Adult to Child Scaling of Frontal Thoracic Impact Response

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Summary

A new series of child dummies is developed by the TNO Crash-Safety Research Centre. The new series is the sequel to the TNO-P dummies, developed in the early seventies, and is extended by adding an 18 month child dummy. The main progress will be a refinement of the anthropometry and an improvement of humanlike behaviour (biofidelity) of the child dummies.

In this report, an attempt was made to provide for a new response requirement for the 18 month child dummy thorax. The response requirement is a scaled version of the adult thorax response requirement (Kroell corridors), since the thoracic response of children is not known. Unlike previously published thoracic scaling requirements, damping of the thorax was included in the scaling procedure.

First, the thorax was modelled by a lumped mass model developed by Lobdell, which is frequently used for dummy thorax design. The model was implemented in the program MADYMO and a variation analysis was conducted. It appeared that the response of the model easily can be adapted to a certain response requirement by varying one or more parameters. Consequently, the visco-elastic element of the model could be left out.

Secondly, the model parameters were scaled by taking into account the differences in geometry and material properties between the thorax of the child and the adult. On these grounds, mass and stiffness were scaled, although not all geometric parameters had been accurate or available. Therefore, the calculated scaling ratios are preliminary and have to be recalculated when more accurate data are available.

Damping was scaled in a more general way, because the relation between the thorax quantities geometry and material properties and the causes of damping is very complicated. The resultant damping scaling ratio was determined by assuming that the material properties of the thorax related to damping are not dependent on age.

Thirdly, the impact response of the Lobdell model with scaled parameters was simulated. The impact response of the child thorax showed a highly overdamped behaviour. The impact response was simulated for a range of damping scaling ratios, the simulations showed that the damping scaling ratio probably is an upper estimate. The new response requirement can be based on the highly overdamped curve, although it might well be that the child impact response shows a more adult-shaped curve. Further research on modelling of the human thorax or if possible child cadaver impactor test should demonstrate which assumption is the most realistic.
1 Introduction

General
In modern society, one of the leading causes of death and disability are injuries sustained by motor vehicle crashes. During a vehicle accident, the human body is exposed to extreme loading conditions. If these conditions exceed a certain level, anatomical damage or failure of normal function might occur. The kind of injury assessed to the car occupant is dependent on which injury mechanism occurs, in other words what type of loading will cause the injury. Three types of loading can be distinguished: compression of the body, viscous loading in the body and inertial loading of internal structures. Dependent on the loading situation and the magnitude of the loading, the injury mechanisms can occur in combination with each other. To prevent or minimize injuries, safer motor vehicles have to be designed. Already in the design phase of a car, safety measures have to be implicated. Main points of attention are the improvement of the crashworthiness of the vehicle and the application and improvement of safety devices like seatbelts and airbags. The objective of these safety systems is plural. First, the occupant motion is controlled through which contact with the vehicle interior is prevented. Secondly, the deceleration of the car occupant is reduced by the interaction with the airbag. The effectiveness of a safety device is tested with human substitutes: the crash dummies. An example of such a dummy is the TNO-10 dummy, a simple anthropometric loading device. The restrained TNO-10 dummy is seated on a sled which is decelerated comparable to a severe car accident. The belt system is approved if the belt system remains intact and the displacement of the dummy does not exceed a fixed value. To determine whether safety devices assess injuries to an occupant during a crash, whole vehicle crash tests are executed. In these crash tests, more sophisticated dummies are used to substitute car occupants. Examples of this kind of dummies are the GM Hybrid III, developed for frontal vehicle crash tests and the EUROSID-I (EUROpean Side Impact Dummy), developed for lateral vehicle crash tests, see Figure 1. These dummies are not only anthropometric but show a humanlike mechanical response (biofidelity). Biofidelity of for instance the thorax of the EUROSID-I is accomplished by demanding that the biomechanical response of the dummy thorax meets a response requirement resulting from a series of lateral human cadaver drop tests. Furthermore, the dummies can be supplied with

Figure 1. The EUROpean Side Impact Dummy EUROSID-I.
measuring instrumentation like accelerometers and force transducers. After the crash test, data from the measurement instrumentation can be compared with injury criteria. An injury criterion is a relation between one or more measured quantities and an injury of a certain severity. The injury criteria are defined on the basis of human cadaver tests and animal tests. An example of a injury criterion is the Thoracic Trauma Index (TTI), which combines the peak lateral thoracic spine acceleration and the peak lateral rib acceleration; the mean value of the two acceleration (=TTI) should not exceed 90 g (g = acceleration of gravity). This limitation value is better known as the injury tolerance level.

**Child dummies**

In a car accident often children are involved. The restraint systems standard present offer no full protection to children, therefore specific safety devices have to be designed for the protection of children. From the early 1960’s, TNO developed testing methods for evaluating child safety devices, together with a series of 4 child dummies; the $P_{3/4}$, $P_3$, $P_6$ and $P_{10}$ child dummies. They represent masses and dimensions of children of the following ages: 9 months, 3 years, 6 years and 10 years. Their design is relatively simple, see Figure 2. To complete the series, in 1984 a fifth dummy has been included, the $P_0$, representing a newborn infant. The infant dummy is of a much simpler design. It consists of a head, torso, arms and legs as one single unit. The P-dummies are used mainly as a loading device for evaluating child restraint systems. The testing procedure includes a dynamical sled test. The requirements described for approval of the system are: maximum head excursion, maximum chest acceleration and no abdominal penetration. However, these injury tolerances are only usable if the biomechanical response of the child dummy is realistic. Therefore, more sophisticated child dummies have to be developed. For the development of adult dummies, biofidelity is accomplished by comparing the biomechanical response of the dummy with response requirements resulting from cadaver tests and volunteer tests. Volunteer tests using children are ethically unacceptable and data from child cadaver tests are very limited. A technique that can be used to derive the missing data is scaling. With this method, child data can be derived by scaling dimensions of adults to children. A scaling ratio is the quotient of a child dimension and the corresponding adult dimension. The child cannot simply be seen as a small adult, because not every body segment grows proportional to a certain dimension. Furthermore, several material properties change during growth. The consequence is that most body segments have to be scaled individually.

Besides the anthropometric data, mass and stiffness can be scaled by assuming corresponding

![Figure 2. Exploded view of a TNO child dummy.](image)
material properties. Then, scaling of biomechanical response can be derived by Mertz's method \[^1\]. Mertz normalised responses of lateral cadaver drop tests with ratios based on physical characteristics and used a simple spring-mass system to represent the motion of the cadaver.

**Objectives**
The new series of child dummies is extended by adding an 18 month child dummy. In this report, the adult thorax responses will be scaled to an 18 month child thorax response. Scaling ratios will be determined on the basis of data of a 50th percentile male adult together with known data of an average 18 month child. They further will be specified as "adult" and "child".

Zandbergen \[^2\] indicates the large influence of damping on the thoracic response and also the improbability of equal damping of the thorax for both the adult and the child. For this reason, he recommends to include damping in further thorax scaling procedures. The simple spring-mass system which Mertz used to model the thorax does not contain a damper. A model of the human thorax, including damping, was developed by Lobdell et al. \[^3\]. The model is used for simulating the biomechanical response of the thorax for frontal blunt impact.

In Chapter 2, the Lobdell model will be explained and a variation analysis will be conducted in order to observe the influence of the model parameters on the response of the model. In Chapter 3, scaling to the child thorax will be accomplished by scaling each Lobdell model parameter separately. Next, the response of the child will be simulated by a computer using the scaled parameters. The simulated child response will be used as basis for an 18 month child thorax response requirement, which will be used as a design parameter for the new 18 month child dummy. Certainty on whether the model of the child thorax will behave lifelike is not guaranteed. The results of the child response should therefore be compared with child cadaver impactor tests. Child cadavers for this kind of tests are very limited and regarding the present situation, it is very unlikely that such experiments ever will be conducted.
2 The Lobdell Model

2.1 Introduction

In the early seventies, Lobdell et al. developed a mathematical model for blunt thoracic impact as an aid in understanding the biomechanical response of the human thorax \[^{3}\]. This thoracic analog of the mathematical model is a lumped mass system, a system consisting of a combination of masses, springs and dampers. The Lobdell model is displayed on page 7, Figure 8. To achieve biofidelity, the simulated response of the model has to correlate with the thoracic impact response of a human being. The thoracic impact response can be determined with the aid of impactor tests. Impactor tests can be performed on volunteers, human cadavers or animals. Volunteer tests are restricted to minor loading levels so Lobdell et al. made use of previously published unembalmed cadaver impactor tests \[^{4}\].

Animal impactor tests cannot be used to determine the human impact response. However, the differences between the impact response of dead animals and anaesthetized animals can be an useful tool in the correct interpretation of data between human cadavers and volunteers.

The test setup of Kroell’s cadaver impactor test is displayed in Figure 3. The cadaver was struck by an impactor mass with a certain velocity. During the test, impactor force and striker displacement were measured and high speed photography was applied to determine the chest deflection. The cadaver impactor tests were performed for two combinations of impactor mass and velocity. Force-time curves and force-total deflection curves were presented as test results. Figure 4 shows the force-total deflection curves of a number of cadavers with the high velocity impactor conditions. The curves show a different dynamic response of each cadaver, due to differences of age, physical characteristics and sex. The cadaver response curves of both impactor conditions were averaged first. Next, the averaged curves were adjusted for muscle tensing and skeletal deflection was determined by substracting the deflection of the superficial tissues of the ribcage from the measured total deflection. The recommended response corridors (Kroell corridors) are a 15% deviation area of the averaged, adjusted response curves. The response curves and the corridors are displayed in Figure 5. Lobdell estimated the model parameters and adjusted them after each simulation, until the simulated response met the Kroell corridors, see Figure 6. The Kroell response corridors can be used for evaluating a dummy thorax.
Figure 4. Dynamic force-total deflection characteristics for unembalmed cadavers using high impactor velocity conditions (23.1 kg with 7.15 mls), FM = male test specimen, FF = female test specimen.

Figure 5. Averaged, adjusted force-skeletal deflection curves (solid lines) and recommended response corridors (dashed lines) for both impactor conditions.

Lobdell et al. evaluated the chest structures of dummies available at that time for frontal blunt impact response. None of the resulting impactor force-skeletal deflection curves met the response corridors. Another approach was made by Neathery and Lobdell, who applied the model as a tool in the development of a new dummy thorax. They produced a mechanical chest which behaves similarly to the model and closely approaches the Kroell response corridors. The ribmodule of the EUROSID-1 has been developed in a rather similar way. Lateral cadaver drop tests have been conducted to obtain a response requirement and a slightly adapted Lobdell model has been used to determine the values of the required parameters.

As mentioned before, the Lobdell model will be used in the scaling procedure of the thorax. Each model parameter has a specific physical meaning, for example stiffness $k_{23}$ represents the ribcage stiffness.
All model parameters will be scaled by comparing the matching physical meaning of the child and the adult. The simulation of the model with the scaled parameters will result in the thoracic impact response of the child. Because the thoracic impact response of a child is not known and the scaling is based on fairly rough assumptions, it is important to know how the different scaling factors effect the impact response.

The next section contains a short description of the thorax anatomy. Further on in this chapter, a variation analysis of the model will be conducted. By comparing the simulated responses, the influence of each parameter will be clarified. The model will be simulated with the program MADYMO.

2.2 Anatomy of the thorax

The thorax or chest is the upper part of the torso, from the base of the neck to the diaphragm, which separates the thorax from the abdomen. The thorax can roughly be subdivided into the following parts:

- the ribcage
- the thoracic spine
- the contents
- the outside layer

The ribcage, see Figure 7, consists of twelve pairs of flat curved bones, the ribs. At the posterior side (backside), all pairs of ribs are connected to the thoracic spine. At the anterior side (frontside), the upper seven pairs of ribs are connected by cartilage to a large, flat bone, the sternum. The next three pairs of ribs are indirectly connected to the sternum and the lower two pairs of ribs (floating ribs) are not connected to the sternum. Further, all ribs are
interconnected by intercostal muscles.
The thoracic spine, the lumbar spine and the cervical spine (neck) together form the vertebral column. The thoracic spine consists of twelve bones (vertebrae) which are interconnected by cartilage intervertebral discs, ligaments and muscles. The spinal cord passes through the vertebral column.
The contents of the thorax can be divided into three segments. The left and the right region contain the lungs and the centre part contains the heart, the large vessels and the trachea.
The outside layer of the thorax consists of skin, fat and a variety of muscles.

2.3 Description of the Lobdell Model
The mechanical analog Lobdell developed is shown in Figure 8, the values and the meaning of the parameters are given in Table I.

![Figure 8. The Lobdell model.](image)

The system has four degrees of freedom, noted by the x-variables in the figure. Lobdell derived four differential equations from the mechanical analog and solved them numerically. The simulation of the system proceeded until the force in spring $k_{12}$ became negative, which in reality cannot occur, as the impactor separates from the thorax.
The recommended force-deflection corridors are specified for impactor force and skeletal deflection. The impactor force has been determined by the force on mass $m_1$, which equals the force in spring $k_{12}$. Skeletal deflection was determined by the difference between displacements $x_3$ and $x_2$. The model parameters were estimated and adjusted after each simulation, until the impact response fell within the Kroell corridors, see Figure 6.
A bi-linear spring stiffness $k_{23}$ was used to satisfy the Kroell corridors at large deflection, see Figure 9. The increasing stiffness at large deflections is due to the non-linear material
Table I. Parameters of the Lobdell model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>low velocity impactor conditions</td>
<td></td>
</tr>
<tr>
<td>impactor mass, impactor velocity</td>
<td>19.5 kg, 4.92 m/s</td>
</tr>
<tr>
<td>high velocity impactor conditions</td>
<td></td>
</tr>
<tr>
<td>sternal effective mass</td>
<td>0.45 kg</td>
</tr>
<tr>
<td>vertebral effective mass</td>
<td>27.2 kg</td>
</tr>
<tr>
<td>stiffness of skin and flesh between sternum and impactor</td>
<td>281 kN/m</td>
</tr>
<tr>
<td>stiffness of ribcage</td>
<td></td>
</tr>
<tr>
<td>primary spring stiffness transition point</td>
<td>26.3 kN/m, 31.8 mm</td>
</tr>
<tr>
<td>secondary spring stiffness</td>
<td>78.8 kN/m</td>
</tr>
<tr>
<td>damping internal thorax</td>
<td></td>
</tr>
<tr>
<td>compression</td>
<td>0.525 kN/m, 1.23 kN/m</td>
</tr>
<tr>
<td>elongation</td>
<td></td>
</tr>
<tr>
<td>visco-elastic behaviour of thorax</td>
<td>13.2 kN/m, 0.18 kN/m</td>
</tr>
</tbody>
</table>

behaviour of the ribcage and the contribution of stiffness by compression of the internal organs. It also had been necessary to set a different damping coefficient $c_{23}$ for elongation and compression in order to satisfy the descending part of the corridors, Figure 10. The descending part of the cadaver response is mainly determined by the fact that the cadaver thorax is unable to move back into the original shape due to rib fractures and because the backflow of blood and air continues after the impactor released from the thorax.

Figure 9. Stiffness characteristic $k_{23}$.

Figure 10. Damping characteristic $c_{23}$. 
2.4 The Lobdell Model in MADYMO

MADYMO is a computer program, designed for the analysis of the dynamical response of the human body and of mechanical systems especially for extreme loading conditions. MADYMO consists of a number of modules, see Figure 11.

![Diagram of MADYMO modules](image)

The multibody module calculates the contribution of the inertia of rigid bodies to the equations of motion. The contributions of specific force elements such as springs and dampers are calculated by other modules. The bodies can be interconnected by kinematic joints and also can be coupled to finite element structures. Finite element modelling (FEM) can be applied to program geometric complex structures and several material models. For more information on MADYMO, the reader is referred to the MADYMO Users' Manual [7].

The Lobdell model is a lumped mass system. In MADYMO it is possible to analyze the motion of several systems of rigid bodies. The Lobdell model will be split into two different systems, respectively the impactor and the thorax. The MADYMO inputfile for the Lobdell model is provided in Appendix A.

To allow separation of impactor and thorax after the impact, the contact force between the impactor and the thorax is generated by a specific contact model. The two masses, programmed as two ellipsoids, cannot deform themselves (rigid bodies), but are allowed to penetrate into each other. The contact force that has to be generated is a function of the penetration of the two ellipsoids and can be described as a spring with stiffness $k_{12}$.

The model will be simulated for the same two different impactor combinations as Lobdell used: impactor mass 19.5 kg with a velocity of 4.92 m/s and impactor mass 23.1 kg with a velocity of 7.15 m/s. Two different impactor combinations are used because the Kroell corridors are based on cadaver tests using these impactor combinations and, therefore, are not valid for other impactor conditions.

The model will be simulated with (original model) and without (simplified model) the visco-elastic element $c_2\cdot v_2^2$. For scaling, it is desired to keep the model as simple as possible. Furthermore, scaling of visco-elastic material behaviour will be very complicated, if not impossible.

The results of the simulations are shown in Figures 12a-d.

Figures 12a and 12b show that the response of the model closely approaches the Kroell
corridors with or without the visco-elastic element, consequently the visco-elastic element will further be left aside. Figure 12c shows the force equilibrium of $m_2$:

$$ m_2 \text{ resultant force} = k_{22} \text{ elastic force} + c_{22} \text{ damping force} = k_{12} \text{ elastic force} + m_2 \text{ acceleration force} $$

where $m_2 \text{ acceleration force} = m_2 \cdot \text{acceleration X2}$ (Figure 12d).

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**Figure 12a.** Original and simplified force-deflection curves with Kroell corridors (dotted lines), $v_{imp}$ 4.92 m/s and $m_{imp}$ 19.5 kg.

**Figure 12b.** Original and simplified force-deflection curves with Kroell corridors (dotted lines), $v_{imp}$ 7.15 m/s and $m_{imp}$ 23.1 kg.

**Figure 12c.** Force-time curves of forces working on $m_2$, $v_{imp}$ 4.92 m/s and $m_{imp}$ 19.5 kg.

**Figure 12d.** Acceleration-time curves of $m_2$ and $m_3$, $v_{imp}$ 4.92 m/s and $m_{imp}$ 19.5 kg.
2.5 Sensitivity analysis

A sensitivity analysis is a technique to study the response of a dynamical system. By varying each parameter over a range, the sensitivity of the system for that specific parameter can be determined on the basis of the alteration of the simulated response. The chosen variation of each parameter is: (0.2, 0.5, 1, 2, 5) - the Lobdell value. The results are displayed in two figures for each parameter, a force-deflection figure and a force-time figure. The low velocity impactor conditions are chosen as unchanged input parameters.

Variation of mass $m_2$

![Diagram of a dynamical system with parameters $m_1$, $k_{12}$, $m_2$, $k_{23}$, $c_{23}$, $m_3$, $x_1$, $x_2$, $x_3$, $x_4$, $x_5$, and displacement $X_3-X_2$.]

---

**Figure 13a. Impact response: force-deflection curves.**

**Figure 13b. Impact response: force-time curves.**

Decreasing the value of $m_1$ mainly affects the initial part of the impact response. Increasing the value of $m_2$ yields a phenomenon called backfiring. This phenomenon occurs because the thorax system becomes underdamped, that is $m_2$ begins to oscillate. Previously published cadaver data [41,81] show no backfiring, therefore the human thorax can be presumed overdamped. It is noted that $m_1$ and $m_3$ are related, the sum of the two masses is the total mass of the thorax. The mass of the thorax is the total of the masses of its parts, they are lungs, heart, ribs, sternum, vertebral column, etc. In fact, dividing the thorax mass into the masses $m_2$ and $m_3$ is a model approach.
Varying the value of $m_3$ affects the location of the point maximum deflection/maximum force of the impact response. A higher value yields a higher force as well as a higher deflection. If $m_3$ is set infinite, the model can be used to simulate a cadaver test in which the back of the subject is rigidly mounted, a restrained back impactor test. A restrained back experiment should be conducted with a lower impactor mass in order to become a similar response, see Figure 14c. Normally, a substantial part of the impactor energy is transformed into whole body movement, whereas in the restrained case the total impactor energy is used in thorax deformation.
Figure 15a. Impact response: force-deflection curves.

Figure 15b. Impact response: force-time curves.

Variation of spring stiffness $k_{12}$

The effect of stiffness $k_{12}$ is large at low deflection. Increasing the stiffness $k_{12}$ yields high force peaks at low deflections and causes oscillating behaviour of the impact response. This oscillating behaviour is mainly caused by the absence of a damper between the impactor and the sternum. On the other hand, the biological structures between the impactor and the sternum are assumed to have some damping material properties. If the value of stiffness $k_{12}$ comes close to the value of stiffness $k_{23}$, also the midrange of the force-deflection curve is influenced. In reality, the stiffness of the tissue is much higher than the stiffness of the ribcage because of its slight thickness. In case the wooden impactor face is covered by a foam, the stiffness of $k_{12}$ is lower and damping is introduced. The effect of the padding has been tested on a GM Hybrid-I dummy by Lobdell et al. and is displayed in Figure 16.
The stiffness of the ribcage itself is highly non-linear, considering its complicated geometry and material properties. Up to the point that rib fractures occur, the stiffness of the ribcage is assumed to increase as a function of skeletal deflection. Lobdell approximated this function by choosing a bi-linear spring stiffness. Varying the value of $k_{23}$ can be accomplished by changing the primary stiffness, the secondary stiffness and the transition point. For the variation analysis, the bi-linear spring stiffness will be substituted by a corresponding exponential relation:

$$F(\Delta X) = a(e^{b\Delta X} - 1)$$

where $a = 700$, $b = 25$.

The variation of stiffness $k_{23}$ is now accomplished by varying parameter $b$ of Eq. (1). To obtain a similar variation range, the following values for parameter $b$ have been derived: $b = [10, 20, 25, 30, 40]$.

An increase of stiffness $k_{23}$ causes a higher maximum force and a lower maximum deflection, and conversely, a decrease of stiffness $k_{23}$ makes the maximum force at high deflection level disappear and causes a higher maximum deflection.
The damping coefficient has been set different for extension and compression to satisfy the Kroell corridors in the descending part of the response curve. The effect of varying the damping coefficient for compression is all over the curve. A lower value of $c_{23}$ causes as well a higher deflection as a higher force in the point maximum deflection/maximum force and the oscillating behaviour appears. A higher value of $c_{23}$ causes an overdamped behaviour, the
characteristic force-deflection curve becomes unrecognizable. For extension, damping only effects the descending part of the force-deflection curve.

2.6 Discussion
The variation analysis shows that all model parameter influence the impact response. As a consequence of this, all thorax parameters have to be scaled, assuming that these parameters change during growth.

Furthermore, the variation analysis shows that each parameter influences a specific part of the response curve. Then, by varying one or more parameters, a wide variety of response curves can be created. A summary of the influences of each model parameter is given in Table II the influenced curve quantities are displayed in Figure 19.

Table II. Summary of the influence of Lobdell parameters on characteristic points of the impact response curves.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Maximum force low deflection</th>
<th>Maximum force high deflection</th>
<th>Maximum deflection</th>
<th>Plateau force</th>
<th>Oscillation</th>
<th>Time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_2$</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$m_3$</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>$k_{12}$</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$k_{23}$</td>
<td>0</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$c_{23}$</td>
<td>+</td>
<td>+/-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

+ : increase-decrease of value model parameter causes increase-decrease of characteristic point
- : increase-decrease of value model parameter causes decrease-increase of characteristic point
+/- : increase and decrease of value model parameter causes increase of characteristic point
0 : increase or decrease have minor or no influence

Figure 19. Characteristic points of the impact response curves.
3 Scaling Procedure

3.1 Scaling in general
Scaling will be applied to derive unknown thorax quantities of an 18 month child. The unknown thorax quantities are mass, stiffness and damping. They will be scaled by taking into account the geometric and material differences between the child and adult thorax. Suppose the child and adult thoraxes are a small and a large cube with the same material properties. All length dimensions of the cubes scale equally, length scaling yields:

\[ \lambda = \frac{l_{\text{small cube}}}{l_{\text{large cube}}} \]  \hspace{1cm} (2)

For all three unknown quantities, considerations are made on how these quantities are scaled in general.

mass
Scaling of mass is based on identical density of the cubes. Mass is the product of density (\( \rho \)) and volume (V), which yields:

\[ m = \rho V \Rightarrow \frac{m_{\text{small cube}}}{m_{\text{large cube}}} = \frac{\rho_{\text{small cube}}}{\rho_{\text{large cube}}} = \lambda^3 \]  \hspace{1cm} (3)

stiffness
Scaling of stiffness is based on identical strain distribution of the cubes. If identical strain (\( \varepsilon \)) distribution is assumed and deformation is proportional to its initial length (l), deformation (d) also scales with \( \lambda \):

\[ \frac{d_{\text{small cube}}}{d_{\text{large cube}}} = \frac{R_{\varepsilon} \cdot l_{\text{small cube}}}{\varepsilon \cdot l_{\text{large cube}}} = \lambda \]  \hspace{1cm} (4)

All material parameters determining the stress (\( \sigma \)) as a function of strain (E, \( \nu \)) are assumed to scale with a stress ratio \( R_{\sigma} \):

\[ \frac{\sigma_{\text{small cube}}}{\sigma_{\text{large cube}}} = R_{\sigma} \]  \hspace{1cm} (5)

Force is defined as the integral of stress over a certain surface. Together with Eq.(2) and Eq.(5), the force scaling factor equals:
\[
\frac{F_{\text{small cube}}}{F_{\text{large cube}}} = R_F = R_\sigma \lambda^2 \quad (6)
\]

Stiffness is the quotient of force and deformation, which yields:

\[
\frac{k_{\text{small cube}}}{k_{\text{large cube}}} = R_k = R_\sigma \lambda \quad (7)
\]

damping

For the cubes, an identical strain-rate (\(\varepsilon'\)) distribution can be assumed at a certain instant. The rate of deformation (\(d'\)) will scale with \(\lambda\):

\[
\frac{d'_{\text{small cube}}}{d'_{\text{large cube}}} = R_{d'} = \frac{\varepsilon'_{\text{small cube}}}{\varepsilon'_{\text{large cube}}} = \lambda \quad (8)
\]

Stress parameters were assumed to scale with \(R_\sigma\), Eq.(5), an identical assumption can be made for damping stress parameters which will be scaled with \(R_\sigma\). Damping force will be scaled with the ratio \(R_\sigma \lambda^2\), Eq.(6), the damping coefficient will be scaled with the ratio \(R_\lambda\), Eq.(7).

In reality, length dimensions of the human thorax will not scale equally for every direction, therefore different scaling factors have to be applied for each direction: \(\lambda_x, \lambda_y, \lambda_z\). However, in the particular case where \(\lambda_x = \lambda_y = \lambda_z\), the scaling factors derived in this section have to be satisfied.
3.2 Scaling of mass

The thorax mass is assumed proportional to the thorax volume $V$, that is the densities of the thorax of the child and the adult are assumed to be equal. To obtain a similar response of child and adult, the impactor impulse has to be scaled. Since the velocity scaling factor is equal to one, the impactor mass will be scaled with the same ratio as the thorax mass.

$$R_m = \frac{m_{\text{child}}}{m_{\text{adult}}} = \frac{V_{\text{child}}}{V_{\text{adult}}}$$  \hspace{1cm} (9)

The volume and the shape of the thorax can be described using the parameters displayed in Figure 20:

**Figure 20.** Human body with coordinate system and thorax geometry parameters.

The geometry scaling ratios of the thorax are chosen to be:

$$\lambda_x = \frac{\text{chest depth}_{\text{child}}}{\text{chest depth}_{\text{adult}}}$$

$$\lambda_y = \frac{\text{chest breath}_{\text{child}}}{\text{chest breath}_{\text{adult}}}$$

$$\lambda_z = \frac{\text{chest height}_{\text{child}}}{\text{chest height}_{\text{adult}}}$$  \hspace{1cm} (10)

The mass scaling factor is:

$$R_m = \lambda_x \cdot \lambda_y \cdot \lambda_z$$  \hspace{1cm} (11)
3.3 Scaling of stiffness

Stiffness $k_{12}$
The elastic modulus of skin and flesh between impactor and sternum is presumed not to differ with increasing age. The scaling of spring $k_{12}$ therefore is only dependent of the impactor face area ($A_{imp}$) and the thickness of skin and flesh (h) between impactor and sternum.

$$R_{k_{12}} = \frac{k_{12,\text{adult}}}{k_{12,\text{adult}}} = \frac{R_{A}}{R_{h}}$$

$$R_{A} = \frac{A_{\text{imp,adult}}}{A_{\text{imp,adult}}} = \frac{\text{sternum height \cdot chest breath}_{\text{child}}}{\text{sternum height \cdot chest breath}_{\text{adult}}} = \lambda_{z} \cdot \lambda_{y} \quad (12)$$

$$R_{h} = \frac{h_{\text{child}}}{h_{\text{adult}}} = \lambda_{z}$$

$$R_{k_{12}} = \frac{\lambda_{z} \cdot \lambda_{y}}{\lambda_{x}} \quad (13)$$

Stiffness $k_{23}$
During growth, the geometry and the material characteristics of the human ribcage will change. Considering the overall geometry of the ribcage, two major differences can be seen:
- The presence of a cartilage part at the posterior and anterior side of the child thorax. The elastic modulus of cartilage is much smaller than the one of bone. During growth, in particular the posterior cartilage part will stiffen.
- The cross-sectional shape of the ribcage of a child is more round than the one of an adult.

The stiffness ratio of the ribcage will be determined by taking the stiffness ratio of just a single rib of the child and a similar one of the adult. It is assumed that the stiffness ratio of the entire ribcage equals the stiffness ratio of a single rib. The stiffness ratio is determined by the ratio of the static stiffness of two curved beams with different boundary conditions. The adult rib will be represented by a fixed circular beam, the child rib will be represented by a simply supported circular beam, see Appendix D. The difference in stiffness caused by the anterior cartilage part is taken care of by factor $\alpha$.

The second remark indicates that $\lambda_{x}$ is not equal to $\lambda_{y}$. The radius can be scaled by one of the cross-sectional parameters, in this case it will be scaled by $\lambda_{y}$. To take into account the different cross-sectional shape, the stiffness scaling factor will be multiplied with a factor $\beta$.

This factor is calculated by comparing the static stiffnesses of elliptical curved beams of child and adult measurements. This calculation is presented in Appendix E.
The stiffness scaling factor of $k_{23}$ is:

$$R_{k_{23}} = \alpha \beta \left( \frac{EI_{\text{child}}}{R^3_{\text{child}}} \right) = \alpha \beta \left( \frac{R_{E_{\text{child}}}}{R_{E_{\text{adult}}}} \right) \left( \frac{R_{I_{\text{child}}}}{R_{I_{\text{adult}}}} \right)$$

$$R_{E} = \frac{E\text{-modulus, bending}_{\text{child}}}{E\text{-modulus, bending}_{\text{adult}}}$$

$$R_{I} = \frac{\text{moment of inertia}_{\text{child}}}{\text{moment of inertia}_{\text{adult}}}$$

$$R_{R} = \left( \frac{\text{radius rib}}{\text{radius rib}} \right)^3_{\text{child}} = \lambda^3_{\text{y}}$$

The moment of inertia of the rib is presented in Appendix F.

3.4 Scaling of damping

Damping coefficient $c_{23}$

Damping of the thorax is mainly caused by the internal structure of the thorax and much less caused by damping in the ribcage itself. The internal structure of the thorax is rather complicated what makes it difficult to determine the exact causes of damping. Some suggestions can be made about phenomena which can be related with damping of the thorax:

- Air is suddenly compressed in the lungs. From impactor experiments with freshly killed swine \cite{9} it can be seen that the pressure in the lungs rises direct after the impact. The air cannot escape immediately by the trachea through which the abdominal contents are pressed down. The overpressure quickly falls down as the air escapes and the abdominal contents distension.

- Muscles for the respiration suddenly become overtensed. It is assumed that muscles under these conditions show a strong visco-elastic behaviour.

- Damping of blood flowing through the arteries is not believed to have a large influence on the total damping of the thorax. The arteries are already under pressure; it is unlikely that the heart is able to pump even more blood through the veins without getting damaged. Rather than considering damping caused by bloodflow through the arteries, the heart itself can be seen as an incompressible medium with internal damping.

The damping coefficient will be scaled in the general way, that is $R_{c_{23}} = R' \lambda$. A numerical value for scaling factor $R'_{\alpha}$ cannot be given, there is no information on stress strain-rates relations of the adult and the child thorax. Presuming an equal stress strain-rate relation of child and adult thorax, $R'_{\alpha}$ will be set equal to one, resulting in the damping coefficient scaling ratio:
3.5 Discussion

All model parameters have been scaled, an overview of the scaling ratios and the anthropometric data used for the calculation of the scaling ratios are given in \(\).

There are two different ways to determine the child impact response requirements. First, the Kroell response corridors can be scaled with the use of Mertz’s method. Mertz \cite{11} derived force, time and deflection ratios for normalizing the force-time response corridors of cadavers, see Appendix B. The force and deflection ratios are:

\[
R_f = \sqrt{R_k R_m}
\]

\[
R_d = \frac{R_m}{R_k}
\]

where \(R_k = R_{k23} = 0.23\) and \(R_m = 0.14\), see Table III. Secondly, a deviation area can be defined on the basis of a simulation of the Lobdell model with the scaled parameters.

The Lobdell model with the scaled parameters is representative of the child thorax, then, the simulation of the impact with the Lobdell model can be seen as the child impact response.

Both methods have been applied, the results are shown in Figure 21. The figure shows a highly overdamped behaviour of the simulated child impact response, the damping coefficient scaling factor probably is an upper estimate. It could well be that the real impact response shows a more adult shaped curve. Figure 21 also illustrates that the scaled Kroell response corridors are only satisfied if the damping coefficient scaling factor \(R_{c23} < 0.2\).

The simulated child response will further be taken as basis of the new child response requirements. The response requirements are determined for two impactor conditions and are displayed in Figure 22, the corresponding data are given in Table III. The deviation area is restricted by 10% deviation of the maximum force and 10% deviation of the maximum deflection.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 18 month child</th>
<th>Adult</th>
<th>Ratio (child/adult)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p14 terone depth axilla</td>
<td>220.0</td>
<td>112.8</td>
<td>0.49</td>
</tr>
<tr>
<td>p13 terone breadth axilla</td>
<td>305.5</td>
<td>162.2</td>
<td>0.53</td>
</tr>
<tr>
<td>p10 suprasternal height</td>
<td>590.9</td>
<td>309.1</td>
<td>0.52</td>
</tr>
<tr>
<td>p11 lower sternum height</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>p17 rib depth</td>
<td>1.48</td>
<td>0.81</td>
<td>0.55</td>
</tr>
<tr>
<td>p17 rib height</td>
<td>1.48</td>
<td>0.81</td>
<td>0.55</td>
</tr>
<tr>
<td>rib E-modulus (kN/mm²)</td>
<td>13</td>
<td>8</td>
<td>0.62</td>
</tr>
<tr>
<td>mass</td>
<td>mₐ₀ = 19.5; mₐ₁ = 23.1</td>
<td>mₐ₀ = 27.3; mₐ₁ = 3.2</td>
<td>0.14</td>
</tr>
<tr>
<td>impactor area (mm²)</td>
<td>1,1804</td>
<td>5,1803</td>
<td>0.38</td>
</tr>
<tr>
<td>stiffness skin and flesh (kN/m)</td>
<td>281</td>
<td>127</td>
<td>0.56</td>
</tr>
<tr>
<td>rib moment of inertia (mm²)</td>
<td></td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>damping (kN/m/s)</td>
<td>compression cₛ = 0.525</td>
<td>compression cₛ = 0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>force</td>
<td>Rₛ</td>
<td></td>
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</tr>
<tr>
<td>displacement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>response coefficient</td>
<td>displacement (Kneel)</td>
<td>force (Kneel)</td>
<td>displacement (Mertz)</td>
</tr>
</tbody>
</table>
| vₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐ₌ₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐₐ$_$
Figure 22. Resulting response requirements for 18 month child dummy thorax. Above: $m_{\text{imp}} = 2.7 \text{ kg}, v_{\text{imp}} = 4.92 \text{ m/s}$. Under: $m_{\text{imp}} = 3.2 \text{ kg}, v_{\text{imp}} = 7.15 \text{ m/s}$. 
4 Conclusions and Recommendations

- A variation analysis of the Lobdell model shows that the response of the model easily can be adapted by varying one or more model parameters. Each model parameter has a specific effect on the simulated response and therefore are considered to be scaled. Furthermore, the Lobdell model can be simplified by erasing the visco-elastic element because the response satisfies the Kroell corridors anyway, with or without the visco-elastic element.

- Scaling of damping was only possible by presuming equal material properties of child and adult. So far, there is no satisfying explanation for the exact causes of damping of the human thorax. There is a need for a more detailed description of the human thorax, a model in which the contents of the thorax (heart, lungs) are modelled and contribute to a better understanding of the impact behaviour of the thorax.

- It is possible to scale all the other model parameters on geometrical differences between the child and adult thorax. Because of the fact that there are no child cadaver tests done before, it is not clear whether the child thoracic cadaver response is highly overdamped or shows a good resemblance to the adult cadaver thoracic response.

- Scaling ratios of rib geometry ($\gamma_x, \gamma_y$) are approximated by interpolating data of a 6 year old child and a 77 year old female, using chest breath as interpolation parameter. More exact rib geometry dimensions have to be searched for. It is possible that the rib geometry is related to the chest geometry, in that case the rib geometry ratios ($\gamma_x, \gamma_y$) equal the chest geometry ratios ($\lambda_x, \lambda_y$).
Appendix A. MADYMO Inputfile of Lobdell model.

Analysis of Thoracic Impact Response

The Lobdell Model
THU 11 November 1993

0.00 0.300
1 0.0001 0.0001 0.001
0 0.5 0.01 0.1

INERTIAL SPACE
  Inertial space
  PLANES
  -1 0.0 -1.0 0.0 1.0 -1.0 0.0 1.0 0 0 0.0 Inertial space
-999

END INERTIAL SPACE

SYSTEM 1
Thorax
CONFIGURATION
  2 1
-999

GEOMETRY
  -1.0 0.0 0.0 0.0 0.0 0.0 Mass3
  -1.0 0.0 0.0 0.0 0.0 0.0 Mass2
-999

INERTIA
  27.2 0.001 0.001 0.001 0.0 0.0 0.0
  0.45 0.001 0.001 0.001 0.0 0.0 0.0
-999

JOINTS
  1 FREE
  2 FREE
-999

ELLIPSOIDS
  1 0.1 0.1 0.1 0.0 0.0 0.0 4 0 0 0.0 Mass3
  2 0.1 0.1 0.1 0.0 0.0 0.0 4 1 1 0.0 Mass2
-999

FUNCTIONS
  2
  0.0 0.0
  1.0 281E03
-999

INITIAL CONDITIONS
  -1.0 0.0 0.0 0.0 0.0 0.0

END SYSTEM 1

SYSTEM 2
Impactor
CONFIGURATION
  1
-999

GEOMETRY
  -2.5 0.0 0.0 0.0 0.0 0.0 Impactor
-999

INERTIA
  19.5 0.001 0.001 0.001 0.0 0.0 0.0
-999

JOINTS
  1 FREE
-999

ELLIPSOIDS
  1 0.05 0.05 0.05 0.0 0.0 0.0 4 0 0 0.0 Impactor
-999

INITIAL CONDITIONS
  -2.5 0.0 0.0 4.92 0.0 0.0

END SYSTEM 2
FORCE MODELS
CONTACT INTERACTIONS
ELLIPSOID-ELLIPSOID
  1 2 2 1 0 0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
-999
END CONTACT INTERACTIONS
KELVIN ELEMENTS
  1 1 0.0 0.0 0.0 1 2 0.0 0.0 0.0 2 0 0.0 0.0 4 0 0 k23c23
-999
FUNCTIONS
  5
  -0.1 -60.0E02
  -3.18E-02 -8.36E02
  0.0 0.0
  3.18E-02 8.36E02
  0.1 60.0E02
  4
  -1.0E03 5.25E02
  -1.0E-03 5.25E02
  1.0E-03 1.23E03
  1.0E03 1.23E03
-999
MAXWELL ELEMENTS
  1 1 0.0 0.0 0.0 1 2 0.0 0.0 0.0 1 0 0.0 0.0 2 1.0 1.0 kve23cve23
-999
FUNCTIONS
  3
  -1.0 -13.2E03
  0.0 0.0
  1.0 13.2E03
  2
  -100.0 0.18E03
  100.0 0.18E03
-999
END FORCE MODELS
OUTPUT CONTROL PARAMETERS
  0 0 0.01 0 0.0
RELDIS
  1 2 0.0 0.0 0.0 1 1 -1.0 0.0 0.0 0 DisplacementX3X2
-999
FORCES
  2 1 0
  3 2 0
-999
END OUTPUT
END INPUT
Appendix B. Mertz' normalizing method for a simple spring-mass system.

Mertz developed a procedure for normalizing impact response data of lateral cadaver drop tests. He noticed that the force-time response curves show a good resemblance to half a sinewave, therefore the dropped cadaver could be modelled by a simple spring-mass system.

First, force and time scaling ratios were determined by an analysis of the differential equations of motion for a simple spring-mass system with mass $M$ and stiffness $K$:

Displacement $x$, velocity $v$ and acceleration $a$ can be expressed as:

\[
x = V_0 \sqrt{\frac{M}{K}} \sin \left(\sqrt{\frac{K}{M}} t\right)
\]

\[
v = V_0 \cos \left(\sqrt{\frac{K}{M}} t\right)
\]

\[
a = -V_0 \sqrt{\frac{K}{M}} \sin \left(\sqrt{\frac{K}{M}} t\right)
\]

$V_0$ is the impact velocity.

Using Newton's second law, the force can be expressed as:

\[
F = -V_0 \sqrt{KM} \sin \left(\sqrt{\frac{K}{M}} t\right)
\]

The duration of the contact with the surface, half a sinewave, is:

\[
t = \pi \sqrt{\frac{M}{K}}
\]

Secondly, Mertz defined mass and stiffness ratios as:

\[
R_m = \frac{m_s}{m_i}
\]

\[
R_k = \frac{k_s}{k_i}
\]

The subscript $s$ refers to the average mass and stiffness of the used cadavers and subscript $i$ refers to the mass and stiffness of the cadaver which impact response data was normalized.

Substituting Eq.(B.4) into Eq.(B.3) yields:

Normalizing factors for displacement, velocity, acceleration and force are obtained by
\[ R_t = \frac{R_m}{\sqrt{R_k}} \]  \hspace{1cm} (B.5)

substituting Eq.(B.4) and Eq.(B.5) into Eq.(B.1) and Eq.(B.2):

\[ R_x = \frac{R_m}{\sqrt{R_k}} \]
\[ R_v = 1 \]
\[ R_s = \sqrt{\frac{R_k}{R_m}} \]  \hspace{1cm} (B.6)
\[ R_f = \sqrt{R_k R_m} \]
Appendix C. Castiglioni's first theorem.

According to [13], Castiglioni's first theorem can be applied on beams under the following requirements:

1. The beam is of homogeneous material that has the same modulus of elasticity in tension and compression.
2. The curvature of the beam is in the plane of bending and the radius of bending is at least 10 times the depth.
3. The cross-section is uniform.
4. The beam has at least one longitudinal plane of symmetry.
5. All loads and reactions are perpendicular to the axis of the beam and lie in the same plane, which is a longitudinal plane of symmetry.
6. The beam is long in proportion to its depth and is not disproportional wide.
7. The maximum stress does not exceed the proportional limit.

Because ratios are taken, the stiffness of the ribcage will be represented by the stiffness of an ideal curved beam. Requirements (1), (3), (4), (5) and (6) are met.

In [14], Hamilton et al. used data concerning dimensions of the cross-sections of as well the adult’s rib as the 6 years old child’s rib. Since the ratio (radius of curvature/rib depth) is larger than 10 and is supposed to decrease with age, requirement (2) is also performed to.

In reality, the maximum stress will exceed the proportional limit, that is ribfractures will occur. Therefore, only nonfractural cases will be considered. In the Lobdell model, plastic deformation is represented by the non-linear behaviour of spring $k_2$.

The deflection of a curved beam under loading can be found by using the following equations:

$$U_f = \int \frac{M^2 ds}{2EI} \quad (\text{strain energy of flexion})$$

$$y = \frac{\partial U_f}{\partial F} \quad (\text{Castigliano's first theorem})$$

An example of bending of a circular curved beam is presented in Figure 23.

![Figure 23. Bending of a circular curved beam.](image)
\[ D_y = \frac{\partial U}{\partial V}, \quad D_x = \frac{\partial U}{\partial H}, \quad \Theta = \frac{\partial U}{\partial M_0} \]  

\[ M(x) = VR\sin x + HR(1 - \cos x) + M_0 \]  

Disregarding shear and axial stress, and replacing \( ds \) by \( R \, dx \), Eq.(C.1) yields:

\[ U_t = \int_0^\pi \frac{[VR\sin x + HR(1 - \cos x) + M_0]^2 R \, dx}{2EI} \]  

Equation Eq.(C.3) is differentiated first and then integrated.
Appendix D. Stiffness ratio with respect to different boundary conditions.

In this section, factor $\alpha$, which takes into account the different boundary conditions, is determined. The derivation is based on Castigliano’s first theorem and stiffnesses will be calculated using $F=kD$.

![Figure 24. Circular beam, upper end simply supported, lower end fixed and free body diagram.](image)

This configuration is not symmetric and statically undetermined, the reaction forces can be determined from the equations of equilibrium and one known displacement.

\[
\sum F_H = 0 \Rightarrow H_A + H_B = 0
\]

\[
\sum F_V = 0 \Rightarrow V_B = P
\]

\[
\sum M_B = 0 \Rightarrow M_B = H_A 2R
\]

\[
M_x = -PR\sin x + HR\cos x ; \frac{\partial U_x}{\partial H} = 0
\]
\[
\frac{\partial U_f}{\partial H} = \frac{1}{EI} \int_0^\pi [HR(1-\cos x) - PR\sin x \cdot R(1-\cos x)R \, dx \quad \Rightarrow \\
0 = \frac{R^3}{EI} \int_0^\pi [H(1-2\cos x + \cos^2 x) - P(\sin x - \sin x \cos x)] \, dx \quad \Rightarrow \\
0 = \left[ H(\frac{3}{2}x - 2\sin x + \frac{1}{4}\sin 2x) - P(-\cos x + \frac{1}{2}\cos^2 x) \right]_0^\pi \quad \Rightarrow \\
\frac{3\pi}{2}H = 2P \Leftrightarrow H = \frac{4}{3\pi} P
\]

\[
M(x) = \frac{4}{3\pi} PR(1-\cos x) - PR\sin x 
\]

\[
\frac{\partial U}{\partial V} = \frac{i}{EI} \left[ \frac{4}{3\pi} PR(1-\cos x) - PR\sin x \right] R\sin x R \, dx \quad \Rightarrow \\
= \frac{PR^3}{EI} \left[ \frac{4}{3\pi} (\sin x - \cos x \sin x) - (\frac{1}{2} - \frac{1}{2}\cos 2x) \right]_0^\pi \quad \Rightarrow \\
= \frac{PR^3}{EI} \left[ \frac{8}{3\pi} - \frac{\pi}{2} \right] = -0.722 \frac{PR^3}{EI}
\]

Using \( F=kD \), the stiffness for the adult will be:

\[
k_{\text{adult}} = 1.39 \frac{EI}{R^3}
\]

The stiffness of the child is obtained by substituting \( a=b \) into equation Eq.(E.2).

\[
k_{\text{child}} = 0.637 \frac{EI}{R^3}
\]

\[
R_k = \frac{k_{\text{child}}}{k_{\text{adult}}} = \alpha \left( \frac{EI}{R^3} \right)_{\text{child}} \quad ; \quad \alpha = 0.46
\]

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Appendix E. Stiffness ratio with respect to different elliptical cross-sections.

In this section, a scaling factor ($\beta$), which takes into account the different elliptical cross-sections, is determined. The derivation is based on Castigliano’s theorem and stiffnesses will be calculated using $F=kD_v$.

![Elliptical beam, both ends simply supported and free body diagram.](image)

**Figure 25.** Elliptical beam, both ends simply supported and free body diagram.

This configuration is symmetric and statically determined, the reaction forces can be obtained from the equations of equilibrium:

\[
\begin{align*}
\sum F_H &= 0 \Rightarrow H_A + H_B = 0 \\
\sum F_V &= 0 \Rightarrow V_B = P \\
\sum M_B &= 0 \Rightarrow H_A \cdot 2R = 0 \Rightarrow H_A = -H_B = 0
\end{align*}
\]  
(E.1)
\[ R(x) = \frac{ab}{\sqrt{b^2 \sin^2 x + a^2 \cos^2 x}} \]

\[ M(x) = -PR(x)\sin x \]

\[ U_r = 2 \int_0^\pi \left[ \frac{PR(x)\sin x}{2EI} \right] R(x) \, dx \]

\[ D_v = \frac{\partial U_r}{\partial V} = -\frac{\partial U_r}{\partial P} = -\frac{2P}{EI} \int_0^\pi R^3(x) \sin^2 x \, dx \]  \hspace{1cm} (E.2)

If the stiffness ratio is determined directly from Eq.(E.2), the radius of the ribcage will be scaled double (see also D.8). In order to exclude the radius of the elliptical beam, the stiffness of the elliptical beam of the adult has to be divided by the stiffness of the elliptical beam of the adult with adapted child dimensions. Since the radius is scaled by \( \lambda_r \), the y-direction parameter (a) must be kept constant, the x-direction parameter (b) will be adapted. The stiffnesses of the adult and the adapted adult can be obtained by numerically solving Eq.(E.2) and using \( F=KD_v \). The concerning parameters are presented in Table IV.

**Table IV.** Rib stiffnesses for child, adult and adapted child.

<table>
<thead>
<tr>
<th></th>
<th>( a ) (m)</th>
<th>( b ) (m)</th>
<th>( k_{\text{sl}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>8.11E-02</td>
<td>5.64E-02</td>
<td>0.31</td>
</tr>
<tr>
<td>Adult</td>
<td>1.53E-01</td>
<td>1.16E-01</td>
<td>2.3</td>
</tr>
<tr>
<td>Adult:\text{adapted}</td>
<td>1.53E-01</td>
<td>1.06E-01</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Factor \( \beta = 2.3/2.1 = 1.09 \)
Appendix F. Moment of inertia of rib.

In this section, the moment of inertia of the rib, which has an elliptical cross-section, is derived. The geometry of the rib and its cross-section are shown in Figure 26.

\[ I_z = \int x^2 dA \]
\[ \frac{x^2}{b^2} + \frac{z^2}{a^2} = 1 \iff z = \pm \frac{a}{b} \sqrt{b^2 - x^2} \]
\[ I_z = \int_{-b}^{b} x^2 (2z dx) \]
\[ I_z = \int_{-b}^{b} x^2 \frac{2a}{b} \sqrt{b^2 - x^2} dx \Rightarrow I_z = \frac{\pi}{4} ab^3 \] (A.1)

The geometric scaling ratios of the rib are chosen to be:

\[ \gamma_x = \frac{b_{\text{child}}}{b_{\text{adult}}} \] (A.2)
\[ \gamma_z = \frac{a_{\text{child}}}{a_{\text{adult}}} \]

The moment of inertia scaling factor is:

\[ I_z = \gamma_x^3 \gamma_z \] (A.3)
5 References


