a

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procedure used to calibrate the amplitude of picometer vibrations measured with a stabilized Michelson interferometer. Such an interferometer is electronically stabilized against drift of thermal origin and can be used to measure vibrational amplitudes as small as 0.08 pm.¹ Usually, the amplitude of such vibrations is determined by comparison with that of vibrations exhibited by a calibrated piezo-electric crystal. Measurement of magnetostrictive effects, however, entails calibration, including the a-c and d-c magnetic fields needed in this case. Although these fields may be measured by other methods, i.e., the Hall effect and the voltage induced in a coil, an alternative method of measurement is valuable as it gives an indication of possible systematic errors. Such calibration, including the magnetic fields, is found by the use of a conducting sample in which eddy currents are generated by the a-c component of the magnetic field. The d-c component of this field exhibits a Lorentz force on these currents. This force causes an a-c deformation of the sample, the magnitude of which depends on its elastic properties. In the case of common values of magnetic-field strength, elastic properties and electrical conductivities of the sample, this deformation can be measured with sufficient accuracy. As it proves that the two methods give the same results and consequently, that systematic errors are negligible, the procedure can be reversed and used to determine the elastic properties of the sample.

The well-known method of determining the elastic properties of solids by measuring the velocity of ultrasonic waves was used in our experiment.

In the first section of this paper, an expression will be derived for the axial displacement of a cylindrical sample in an axial a-c and d-c magnetic field. The second section is devoted to the comparison of calculated and measured displacements.

The influence of the skin effect will then be discussed since it attempts to disturb the eddy-current distribution in the sample. Finally the merits of the method will be discussed.

It is assumed that the principle of the magnetostriction measurement with the stabilized interferometer is known. Details can be found in Ref. 2.

The Displacement of a Cylinder in an Axial Magnetic Field

The geometrical configuration of the sample in the field is given in Fig. 1. Cylinder coordinates are used as indicated in the figure. In calculating the displacement in the z -direction, an infinitesimal element of the cylinder

Fig. 1—The position and suspension of the sample relative to the magnetic fields

indicated in Fig. 2 is considered. An expression for the body force F on this element is derived and then the system of equations formed by (1) the equilibrium equations; (2) the strain-displacement relations; and (3) the constitutive equations is solved.

To find an expression for the body force, the integral form of the law of electromagnetic induction is applied to the circular circuit, with radius r drawn about the center axis of the cylinder and shown as a shaded area in Fig. 1.

$$\int_0^{2\pi} E_\theta r d\theta = - \frac{\partial}{\partial t} \int_0^{2\pi} \int_0^{r_0} B_1 r dr d\theta \quad (1)$$

Here, E_θ is the circumferential field strength induced by the axial a-c magnetic induction B_1 . Inside the circle

Fig. 2—Infinitesimal element of the cylindrical sample and the relevant forces

($r < r_0$), the field strength equals

$$E_\theta = - \frac{r}{2} \frac{\partial B_1}{\partial t} \quad (2)$$

and thus the current density J_θ defined by $\vec{J}_\theta = J_\theta \vec{e}_\theta$, where $\vec{e}_\theta$ is the unit vector in the θ -direction, is given by

$$J_\theta = - \frac{r}{2\varrho} \frac{\partial B_1}{\partial t} \quad (3)$$

where ϱ is the resistivity of the sample. The Lorentz force $\vec{F}_r$ on an element with volume $r dr d\theta dz$, resulting from the action on the current of the d-c magnetic field $\vec{B}_0$ defined by $\vec{B}_0 = B_0 \vec{e}_z$, where $\vec{e}_z$ is the unit vector in the z -direction, is

$$\vec{F}_r = \vec{J}_\theta \times \vec{B}_0$$

and thus

$$F_r = - \frac{r}{2\varrho} B_0 \frac{\partial B_1}{\partial t} \quad (4)$$

while F_θ and F_z are both zero.

The relevant equations of equilibrium in cylinder coordinates with axial symmetry can be written as

$$\begin{aligned} \frac{\partial \sigma_{rr}}{r} + \frac{\partial \sigma_{rz}}{z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + F_r &= 0 \\ \frac{\partial \sigma_{rz}}{r} + \frac{\partial \sigma_{zz}}{z} + \frac{\sigma_{rz}}{r} &= 0 \end{aligned} \quad (5)$$

The relevant expressions of Hooke's law are

$$\begin{aligned} \epsilon_{rr} &= \frac{1}{E} \{ \sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz}) \} \\ \epsilon_{\theta\theta} &= \frac{1}{E} \{ \sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz}) \} \\ \sigma_{rz} &= G \epsilon_{rz} \end{aligned} \quad (6)$$

while the strain-displacement relations are

$$\begin{aligned} \epsilon_{rr} &= \frac{\partial u_r}{\partial r} & \epsilon_{rz} &= \frac{1}{2} \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \\ \epsilon_{\theta\theta} &= \frac{u_r}{r} \\ \epsilon_{zz} &= \frac{\partial u_z}{\partial z} \end{aligned} \quad (7)$$

In these equations, σ_{ij} and ϵ_{ij} are respectively the stress and the strain acting on the plane i in the direction j . The displacements in the radial, tangential and axial directions are indicated by u_r , u_θ and u_z , respectively.

Solving the equations for a plane-strain situation ($\epsilon_{zz} = 0$), the displacement in the axial direction is

$$u_z = - \frac{\nu}{4\varrho E} B_0 \frac{\partial B_1}{\partial t} \left(1 + \frac{a^2}{b^2} \right) b^2 l \quad (8)$$

for the case of a hollow cylinder with inner and outer radius a and b , respectively.

For a solid cylinder the displacement is

$$u_z = - \frac{\nu}{4\varrho E} B_0 \frac{\partial B_1}{\partial t} b^2 l \quad (9)$$

In the case of plane stress ($\sigma_{zz} = 0$), the displacement of

a hollow cylinder is

$$u_z = -\frac{\nu(\nu+3)}{8\rho E} B_o \frac{\partial B_1}{\partial t} b^2 l \left\{ 1 + \frac{a^2}{b^2} - 2 \frac{\nu+1}{\nu+3} \frac{r^2}{b^2} \right\} \quad (10)$$

and, for the solid cylinder,

$$u_z = -\frac{\nu(\nu+3)}{8\rho E} B_o \frac{\partial B_1}{\partial t} b^2 l \left\{ 1 - 2 \frac{\nu+1}{\nu+3} \frac{r^2}{b^2} \right\} \quad (11)$$

Assuming that the mirror glued to the sample shows the mean displacement over the surface, eq (10) gives

$$\langle u_z \rangle = \frac{\int_0^{2\pi} \int_a^b u_z r dr d\theta}{\int_0^{2\pi} \int_a^b r dr d\theta} = -\frac{\nu}{4\rho E} B_o \frac{\partial B_1}{\partial t} \left(1 + \frac{a^2}{b^2} \right) b^2 l \quad (12)$$

and, for the solid cylinder, eq (11) gives

$$\langle u_z \rangle = -\frac{\nu}{4\rho E} B_o \frac{\partial B_1}{\partial t} b^2 l \quad (13)$$

For evaluating the dependence of u_z on r in the plane-stress condition, a plot of the ratio

$$\frac{u_z}{\langle u_z \rangle} = \frac{\nu+3}{2} \left\{ 1 - \frac{2 \frac{\nu+1}{\nu+3} \frac{r^2}{b^2}}{1 + \frac{a^2}{b^2}} \right\} \quad (14)$$

from eqs (10) and (12) is made as a function of r/b for different values of a/b (Fig. 3).

Experimental Evaluation of the Formulas

To verify the formulas, the displacements u_z are measured of four different metals in four different configurations and at ten different values of the product $B_o B_1$.

The materials used were copper, gold, tin and aluminum. The four different configurations are shown in Fig. 4. Standard dimensions were length $l = 10$ mm, outer dia-

meter $b = 5$ mm, inner diameter $a = 4$ mm, height of stud or rim $l_2 = 1$ mm, thickness of mirror $d = 1$ mm. Apart from the displacements, the a-c and d-c magnetic fields, the values of Young's modulus and Poisson's ratio and the electrical resistivity were measured. The axial displacement is calculated from these last five measured

Fig. 3—Plot of the ratio of the axial displacement u_z and the mean displacement $\langle u_z \rangle$ as a function of radius r . Curves 1, 2 and 3 refer to eq (14) with different values of a/b , i.e., 0.0, 0.5 and 0.8. Curve 4 gives the results of a finite-element calculation for a solid cylinder. Curve 5 is the curve for constant strain. In all cases, Poisson's ratio equals 0.35

Fig. 4—Configurations of the different samples with mirrors. A: solid cylinder; B: hollow cylinder; C: solid cylinder with rim; D: solid cylinder with stud

values and from the dimensions of the sample and compared to the measured value.

The value of the d-c field was determined by a Hall-effect gaussmeter. The value of the a-c field was determined by placing a coil of the same dimensions as the sample in place of the sample and measuring the induced voltage with a lock-in amplifier.

Ten different values of the product $B_0 B_1$ were obtained by changing the distance between the poles of the electromagnet used at constant a-c and d-c current. The resulting values of B_0 and B_1 are given in Table 1. Position 1 of the poles corresponds to a pole distance of 34 mm. The decrease in distance from one position to the next is 1 mm. The frequency of the a-c field is 66 Hz.

The value of Young's modulus and Poisson's ratio for copper were determined by the ultrasonic pulse-echo-overlap method. This was done because quite considerable differences are found in literature on values of Young's modulus.

The sample used for this measurement was a copper cylinder turned from the same piece of material as the sample used for the displacement measurement. It was 49.5 mm in length and 30 mm in diameter.

The frequency of the longitudinal waves was 2.3 MHz and their velocity was 4760 ms⁻¹. The frequency of the transverse waves was 4.2 MHz and their velocity 2260 ms⁻¹. Young's modulus and Poisson's ratio were calculated using the formulas

$$E = 4 \rho_m v_t^2 \frac{\frac{3}{4} v_l^2 - v_t^2}{v_l^2 - v_t^2} \quad (15)$$

$$\nu = \frac{\frac{1}{2} v_l^2 - v_t^2}{v_l^2 - v_t^2} \quad (16)$$

where ρ_m is the mass density of the solid. The accuracy in Young's modulus due to errors in the determination of sample length, specific mass and travel time of the waves was estimated to be better than 0.5 percent. The electrical resistivity was determined by a four-point probe method. The values found did not differ from those in literature.

The dependence of the results on the pole distance is given in Table 2 for the copper sample of Fig. 4(d) but with the stud removed. The length of this sample was 9.04 mm. The values of ρ , E and ν are as given in Table 3. The results for other samples are given for the value of $B_0 B_1$, corresponding to position 1 of the poles.

The measured displacements of copper, gold, tin and aluminum samples of configuration 4(a) are compared with the values calculated from eqs (9) or (13). The results are tabulated in Table 3.

The dependence of u_z on the frequency of the modulating field was examined by repeating the measurements at a frequency of 33 and 132 Hz. The results are shown in Fig. 5. They are measured for a single crystal of copper with the configuration shown in Fig. 4(a) and with the $\langle 111 \rangle$ axis oriented along the axis of the cylinder. These figures apply to position 1 of the pole distance of the electromagnet.

If experiments on configuration 4(b) are compared to calculations according to eq (12), the agreement is not good. The same applies to experiments on configuration 4(c) and 4(d) compared to calculations according to eq (11).

In the calculations it is assumed that, in configuration 4(c), the mean displacement of the rim of the cylinder will be measured and the same assumption applies to the mean

TABLE 1—VALUES OF MAGNETIC INDUCTION AS A FUNCTION OF POLE DISTANCE

Position Poles	B_1 (T)	B_0 (T)	$B_0 B_1$ (mT ²)
1	0.0145	0.581	8.42
2	0.0149	0.599	8.93
3	0.0154	0.617	9.50
4	0.0159	0.638	10.1
5	0.0165	0.660	10.9
6	0.0170	0.681	11.6
7	0.0176	0.706	12.4
8	0.0182	0.731	13.3
9	0.0188	0.758	14.3
10	0.0195	0.788	15.4
11	0.0201	0.819	16.5

TABLE 2—COMPARISON OF CALCULATED AND MEASURED DISPLACEMENT OF A COPPER CYLINDER AS A FUNCTION OF POLE DISTANCE

Position Poles	Axial Displacement (pm)		
	Measured	Calculated	Ratio
1	8.05	8.27	1.03
2	8.53	8.77	1.03
3	9.01	9.33	1.04
4	9.68	9.92	1.02
5	10.4	10.7	1.03
6	11.1	11.4	1.03
7	12.0	12.2	1.02
8	12.6	13.1	1.04
9	13.7	14.0	1.03
10	14.8	15.1	1.02

TABLE 3—COMPARISON OF CALCULATED AND MEASURED DISPLACEMENT OF SOLID CYLINDERS OF DIFFERING MATERIALS

	ρ ($10^{-8} \Omega \text{m}$)	E (10^{10} Nm^{-2})	ν	Axial Displacement (pm)		
				Measured	Calculated	Ratio
Cu 1.67	12.4	0.35	9.16	9.35	1.02	
Au 2.19	7.9	0.42	5.07	4.77	0.94	
Sn 11.5	5.0	0.33	3.40	3.13	0.92	
Al 2.65	7.02	0.34	10.5	10.0	0.95	

displacement of the central part of the cylinder in configuration 4(d).

In the case of configuration 4(b), the discrepancy is mainly due to the elastic properties of the mirror, while in configurations 4(d) the assumptions of plane strain or plane stress are the main origins of discrepancy between calculation and measurement. To overcome this difficulty, a finite-element calculation is made of the displacement u_z as a function of the radius r of the solid cylinder of configuration 4(a). The results are shown in Fig. 3 and Table 4. The displacement in the case of configuration 4(b) is also calculated by the finite-element method. In this analysis, the elastic properties of the mirror are also

Fig. 5—Frequency dependence of the axial displacement u_z of a cylindrical single crystal of copper

taken into account. These are: $E = 6 \times 10^{10} \text{ Nm}^{-2}$ and $\nu = 0.24$. The results are shown in Table 4.

Influence of the Skin Effect

The skin effect is the limitation of the penetration of electromagnetic waves in a conducting material. As a result, the field strength and, thus, the currents induced in the cylindrical sample will not show a linear dependence on radius r .

In many cases, the skin effect is expressed in terms of the skin depth d , giving the decay of an electromagnetic wave to its e^{-1} value where it penetrates a conducting medium.³ The skin depth $d = \sqrt{\frac{2\rho}{\omega\mu}}$ for copper at a frequency of 66 Hz equals 14 mm. As the dimensions of the sample are of the same order of magnitude, an accurate calculation is necessary. The nonlinearity in the field strength E_θ will be determined by calculating the deviation of E_θ at the surface of the cylinder from its value found in eq (2).

Maxwell's equations read:

$$\begin{aligned} \nabla \times \vec{H} &= \frac{\vec{E}}{e} & \nabla \cdot \vec{E} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{H}}{\partial t} & \nabla \cdot \vec{H} &= 0 \end{aligned} \quad (17)$$

TABLE 4—COMPARISON OF CALCULATED AND MEASURED DISPLACEMENT OF COPPER IN DIFFERENT CONFIGURATIONS

Configuration	Axial Displacement (pm)	
	Measured	Calculated
4b	24.1	24.07
4c	9.37	9.42
4d	11.3	11.58

The divergence of $\vec{E}$ and $\vec{H}$ are chosen as zero because a solution is sought inside a homogeneous conductor having no free electric or magnetic charges.

Cylinder coordinates are used as defined in Fig. 1. The r -, θ - and z -components of the Maxwell equations in cylinder coordinates are:

$$\begin{aligned} \frac{1}{r} \frac{\partial H_z}{\partial \theta} - \frac{\partial H_\theta}{\partial z} &= \frac{E_r}{e} & \frac{1}{r} \frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z} &= -\mu \frac{\partial H_r}{\partial t} \\ \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} &= \frac{E_\theta}{e} & \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} &= -\mu \frac{\partial H_\theta}{\partial t} \\ \frac{1}{r} \left\{ \frac{\partial}{\partial r} (r H_\theta) - \frac{\partial H_r}{\partial \theta} \right\} &= \frac{E_z}{e} \\ \frac{1}{r} \left\{ \frac{\partial}{\partial r} (r E_\theta) - \frac{\partial E_r}{\partial \theta} \right\} &= -\mu \frac{\partial H_z}{\partial t} \end{aligned} \quad (18)$$

From symmetry, we find $\frac{\partial}{\partial \theta} = 0$ and, if we neglect the influence of the finite length of the sample, we can state $\frac{\partial}{\partial z} = 0$. The set of equations reduces to

$$\begin{aligned} E_r &= 0 & H_r &= 0 \\ -\frac{\partial H_z}{\partial r} &= \frac{E_\theta}{e} & + \frac{\partial E_z}{\partial r} &= \mu \frac{\partial H_\theta}{\partial t} \\ \frac{1}{r} \frac{\partial}{\partial r} (r H_\theta) &= \frac{E_z}{e} & \frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) &= -\mu \frac{\partial H_z}{\partial t} \end{aligned} \quad (18a)$$

In order to find the dependence of E_θ on H_z , the second and sixth equation are combined. Eliminating E_θ and assuming a purely sinusoidal behavior of the field strength leads to the differential equation

$$\begin{aligned} \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} - \frac{j\omega\mu H_z}{e} &= 0 \text{ and if we put } p^2 = -\frac{j\omega\mu}{e} \\ \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} + p^2 H_z &= 0 \end{aligned}$$

This is a zero-order Bessel function with argument pr ,

so that the general solution is

$$H_z = CJ_0(pr) \quad (19)$$

and, as $E_\theta = -\rho \frac{\partial H_z}{\partial r}$

$$E_\theta = -\rho C \rho J_1(pr) \quad (20)$$

The integration constant C may be found from the consideration that $H_z = H_1$ if $r = r_o$, so that

$$H_1 = CJ_0(pr_o) \text{ and thus} \\ H_z = H_1 \frac{J_0(pr)}{J_0(pr_o)} \quad (19a)$$

and

$$E_\theta = -\rho \rho \frac{J_1(pr)}{J_0(pr_o)} \quad (20a)$$

Expecting small values for the argument pr , a power-series expansion for $J_0(pr)$ and $J_1(pr)$ can be used.³

$$J_0(pr) = 1 - \frac{(pr)^2}{4} + \frac{(pr)^4}{64} - \dots \\ J_1(pr) = \frac{pr}{2} \left\{ 1 - \frac{(pr)^2}{8} + \frac{(pr)^4}{192} - \dots \right\}$$

E_θ can now be written as

$$E_\theta = -\frac{\rho^2 r \rho}{2} \frac{1 - \frac{(pr)^2}{8} + \frac{(pr)^4}{192}}{1 - \frac{(pr_o)^2}{4} + \frac{(pr_o)^4}{64}} H_1 \quad (20b)$$

If only the zero-order terms of this expansion are taken, then

$$E_\theta = -\frac{\rho^2 r \rho}{2} H_1 = \frac{j\omega\mu}{2} r H_1 = \frac{\mu r}{2} \frac{\partial B_1}{\partial t},$$

as expected.

Using numerical values:

$$\omega = 2\pi \times 66 \text{ rads}^{-1} \\ \mu = 4\pi \times 10^{-7} \text{ As V}^{-1} \text{ m}^{-1} \\ \rho = 1.67 \times 10^{-8} \Omega \text{ m} \\ r_o = 2.5 \times 10^{-3} \text{ m}$$

As the error is greatest at $r = r_o$, we calculate the value of E_θ in this point:

$$E_\theta = \frac{j\omega\mu r_o H_1}{2} \left\{ \frac{1 + j0.0244 - 1.98 \times 10^{-4}}{1 + j0.0488 - 5.94 \times 10^{-4}} \right\} \\ E_\theta = \frac{j\omega\mu}{2} r_o H_1 \times 0.9988$$

From this result it can be concluded that the skin effect is of no influence on the previous comparison between measured and calculated values.

Discussion and Conclusion

From the foregoing sections, it follows that the interferometric measurement of strains resulting from eddy currents offers a new way to determine the elastic properties of conductive materials. Although the ultra-

sonic-velocity method of measuring elastic properties is very well established and a great deal of work has been done to refine and optimize it, a comparison of this new method with the ultrasonic one is made as both of them have the advantage that only small pieces of materials in the order of 10^{-6} m^3 are needed. It must be admitted that the absolute accuracy obtainable with the ultrasonic method is much better. It will be quite difficult to increase the accuracy of the new method to better than 1 percent due to the number of variables involved and the required accuracy of their measurement. A second drawback of the new method is that it measures the quotient of Poisson's ratio and Young's modulus. This may be overcome by looking for a configuration where these constants appear in different combinations in the formulas. From the plane-strain calculation it follows that for a solid cylinder the radial displacement

$$u_r = -\frac{(1+\nu)(1-2\nu)}{8(1-\nu)E} B_o \frac{\partial B_1}{\partial t} b^2 l,$$

implying that a measurement of u_r gives a second independent relation between ν and E . This, however, is not satisfactory because the plane strain calculation is an approximation. From the finite-element method it was found that the radial displacement u_r at the free end of the solid cylinder is proportional to $1/\nu E$. Both Poisson's ratio and Young's modulus can be determined in this way, although the measurements were not carried out in the present instance.

The main advantage of the new method is that the demands on surface flatness are quite low.

While the surfaces of the ultrasonic sample have to be flat and parallel to within at least one tenth of the ultrasonic wavelength, which equals $15 \mu\text{m}$ for 30 MHz longitudinal waves in copper, the new method demands only that the dimensions of the sample can be determined to an accuracy of about 1 percent.

A second advantage of the new method compared to the ultrasonic one, is in the use of low frequencies. Although, in general, no dispersion takes place, the difficulties encountered with the ultrasonic method should be mentioned in the case of ferrites, where the movement of Bloch walls affects the elastic properties, these walls being too slow to respond to the ultrasonic signals.

A general conclusion is that the new method can be used in those special cases where the ultrasonic method is not applicable.

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