Inhomogeneous compression of a circular cylinder

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### Samenvatting

Het rapport is een beknapt uitvoering van rapport 0191, bestemd voor externe publicatie.

### Prognose

Datum: 11-3-'68

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Inhomogeneous compression of a circular cylinder.
E. Mot.*

Summary.
By a method, very similar to the "Bridgman correction" of the stress in the neck of a tensile test specimen, a formula is derived for the load, needed to compress a circular cylinder. Experimental results are given for four materials.

List of symbols.
- a radius of central plane during straining
- a₀ initial radius of cylinder
- b half height of cylinder during straining
- b₀ initial half height of cylinder
- c effective stress if \( \delta = 1 \)
- m strain hardening exponent
- R radius of curvature of the deformed cylinder profile near central plane
- (delta) \( \delta \) logarithmic strains
- \( \overline{\delta} \) effective logarithmic strain
- (theta) \( r, z, \theta \) coordinates (suffixes)
- (sigma) \( \sigma \) normal stresses
- \( \overline{\sigma} \) effective stress
* refers to tensile test

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It is well known that Bridgman derived a correction formula for the stress in the neck of a tensile test bar. By integration he found the external load, \( L^* \), \([1]\)

\[
L^* = \pi a^*(a^* + 2R^*) \ln \frac{2R^* + a^*}{2R^*}
\]

(1)

in which \( a^* \) is the radius of the neck of the test specimen and \( R^* \) the radius of curvature of the neck profile.

It can be proved in a perfectly identical way that the same formula holds for the load needed for inhomogeneous compression of a circular cylinder. However, in that case the sign of \( R^* \) must be chosen negative. Introducing \( R = -R^* \), we find \([2]\)

\[
L = \pi a(2R-a) \ln \frac{2R}{2R-a}
\]

(2)

Now we introduce the exponential strain-hardening model

\[
\bar{\sigma} = \sigma \bar{e}^m
\]

(3)

in which

\[
\bar{\sigma}^2 = \frac{2}{3} (\bar{\sigma}^2 + \bar{\sigma}_r^2 + \bar{\sigma}_\theta^2)
\]

(4)

Fig. 1. Initial and ultimate shape of the cylinder.

If the strain is uniform near the central plane, we have

\[
\bar{\sigma}_r = \ln \frac{a}{a_0}
\]

\[
\bar{\sigma}_\theta = \ln \frac{a}{a_0}
\]

(5)

\[
\bar{\sigma}_z = -2 \ln \frac{a}{a_0}
\]

Since the strain path is straight, (4) and (3) are valid. We find

\[
\bar{\sigma} = 2 \ln \frac{a}{a_0}
\]

(6)
Hence,

\[ L = \pi ac(2\ln \frac{a}{a_0})^n(2R-a)\ln \frac{2R}{2R-a} \]  \hspace{1cm} (7)

Now the cylinder was deformed according to Fig. 1. This was accomplished by covering the outer planes with a plate according to Fig. 2, forcing the radius \( a_o \) of these planes to remain unchanged.

Fig. 2. Shape of plates at either side of the cylinder.

It was assumed that the curvature of the initially straight cylinder was approximately circular. The radius of this circle follows from the constancy of volume. From geometrical considerations, it can then be proved that

\[ (aR^2-bR\cos\alpha)(a_o-R\cos\alpha)+\frac{2}{3}b^3-a_o^2b = a_o^2b_0^2 \]  \hspace{1cm} (8)

in which

\[ \alpha = \arcsin \frac{b}{R}, \]  \hspace{1cm} (9)

while

\[ a = a_o + R(1-\cos\alpha) \]  \hspace{1cm} (10)

For known values of \( a_o, b_o \) and \( b \), we can find \( R \) by successive approximation.

Together with the values of \( c \) and \( m \), which were independently calculated from a tensile test, the load can now be calculated from (10) and (7).

Experimental verification.

The value of the load \( L \) was measured as a function of the height of the cylinder. The agreement seems to be very good, except for the brass specimen (Ms 58).
This was the only material, however, which had a coarse crystal structure, hence the discrepancy may be due to anisotropy.

<table>
<thead>
<tr>
<th>b(mm)</th>
<th>a(mm)</th>
<th>R(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>15.5</td>
<td>622</td>
</tr>
<tr>
<td>23</td>
<td>16.0</td>
<td>278</td>
</tr>
<tr>
<td>22</td>
<td>16.5</td>
<td>165</td>
</tr>
<tr>
<td>21</td>
<td>17.0</td>
<td>110</td>
</tr>
<tr>
<td>20</td>
<td>17.6</td>
<td>77.7</td>
</tr>
<tr>
<td>19</td>
<td>18.2</td>
<td>57.2</td>
</tr>
</tbody>
</table>

Table I. Calculated values of R

For b = 19.2 mm, a radius R = 97 mm was measured, the theoretical value being of the order of 60 mm (Table I). Thus, as was to be expected, the surface does not exactly get the shape as indicated in Fig. 3. However, since the radius is still large relative to the radius a, the above discrepancy does not substantially influence the results.

The importance of this theory does not in the first place lie in the fact that it allows for introducing a stress distribution near the central plane from which the load can be calculated.

The advantage of the present approach is that the process takes place without the introduction of frictional forces at the surface of the specimen.

The geometrical data of the specimen were \( a_o = 15 \) mm, \( b_o = 25 \) mm; the material data were according to the following specification:

**St C 22.** 0.22% C; P < 0.06%; S < 0.06%, Normalised (perlite + ferrite structure) \( c = 849 \) N/mm\(^2\), \( m = 0.230 \).

**Ma 58.** 58% Cu; 25% Pb; 39.5% Zn.

Annealed, 3 hours at 400°C

\( c = 720 \) N/mm\(^2\); \( m = 0.292 \).
Cu (electrolytic). 99.92 % Cu; 0.05 % O₂.
Annealed, 3 hours at 500°C
c = 459 N/mm²; m = 0.401

Al St 51. 98 % Al; 1 % Si; 0.6 % Mg.
Annealed, 3 hours at 350°C
c = 210 N/mm²; m = 0.120.

Fig. 3 shows the calculated and measured values of the load as functions of the displacement.

Fig. 3. Comparison of theoretical and experimental results of the compression test.

Conclusion, prospects and limitations.
We have shown that the Bridgman correction can successfully be applied to inhomogeneous compression of a circular cylinder.
For industrial purposes this gives the possibility of a reversed procedure, viz. the determination of c and m by means of a compression test. In that case, a force-displacement graph is taken and the values of c and m which make formulae (10) and (7) fit best to the graph are determined with the help of a set of standard graphs.
A limitation is, however, that the strains must be kept below approx. 20 % due to the decreasing value of R. However, within this range, the test would have considerable advantages compared with the tensile test, since the specimens are easier to make and since continuous measurement is easier to realise.
Finally, it is interesting to notice that this experimental result is essentially a proof that for the materials investigated for moderate strains no Bauschinger effect exists.
References.


Fig. 1