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Steady Newtonian and non-Newtonian flow in a curved tube: LDA-experiments

H. Brouwers

WFV report 96.069

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May 1996
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Chapter 1

Introduction

Atherosclerosis is a complicated disease that causes progressive occlusion of the lumen of arteries. The effects of this disease are one of the most important causes of mortality in our post industrial societies, so that explains that the investigations based on it are quite numerous. Atherosclerosis preferentially develops in bends and bifurcations in the larger arteries, e.g. the carotid bifurcation. Atherosclerosis in the carotid bifurcation is the major cause of ischemic attacks. The bifurcation consists of a main branch, the common carotid artery, which asymmetrically divides in two branches, the internal carotid artery and the external carotid artery.

In the proximal part of the internal carotid artery bifurcation, a small widening exists, named the carotid sinus. From clinical practice it is known that the non-divider side of this sinus is very sensitive to the development of atherosclerotic lesions. The local nature of the genesis of the atherosclerotic disease is assumed to be related to local characteristics of the flow field, such as low shear rates and reversal flow [Ku et al (1985)]. The study of the flow in this bifurcation is therefore of great clinical interest with respect to both the genesis and the diagnostics of atherosclerotic disease.

Blood is a concentrated suspension of blood cells in plasma. It is well established that blood exhibits non-Newtonian behaviour [Thurston (1979)]. Most of the numerical and experimental studies on blood flow in larger arteries however employ a Newtonian fluid as a model for blood. Some studies suggest that the non-Newtonian properties of blood cannot be neglected. Significant changes between Newtonian and non-Newtonian flows behaviour were observed experimentally by [Liepsch and Movarec (1984)]. In their experiments they employed a polyacrylamide solution as a blood analog fluid with a good match for the shear thinning properties but an elastic component of the complex viscosity that was too high. [Zuidervaart (1995)] presented a detailed analysis of the velocity distribution in a rigid model of the carotid bifurcation under steady flow conditions. He also found a significant influence of the non-Newtonian fluid properties on the velocity distribution. The strength of the secondary velocity distribution was greatly reduced due to the viscoelasticity of the blood analog fluid. Zuidervaart used a solution with Xanthan gum as a blood analog fluid, showing a good match to the macroscopic properties of blood.

The complexity of the flow field in the carotid bifurcation prevents a detailed analysis of the influence of the non-Newtonian properties of the blood analog fluid on the velocity distribution. It therefore seems appropriate to study the flow of a viscoelastic blood analog fluid in a more simple geometry to be able to evaluate the wall shear rates and stresses using an appropriate blood analog fluid. For this purpose, steady flow in a 90° curved tube will be investigated experimentally by means of Laser Doppler Anemometry. The geometry of a 90° curved tube is simpler than the carotid bifurcation but still is physiologically relevant [Olson (1971)]. A detailed comparison of the complete velocity field for a Newtonian and a non-Newtonian blood
analog fluid will be presented. Furthermore, the results of this study can be used to validate numerical codes that incorporate viscoelasticity in complex flows.
Chapter 2

Experimental methods

In this chapter, the experimental methods that were used will be discussed. In section 2.1 two models of the 90-degree curved tube are dealt with. Then a description of the rheological properties of the Newtonian and the non-Newtonian fluids that were used is given (section 2.2). In section 2.3 the experimental setup is described, containing a description of the fluid circuit (2.3.1), the steady flow which was used (2.3.2), the way in which the L.D.A.-measurements were performed (2.3.3) and the measuring grid which was used (2.3.4).

2.1 The 90-degree curved tube models

The experiments were performed in two Perspex models of a 90-degree curved tube. The internal radius \(r\) was 4 \(mm\) and the radius of curvature \(R\) was 24 \(mm\), yielding a curvature ratio \(\delta = r/R = 1/6\). Two models were used, because the measurements of the three velocities components, using the same model, is impossible due to construction's characteristics (see Figure 2.1). One model consisted of two halves of Perspex, split at the plane of symmetry, and was used to measure the axial velocity component and the secondary velocities component which is parallel to the plane of symmetry of the model. The other model is divided along the curve, and was used to measure the secondary velocities component which is perpendicular to the plane of symmetry of the bend. In the following of this report, the first measurement model will consist in the model which is used to measure the axial and the (parallel) secondary velocities components and the second one is used for the (perpendicular) secondary velocities component. The two models are identical to the models used by [Bovendeerd (1987)], [Rindt (1989)] and [Van de Vosse (1989)].
2.2 The rheological properties of the fluids

In order to be able to perform LDA measurements in the Perspex's models, the Newtonian and non-Newtonian fluid should meet the following requirements:
- It should be transparent to facilitate LDA measurements;
- It should have a refraction index close to that of perspex, \( n = 1.491 \).
- The non-Newtonian fluid properties should match those of blood.

Matching of the refraction index is necessary in order to prevent an unwanted shift and deformation of the measurement volume.

2.2.1 Newtonian fluid

To satisfy the conditions and to get Newtonian fluid behaviour, a concentrated solution of 71 weight % KSCN (potassium thiocyanate) in water is used. This KSCN solution shows Newtonian fluid behaviour with a dynamic viscosity \( \eta = 2.9 \, \text{mPa.s} \) and a density \( \rho = 1410 \, \text{kg/m}^3 \) at a temperature of 37.5 °C. The viscosity measurements were carried out in a parallel-plate viscosimeter (Rheometrics - RFS II).

2.2.2 Non-Newtonian fluid

In order to satisfy the two first conditions and to get visco-elastic and shear thinning properties close to those of blood, an additive for the KSCN solution is necessary. A number of additives are known to change the rheological viscometric properties of a Newtonian fluid into non-Newtonian blood-like properties ([Thurston (1979)]), of which one is Xantan-gum. An old KSCN+XG solution which was used by Zuidervaart in 1995 was taken and after measuring its rheological properties it was clear that its viscosity was lower than those of blood. Approximately 75 weight ppm were added to the previous solution in order to get a solution which behaves as close as possible to Thurston's rheological measurements of blood ([Thurston (1979)]). The non-Newtonian viscosity in steady flow and the shear rate results of the new solution seemed to be the same as those of a 275 weight ppm KSCN+XG solution which was separately studied. Zuidervaart (1995) used a Couette device to measure the visco-elastic properties, resulting in an overestimation of the elastic component of the complex viscosity. The 71 weight % KSCN + 275 weight ppm XG solution showed the best compromise between
the shear thinning properties that are slightly higher than those of blood and the frequency
dependence of the viscoelastic properties in oscillatory flow that were lower than those of blood
(Figure 2.2 and Figure 2.4). In order to compare two non-Newtonian fluids, the most important
parameter is, in the first place, the non-Newtonian viscosity in steady flow. The shear thinning
properties of the KSCN+XG solution were very close to those of Thurston's measurements, so
therefore the fluid behaves almost like blood. In the steady shear experiments, the normal stress
differences were measured as well. Due to the low values of the normal stress difference and
evaporation of the fluid, these measurements only give qualitative information.

Figure 2.2: Shear-thinning of a 71% KSCN solution with approximately 275 ppm XG (x=KSCN + XG, o=Thurston-1979)
Figure 2.3: Normal stress difference vs. shear rate
Figure 2.4: Dynamic visco-elastic properties of the non-Newtonian solution
2.3 Experimental setup

2.3.1 The fluid circuit

The fluid circuit which was used for the steady experiments is presented in Figure 2.5.

Two constant head tanks were used to eliminate the cyclic disturbances of the pump and to get a stationary flow. This two constant head tanks, which were placed at a different level, caused a constant pressure gradient, which was responsible for the stationary flow through the bend. The fluid was pumped back from the lower head tank towards the upper head tank by a progressing cavity pump (Robbins & Myers inc., Moyno products, model 30105). The pump parts which were in contact with the fluid were non-metallic in order to prevent corrosion due to the salt solution. The upper constant head tank was surrounded by a water-filled container, which was kept at a constant temperature (37.5 °C) by means of a temperature controller. When the fluid flowed out of the upper head tank, it reached an inlet tube of length 150·D where D represents the diameter of the inlet length and the bend, D = 8 mm. This inlet length was used to be sure that the flow was stable and fully developed when it reached the entrance of the bend. Between the upper head tank and the inlet length, a small tap was inserted to adjust the flow. Further, the fluid flows through the 90-degree curved tube, our measuring section, and downstream of this part, the flow was measured by a flow sensor (Transflow 601, Skalar Instruments) and
transported back to the lower head tank.

2.3.2 The steady flow

The steady flow in a bend can be characterised by two dimensionless numbers:

\[ Re = \frac{\rho \cdot \bar{U} \cdot D}{\eta_\infty} \]

where \( Re \) represents the Reynolds number, and

\[ \kappa = \sqrt{\frac{a}{R} \cdot Re} \]

where \( \kappa \) represents the Dean number,
\( \rho = 1410 \text{ kg/m}^3 \) (the density of the fluid),
\( D = 8 \text{ mm} \) (the diameter of the tube),
\( \bar{U} \) is the mean axial velocity in the tube,
\( \eta_\infty \) is the high shear rate limit of the viscosity of the fluid
and \( \frac{a}{R} = 1/6 \) (the curvature ratio of the bend).

For the Newtonian fluid, the Reynolds number was \( Re = 300 \)
and the corresponding Dean number \( \kappa = 122 \),
with a flow rate of \( Q = 3.87 \cdot 10^{-6} \text{ m}^3/\text{s} \),
a mean velocity of \( \bar{U} = 0.077 \text{ m/s} \)
and a maximum velocity of \( 2\bar{U} = 0.154 \text{ m/s} \).

The value \( Re = 300 \) was chosen so that the measurements could be compared to those of [Van de Vosse et al (1989)] (\( Re = 300 \)) and [Zuidervaart (1995)] (\( Re = 270 \)). For the non-Newtonian fluid, the high shear rate limit of the viscosity \( (\eta_\infty = 2.9 \text{ mPa} \cdot \text{s}) \) was used for the definition of \( Re \). The high shear rate limit of the viscosity of the KSCN+XG-solution is identical to the viscosity of the KSCN-solution (Figure 2.2). As a consequence, the flow rate and the mean velocity were identical for the two fluids.

2.3.3 The L.D.A.-measurements

For the velocity measurements a two components fibre optics L.D.A. (Laser-Doppler Anemometry) system in backscatter mode was used in combination with a Flow Velocity Analyzer (58N20, Dantec) with a 300 mV argon-ion laser (5500 \( \lambda \), Ion Laser Technology). Through glass fibres, the green beam and for the first model also the blue beam were transmitted to the measuring probe. A front lens with focal length of 80 mm was used to focus the laser beams, so in the focus point an elliptical measuring volume is created, which dimensions are dependent on the angle between the laser beams and the wavelength of the laser beams. So for the green light there was an elliptical measuring volume of \( dx \ast dy \ast dz = 168 \ast 38.82 \ast 39.9 \mu m^3 \) and for the blue light it was \( dx \ast dy \ast dz = 159.3 \ast 36.82 \ast 37.84 \mu m^3 \). To measure the three velocity components (axial velocities and the two components of the secondary velocities), two different models for the bend were used. With the first model the axial velocities and the first component of the secondary velocities were measured simultaneously with two green laser beams (\( \lambda = 514.5 \text{ nm} \)) and two blue laser beams (\( \lambda = 488 \text{ nm} \)) respectively. With the second model only the green laser beams were used to determine the second component of the secondary velocities. The lower frequency bandwidth (120 kHz) which was used in the second model is due to the fact that the secondary velocities which were measured were expected to be very small. In the first model
this lower frequency cannot be used because the measuring probe in relation to the bend was not rotated; so in plane 0 the axial velocities were measured with the green beam and in plane 90 the axial velocities were measured with the blue beam, so the (relative high) axial velocities were measured with the green and blue beam, and for that a light bandwidth of 400 kHz was needed.

To be able to do L.D.A.-measurements, the two fluids which were used were seeded with crystals (Titanium dioxide, Iodine 111, Merck), which were smaller than 15 μm. The accuracy which was reached with the measurements depended on the seeding concentration. For that reason this seeding concentration was optimized in order to get results as accurate as possible, (20 till 30 mg/l).

2.3.4 The measuring grid

The velocities were measured in seven different planes, perpendicular to the cross section of the bend (Figure 2.6). The first plane was the plane \( \theta = 0 \) degrees, in the beginning of the bend. Then plane \( \theta = 15, 30, 45, 60, 75 \) and 90 degrees were measured, the last of which was the end of the bend. In each plane a square grid of 10 by 10 mm was initially generated, with a distance between each point of 0.5 mm (so 441 points) and a circle with a radius of 5 mm was determined in it. Then all the points of the square that were outside this circle were deleted and as a result of this only 317 points were left.
Figure 2.6: Experimental steady measurements planes ($\theta = 0$ to 90 degrees) and the measuring grid in them
Chapter 3

Experimental results

3.1 Results analysis

In the presentation of the results, all the coordinates inside the bend are made dimensionless by the inner radius \( a \) of the bend, so all the \( Y \)-coordinates in the horizontal plane go from \( y/a = y' = -1 \) at the outer bend to \( y/a = y' = 1 \) at the inner bend and all the \( Z \)-coordinates go from \( z/a = z' = -1 \) at the top of the bend to \( z/a = z' = 1 \) at the bottom of the bend. The axial velocities and secondary velocities are made dimensionless with the mean axial velocity for each plane. The iso(axial)velocities distribution shows a level difference of \( \frac{\Delta U}{U} \approx 0.2 \) between two level lines.

For a better understanding of the following measurements plots and analyses, the most important measurements data are collected in Table 3.1:

<table>
<thead>
<tr>
<th>Plane (°)</th>
<th>max. axial velocity</th>
<th>Y location of this max.</th>
<th>(Y,Z) vortices coordinates</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>2.02</td>
<td>1.86</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>2.00</td>
<td>1.83</td>
<td>-0.12</td>
</tr>
<tr>
<td>30</td>
<td>1.91</td>
<td>1.81</td>
<td>-0.37</td>
</tr>
<tr>
<td>45</td>
<td>1.85</td>
<td>1.74</td>
<td>-0.61</td>
</tr>
<tr>
<td>60</td>
<td>1.82</td>
<td>1.73</td>
<td>-0.67</td>
</tr>
<tr>
<td>75</td>
<td>1.75</td>
<td>1.71</td>
<td>-0.62</td>
</tr>
<tr>
<td>90</td>
<td>1.74</td>
<td>1.78</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

Table 3.1: Most important measurements data
Figure 3.1: Newtonian and non-Newtonian flow behaviour in the plane O
Figure 3.2: Newtonian and non-Newtonian flow behaviour in the plane 15
Figure 3.3: Newtonian and non-Newtonian flow behaviour in the plane.
Figure 3.4: Newtonian and non-Newtonian flow behaviour in the plane.
Figure 3.5: Newtonian and non-Newtonian flow behaviour in the plane 60
Figure 3.6: Newtonian and non-Newtonian flow behaviour in the plane 75
Figure 3.7: Newtonian and non-Newtonian flow behaviour in the plane 90°
Figure 3.8: First velocities profile along the horizontal center line (solid line = Newtonian; dashed line = NonNewtonian)
Figure 3.9: First velocities profile along the vertical center line (solid line = Newtonian; dashed line = NonNewtonian)
3.1.1 Bend entrance

If the axial velocity profiles of the Newtonian and the non-Newtonian flow in Figure 3.1 and Figure 3.10 are compared, the Newtonian fluid shows a parabolic axial velocity distribution. The measured velocities compare well to the theoretically expected velocity distribution. The profile for the non-Newtonian flow is more flattened at the entrance of the curved tube. This aspect is due to the shear thinning behaviours of the non-Newtonian fluid: the fact that at high shear rate (here close to the wall of the tube) the viscosity has a lower value, causes a flattened axial velocity profile. As a result of this flattening, the axial velocity gradients along the wall of the tube are slightly higher for the non-Newtonian fluid. One important remark is, looking at Table 3.3 and Table 3.4, that the flow in the non-Newtonian case seems to be slightly higher compared to the Newtonian one. In the bend entrance, if we just look at the axial velocity profile, no influence of the bend is observed.

Figure 3.10: Axial velocities profile along the Y-axis (dashed line) and the Z-axis (solid line) compared to the theoretical parabolic profile (dotted line) in the plane O

The secondary velocities vector plot shows that the secondary velocities are parallel to the horizontal plane for both fluids in the center of the tube and are relatively small. All of the
secondary velocities are directed from the outer bend towards the inner bend, implying that the Z-axis secondary velocities are almost equal to zero. These secondary velocities are caused by an upstream effect of the curved tube.

### 3.1.2 Downstream the curved tube

Downstream the entrance of the curved tube, the flow sustains a combination of four forces which can work on the fluids. First there are centrifugal forces $F_c$ which are caused by the flow motion in the curvature. These centrifugal forces are directed towards the outer bend and they are more important in the regions where the axial velocities are high, so in this case in the central core of the tube. Second there are viscous forces $F_v$ which are a result of the viscosity of both fluids. When one specific cross-section of the tube is observed, these viscous forces are always directed in the opposite direction of the secondary velocities in that cross-section. Third an inertia effect is apparent due to the momentum of the fluids. Last but not least the pressure gradient forces $F_p$ are playing a role and these forces are directed from the outer bend towards the inner bend, so in the opposite direction of the centrifugal forces (Figure 3.11). In this study, the gravity forces are considered neglectable compared to the other forces acting on the fluid.

![Diagram of forces and velocities in a cross-section plane](image)

**Figure 3.11:** Forces which act in the cross-section plane

In order to compare the secondary flow which experiences in the tube, it was quantified by its mean axial vorticity ($\xi$) defined as

$$\xi = \frac{\Gamma}{A} = \frac{1}{A} \oint_S v \cdot ds$$  \hspace{1cm} (3.1)

where $S$ is taken in a plane of constant $x$, surrounding a region with surface $A$ and $v$ represents the secondary velocity component. Taking $S$ along the horizontal center plane and the upper pipe wall, the mean axial vorticity $\xi_c$ in the central core is found. Following the Olson and Snyder’s procedure, it is the best to use this quantity in the dimensionless form:

$$\xi_c = \frac{\xi a}{U^{3/2}}$$  \hspace{1cm} (3.2)
with $a$ the radius, $\bar{U}$ the mean axial velocity and $\delta$ the curvature ratio.

The direction of the secondary velocities was from the outer bend towards the inner bend in plane 0 and is changed in the following planes as a result of the forces previously described. In the central region of the tube, the secondary flow is directed towards the outer bend due to the predominance of the centrifugal forces over the pressure gradient forces. Therefore the maximum of the axial profile is shifted along the horizontal center line in the direction of the outer bend with a maximum shift in plane 60. Further downstream, this shift decreases slightly because the secondary flow (the mean axial vorticity in the central core) is decreasing (Figure 3.13). Near the wall of the tube, the secondary velocities are directed towards the inner bend as a result of the bigger pressure gradient forces compared to the centrifugal forces, the latter of which are relatively small near the wall.

For the non-Newtonian fluid the shift of the maximum axial velocity locations is smaller compared to the Newtonian one, resulting from its more important viscous forces (relatively low shear rate) in the central part of the tube, which reduce the axial velocities in this region (Table 3.1). The centrifugal forces, acting on the non-Newtonian fluid, will therefore be less dominant up till plane 60, leading to a lower mean axial vorticity ($\xi_e$ – Figure 3.13). This results in a less pronounced shift of the position of the maximum axial velocity (Figure 3.12). Downstream plane 60, the fluid is more spread along the outer bend, especially for the Newtonian fluid, and the centrifugal forces are lower and the mean axial vorticity therefore decreases. The mean axial vorticity for the non-Newtonian fluid is slightly higher than the Newtonian for two reasons. At first, the difference between the maximum axial velocity of both fluids decreases, and in plane 90, the non-Newtonian axial velocity maximum is higher than the Newtonian one. Also the application point of the non-Newtonian maximum axial velocity is close to the central core, so the centrifugal effects on this fluid are increased.

Caused by the opposite directions of the secondary velocities in the central region and near the wall of the tube, two vortices called Dean vortices (the hemispheres) appear in the secondary velocities vector plots for the Newtonian and the non-Newtonian flow. The centers of the Dean vortices shift and their strength change as a function of the downstream position in the tube (Figure 3.14 and Figure 3.13 respectively). In fact, in the center of the vortex, the vectorial sum of all those forces is equal to zero, so in those points the secondary velocities are zero. The shift of the centers of the vortices are less for the non-Newtonian visco-elastic flow compared to the Newtonian flow (Figure 3.14).

The maximum secondary velocity along the horizontal center line shifted with the same amount and in the same direction as the maximum of the axial velocity, so towards the outer bend in both cases (Newtonian and non-Newtonian) till plane 45. Further downstream this maximum along the horizontal center line is decreasing and moving in reversed direction, so towards the inner bend for both cases. In the Newtonian case, the secondary velocities increase in the neighbourhood of the upper and the lower wall downstream plane 0 and become even higher than the secondary velocities along the horizontal center line in plane 45 till plane 90. In the non-Newtonian case, this is not clearly observed. In plane 90 there appears some sort of tail (where the secondary velocity is almost zero) close to the Dean vortices for the Newtonian fluid. This tail is probably caused by the fact that the fluid particles with relative low axial and secondary velocities near the center of the tube are not able to penetrate into the region with high axial velocities near the outer wall.

Another phenomena is apparent in plane 45 and further downstream in the Newtonian case.
The isoaxial velocity lines show 'C-shaped' curves. These 'C-shapes' become more accentuated as the axial distance increases (plane 60 and plane 75) and then decrease slightly again in plane 90. In the non-Newtonian case, the formation of the 'C-shapes' appears in plane 60, 75 and 90 but is much less pronounced if compared to the Newtonian case (Figure 3.4 till Figure 3.7 and Figure 3.9). The ends of the the 'C-shaped curves' are related to the position of the centers of the Dean-vortices. Coupled with this phenomena in the Newtonian case, a sort of plateau in the central core of the tube is described by the mean axial velocities (Figure 3.4 till Figure 3.7 and also Figure 3.8).
Figure 3.12: maximum axial velocity location along the horizontal center line vs. angle (solid line: Newtonian fluid, dashed line: Non-Newtonian fluid)

Figure 3.13: Mean axial vorticity in the central core vs. angle (solid line: Newtonian fluid, dashed line: Non-Newtonian fluid)

Figure 3.14: Vortex centers location
The effect of the non-Newtonian fluid on the axial wall shear rates in the symmetry plane at outer and inner bend is presented in (Table 3.2).

<table>
<thead>
<tr>
<th>Plane (°)</th>
<th>Outer bend</th>
<th>Inner bend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Newt.</td>
<td>nNewt.</td>
</tr>
<tr>
<td>0</td>
<td>72</td>
<td>70</td>
</tr>
<tr>
<td>15</td>
<td>78</td>
<td>98</td>
</tr>
<tr>
<td>30</td>
<td>134</td>
<td>126</td>
</tr>
<tr>
<td>45</td>
<td>174</td>
<td>154</td>
</tr>
<tr>
<td>60</td>
<td>172</td>
<td>176</td>
</tr>
<tr>
<td>75</td>
<td>136</td>
<td>152</td>
</tr>
<tr>
<td>90</td>
<td>146</td>
<td>148</td>
</tr>
</tbody>
</table>

Table 3.2: Wall shear rate along the horizontal center line close the outer and inner bend

This table describes the value $\frac{du}{dr}$ when $r$ is reaching $R$ and where $u$ represents the axial velocities (close to the wall). The values for the shear rates near the wall give just an indication of the real shear rates in this region, because $du$ is calculated by linear interpolation between $u(r = \pm 4 \text{ mm})$ and $u(r = \pm 3.5 \text{ mm})$, so especially for the inner bend the values of the shear rates close to the wall are probably overestimated. Looking at Table 3.2, it can be concluded that the shear rates near the outer wall are significant higher than near the inner wall for all planes, except plane 0, and this is true for both cases (Newtonian and non-Newtonian). This fact is caused by the downstream shift of the axial velocity, first towards the outer bend and downstream plane 60 towards the inner bend. Further it can be observed that for the outer bend the Newtonian shear rate reaches its maximum in plane 45, while the non-Newtonian shear rate has its maximum in plane 60 near the outer wall and downstream this plane, the non-Newtonian values are higher than the Newtonian one, in contradiction to the values before this plane, where the Newtonian values are higher (except for plane 15). For the inner bend, no significant differences between the Newtonian and the non-Newtonian shear rates are observed.

### 3.2 Errors

#### 3.2.1 Errors in flow rate

In order to be able to use the experimental results, one important aspect is that the flow in the circuit is exactly the same as the flow which was related to the specified Reynolds number (3.87 $\cdot 10^{-6} \text{ m}^3/\text{s}$). So the first step after collecting the measurements is to recalculate the flow from the measured velocities.

This flow can be recalculated in the following way:

$$Q = \int_{r'=0 \text{ mm}}^{r'=r} U(r')2\pi r' dr'$$

where $r = 4 \text{ mm}$, $r'$ is the radial distance from the center and $U$ is the measured axial velocity. Discretization of this integral yields:
\[ Q = \sum_{i=0}^{n} \pi [((i+1)\frac{r}{n})^2 - (i\frac{r}{n})^2]\bar{U}(i) \]  

(3.4)

where \( \bar{U}(i) \) is the mean velocity of the velocities which are measured in the crown with an internal radius of \( i\frac{r}{n} \) and an external radius of \( (i+1)\frac{r}{n} \).

This computation was realised for the seven planes and the results for the Newtonian and for the non-Newtonian measurements are presented in Table 3.3 and Table 3.4 respectively.

<table>
<thead>
<tr>
<th>plane ( \theta )</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error/specified flow(%)</td>
<td>-0.1299</td>
<td>0.9091</td>
<td>1.39</td>
<td>0.1299</td>
<td>-1.4286</td>
<td>1.1688</td>
<td>1.0390</td>
</tr>
</tbody>
</table>

Table 3.3: Results for the Newtonian fluid

<table>
<thead>
<tr>
<th>plane ( \theta )</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computed flow ((10^{-6}m^3/s))</td>
<td>3.9834</td>
<td>4.0431</td>
<td>3.8952</td>
<td>3.9631</td>
<td>3.9439</td>
<td>4.0981</td>
<td>3.9074</td>
</tr>
<tr>
<td>Error/specified flow(%)</td>
<td>2.93</td>
<td>4.4156</td>
<td>0.6494</td>
<td>2.3377</td>
<td>1.9481</td>
<td>5.8442</td>
<td>0.9091</td>
</tr>
</tbody>
</table>

Table 3.4: Results for the non-Newtonian fluid

The errors made by determining the flow can be divided in two independent groups:
First the flowmeter itself has an error of about 2\% due to a zero-drift. Also with the calibration of the flowmeter an error of about 2\% is apparent.
Second, in computing the flow rate from the velocity measurements, errors were made in the velocity measurements themselves and in the computation of the discretized integral. The differences between the computed flow and the measured flow are given in Table 3.3 and Table 3.4. These fluctuations obtained in the Newtonian flow have a maximum of about 1.4\%, while the fluctuation maximum for the non-Newtonian flow is about 5.8\%. Even if some differences are found between the specified flow and the recalculation made, no significant errors appear. The results which will be shown are still valid. This remark is more inforced by the fact that the velocities components are made dimensionless by the mean axial velocity in this presentation. So the slight flow differences have no importance in the way of explanation because the results are directly comparable.

3.2.2 Errors in velocity measurements

In order to be able to check the accuracy of the L.D.A. measurements, the relative error of each velocity component (its root mean square – R.M.S.) was calculated for each measuring point.

\[ R.M.S. = \sqrt{\frac{\sum_{i=1}^{N} (u_i - \bar{U})^2}{N}} \]
where

\( u_i \) = velocity component of \( i^{th} \) particle along one direction, \( \bar{U} \) = mean velocity value and \( N \) = total number of samples at this measuring point.

The velocities errors are in fact due to several causes:
- presence of a velocity gradient in the measuring volume.
- random noise in the velocity signal.
- velocity values computation.

In order to see clearly the measurements errors due to the L.D.A. setup, the axial velocities profile and its measurements errors were plot (Figure 3.15 till Figure 3.18). The largest root mean square are observed in the points where the velocity gradient \( \left( \frac{du}{dx} \right) \) is high. Also the comparison between the measurements along the two center lines shows that the most important errors are located along the vertical center line. This phenomena is due to the measurement volume shape, this one is an ellipse which has its main radius along the vertical axis. So the largest error are located where the flow experiences an important pressure gradient along this main radius (here along the vertical axis). Even if some important errors exist in some measurements points, those figures are explicit, so the results are definitely reliable.
Figure 3.15: Axial velocities profile and its measurements errors (left: Newtonian, right: NonNewtonian) along the horizontal center line.
Figure 3.16: Axial velocities profile and its measurements errors (left: Newtonian, right: NonNewtonian) along the horizontal center line.
Figure 3.17: Axial velocities profile and its measurements errors (left: Newtonian, right: NonNewtonian) along the vertical center line.
Figure 3.18: Axial velocities profile and its measurements errors (left: Newtonian, right: NonNewtonian) along the vertical center line
3.2.3 Errors in alignment

As a result of the error which is made by determining the relative angle between the laser and the bend, a small error appears in the measured velocities. For example, when the angle error $\alpha = 1^\circ$, we get (Figure 3.19):

$$V' = \tan(1^\circ) \cdot U'$$

and in the most unfavourable case, with the maximum axial velocity $U' = 0.154 \text{ m/s}$, $V' = 2.688 \cdot 10^{-3} \text{ m/s}$

It appears that $V_{\text{measured}}' > V'$. That means that even if the measurements have an error due to a possible incorrect relative angle between the probe and the laser, it can despite be observed that the secondary velocities along the Y-axis are still directed towards the inner bend.
Chapter 4

Discussion and conclusions

4.1 Secondary flow in the entrance plane

The secondary flow orientation in the plane $O$ is the only effect of the bend on our flow in this entrance plane, even if the secondary $Y$-axis velocities are very low compared to the other planes ($< 5 \cdot 10^{-3} \, \text{m/s}$). This indicates a sort of upstream effect of the bend on the particles which want to go straight through. Those are already orientated to the inner bend due to "the continuity” of the fluid between the particles which are already in the first degrees of the bend curvature and those which are just at the entrance. A sort of ring appears along the wall (non-Newtonian flow), in which the secondary velocities are close to zero.

4.2 Comparison with previous studies

Looking at the results of this study, it can be observed that they are comparable to those of [Rindt (1989)], [Van de Vosse (1989)] and [Bovendeerd et al (1987)] for the Newtonian case and [Zuidervaart (1995)] for both cases, with the remark that the non-Newtonian fluid behaviour is closer to that of blood than the one of Zuidervaart. The fact that the axial velocity profile is flattened in the non-Newtonian case is probably caused by the shear-thinning behaviour of the non-Newtonian fluid. At lower shear-rates, e.g. in the central part of the tube, the viscosity of the non-Newtonian fluid is higher than the Newtonian one, which results in a more flattened (axial) profile in the neighbourhood of the maximum velocity. Also the non-Newtonian secondary flow is different compared to the Newtonian. In the Newtonian case, the mean axial vorticity found agrees well with previous results. A qualitative comparison with the results of [Bovendeerd (1987)] ($Re = 700$), and a comparison with the ones from [Van de Vosse et al (1989)], experienced at $Re = 300$, gives that the Newtonian mean axial vorticity obtained follow accurately those results. The appearance of the Newtonian plateau in the axial velocity profile after plane 45 due to the secondary flow near the the plane of symmetry, which leads to an expansion of the region of low axial velocity towards the center of the tube, can not be observed in the non-Newtonian case, probably caused by the difference in the secondary velocities compared to the Newtonian ones.

In the models of the 90 degree curved tube, detailed LDA-measurements of all the velocity components reveal significant differences between Newtonian and non-Newtonian flow fields. The macroscopic viscometric behaviour of the non-Newtonian fluid showed a close agreement to the properties of blood (flattened axial velocity, lower secondary flow).
For the Newtonian fluid, the general features (Dean vortex and C-shaped isoaxial velocity distribution) agree well with those found in previous studies. The secondary flow towards the inner bend in the entrance plane is confirmed experimentally (Bovendeerd) and numerically (Van de Vosse). The vortex tail in plane 90 and the vortex strengths are also in good agreement (Bovendeerd).

For the non-Newtonian fluid, the flattened axial velocity profile and the lower components of the secondary velocities were also found by Zuidervaart in the model of the carotid artery bifurcation. As the number of parameters in the 90 degree bend are reduced in comparison to the model of the carotid artery bifurcation, it is possible to make a better physical explanation for the behaviour of both flows than Zuidervaart did. With the visco-elastic fluid, more investigations can be done, for example measurements for an unsteady flow in a 90-degree bend can be performed. Also these steady experiments could be used as a first comparison with the unsteady measurement results, which will follow in the future.
Chapter 5

References


Thurston G.B. (1979), Rheological parameters for the viscosity and thixothropy of blood, 3rd congress of biorheology, Biorheology 16, pp. 149-162.
