Analysis of errors in a linestandard measurement

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The line standard was measured on a special interferometer. A description of the interferometer and associated equipment is given. Two sources of systematic errors were analysed and evaluated by comparing series of measurements in different mechanical arrangements. After correcting for these systematic errors, random error was evaluated.
Fotoelektrische microscoop
gij Neemt streep op liniaal waar
bij Bepaalt plaats van wagen met
liniaal m.b.v. servomechanisme.

Laserinterferometer
Deze meet de verplaatsing van
de wagen en daardoor de afstand
van de strepen op de Liniaal.
FIG. 1

SCHEMATIC DIAGRAM OF THE APPARATUS
I. General Description

Fig 1 shows the schematic diagram of the set up used to measure a line standard. The method uses a photodetector microscope to locate the line standard. An optical interference system is used in conjunction with a counting system which counts the movement of the line standard to zero line, which has been positioned under the microscope, in terms of wavelength of the laser used.

The system uses a He-Ne laser with a constant wavelength output. The coherence length of a laser is sufficient to permit observations of interference fringes over the entire scale length. An oscilloscope is used to adjust the microscope on the zero line of the standard, using servo-mechanism II. The counting system I is adjusted to zero.

The cycle of measurement is started by setting in the time relay which puts the counting system to zero, and causes the counting system I to give the number of counts to the memory which is transmitted to punch which transmit the
reading on tape. The counting system II is connected to the relais which is put by the time-relais in such a position that the voltage from the source can pass through the relais to servomechanism I which moves the standard until the counting system II has counted 12600 counts. The relais has two positions, one actuating servomechanism I and the other to actuate servomechanism II. When the counting system II has counted the 12600 counts (this means roughly 1 mm) the relais changes its position stopping servomechanism I and starting servomechanism II. If there is a signal from the microscope, the servomechanism II moves until the line is positioned under the microscope. During this time the counting system I is continually counting the movement of the line standard in terms of \( \frac{A}{Q} \) and when the servomechanism II has stopped, the cycle begins at the expiry of the adjusted time period of the time-relais by the counting system II sending the number of counts to the memory.
I.1 Optical system

Fig. 2.
Schematic diagram of the optical system.

A: He-Ne Laser
B: Kösters prism
C: Auxiliary prism
D, D': Photocell
E: Moving mirror
E': Fixed mirror

A laser beam issuing from the laser enters the kösters prism by means of the auxiliary prism. The beam is split at the plane $F$ of the kösters prism in two beams, one falling on the mirror $E$ attached to the line standard and the other on the mirror $E'$ fixed to the body of the microscope. The mirrors $E$ and $E'$ are triple mirrors. Each of the returning beams is split into two beams at the plane $F$, one falling on
photocell D and the other on the photocell D'. Thus there are two incident beams on each photocell which interfere.

When the line standard is moving, the intensity of light falling on each photocell varies and becomes maximum when the path difference between the two beams incident on each photocell becomes a multiple of \( \frac{\lambda}{2} \).

The voltage induced in each cell due to light energy of two interfering light beams, takes the form shown in the figure.

A trigger mechanism is used to convert this waveform into pulses which are received by the counting system.

For each \( \lambda \) path difference, the counting system receives two pulses from each photocell. The fringes on the photocells differ in phase by \( \frac{\lambda}{2} \).

It may be seen from the arrangement that \( \lambda \) path difference is produced by \( \frac{\lambda}{2} \) movement of the line standard. Thus \( \frac{\lambda}{2} \) movement of the line standard gives rise to 4 pulses and each
count of the counting system is equivalent of \( \frac{\lambda}{\beta} \) movement of the line standard.
I.2. The microscope

The line standard is illuminated by light from the bulb G. The image of the line is projected in the plane of the slit by the optical system. The slit is made to vibrate at 50 Hz. The light passing through the slit falls on the photoelectric cell by means of a lens D. Light of varying intensity falls on the photocell due to the position of the slit.

The voltage induced in the photocell varies with respect to time as shown in the figure.

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\[ V \sim T \]
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A magnifier receives this signal and sends this magnified signal to the servomechanism. If the line on the line standard is not positioned symmetrically with respect to the axis of vibration of the slit, the pulses received by the servomechanism will not vary uniformly with respect to time. In this case a voltage component of 50 Hz is generated which causes the servomechanism to move the line standard until the line is positioned symmetrically with respect to the axis of vibration of the slit. At this time, the pulses will vary uniformly with respect to time and then a voltage component of 100 Hz is generated in the servomechanism and no component of 50 Hz is generated, the servomechanism then stops.
Calculation of the actual wavelength.

As explained before the results are in counts and one count \( \frac{1}{\beta} \).

The wavelength of the laser used:

632.81984 nm.

This under the standard conditions:

20°C; 760 mm Hg; 59% R.H.; 0.03% CO₂.

The wavelength of the laser depends upon the refraction index of air which is influenced by the temperature, pressure and humidity. The value of \( \frac{\Delta \lambda}{\lambda} \) is corrected for the influence of the change in ambient conditions by the following formulas:

Pressure: \( \frac{\Delta \lambda}{\lambda} = -0.0000283 (P_0 - P) \)

Temperature: \( \frac{\Delta \lambda}{\lambda} = +0.0000735 (T - T_0) \)

Humidity: \( \frac{\Delta \lambda}{\lambda} = +0.0000044 (F - F_0) \)

\( P_0, T_0 \) and \( F_0 \) are the standard conditions.

The change in length of the line standard due to temperature drift can be considered in the form of a correction applied to wavelength in the following expression:

\( \left( \frac{\Delta \lambda}{\lambda} \right)_E = -\alpha (T - T_0) \left( \frac{\Delta \lambda}{\lambda} \right) P_0 \) where \( \alpha \) denotes
The coefficient of linear expansion of the line standard and \( \frac{\Delta}{\phi} \) denotes the actual \( \frac{\Delta}{\phi} \) of the measured line standard = 0.8 \times 10^{-6} \text{ (Invar)}.
Statistical analysis of measurements on the 1 meter line standard.

The line standard was measured by an interferometer, described in the general description. The photomicroscope used in the interferometer can be adjusted in two positions, 'west' and 'east'.

The scale was measured twice with the microscope in west position and twice with the microscope in east position. During each measurement, the scale was moved from one line to the next line continuously first in the forward direction and then in the backward direction. Observations were made at every line. Thus four pairs of forward and backward observations were obtained for each line of the scale.

The observations give for each line the number of counts recorded. Each count corresponds to $\frac{1}{9}$ of the laser used. The value of $\frac{1}{9}$ was calculated for each set of forward and backward measurements at the average temperature, pressure and humidity prevailing during the measurement. The product number of counts $\times \frac{1}{9}$ minus the nominal value of the line was obtained for each line i.e.,
the deviation of each line from its nominal value was obtained. The deviations thus obtained were analysed for random errors. Two principal sources of variation of readings were considered to be temperature drift during the measurement and the assymetric of the microscope. The temperature drift is expected to cause a difference between the forward and backward readings and the assymetry of the microscope causes the readings in two positions of the microscope to be different. An attempt was made to correct the readings for temperature drift. It is usually assumed that the temperature drift is linear during such experiments. With a linear temperature drift it is seen that the difference $F-B$ (Forward-Backward readings) is given by a relation of the form $F-B = A + Bn + Cn^2$ where $n$ means the reading of the $n$th line.

The difference $F-B$ was plotted against $n$ for four pairs of $F$ and $B$ readings. The resulting graphs suggested a nearly linear relation rather than a parabolic in three graphs and a parabolic relation for one graph.
In one graph an attempt was made to fit a straight line by taking a sample of 7 and 13 readings of 50 lines and calculating the regression line. In the same graph a straight line was drawn by inspection. It was found that the coefficients of the calculated straight line and of the straight line drawn by inspection do not differ appreciably. For the three remaining graphs, two straight lines and one parabola were fitted by inspection. The readings were corrected for temperature drift by the relations $F' = F - \frac{y}{2}$, $B' = B + \frac{y}{2}$, where the prime denotes the corrected readings and $y$ is the difference between forward and backward readings given by the regression curve.

To eliminate the effect of asymmetry of the microscope, the east and west, $F'$, $B'$ readings were considered separately. The within group standard deviation for groups of 4 west readings was found to be 1.46 nm, while a value of 0.44 nm was obtained for east readings. These were unexpectedly significantly different. Further analysis revealed systematic differences
present in two measurements in West position. It was expected that correction for these differences will improve the within group standard deviation of West readings. This was supported by the result of an analysis performed on West readings corresponding to 20 lines selected out of 1000 lines. For want of time it was not possible to carry out an analysis of variance on the full 8,000 corrected (i.e. $r$, $b$) observations.

**Conclusion.**

The analysis shows that systematic errors other than due to temperature drift and asymmetry of microscope are present. The random error in the measurement is not more than 300 nm. It is proposed that the analyses of variance may be carried on the readings corrected for temperature drift. It is expected to give a better estimate of random error. It is also proposed that the possibility of carrying out designed experiments may be considered for the analysis of systematic influences.
Appendix:

The formula used to calculate the within variance of the east and the west readings:

\[ S^2 = \frac{\sum \left( x_{i,j} \right)^2 - \left( \sum x_{i,j} \right)^2}{N(n-1)} \]

The variance of the entire measurements can be calculated by the formula:

\[ S_{\text{ew}}^2 = S_{\text{w}}^2 + S_{\text{e}}^2 \]

- \( S_{\text{w}}^2 \): west variance
- \( S_{\text{e}}^2 \): east variance

The variances have been calculated by the EL-X-0 computer.

The programs used by the computer to calculate were made by Drs. N.A.L. Touwen.