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A simplified quasi-static model of the human knee joint based on a general two-body-system theory.

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Summary

In order to analyse the mechanical behaviour of the human knee joint Wismans [1] developed a three-dimensional statically indeterminate knee-joint model. In the present paper a more general model is presented in which some essential limitations of Wismans model have been eliminated. The general model can be used to develop knee joint models similar to the one of Wismans. This is demonstrated by means of a highly simplified knee joint model in which the complex geometry of the articular surfaces has been replaced by easily handled geometrical shapes. It turns out that such a model is able to predict a number of features of the mechanical behaviour of the human knee joint.
Introduction

In order to study the forces and relative motions in the human knee joint Wismans et al. [1] developed a three dimensional statically indeterminate knee joint model. In this model two rigid bodies, representing femur and tibia, are connected by contact points and non-linear elastic springs, representing the ligaments and part of the capsule. The articular surfaces are represented by polynomials and the friction between the surfaces is neglected. The menisci are not represented in this model. For a given external load the relative joint position can be calculated as a function of the flexion angle. Contact forces, locations of the two contactpoints and the strain in the ligaments result from these calculations. Although this model is able to describe many aspects of the mechanical behaviour satisfactorily, the need was felt to eliminate some limitations. Therefore a more general model has been formulated similar to the one developed by Wismans. In the following sections we will, briefly, highlight some of the essential elements in this general model and, as an illustration, discuss some results obtained with a simplified knee joint model based on the general model, which has been developed to find some of the essential features of the actual complex geometry of the articular surfaces.
Some essential elements of the general model

In the general model we consider the relative motions of two rigid bodies, of which one will be space fixed (Fig. 1). The motion of the moving body is described by means of 6 independent kinematic parameters (e.g.: 3 translation- and 3 rotation-parameters). In order to obtain a particular motion pattern it is possible to introduce M independent kinematic constraints, so that it is possible to prescribe virtually any desired kinematic parameter. The two rigid bodies can make contact in N contact points, situated on the surfaces of the rigid bodies. The geometry of these surfaces can be chosen almost arbitrarily with the only restriction that all curvatures must be continuous along the surfaces. The position vector of an arbitrary point on each surface must be specified as a bijective vector function of two independent surface-coordinates. The number of contact points, N, can vary during the relative motion of the rigid bodies. Both contact points and kinematic constraints constitute a limitation of the number of degrees of freedom of the moving body. The sum of the number of contact points and kinematic constraints has been limited to 5 (0 ≤ N+M ≤ 5), because otherwise there would be no degrees of freedom left.

External loads, acting on the moving body, can be a function of the position and orientation of the moving body which makes it possible to introduce follower forces, for example. Furthermore, the two bodies can be coupled with an unlimited number of elastic springs with arbitrary force-length relationships. Force-length relationships and insertion points of the springs must be specified.

The governing set of equations contains the 6 equations of equilibrium, M kinematic constraints and 5*N contact conditions which account for the contact between the two rigid bodies. From these equations we have to solve the 6+M+5*N unknowns: 6 kinematic parameters, N contact forces, 4*N surface-coordinates in the contact points and M Lagrange-multipiers which are introduced to account for the kinematic constraints and are closely related.
to loads required to prescribe certain kinematic parameters. In general this set of equations will be highly non-linear and therefore has to be solved numerically. We have chosen for a numerical solution by means of the Newton-Raphson method, implemented in a Fortran-program. Successive positions of the moving body can now be obtained by successive solution of the governing set of equations with different kinematic constraints or external loads. The number of kinematic constraints can be different for each position. Furthermore the number of contact points can vary as during the solution process points in which the contact force becomes negative automatically are eliminated. Based on this general model we are able to develop knee joint models like Wismans' model but with a number of extensions. The major differences between the general model and Wismans' model are shown in Fig. 2.

In formulating a (simplified) knee joint model there are two major problems we have to deal with. First of all the contact zones on tibia and femur have to be described mathematically. In Wismans' model the surfaces are described with polynomials in space, which were fit on points measured on the articular surfaces of a knee joint. Furthermore we have to describe the relevant ligamentous structures which have to be replaced by elastic springs. In Wismans' model the cruciates, collaterals and part of the capsule are replaced by seven non-linear elastic springs. Stiffness and untensed length of each spring are gained from literature and a trial-and-error process. In order to avoid the problem first mentioned we have started to analyse a highly simplified knee joint model. This analysis served two purposes: first of all we wanted to try out the capabilities of the general model and secondly we wanted to try and find some of the essential features of the actual complex geometry of the articular surfaces. This simplified model will be discussed in the next section.
A highly simplified knee joint model

In the simplified knee joint model the complex surfaces of tibia and femur have been replaced by easily handled plane surfaces and tori, respectively (Fig. 3). The radii of each torus were taken to be 15 and 5 mm., where the largest radius is to be taken in the sagittal plane. To account for the ligamentous structures seven springs with non-linear elastic properties were introduced. These springs represent the anterior and posterior parts of the anterior- (AAC and PAC, resp.), and posterior cruciate (APC and PPC, resp.), the lateral collateral (LC) and the anterior and posterior part of the medial collateral (AMC and PMC, resp.). In extension the anterior part of the cruciates and the medial collateral are loose while the other springs are tensed in extension. Insertion points of the springs were taken from measurements carried out in Nijmegen [2], whereas their constitutive behaviour was taken the same as in Wismans' model, relating force $F$ and strain $\varepsilon$ as:

$$F(\varepsilon) = K \varepsilon^2$$  \hspace{1cm} (1)

with

$$\varepsilon = (L - L_0)/L_0.$$  \hspace{1cm} (2)

$L_0$ is the untensed length of the spring which can be determined from the length $L_r$ and the assumed strain $\varepsilon_r$ of the spring in extension:

$$\varepsilon_r = (L_r - L_0)/L_0.$$  \hspace{1cm} (3)

Values for the constants $\varepsilon_r$, $K$ and $L_r$ (relation (1) and (3)) for the different springs are given in Table 1.
The motion of the femur with respect to the tibia was analysed by prescribing different flexion angles. Flexion, ab-adduction and exo-endorotation are defined as rotations about the axes of the body-fixed vectorbase connected to the tibia (Fig. 3).

No external load is working on the joint except for a small torque, ranging from -1 Nm in extension to 0.5 Nm at a flexion angle of 70 degrees, needed to prescribe the desired flexion angle. When we come to the presentation of some results of model calculations we must keep in mind that some results may disagree with experimental observations because of the simplifications we brought in. For example, it turned out that at flexion angles higher than 70 degrees the results become very unrealistic, which may be due to the chosen geometry of the articular surfaces. The results in this range have therefore been omitted.

As a first result in Fig. 4 it is shown that there is a small amount of exorotation during flexion. From literature [2] it is known that this kind of coupling strongly is influenced by external loads. Since in the present model external loads are relatively small, we must be careful in interpreting this result. This aspect needs further attention in forthcoming model calculations. Furthermore, the lengths of the springs as a function of the flexion angle were obtained from the calculations. In Fig. 5 the strain in the two springs representing the anterior cruciate is given as a function of the flexion angle. Strain here is defined as the elongation of the spring with respect to its length in extension, which makes it possible to compare these strains with results from measurements carried out in Nijmegen [2] represented by the dotted lines. We see that there is a good qualitative agreement. Results obtained for the springs representing the posterior cruciate and the collaterals agree in a similar way. As a last result the displacements of the contact points in posterior direction are shown in Fig. 6. We see that the shift of the contact points on femur and tibia is not much different which means that mainly rolling will take place. As flexion proceeds the amount of sliding increases which is in agreement with results obtained by Wismans [1].
Discussion

In the foregoing a mathematical model is presented to evaluate the mechanical behaviour of the human knee joint, which has been demonstrated by means of a highly simplified knee joint model. The most important simplifications are 1) the representation of the complex geometry of the articular surfaces by simple geometrical shapes and 2) the absence of menisci and patella. It turns out that such a highly simplified model is capable of predicting a number of phenomena concerning the mechanical behaviour of the human knee joint within a certain range of motion. The actual value of this type of model as a tool for gaining insight into knee joint mechanics will become more evident after investigation of the importance of the chosen geometry, initial strains and locations of the insertion points of the springs. These aspects will be subject for further research.

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References


Fig. 1

m kinematic constraints

external load

elastic springs

n contactpoints
<table>
<thead>
<tr>
<th>Wismans' model</th>
<th>General model</th>
</tr>
</thead>
<tbody>
<tr>
<td>* 2 rigid bodies</td>
<td>* 2 rigid bodies</td>
</tr>
<tr>
<td>* 1 kinematic constraint</td>
<td>* M kinematic constraints ( O \leq N + M \leq 5 )</td>
</tr>
<tr>
<td>* 2 contactpoints</td>
<td>* N contactpoints</td>
</tr>
<tr>
<td>* surfaces: polynomials</td>
<td>* arbitrary surfaces</td>
</tr>
<tr>
<td>* external loads not a function of position and orientation of moving body</td>
<td>* arbitrary external loads</td>
</tr>
<tr>
<td>* fixed constitutive behaviour of springs</td>
<td>* arbitrary constitutive behaviour</td>
</tr>
</tbody>
</table>

Fig. 2 A comparison between Wismans' model and the general model.
Fig. 5