Selection of sensors and actuators for an active suspension control problem

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Selection of Sensors and Actuators for an Active Suspension Control Problem

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Abstract

A new method to select sensors and actuators for linear control systems is presented. The key idea is to eliminate candidate sensor/actuator combinations for which a controller achieving a desired level of nominal performance and/or robust stability against unstructured uncertainties cannot be designed. All combinations are subjected to six viability tests, which are necessary conditions for existence of stabilizing controllers meeting a required $\mathcal{H}_\infty$ norm bound. The new selection method is used for an active suspension for a truck. This application illustrates the need for a method dealing with structured uncertainties.

1 Introduction

Preceding controller design, an appropriate number, place, and type of actuators and sensors must be selected. This will be called Input Output (IO) selection, where "input" refers to a manipulated variable and "output" to a measured variable. Compared to modeling and controller design techniques, little attention has been paid to IO selection. Nevertheless, it is of crucial importance. First, the IO set may put fundamental limitations on the system's performance, *e.g.*, it may introduce right-half-plane zeros, which impose restrictions on the achievable bandwidth, regardless of the controller type [4]. Second, the IO set partially determines aspects such as the system's complexity, hardware expenses, and maintenance effort. Due to the combinatorial nature of the IO selection problem, the number of "candidate" IO sets may be huge and favorable ones are easily overlooked. So, an efficient and effective IO selection is desired.

In [14], various IO selection methods are mentioned. Three limitations are commonly encountered. First, IO selection is often restricted to systems with an equal number of inputs and outputs. Second, the controlled and measured variables are not always treated separately; instead, it is frequently assumed that controlled variables can either directly be measured, or suitably be represented by measured variables. Third, quantitative performance specifications and uncertainty characterizations (if employed at all) are usually restricted to one particular frequency (range).

A goal for IO selection is the following: *minimize the number of inputs and outputs, subject to the achievement of a specified Robust Performance (RP) level.* Thus, with the IO set it must be possible to design a controller which stabilizes the system and which meets the performance specifications in the presence of a particular class of uncertainties. Unfortunately, the method presented here is not well-suited to address RP, due to its conservativeness.

The main contribution of this paper is the proposal of an IO selection method for *linear* control systems, that avoids the three limitations mentioned above. It employs criteria based on Nominal Performance (NP) and Robust Stability (RS), which are subproblems necessary for RP. Candidate IO sets for which it is impossible to design a controller that guarantees a specified NP and/or RS level are eliminated. For a manageable number of remaining candidates, the ability to achieve RP could be studied by designing controllers via μ-synthesis.

The paper is structured as follows. After some preliminary aspects regarding $\mathcal{H}_\infty$ control have been summarized, the IO selection is discussed. This is followed by an illustrative application to an active suspension control problem for a tractor-semitrailer. Finally, the IO selection method is evaluated and directions for further research are given. A more extensive treatment of both theory and application is given in [13].

2 The IO Selection Method

Since the basic ideas for the IO selection stem from $\mathcal{H}_\infty$ control theory, Section 2.1 first discusses the standard control system set-up and, second, proposes three desirable closed-loop properties. Section 2.2 provides the actual IO selection tools.

2.1 Desirable Closed-Loop Properties

This paper considers finite dimensional, linear, time-invariant control systems in the "standard" set-up of Fig. 1. The following signals play a role: the manipulated variables u (inputs); the measured variables y (outputs); the controlled variables $z = \binom{q}{z^*}$, which are ideally zero; the exogenous variables $w = \binom{p}{w^*}$, such as reference signals and disturbances (in w^*). The generalized plant G not only includes nominal system data P, but also performance specifications and uncertainty characterizations via the filters V and W. The block $\Delta = \text{diag}(\Delta_u, \Delta_p)$ is assumed stable and serves two purposes. First, it extracts model uncertainties from the nominal plant (Δ_u). Second, it transforms performance specifications into stability specifications (Δ_p). In the sequel, V and W are chosen such that $\|\Delta\|_\infty \leq 1/\gamma$, with γ a positive scalar. For a more detailed discussion on control problem formulations in the framework of Fig. 1, see, *e.g.*, [15].

Based on the small gain theorem, conditions for three de-

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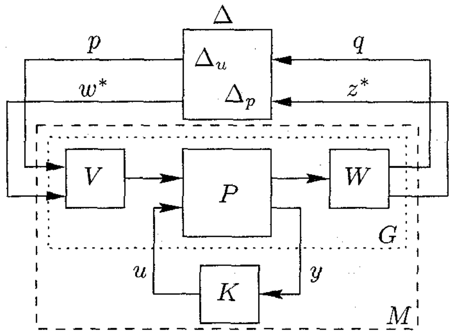

P	:	nominal plant model
G	:	generalized plant
K	:	controller
$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$	:	generalized closed-loop
Δ	:	scaled uncertainty block
V, W	:	design filters

Figure 1: Standard control system set-up

sirable closed-loop properties will be given. The motivation is to test if IO sets can achieve these properties. Essentially, the small gain theorem states, that the interconnection of two stable operators remains stable if the product of the operator gains is smaller than unity [15, Section 9.2]:

> *Necessary and sufficient condition for stability under unstructured* Δ*:* Consider Fig. 1 with Δ and M stable. Stability of the closed-loop under *all unstructured* blocks Δ with $\|\Delta\|_\infty \leq 1/\gamma$ is achieved if and only if $\|M\|_\infty < \gamma$.

In case performance is not specified ($M = M_{11}$, $\Delta_p = 0$), the above gives a necessary and sufficient condition for Robust Stability (RS) against all unstructured uncertainties Δ_u. Alternatively, a necessary and sufficient condition for Nominal Performance (NP) results if uncertainties do not play a role ($M = M_{22}$, $\Delta_u = 0$). If Robust Performance (RP) is the objective (M, $\Delta_u \neq 0$, $\Delta_p \neq 0$), the block Δ is structured, even if Δ_u and Δ_p are unstructured, and the condition is only *sufficient* to guarantee RP. In analogy, the condition for RS becomes only sufficient if Δ_u itself is structured. Hence, due to the inability of the small gain theorem to account for structure in Δ, the closed-loop analysis may yield conservative results. This can be circumvented by the introduction of the so-called structured singular value μ defined in [11]:

> *Necessary and sufficient condition for stability under structured* Δ*:* Consider Fig. 1 with Δ and M stable. Stability of the closed-loop under *all structured* blocks Δ with $\|\Delta\|_\infty \leq 1/\gamma$ is achieved if and only if $\|M\|_\mu < \gamma$, with $\|M\|_\mu := \sup_\omega \mu_\Delta\{M(j\omega)\}$.

Since $\|M\|_\mu$ incorporates knowledge on the block-diagonal structure of Δ, it displays less conservatism than $\|M\|_\infty$ ($\|M\|_\mu \leq \|M\|_\infty$). See [11] or [15, Chapter 11] for more information on μ. Controller design aimed at restricting or minimizing $\|M\|_\mu$ is called μ-synthesis. Unfortunately, $\mathcal{H}_\infty$ controller design and the IO selection method are *not* able to account for structure in Δ.

2.2 Viability Conditions for IO Sets

The generalized plant G in Fig. 1 can be represented in the following state-space form:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u, \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^n$ the state vector and the inputs and outputs as defined previously: $w \in \mathbb{R}^{n_w}$, $u \in \mathbb{R}^{n_u}$, $z \in \mathbb{R}^{n_z}$, and $y \in \mathbb{R}^{n_y}$. $\mathcal{H}_\infty$ controller design for (1) aims at computing an asymptotically stabilizing controller achieving $\|M\|_\infty < \gamma$. A state-space parametrization of such controllers is provided in [5] and for $D_{11} = 0$, $D_{22} = 0$ also in [3]. In the development of this controller parametrization, six assumptions on (1) are made, which must be satisfied for $\mathcal{H}_\infty$ controller design *and* for the IO selection:

1,2. (A, B_2) is stabilizable and (C_2, A) detectable,

3,4. D_{12} has full column rank and D_{21} has full row rank,

5,6. $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank $\forall\ \omega$ and $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank $\forall\ \omega$.

For the $\mathcal{H}_\infty$ controller parameterization, u and y are scaled and unitary transformations on w and z are performed; this is also done preceding the IO selection. Without loss of generality, these manipulations are such, that

$$D_{12} = \begin{bmatrix} 0_{(n_z - n_u) \times n_u} \\ I_{n_u} \end{bmatrix}, \tag{2}$$

$$D_{21} = \begin{bmatrix} 0_{n_y \times (n_w - n_y)} & I_{n_y} \end{bmatrix}, \tag{3}$$

$$D_{11} = \begin{bmatrix} D_{11_{11}} & D_{11_{12}} \\ D_{11_{21}} & D_{11_{22}} \end{bmatrix}, \tag{4}$$

with

$$\begin{aligned} &D_{11_{11}} \in \mathbb{R}^{(n_z - n_u) \times (n_w - n_y)}, && D_{11_{12}} \in \mathbb{R}^{(n_z - n_u) \times n_y}, \\ &D_{11_{21}} \in \mathbb{R}^{n_u \times (n_w - n_y)}, && D_{11_{22}} \in \mathbb{R}^{n_u \times n_y}. \end{aligned}$$

For more details on these transformations, see, *e.g.*, [15, Section 17.2]. To construct a stabilizing controller achieving $\|M\|_\infty < \gamma$, two Riccati equations must be solved, see [5] for their documentation. One of them is related with a state-feedback problem ("state-feedback Riccati"), the other with an observer problem ("observer Riccati"). The following lays the foundation for the IO selection method [3, 5]:

There exists a stabilizing controller such that $\|M\|_\infty < \gamma$ *if and only if the following six conditions hold:*

1. $\max\{\overline{\sigma}(D_{11_{11}}, D_{11_{12}}), \overline{\sigma}(D_{11_{11}}^T, D_{11_{21}}^T)\} < \gamma$,

2,3. The two Hamiltonians associated with the state-feedback Riccati and the observer Riccati do not have $j\omega$-axis eigenvalues. Moreover, $\mathrm{Im}\begin{pmatrix} 0_{n \times n} \\ I_n \end{pmatrix}$ is complementary to each Hamiltonian's subspace corresponding to the stable eigenvalues.

4,5. The solutions X_∞ and Y_∞ to the state-feedback and observer Riccati respectively are positive semi-definite: $X_\infty \geq 0$ and $Y_\infty \geq 0$

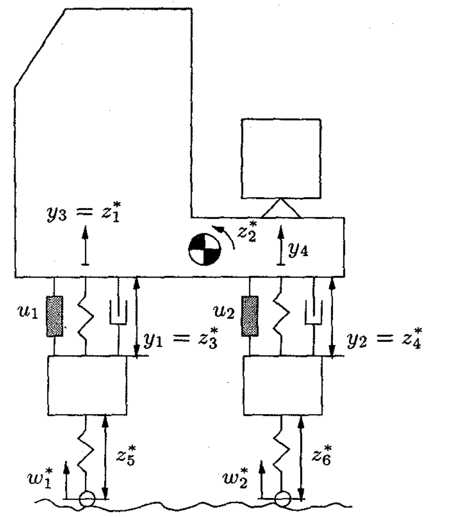

Figure 2: 4 DOF tractor-semitrailer combination

6. The spectral radius of the product $X_\infty Y_\infty$ is smaller than γ^2: $\rho(X_\infty Y_\infty) < \gamma^2$.

These six conditions will be referred to as the "viability conditions" for IO selection.

The key idea of the IO selection is as follows. First, the requirements are expressed by the design filters V, W, so the problem of computing a stabilizing controller achieving $\|M\|_\infty < \gamma$ makes sense. Second, *all* candidate IO sets are subjected to the viability conditions; as soon as one fails, the rest need not be checked. In this way, the time-consuming process of controller design and closed-loop evaluation for each individual IO set is circumvented. The viability conditions are implemented in MATLAB, for which programs from the μ-Toolbox [1] (such as **hinfsyn** and functions called therein) serve as the basis.

Though the "ultimate" design goal is RP, the IO selection is not able to address RP *non-conservatively*. Nevertheless, it is useful for initial screening of IO sets in the following way. To start with, all candidates are tested for their ability to achieve NP, *i.e.*, for an unstructured block Δ_p, the ability to achieve $\|M_{11}\|_\infty < \gamma$ is tested. Next, all remaining IO sets can be checked for their ability to achieve RS against each *individual* unstructured diagonal block Δ_{u_i} of Δ_u, *i.e.*, $\|M_{22_i}\|_\infty < \gamma$ is tested, with M_{22_i} the i-th diagonal block of M_{22} corresponding to Δ_{u_i}. Since NP and RS against each Δ_{u_i} are necessary for RP, the remaining IO sets are still candidates for RP. If their number is manageable, it could be decided to design controllers via μ-synthesis as an ultimate viability test.

3 Active Suspension Control Problem

IO selection is investigated for an active suspension applied in the 4 Degree-Of-Freedom (DOF) model of the vehicle in Fig. 2. Upon request, a MATLAB file generating this model can be obtained from the first author. Two actuators (u_1, u_2) placed between the axles and the tractor chassis are proposed as candidate inputs, while measurements of the suspension deflections (y_1, y_2) and the chassis accelerations (y_3, y_4) are suggested as candidate outputs. This yields 45 candidate IO sets, among which the 4×2 full IO set $y_1 y_2 y_3 y_4 / u_1 u_2$ and eight 1×1 IO sets. Next, the performance specifications and uncertainty characterizations are discussed.

3.1 Performance Specifications

The exogenous input w^* contains the excitation by the road surface (w_1^*, w_2^*) and measurement noises ($w_3^*, \ldots, w_6^*$). Assuming the road input to be lowpass-filtered white noise, the corresponding shaping filter in V is chosen as:

$$V_{1,2} = \frac{v_0}{s/\omega_0 + 1}, \tag{5}$$

see also Fig. 3. For a fair motorway and a forward vehicle speed of 25 m/s, $\omega_0 = 2\pi \cdot 0.25$ rad/s and $v_0 = 8.0 \cdot 10^{-3}$ are representative choices [2].

The measurements are assumed to be disturbed with zero-mean, white noises with an intensity of θ times the measurements' magnitude. For the road surface modeled by (5), this results in the following shaping filters in V:

$$V_{3,4} = 2.6 \cdot 10^{-3}\theta \qquad \text{(suspension deflections)}, \tag{6}$$

$$V_{5,6} = 150 \cdot V_{3,4} \qquad \text{(chassis accelerations)}. \tag{7}$$

Parameter θ can be interpreted as the error fraction in the measured variables. Here $\theta = 0.1$, which is relatively large to clearly see the effect of measurement noise ("y-noise").

Four main design goals are distinguished [8]. First, good driver comfort must be guaranteed (also cargo comfort, but this is not considered here). The tractor's vertical acceleration at the front z_1^* and the rotational acceleration z_2^* quantitatively represent driver comfort. Though the limitation of the accelerations' power spectra due to the stochastic road surface actually involves limiting the $\mathcal{H}_2$ norm [8], this goal must as well as possible be transformed in the $\mathcal{H}_\infty$ norm setting by the choice of suitable filters in W. These are based on human sensitivity plots for vertical and *horizontal* accelerations provided in [8]. However, since the driver's horizontal acceleration can be approximated by a constant times z_2^*, the horizontal sensitivity is used to represent rotational sensitivity. The sensitivity contour for z_1^* is approximated by the magnitude of W_1:

$$W_1 = \rho_1 {\omega_1}^2 \frac{s/\omega_2 + w_{1_0}}{s^2 + 2\zeta\omega_1 s + {\omega_1}^2}, \tag{8}$$

with $w_{1_0} = 0.4$, $\zeta = 1$, $\omega_1 = 2\pi \cdot 10$ rad/s, and $\omega_2 = 2\pi \cdot 5$ rad/s. Via ρ_1, the attenuation of the most crucial accelerations can be specified; here, $\rho_1 = 10$. The sensitivity contour for z_2^* is approximated by the magnitude of W_2:

$$W_2 = \rho_2 \frac{w_{2_0}}{s/\omega_3 + 1}, \tag{9}$$

with w_{2_0}=1, $\omega_3 = 2\pi \cdot 2$ rad/s, and $\rho_2 = 10$. The acceleration weights in (8) and (9) are also depicted in Fig. 3.

The second and third design goal are limiting the suspension deflections (due to space limitations) and the tire deflections (for good handling and minimum road surface damage) respectively. Since for these design goals one is actually interested in restricting the $\mathcal{L}_1$ norm, suitable weights in the $\mathcal{H}_\infty$ norm setting are hard to give. Here, the front and rear weights are chosen equal and constant:

$$W_{3,4} = \rho_3 \qquad \text{(suspension deflections)}, \tag{10}$$

$$W_{5,6} = \rho_4 \qquad \text{(tire deflections)}. \tag{11}$$

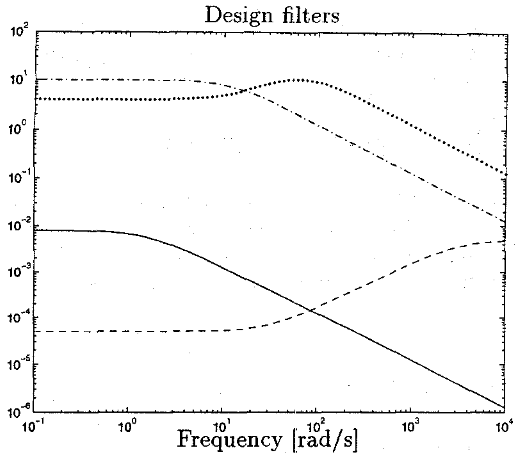

Figure 3: $V_{1,2}$ (-), W_1 ($\cdots$), W_2 (-·), and $W_{7,8}$ (--)

Though the suspension and tire deflection limits will normally not be exceeded for stochastic road surfaces [8], this might happen for deterministic ones. To account for deterministic surfaces, the weights are chosen rather large: $\rho_3 = \rho_4 = 250$.

Finally, weights for the inputs u ("u-weights") must be formulated. In general, the bandwidth of actuators is limited and high-frequency inputs cannot be realized. Therefore, the following bi-proper weighting filter for $z_7^* = u_1$ and $z_8^* = u_2$ is used (see also Fig. 3):

$$W_{7,8} = \rho_5 \frac{s/\omega_4 + 1}{s/\omega_5 + 1}. \tag{12}$$

It is assumed that the bandwidth of the actuators is 5 Hz, *i.e.*, $\omega_4 = 2\pi \cdot 5$ rad/s. Furthermore, it is decided to set $\omega_5 = 100\,\omega_4$ ("$\omega_5 \gg \omega_4$") and $\rho_5 = 5 \cdot 10^{-5}$ (so u-weights are neither negligible nor dominant).

3.2 Uncertainties

In the 4 DOF vehicle model, various uncertain parameters play a role. In this paper, only uncertainties in the spring and damper parameters are accounted for:

- k_{tf}, k_{tr}: front and rear tire stiffnesses,
- k_{sf}, k_{sr}: front and rear suspension stiffnesses,
- b_{sf}, b_{sr}: front and rear suspension dampings.

Suppose a is the nominal value of an uncertain parameter a'. The relation $a' = a(1+w_a\delta_a)$, $\|\delta_a\|_\infty \leq 1$ is used to model the uncertainties. The δ_a's for the uncertain springs and dampers together build the uncertainty block, resulting in a 6×6 *structured*, diagonal Δ_u; the w_a's express the relative amount of uncertainty and are included in W.

With respect to the uncertainty model, an important remark is made. Though in practice the uncertainty in springs and dampers is *real parametric*, both the IO selection and the controller design in this paper account for *complex* uncertainty and hence for a larger class of uncertainties, introducing conservativeness. So, w_a is better interpreted as a specification for the maximally occurring *complex* uncertainty in the real nominal parameter a. For the truck this means, that parametric *and* dynamic uncertainties in springs and dampers are taken into account.

Table 1: Optimal γ's for typical IO sets under different control objectives; u-weights (z_7^* and/or z_8^*) included

IO set	Outputs	Inputs	$z_1^*,\ z_2^*$	$z_3^*,\dots,z_6^*$	$z_1^*,\dots,z_6^*$
1	$y_1\,y_2\,y_3\,y_4$	$u_1\,u_2$	0.67	0.26	0.84
2	$y_1\,y_2\,y_3\,y_4$	u_1	1.06	0.65	1.07
3	$y_1\,y_2\,y_3\,y_4$	u_2	0.93	0.51	1.02
4	$y_1\,y_2$	$u_1\,u_2$	0.76	0.26	0.86
5	$y_3\,y_4$	$u_1\,u_2$	0.67	0.27	0.84
6	y_1	$u_1\,u_2$	0.81	0.57	0.86
7	y_2	$u_1\,u_2$	1.11	0.48	1.21
8	y_3	$u_1\,u_2$	0.67	0.64	0.84
9	y_4	$u_1\,u_2$	1.03	0.51	1.14

4 IO Selection Results

This section starts with the results for the NP-based IO selection, followed by a discussion on RS-based IO selection. Finally, μ-synthesis aimed at RP is performed for IO sets remaining from the NP-based IO selection. The MATLAB μ-Toolbox [1] is employed for controller design.

4.1 NP-Based IO Selection

Preceding the IO selection for all 45 candidates, $\mathcal{H}_\infty$ optimization is performed for nine "typical" IO sets. The aim is to gain insight in the importance of the candidate actuators and sensors. Moreover, the influence of the controlled variables z^* is studied via three cases: 1) z^* only includes the accelerations z_1^*, z_2^* and the inputs u (z_7^*, z_8^*), 2) z^* consists of the suspension and tire deflections $z_3^*,\dots,z_6^*$ and the inputs u, and 3) z^* includes all eight controlled variables. The γ values in Table 1 represent the best achievable NP levels.

A first observation from Table 1 is, that control of the accelerations is dominant over control of the suspension and tire deflections, since the best achievable γ's are larger for acceleration control.

Second, NP significantly deteriorates if only one actuator is used (compare IO sets 1, 2, 3). For the overall control problem, IO sets 2 and 3 employing one actuator are nonviable; IO set 2 based on u_1 is even nonviable if displacements $z_3^*,\dots,z_6^*$ are discarded. Increasing the u-weights, the advantage of using two actuators over one becomes smaller. For the overall control problem with large input weights ($\rho_5 > 3 \cdot 10^{-2}$), $\gamma = 1.24$ even equals the $\mathcal{H}_\infty$ norm of the *uncontrolled* system, irrespective of the input set. The third observation from Table 1 is, that NP is more easily met with the rear actuator u_2 than with the front one u_1.

It also appears, that measurements which best match the control goals are preferable, which is also felt intuitively (compare IO sets 1, 4, 5). If accelerations z_1^*, z_2^* are the focus, acceleration measurements y_3y_4 are preferred to displacement measurements y_1y_2 and they give the same NP level as $y_1y_2y_3y_4$. In analogy, if control is focussed on displacements $z_3^*,\dots,z_6^*$, output set y_1y_2 gives slightly better results than y_3y_4, while it is equally best as $y_1y_2y_3y_4$. For the overall control problem, acceleration measurements are preferred to displacement measurements.

Table 2: IO sets viable for NP with results for optimal controller design; *all* IO sets employ u_1 and u_2

IO set	Outputs	$\|M\|_\infty$ for NP	$\|M\|_{\tilde{\mu}}$ for RP
1	y_1	0.86	1.02
2	y_3	0.84	0.99
3	$y_1\, y_2$	0.86	1.01
4	$y_1\, y_3$	0.84	0.99
5	$y_1\, y_4$	0.85	1.00
6	$y_2\, y_3$	0.84	0.99
7	$y_3\, y_4$	0.84	0.99
8	$y_1\, y_2\, y_3$	0.84	0.99
9	$y_1\, y_2\, y_4$	0.85	0.99
10	$y_1\, y_3\, y_4$	0.84	0.99
11	$y_2\, y_3\, y_4$	0.84	0.99
12	$y_1\, y_2\, y_3\, y_4$	0.84	0.99

Another observation from Table 1 is, that control with two sensors in general gives better results than control with one sensor (compare IO sets $4, \ldots, 9$). Exceptions are IO sets 6 and 8, which are equally best as IO sets 4 and 5 respectively, in case of the overall control problem. On the other hand, discarding displacement measurements $y_1 y_2$ from the full IO set does not degrade performance if acceleration control z_1^*, z_2^* or the overall problem is the focus, while discarding acceleration measurements $y_3 y_4$ does not degrade performance in case of displacement control $z_3^*, \ldots, z_6^*$. It was also observed, that IO sets 1, 4, 5, 6, and 8 achieve the same $\gamma = 0.84$ if the measurement noise is negligibly small.

Two last observations are the following (compare IO sets $6, \ldots, 9$). In case of z_1^*, z_2^*, measurements at the front (y_1, y_3) give better results than measurements at the rear (y_2, y_4), due to the fact that the front acceleration z_1^* is more difficult to control in the crucial frequency region than the rotational acceleration z_2^* [13]. Front measurements also give the best results for $z_1^*, \ldots, z_6^*$. Contrary to this, rear measurements are preferred in case of $z_3^*, \ldots, z_6^*$. Note, that IO sets 7 and 9 employing only one rear measurement are nonviable for the overall NP problem.

Next, for the overall control problem all 45 candidate IO sets are subjected to the viability tests with $\gamma = 1$. The 12 accepted IO sets are listed in Table 2. As could be foreseen from Table 1 and the discussion above, all 30 IO sets based on one actuator are eliminated, as well as y_2/u_1u_2 and y_4/u_1u_2 employing one rear measurement. In addition, y_2y_4/u_1u_2 is nonviable (lowest $\gamma = 1.02$). Notice, that the viable IO sets achieve approximately the same *optimal* NP levels between 0.84 and 0.86. Based on the IO selection goal stated in the Introduction, y_1/u_1u_2 or y_3/u_1u_2 could be used for control if NP is the objective.

4.2 RS-Based IO Selection

As noted in Section 2.2, the IO selection is not well-suited to deal with *structured* blocks Δ. Therefore, IO selection based on RS against the uncertainties proposed in Section 3.2 is not directly possible. As an alternative, the six unstructured diagonal entries of Δ_u are considered one at a time, since RS against each *individual* uncertainty is *necessary* for RS against their combination.

It is emphasized, that the RS problem *in itself* is somewhat artificial for the tractor-semitrailer, since the *uncontrolled* system will remain stable for all spring and damper uncertainties occurring in practice. More specific, RS is achieved with $K(s) = 0$ if the spring and damper parameters are time-invariant and positive. So, RS is certainly achieved with $K(s) = 0$ for all time-invariant *real* δ_a, $\|\delta_a\|_\infty \leq 1$ and $w_a < 1$. However, since the IO selection accounts for time-invariant *complex* uncertainties (see also Section 3.2), a controller might already be needed for $w_a < 1$.

For each complex uncertainty, w_a values can be computed below which the uncontrolled system is guaranteed to be stable. The resulting values are: $w_{k_{tf}} = 0.82$, $w_{k_{tr}} = 0.58$, $w_{k_{sf}} = 0.78$, $w_{k_{sr}} = 0.67$, $w_{b_{sf}} = w_{b_{sr}} = 1.00$. Increasing these values with 0.10, controllers are needed for RS against one uncertainty and RS-based IO selection becomes "useful." It then appears, that IO sets 3, 8, 9, and 12 in Table 2 achieve RS against each *individual* uncertainty of the considered magnitude. Note, that only IO sets employing both y_1 and y_2 are viable, indicating the need to perform measurements which are physically "close" to the uncertainty sources.

To investigate the viability of these four IO sets with respect to RS against the *combination* of uncertainties, controllers are designed via μ-synthesis: if a stabilizing controller achieving $\|M\|_\mu < 1$ exists, the IO set is viable. In fact, the *upper bound* "$\|M\|_{\tilde{\mu}}$" for $\|M\|_\mu$ is computed (see Section 5), so IO sets yielding $\|M\|_{\tilde{\mu}} > 1$ may still be viable. Computational experimentation has shown, that $\|M\|_{\tilde{\mu}}$ and $\|M\|_\mu$ are in most cases almost equal [11].

The μ-synthesis is performed with the so-called *D-K* iteration ("approximate μ-synthesis," see, *e.g.*, [15, Chapter 11]). To reduce numerical problems, `sysbal` was used to balance P, V, W, and G successively. The tolerance in the $\mathcal{H}_\infty$ optimization of *D-K* iteration was limited to 10^{-2}, to avoid numerical problems which may occur if the optimum is approached too closely. The function `musynflp` was used to generate the D-scale approximations (third order or less), because `musynfit` seemed to be less reliable. The *D-K* iteration was stopped if the consecutive μ-plots are close and if the reduction of consecutive $\|M\|_{\tilde{\mu}}$'s was no more than 10^{-2}, which is the γ-iteration tolerance for the involved $\mathcal{H}_\infty$ optimization. Though only RS is considered, negligibly small u-weights ($\rho_5 = 5 \cdot 10^{-7}$) and y-noise ($\theta = 10^{-6}$) make up Δ_p to meet assumptions 3 and 4 in Section 2.2.

It appeared, that IO set 3 in Table 2 may be nonviable with respect to RS ($\|M\|_{\tilde{\mu}} = 1.52$), while IO sets 8 ($\|M\|_{\tilde{\mu}} = 0.93$), 9 ($\|M\|_{\tilde{\mu}} = 0.93$), and 12 ($\|M\|_{\tilde{\mu}} = 0.92$) are viable. Apparently, at least one acceleration must be measured.

4.3 RP-Based IO Selection

Though RS in itself might be artificial for the considered application, RP is not: due to uncertainties, the performance may degrade significantly and the *controlled* system is not necessarily stable. This underlines the need for an IO selection dealing with structured Δ, see also Section 5.

As ultimate viability test for the 12 IO sets achieving NP, controllers are designed via μ-synthesis, using second order D-scale approximations constructed with `musynflp`. Of course, this "brute force" approach is undesirable for a large number of IO sets. The previous NP specifications are used and $w_a = 0.04$ for all six uncertainties. This small w_a

may not be very realistic, but for larger values ($w_a \geq 0.05$) the overall IO set is nonviable in combination with the NP requirements; hence, so are all other IO sets, since eliminating sensors or actuators will never improve control.

From Table 2 it is observed, that nine IO sets are viable for RP and that the $\|M\|_{\bar{\mu}}$ values are very close and between 0.99 and 1.02. Among the viable IO sets, y_3/u_1u_2 employs the minimum number of inputs and outputs and so is preferable based on the suggested IO selection goal.

5 Discussion and Future Directions

A new approach for IO selection was studied, that eliminates candidate IO sets for which a stabilizing controller achieving a specified level of NP and/or RS cannot be designed. The main shortcoming is the inability to account for *structured* uncertainty and thus RP. Our future research will therefore be aimed at developing an *effective* approach dealing with structured Δ's.

One option relies on the μ upper bound (assuming square complex diagonal blocks in Δ only for notational ease):

$$\mu_\Delta\{M(j\omega)\} \leq \inf_{D(j\omega)} \bar{\sigma}\{D(j\omega)M(j\omega)D^{-1}(j\omega)\}, \quad (13)$$

with $D(j\omega)$ a diagonal, frequency-dependent scaling matrix. The bound in (13) is also employed in D-K iteration: for a given closed-loop M, D is optimized and replaced by a rational approximation $\hat{D}$, followed by an $\mathcal{H}_\infty$ optimization $\min_K \|\hat{D}M(G,K)\hat{D}^{-1}\|_\infty$. Preceding the IO selection, optimal D-K iteration is performed for the *full* IO set. Since IO sets which are "as good" as the full IO set yield "the same" optimal M and corresponding D-scale, the key idea for IO selection is to check if a stabilizing controller can be designed achieving $\|\hat{D}M\hat{D}^{-1}\|_\infty < \gamma$, where the rational $\hat{D}$ follows from the final D-K iteration step for the *full* IO set. Again, the six viability conditions are checked for each IO set's original G *extended* with D-scales $\hat{D}$, $\hat{D}^{-1}$. It is emphasized, that $\hat{D}$ might only be optimal for the full IO set, since for each IO set:

$$\min_K \inf_D \|DMD^{-1}\|_\infty \leq \min_K \|\hat{D}M\hat{D}^{-1}\|_\infty. \quad (14)$$

Thus, this IO selection is based on *sufficiency* and viable IO sets may be eliminated.

In [9], another IO selection accounting for structured Δ is proposed. It is based on *necessity*, due to dropping the requirement of the controller being stabilizing. In [12], this method and the one based on D-scales are discussed and applied to an active suspension control problem with uncertain semitrailer weight.

In the current implementation, all candidate IO sets are subjected to the viability conditions. This is not necessary, since IO sets made up of sensors and actuators from a larger but nonviable IO set are also nonviable. Using this notion, efficiency can be considerably improved by starting with the full IO set and only studying smaller IO sets which are subsets of larger viable IO sets ("subset implementation"). The complementary "superset implementation" could also be used, by starting with 1×1 IO sets and employing the notion that adding sensors or actuators to a small viable IO set yields larger viable IO sets.

In [7], necessary and sufficient conditions for the existence of an $\mathcal{H}_\infty$ controller are given in terms of three Linear Matrix Inequalities (LMI's). Future research must reveal if using these LMI's for IO selection instead of the six viability conditions is advantageous, *e.g.*, with respect to efficiency and numerical reliability. The LMI-based IO selection would again only account for unstructured Δ's.

Finally, it will be investigated to which extent the IO selection method can be applied successfully to nonlinear systems for *linearization(s)* in a (sequence of) interesting operating point(s). The results of such an approach must be carefully interpreted, because crucial features of the original nonlinear system can be lost. Also, $\mathcal{H}_\infty$ theory has been extended to *nonlinear* system descriptions (see, *e.g.*, [6]) and therefore it will be investigated if associated controller existence conditions (preferably "global" ones as in [10]) can be used for IO selection.

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