Fracture analysis of dual mass flywheel arc springs

Citation for published version (APA):

Document status and date:
Published: 01/01/2002

Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
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2002.78

Fracture Analysis of Dual Mass Flywheel Arc Springs

Christophe De Metsenaere
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Preface

Due to the enormous development of internal combustion engines the output torques of new generation engines are increasing rapidly. This leads to vibrations, which are in contrast with the demand for more comfort in the vehicle. Therefore LuK GmbH&Co has invented the Dual Mass Fly-wheel (DMFW), which uses a flywheel consisting of two parts connected with weak arc springs. For these springs there is a trade-off between performance and durability, weak springs lead to better isolation but have to endure more impacts due to the decrease in capacity. For better starting behaviour the two-stage spring is used which has a bad durability in lifetime tests. To understand this problem a model is made to simulate arc spring behaviour. With this model some insight is acknowledged to perform experiments, with which the fail mechanism is defined. With these results the springs can be redesigned to improve the performance.
1 Theory

1.1 Theory DMFW

With each new generation cars are getting faster, but also heavier by the growing amount of safety constructions and luxury devices. To improve acceleration the engines have to become more powerful, but the economy should be equal or better. Therefore engines with more torque are designed, but larger torque also leads to a larger irregularity. Especially for diesel engines the new developments have been of such a magnitude that engine torque has doubled within ten years. Engine irregularities lead to vibrations, and thereby the car is less comfortable. To understand this phenomenon we have a closer look at the vehicle drive train. Schematically we have a vehicle model like this:

![Vehicle model diagram]

Simplified to a mass-spring model we get the following system:

![Vibration model]

This system has two important eigenmodes:

Mode 1 Surging

Mode 2 Rattle

In mode 1, the surging mode, the car is in resonance with the engine, these coach-work vibrations are typically in the range of 2-10Hz. This type of vibration occurs when the driver has chosen a higher gear than acceptable, resulting in a low engine speed. After a tip-in with a too high gear, the driver hears and feels a boom.

Mode 2 is typical for vibrations within the 40-80Hz range. This gear resonance occurs at higher engine speeds and can be heard as noise.
Conventional systems use torsion dampers, but the eigenfrequencies are still in the engine's drive range. To solve this the friction can be increased, leading to better isolation at the eigenfrequency, but worse at higher frequencies. The best performance that can be accomplished is an irregularity equal to the engine.

By decreasing the spring rate it is possible to increase isolation. The reverse of this is that the springs are weaker and thus have less capacity. This means the isolation at higher engine torque is worse, and the spring will endure more impacts.

Arc springs are the solution to this problem while they utilise the whole perimeter of the clutch. This means the spring can be longer which enables a lower spring rate. Besides that the friction increases with larger engine torque leading to a higher resistance for impacts. The influence of the DMFW is shown in the figure below.

In the figure above it resembles that there isn't any eigenfrequency. It's still present, not in the drive range anymore, but below the idle engine speed. Besides the spring rate also the transmission's inertia influences the eigenfrequency. Of course no vehicle manufacturer would like to increase the mass and dimensions of its transmission. Therefore the flywheel
mass is split in a primary and a secondary one. Below a comparison is made between a 
standard clutch and the DMFW principle.

<table>
<thead>
<tr>
<th>Model</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine + Flywheel</td>
<td>Engine</td>
</tr>
<tr>
<td>Transmission</td>
<td>Transmission</td>
</tr>
<tr>
<td>Vehicle</td>
<td></td>
</tr>
<tr>
<td>Clutch Disc</td>
<td>Clutch Disc</td>
</tr>
</tbody>
</table>

Besides the benefits mentioned above, better isolation of drive train noise, the DMFW has 
more advantages. While the transmission endures smaller vibration amplitudes it has a 
longer lifetime, and the construction can be more economical. The vibration amplitudes for 
the engine have increased by the DMFW, but the primary flywheel inertia has decreased. 
This smaller inertia leads to smaller torsion and bending forces for the crankshaft, which of 
course enlarges the engine lifetime. The influence of the DMFW on the crankshaft forces 
is shown below.

![Graph showing the comparison between conventional and DMFW systems](image-url)
1.2  Arc springs

As mentioned above the isolation of the vibrations is mainly influenced by two elements, the division of the inertia and the spring characteristic of the DMFW. With the division of the inertia some margin is left, but of course there are guidelines for this. The DMFW spring characteristic (appendix I) is mainly defined by the arc spring characteristic (appendix 2).

This characteristic has a certain starting angle, which is in this case zero, and a block angle, at which the spring is at the end of its capacity and behaves as a rigid body. Besides that the spring characteristic involves friction, which can be seen in the diagram as the energy dissipation. A low spring rate is best for isolation, but it is limited by the length of the spring. A compromise should be made between good driving isolation and following aspects.

With growing engine torque of modern cars the capacity of the spring should be increased, but the installation size cannot enlarge. Thereby the springs need to have a higher spring rate, while otherwise the block angle is reached in normal drive range. These impacts lead to unwanted effects in operation, like noise and shocks.

A larger influence on the spring rate involves the starting behaviour. As stiffness is introduced in the drive train, problems occur with eigenfrequencies. As we have seen before, the eigenfrequency of the secondary inertia is still there, but moved under idle engine speed. Modern high torque diesel engines have powerful combustions; combined with a 4-cylinder irregularity this can lead to starting problems. If the DMFW combines a low spring rate with low friction, low power starter engines cannot speed up the engine fast enough to break through the eigenfrequency of the secondary inertia. The difference between a good and bad starting behaviour is shown below.
As can be seen in the left figure a good starting behaviour involves a quick speed up till idle engine speed is accomplished. In the right one the engine and transmission are stuck in a resonance, and idle speed isn’t reached.

A quick speed up of the engine requires a stiff spring because the eigenfrequency is outside the starting range, but for good isolation a very weak spring is preferable. High friction is also preferred while the DMFW than acts as a rigid connection during start, but means bad isolation.

By the design of the DMFW the friction increases when the displacement angle goes up, this can be seen in the spring characteristic. To provide extra friction during start a friction control plate is invented. This plate increases the friction when acceleration changes sign. A small angle displacement is possible while some play is present; this eliminates the extra friction while driving with constant speed.

While the displacement angles at start and at impact are high it is good to have a high stiffness at large displacement. On the other hand in normal driving a low stiffness provides a better isolation. To combine these two the double stage spring is developed, an example of the series type characteristic is shown below.

![Spring Characteristic](image)

With this characteristic it is possible to have a good isolation and a good starting behaviour, several physical possibilities are shown in appendix 3. In this report the double stage parallel spring is accented. The spring characteristic can be found in appendix 4.
1.2 Modelling

To understand arc spring behaviour it is necessary to have a model for simulation. The forces acting on an arc spring can be visualised:

For one coil the forces acting on it look like this:

With:

- \( F_i \) = Spring force [N]
- \( \psi_i \) = Coil angle [°]
- \( \psi_i^* \) = Coil angle at last position [°]
- \( F_R \) = Friction force [N]
- \( F_{Fi} \) = Centrifugal force [N]
- \( N_i \) = Normal force [N]
- \( \mu \) = Friction coefficient
- \( m_w \) = Coil mass [kg]
- \( r \) = Working radius [m]
- \( \omega \) = Angular velocity [1/s]
For load we have following formulas:

\[
\begin{align*}
\psi_i &= \psi_i^* - \frac{F_{i+1} \cdot r}{c_w} \\
(F_{i+1} - F_i) \cdot \cos \frac{\psi_i}{2} &= F_{R,i} \\
(F_{i+1} + F_i) \cdot \sin \frac{\psi_i}{2} + F_{fl} &= N_i \\
F_{R,i} &= \mu \cdot N_i \\
F_{fl} &= m_w \cdot r \cdot \omega^2
\end{align*}
\]

\[\Rightarrow F_i = \frac{\left(\cos \frac{\psi_i}{2} - \mu \cdot \sin \frac{\psi_i}{2}\right) \cdot F_{i+1} - \mu \cdot F_{fl}}{\left(\cos \frac{\psi_i}{2} + \mu \cdot \sin \frac{\psi_i}{2}\right)}\]

And for unload the friction force has of course changed sign:

\[
\begin{align*}
\psi_i &= \psi_i^* - \frac{F_{i+1} \cdot r}{c_w} \\
(F_i - F_{i+1}) \cdot \cos \frac{\psi_i}{2} &= F_{R,i} \\
(F_i + F_{i+1}) \cdot \sin \frac{\psi_i}{2} + F_{fl} &= N_i \\
F_{R,i} &= \mu \cdot N_i \\
F_{fl} &= m_w \cdot r \cdot \omega^2
\end{align*}
\]

\[\Rightarrow F_i = \frac{\left(\cos \frac{\psi_i}{2} + \mu \cdot \sin \frac{\psi_i}{2}\right) \cdot F_{i+1} + \mu \cdot F_{fl}}{\left(\cos \frac{\psi_i}{2} - \mu \cdot \sin \frac{\psi_i}{2}\right)}\]

With Matlab it is possible to compose a program that simulates a characteristic. When this characteristic is compared to a measured one it can be seen that the model is realistic. The Matlab script for the following spring characteristic can be found in appendix 5.
2 Research description

Two-stage screw threat coils have shorter life spans than comparable one-stage. Goal is to define the causes of the fractures that occur during life tests.

3 Problem description

For the design of the coils that are used in the DMFW (Dual Mass Fly-wheel), one needs to know the maximal stress that a coil can resist during its lifetime. These data can be obtained from Wöhler diagrams, in which the stress is shown as a function of the load cycles. The results of the screw thread coils life test is shown below:

When the data for two-stage coils are compared to one-stage, the first have less load cycles before fracture than the second with the same stress. Besides that it can be easily seen that the data is very staggered. To obtain a balanced guideline for design, the boundaries of the Wöhler-diagram have to be moved to a lower stress level as can be seen below:
The maximum design stress is lowered by 200N/mm², which means the springs have to be stiffer, resulting in a worse isolation of the engine vibrations.

When the causes for the fractures can be identified, a more detailed guideline can be produced which can lead to higher design stresses, meaning a higher performance for the DMFW.
4 Hypothesis

Several hypotheses exist for the fractures in two stage arc springs. The reason for the difference in durability should be fatigue, while the wear of arc springs is limited. The springs are shot-peened and hardened, resulting in a minor wear, which is proven in practice. A repeating load at a low stress rate causes fatigue. In case of compression spring there's only shear present. In a Smith-diagram the shear can be visualised. In this case the load is dynamical expanding, meaning the stress starts at zero and increases till a maximum is reached. This can be seen in figure the below as the black trajectory.

There are two options for shear increase, an increase in the compression direction, and one in tension direction. The first one is visualised by the blue curve, and the second one by the green curve. Of course a combination of these two is also possible. The possible mechanisms for shear increase are listed below.

4.1 Tension

Force transfer: Tension can only occur with influence from the outer spring, because the innerspring is connected at one side. Therefore there should be a force from the outer spring onto the inner spring.

Friction increase: To conduct this force a friction should be present, which is caused by the normal force of the spring, in combination with centrifugal force. Because the geometry of the contact surface is not flat this friction force can be higher than expected. In extreme cases this phenomenon can result in a form lock-up.
4.2 Compression

Blocked coils: For compression the same mechanisms as for tension are present. Besides these the shear increase is caused by a blocked condition of the inner spring windings. If the forces on a winding are large enough it is pressed together until the coils are in contact and conduct the force without increase of resistance.

4.3 Test matrix

To ease comparison a test matrix is made, in which the results of the tests of all hypothesis are compared.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Lifetime tests</th>
<th>Measurements</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Test</td>
<td>Test</td>
</tr>
<tr>
<td>Tension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Force transfer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Friction increase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Force transfer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Friction increase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Blocked coils</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5 Tests

To test the hypothesis several tests stands are present, the used ones are listed below. Details of the springs can be found in appendix 4.

5.1 Lifetime test

The first test method to be performed is the already mentioned lifetime test. With this test stand four springs together are swung by a given torque path or angle path (for details see appendix 6). While the torque path is better controllable this one is preferred.

To obtain this torque path it is necessary to perform a static trajectory test, which gives the displacement of the two-stage spring as a function of the input torque. This test gives the relation between angle and torque for a hysteresis loop, with which it is possible to define the torque range for the measurements (see appendix 2). The torque range that is to be used is defined by the breakpoints in the hysteresis loop. To test the hypothesis mentioned in chapter 3 following tests are carried out.

1. The first test is carried out with the complete spring set. Here we test the spring by its full range to compare these results to the ones of test 2. In this case the possibility for tension is present and this should influence the results.

2. To distinguish the tension and compression hypothesis a test is performed with the complete spring set. The spring set is tested in 2\textsuperscript{nd} stage to exclude the secondary hypothesis, while in this case the spaces between the windings are smaller, so form lock-up is avoided.

3. To research the influence of the springs on each other, the inner spring is tested alone. If other elements are involved that don’t fit the hypothesis this should be visible in the test results. Besides that we have to be sure the springs on it’s own have a longer lifetime than in a set.

The details of the tests are shown in the matrix below:

<table>
<thead>
<tr>
<th>Test</th>
<th>Stage</th>
<th>Spring</th>
<th>Torque Range [Nm]</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1+2</td>
<td>is+os</td>
<td>10 – 269</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>is+os</td>
<td>43 – 269</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1+2</td>
<td>is</td>
<td>10 – 98</td>
<td>$T&gt;T_{\text{practice}}$</td>
</tr>
</tbody>
</table>

The springs are checked for fractures with an interval of 50,000 load changes. If a fracture occurs a new one replaces the spring, lifetime and number and places of the fractures are noted down. The data of the test can be plotted in a Wöhler diagram. The normal stress set on the y-axis is defined with the formulas described.
in appendix 7. With this stress the spring lifetimes can be plotted in the Wöhler diagram below.

![Wöhler Diagram for inner springs](image)

From the diagram it can be seen that the lifetime of the springs in test 3 is about two times longer than in test 1 and 2. This means the lifetime of an innerspring alone is twice the lifetime of an innerspring in a spring set. As noted in the problem description the durability of 2-stage springs is worse than for normal springs. The difference between test 1 and 2 is very small, and therefore we can conclude that the durability is not influenced by tension forces.

### 5.2 Electron Microscope

To identify the fractures that occur in the springs, the surfaces are examined with an electron microscope. With this microscope the following fracture surface appears:
The fracture started at the inside of the spring corresponding to the yellow arrow, and extended in radial direction into the wire.

Here the yellow arrows correspond to the line indicated in the previous figure. This fracture arose as a result of the present shallow axial lines, which are caused by the wire production process. This extended axial line is showed enlarged in the following figure. The green arrows indicate the continuation of the fracture from the axial line.

The penetration of the axial line is about 100μm below the fracture surface.

The fracture image described above is typical for fatigue; the axial lines caused by manufacturing starts a fracture that enlarges with increasing load cycles. The fracture line decreases the area of force conduction, resulting in a higher stress, which causes an impact fracture.

The position of the fracture initiation can be declared from the stress distribution in the wire:

By the difference in arc length between the inside and the outside of the wire the stress at the inside is greater than the stress at the outside, causing the wire to break at the inside.
5.3 Stress

Within LuK a file exists, based on excel, to simulate dynamic arc spring behaviour. This file uses the static equilibrium of forces of each coil, as shown in the theory in chapter 1.3. Originally this file was designed to compute outer spring behaviour, especially partial loops but it can also be used for simplified inner spring models.

The contact geometry is neglected; the model only uses the spring-cup contact, which is a good model for outer spring simulation. With the file it is possible to compute the displacement of each coil of the spring. This can be done for several relevant torque and friction coefficients. This gives the results shown below. These results should define the probability of the blocked coils hypothesis, and give insight in the stress distribution of the springs.

The test in paragraph 4.2 gives the displacement of each coil separately, for different torque and friction coefficients. If the displacement is set out as a function of the winding the following plot can be created.

As can be seen in the diagram, when the torque and matching displacement angle are large enough, some coils are blocked. This occurs at the front of the spring, while the force at the first springs is larger because of the friction in the spring, see theory chapter 1.3. We can also see that in this situation with a friction coefficient of 0.15 a block situation for a coil can't occur. This is while the block angle can't be reached as a result of the block situation of the outer spring.

The block angle $\alpha_{\text{block}}$ is 21.4° as can be computed from the spring data in appendix 4 and the torque needed for this displacement is 88.8 Nm. From the theory in appendix 7 we can see that the stress is a linear function of the displacement angle $\phi$. If we know the displacement for each coil, it is possible to compute its stress. While the stress at block situation is known it is easy to define the stress of the windings:

$$\sigma_i = \frac{\sigma_{\text{block}}}{\phi_i} \cdot \phi_i$$

With $\sigma_{\text{block}}=927$ N/mm² and $\phi_i,\text{block}=21.4°$.
With this relation the stress in the coils can be computed, this gives the following diagram:

The stress in the coils has a maximum of 1200 N/mm² when a winding is in blocked situation. At the front side where the load is placed the stress is higher than at the end, while the force present in the coils decreases due to the friction.

The maximum stress is 927 N/mm², this can be concluded from the foregoing diagrams. This is true for a system with friction coefficient 0.15. But what happens when the friction coefficient is larger? As can be seen in the theory the geometry of the contact surface of the spring results in a larger friction. The results for five different friction coefficients are shown below.
Apparently the stress can be higher than 927 N/mm². A higher friction coefficient results in a higher stress at the end coils. Although the mean stress is the same the maximum stress is larger.

5.4 Measurements

With the method above it is possible to compute stresses in single springs. To compute stresses in 2-stage arc springs the situation is more difficult. In principle the computation of 2-stage springs is as easy as the super position of both single springs. The sum of both forces is equal to the force of a 2-stage arc spring set. This would be the case if no contact between inner and outer spring were present. The influence of the friction between inner and outer spring will appear if the measurements of both are super positioned and compared to the measured characteristic of the set.

These two characteristics are equal for first stage, and until 107° in the second stage. For the second part of second stage we can see that there is an increase in torque. With a closer look we can see the difference at block is about 40Nm.
This difference in torque can be explained by two elements:

1. If we take a look at the relative displacement of inner and outer spring coils on a test stand we can see there is a sign change in mid 2nd stage. This is caused by the difference in relax angle of inner and outer spring, when the 2nd stage starts the outer spring is already loaded. Below the relative displacement is visualised. For part 1 the change in coil angle is larger for inner than for outer spring coils. At part 2 it's the other way round. Physically we can imagine this in the following way:
   - In part 1 the inner spring has a resistance from the outer spring, the friction force works in negative direction. Overall the inner spring coils are relatively moving in compression direction.
   - In part 2 the inner spring is pressed by the outer spring, this is caused by the blocking effect that the outer spring has to protect the set from overload. By this effect the coils are turned off one by one, leading to a quick relative movement of the outer spring in compression direction. The friction force now works in positive direction, which presses the inner spring onto the impact stop.

2. When the displacement angle is equal to the block angle the coil angles for inner and outer spring are very similar, by this effect the coils have a sort of form lock, creating a large friction.

The complete phenomenon of stress increase can be described by the following. When the spring set is loaded it has a point in 2nd stage where the overall relative coil displacement of the inner spring changes from positive to negative. This means that in part 1 the inner spring coils are relatively moving forward, and in part 2 relatively backward. While friction always has the opposite sign of the displacement.
direction the friction is in part 1 negative and in part 2 positive. Thereby in part 1 the coil force is decreased by friction and in part 2 increased.

The spring wires are winded in opposite direction, but at maximum load this difference is decreased, leading to a geometry that enables a kind of form lock. The friction force is increased, leading to a large transfer of coil force from the outer spring into the inner spring. The impact stop at the free end and the connection at the trumpet end are the boundaries for the spring, and thereby the form lock is limited.

The form lock area is a resistance for the coils at the trumpet side. When the outer spring blocking area moves on forward, the front inner spring coils have a displacement in compression direction, pushing against the form lock resistance. This leads to a stress increase at the trumpet side, and by the construction of this trumpet there is a weak spot where fractures first occur.

For the free end coils the relative displacement of the outer spring pushes the form locked region, causing an extra force on these free end coils. This extra force can be equal to the friction force of the form locked region at most. Schematically the system looks like this:

When in the end of the 2nd stage the outer spring blocking occurs, the blue area is enlarged, resulting in a relative displacement of these coils. Because the right red region has still some displacement left it is pressed against the form locked grey coils. The grey region on his turn presses the left red coils, leading to a higher stress.

The division of the stress can only be visualised qualitative because it has many Degrees Of Freedom.

The spring has two maximums, one at the trumpet side and one at the free end. The place where these two maximums occur cannot be precisely predicted; this should be possible with the simulation model explained in chapter 5.6. The maximum stress that can occur in the spring is equal to the normal stress plus the stress caused by the friction force input of the outer spring. The magnitude of the stress increase is therefore equal to the friction coefficient between inner and outer spring. With single springs the stress decreases with the friction coefficient, which means it decreases with 15%, because the friction coefficient \( \mu \) is equal to 0.15.
Here we have an increase of about 40 Nm maximum at block angle, with a theoretical end torque of 179.1 Nm as can be seen in the spring data appendix 4. This means the friction coefficient between inner and outer spring at that point is equal to 0.22. Thus the stress of the inner spring increases with about 22% due to the increase in friction combined with the friction force sign change.

With this in mind we can compute the stress of the inner spring in praxis, with an increase of 22% the spring has a stress increase of 204 N/mm², resulting in a stress of 1131 N/mm². If we combine this result with the one from the durability tests this leads to the following points in the Wöhler diagram.

These results correspond to the ones for standard springs made from the same material.

### 5.5 Geometry

The displacement of the spring coils is different for each coil, and for each load case. This leads to a difference in geometry, which causes difference in friction factors. To understand the influence of the geometry with CAD software a measurement is made under conditions of equal coil distances for inner and outer spring. Appendix 8 shows that the influence of equal distances is large; the inner spring coils seriously hook up with outer spring coils although they are opposite winded.
5.6 Simulation

To understand the influence of the forces that are present in the arc spring coils it is significant to simulate the load and unload behaviour. Therefore the following model is made for one coil:

The forces working on inner and outer spring coil become:

Forces on outer spring coil:
- $F_{OS,i-1}^{OS}$ = Interaction force outer spring $i \leftrightarrow i-1$
- $F_{OS,i}^{OS}$ = Interaction force outer spring $i \leftrightarrow i+1$
- $F_{fr}^{OS}$ = Centrifugal force outer spring coil $i$
- $N_{OS,i}^{OS}$ = Normal force outer spring coil $i$
- $N_{IS,i}^{IS}$ = Normal force inner spring coil $i$
- $R_{OS,i}^{OS}$ = Friction force outer spring coil $i$
- $R_{IS,i}^{IS}$ = Friction force inner spring coil $i$

Forces on inner spring coil:
- $F_{IS,j-1}^{IS}$ = Interaction force inner spring $i \leftrightarrow i-1$
- $F_{IS,i}^{IS}$ = Interaction force inner spring $i \leftrightarrow i+1$
- $F_{fr}^{IS}$ = Centrifugal force inner spring coil $i$
- $N_{IS,i}^{IS}$ = Normal force inner spring coil $i$
- $R_{IS,i}^{IS}$ = Friction force inner spring coil $i$
These forces have the following relationships:

\[
\begin{align*}
F_{i}^{OS} &= F_{i-1}^{OS} + R_{i}^{OS} + R_{i}^{IS} \\
F_{i}^{IS} &= F_{i-1}^{IS} + F_{i}^{IS} \\
N_{i}^{IS+OS} &= N_{i}^{OS} + N_{i}^{IS}
\end{align*}
\]

According to chapter 1.3 The forces described above can be expanded:

\[
\begin{align*}
N_{i}^{i} &= (F_{i-1}^{IS} + F_{i}^{IS}) \cdot \sin\left(\frac{\phi_{i}^{IS}}{2}\right) + F_{i}^{IS} \\
N_{i}^{IS+OS} &= (F_{i-1}^{OS} + F_{i}^{OS}) \cdot \sin\left(\frac{\phi_{i}^{OS}}{2}\right) + N_{i}^{IS} + F_{i}^{OS} \\
R_{i}^{IS} &= (F_{i-1}^{IS} - F_{i}^{IS}) \cdot \cos\left(\frac{\phi_{i}^{IS}}{2}\right) \\
R_{i}^{OS} &= (F_{i-1}^{OS} - F_{i}^{OS}) \cdot \cos\left(\frac{\phi_{i}^{OS}}{2}\right) \\
R_{i}^{IS} &= f \cdot N_{i}^{IS} \cdot \text{sign}(v_{i}^{IS} - v_{i}^{OS}) \\
R_{i}^{OS} &= \mu \cdot N_{i}^{IS} \cdot \text{sign}(v_{i}^{OS}) \\
F_{i} &= m_{w} \cdot r \cdot \sigma^{2}
\end{align*}
\]

This leads to an interaction force for inner and outer spring:

\[
F_{i}^{IS} = \frac{\left(\cos\left(\frac{\phi_{i}^{IS}}{2}\right) - f \cdot \text{sign}(v_{i}^{IS} - v_{i}^{OS}) \cdot \sin\left(\frac{\phi_{i}^{IS}}{2}\right)\right) \cdot F_{i-1}^{IS} - f \cdot \text{sign}(v_{i}^{IS} - v_{i}^{OS}) \cdot F_{i}^{IS}}{\left(\cos\left(\frac{\phi_{i}^{IS}}{2}\right) + f \cdot \text{sign}(v_{i}^{IS} - v_{i}^{OS}) \cdot \sin\left(\frac{\phi_{i}^{IS}}{2}\right)\right)}
\]

\[
F_{i}^{OS} = \frac{\left(\cos\left(\frac{\phi_{i}^{OS}}{2}\right) - \mu \cdot \text{sign}(v_{i}^{OS}) \cdot \sin\left(\frac{\phi_{i}^{OS}}{2}\right)\right) \cdot F_{i-1}^{OS} - \mu \cdot \text{sign}(v_{i}^{OS}) \cdot F_{i}^{OS} - (f \cdot \text{sign}(v_{i}^{OS} - v_{i}^{IS}) + \mu \cdot \text{sign}(v_{i}^{OS}) \cdot N_{i}^{IS}}{\left(\cos\left(\frac{\phi_{i}^{OS}}{2}\right) + \mu \cdot \text{sign}(v_{i}^{OS}) \cdot \sin\left(\frac{\phi_{i}^{OS}}{2}\right)\right)}
\]

With:

- \( OS, IS \) = outer respectively inner spring
- \( i, i-1 \) = coil i respectively coil i-1
- \( \psi \) = coil angle [°]
- \( \mu \) = friction coefficient outer spring – cup [-]
- \( f \) = friction coefficient inner spring – outer spring [-]
- \( v \) = relative coil velocity

positive/negative velocity is respectively compression/tension.

The change of the friction factor f can be computed but is a simplification of a 3D model, and presumably it needs some empirical correction. This is beyond the purpose of this research.
5.7 Test matrix

The results of the tests can be qualified in the already mentioned test matrix. A green color means a conformation of the theory, a red one a rejection. Yellow means nothing can be said about the theory.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Lifetime tests</th>
<th>Measurements</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
<td>Test 3</td>
</tr>
<tr>
<td>Tension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Force transfer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Friction increase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Force transfer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Friction increase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Blocked coils</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The result of the matrix is clear, we should have an increase in friction, together with a force transfer. The theory should be quantified to make guidelines for design, which can be done by the development of a simulation as explained in chapter 5.6.
6 Conclusion

The Wöhler diagram shows that the durability for double stage springs is lower than for single stage ones. This must be caused by an increase in stress by a rise in tension or compression forces, while wear has not occurred. From the results of durability test 1 and 2 we can conclude that there is no difference between testing in 2nd stage and testing in the complete range. This means the influence of tension forces can be neglected in the durability tests. We can conclude that the mechanism that causes the fractures is an increase of shear in compression direction.

A comparison of a measured characteristic with a super positioned one shows the effect that causes the stress increase. At the end of the 2nd stage the stiffness increases as a result of larger friction. This larger friction is caused by the geometry of the springs; comparable coil distances lead to a blocked region where the position of the coils is maintained. Therefore the spring capacity is decreased and the coils that are moveable absorb the input torque. In combination with the friction direction change this leads to an increase in stress for the coils at the trumpet and at the free end.

The stress can increase until it has reached the block stress, 1200 N/mm². The springs are designed to endure 927 N/mm², but have an increase which magnitude is influenced by the friction factor. Because the difference between design stress and operation stress is about 200 N/mm² larger than the difference for one-stage springs, the two stage springs have a shorter lifetime.
Appendix 3 Different springs

1. Normal arc spring

2. Series springs
   - 2-stage characteristic
   - less friction

3. 1-Stage Parallel springs
   - more capacity

4. 2-Stage Parallel springs
   - more capacity
   - 2-stage characteristic
   - more friction

5. Damping springs
   - Better impact behaviour
%simulation of a spring characteristic
%praktikum 2st. bogenfedern

%%%clear all existing variables, memory
\texttt{clear all}

%%%constants
\texttt{r=0.1145; \%radius [m]}
\texttt{m=0.00264; \%coil mass [kg]}
\texttt{ifed=50; \%number of coils [-]}
\texttt{c=3.64; \%spring stiffness [Nm/°]}
\texttt{WO=120.6; \%relaxed spring angle [°]}
\texttt{Wbis=89.1; \%inner spring angle at block [°]}
\texttt{Wbos=96.2; \%outer spring angle at block [°]}
\texttt{sigmab=927; \%inner spring stress at block [N/mm²]}
\texttt{beta=107}
\texttt{mu=0.15; \%friction coefficient [-]}
\texttt{M=150; \%Torque on one spring [Nm]}
\texttt{rpm=0; \%RPM [1/min]}

%%%conversion of constants/variables
\texttt{psi0=(WO*2*pi)/(360*ifed); \%relaxed coil angle [rad]}
\texttt{psib=(Wbis*2*pi)/(360*ifed); \%block coil angle [rad]}
\texttt{F0=M/r; \%spring force [N]}
\texttt{omega=2*pi*rpm/60; \%speed [rad/s]}
\texttt{Ff=m*r*omega^2; \%centrifugal force [N]}
\texttt{cwrad=ifed*360*c/(2*pi); \%spring stiffness per coil [Nm/rad]}

%%%computation of coilforces
\texttt{a=50; \%Computation number}

%%%vectors for collecting variables
LuK GmbH & Co.
de Metsenaere Christophe / EZV

\( F = [1] \);  \%force vector
\( n = [1] \);  \%number vector
\( \psi = [1] \);  \%angle vector
\( \text{Fr} = [1] \);  \%friction force vector
\( N = [1] \);  \%normal force vector
\( \text{sumphi} = [1] \);  \%sum phi
\( K = [1] \);  \%Torque vector

%load loop
for \( k = [0:\text{F0/a:F0}] \)
  %begin conditions load
  \( \psi_i = \psi_0 \);  \%begin angle
  \( \text{Fi} = k \);  \%begin force
  \( \text{sumphi} = 0 \);  \%sum coildisplacement

  for \( i = [1:1:\text{ifed}] \)
    \( \psi_i = \psi_0 - \text{Fi} \times r / \text{cwrad} \);  \%physical boundary for coil angle
    if \( \psi_i \geq \psi_b \)
      \( \psi_i = \psi_i \);
    else
      \( \psi_i = \psi_b \);
    end
    \( \text{Fpi} = \text{Fi} \);  \%\( \text{Fpi} = \text{previous force (Fi-1)} \)
    \( \text{Fi} = ((\cos(\psi_i/2) - \mu \times \sin(\psi_i/2)) \times \text{Fi} - \mu \times \text{Ff}) / (\cos(\psi_i/2) + \mu \times \sin(\psi_i/2)) \);  
    \( \text{Fri} = (\text{Fpi} - \text{Fi}) \times \cos(\psi_i/2) \);  \%friction force computation
    \( \text{Ni} = (\text{Fpi} + \text{Fi}) \times \sin(\psi_i/2) + \text{Ff} \);  \%normal force computation

  end
  \( \text{F} = [F, \text{Fi}] \);
  \( \text{n} = [n, i] \);
  \( \psi = [\psi, \psi_i] \);
  \( \text{Fr} = [\text{Fr}, \text{Fri}] \);
  \( \text{N} = [N, \text{Ni}] \);
  \( \text{sumphi} = \text{sumphi} + \psi_i \);  \%spring displacement=sum coil displacement
end

\text{Progress_Load} = \frac{k}{\text{F0}} \times 100  \%Progress
% unload loop
for k=[F0:-F0/a:0]

% begin conditions unload
psii=psi0;   % psii begin unload=psii load
Fi=k;
sumphii=0;
% computation loop for unload
for i=[1:1:ifed]
    psii=psi0-Fi*r/cwrad;   % physical boundary for coil angle
    if psii>=psib
        psii=psii;
    else
        psii=psib;
    end
    Fpi=Fi;   % Fpi=previous force (Fi-1)
    Fi=(((cos(psii/2)+mu*sin(psii/2))*Fi+mu*Ff)/(cos(psii/2)-mu*sin(psii/2)));
    Fri=(Fi-Fpi)*cos(psii/2);   % friction force computation
    Ni=(Fpi+Fi)*sin(psii/2)+Ff;   % normal force computation

% collecting coil variables in vectors
[F]=[F,Fi];
[n]=[n,i];
[psi]=[psi,psii];
[Fr]=[Fr,Fri];
[N]=[N,Ni];
sumphi=sumphi+psi;  % spring displacement = sum coil
end

Progress_Unload=100-k/FO*100  % Progress

[K]=[K,k];

sumphi=[sumphi,sumphi];
end

F;
n;
psi;
Fr;
N;

Torque=K*r;

angle=sumphi*360/(2*pi);

plot(angle,Torque)
xlabel('Angle [°]')
ylabel('Torque[Nm]')
Title('Inner spring characteristic')
grid
Appendix 6 Durability test

Test rig: Arc spring test rig  Test rig – No.: 30

Test rig schematic:

Technical data:
- swing angle: 0 – 120°
- frequency: 0 – 10 Hz
- torque: 0 – 1200 Nm
- number of springs: max. 4

Applicability

Part 1: Part 1 is for arc springs with a one stage linear characteristic. The arc springs may comprise an inner spring. The inner spring is not allowed to be fixed to the outer spring.

Test conditions

Part 1: The tests are carried out torque controlled between a low of 10 Nm and a high of 1.5 x $T_{th}$ (theoretical stop torque, see drawing). Springs being part of a set have to be tested as a set between a low of 10 Nm and a high of 1.5 x $T_{th}$ (outer spring) + 1.0 x $T_{th}$ (inner spring).

4 or 8 arc springs have to be tested until $1.0 \times 10^6$ cycles. Visual checks for breakage are carried out at about 200,000; 500,000; 800,000 and $10^6$ cycles.

Testing frequency is 6 Hz.
Applicability

Part 2:

Part 2 is for the springs listed in the following table.

<table>
<thead>
<tr>
<th>Sort</th>
<th>example</th>
<th>low torque for one spring [Nm]</th>
<th>high torque for one spring [Nm]</th>
<th>spring position</th>
<th>pressure piece at lever arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>spring with two stage characteristic (with shorter inner spring)</td>
<td>9 961 042 02</td>
<td>10</td>
<td>To be determined from graphs, see figure 1. All windings have to blocked. The high torque has to be at least higher than 2 x Tn (OS) + 1 x Tn (IS)</td>
<td>standard</td>
<td></td>
</tr>
<tr>
<td>one stage damping spring</td>
<td>9 961 060 00</td>
<td>10</td>
<td>1.5 x Tcontact (OS) + 1 x Tn (IS) (contact torque, see drawing)</td>
<td></td>
<td>standard</td>
</tr>
<tr>
<td>two stage damping spring</td>
<td>9 961 088 01</td>
<td>10</td>
<td>2.0 x Tcontact (OS) + 1 x Tn (IS) (contact torque, see drawing)</td>
<td>The fixation area between outer and inner spring is at the lever arm</td>
<td>standard</td>
</tr>
<tr>
<td>arc spring with coast segment, one or two stage</td>
<td>9 961 017 16 9 961 088 00</td>
<td>10</td>
<td>To be determined from graphs, see figure 1. All windings have to blocked. The high torque has to be at least higher than 2 x Tn (OS) + 1 x Tn (IS)</td>
<td>coast segment at lever arm</td>
<td>Nose pressure piece, is specific for every spring (see figure 2)</td>
</tr>
<tr>
<td>one stage damping spring with coast segment</td>
<td>9 961 088 03</td>
<td>10</td>
<td>2.0 x Tcontact (OS) + 1 x Tn (IS) (contact torque, see drawing)</td>
<td>coast segment at lever arm</td>
<td>Nose pressure piece, is specific for every spring (see figure 2)</td>
</tr>
<tr>
<td>two stage damping spring with coast segment</td>
<td>9 961 088 02</td>
<td>10</td>
<td>2.0 x Tcontact (OS) + 1 x Tn (IS) (contact torque, see drawing)</td>
<td>coast segment at lever arm</td>
<td>Nose pressure piece, is specific for every spring (see figure 2)</td>
</tr>
<tr>
<td>series connected arc spring</td>
<td>9 961 101 00</td>
<td>10</td>
<td>1.0 x Tcontact (set)</td>
<td>drive spring at lever arm, fixation area in the middle between the springs</td>
<td>standard</td>
</tr>
</tbody>
</table>
Figure 1: Determiniation of the high torque for a two stage characteristic.

Figure 2: Drive spring for the lever arm at coast springs

Test criteria:
- breakage of arc spring (average life = 600,000 load cycles)
- function
- set less
The following conditions must be met:

=> Test: 4 springs are cycled up to $10^6$ lines.

- min. requirements: no breakage below 200,000 cycles.

- In case 3-4 specimen show no breakage: OK.

- In case 1-2 specimen show no breakage: Test has to be repeated. Further testing of 4 springs

- In case of zero specimen show no breakage: N.G.

There is a total of 8 results after the repetition.

- In case at least 4 specimen show no breakage: OK.

=> set losses: The relative loss must not exceed 1% after $10^6$ cycles. This has to be checked especially for load set springs. For heat set springs this limit was never reached. Therefore a check is not necessary.

Test time:

- 3 days for 4 springs

<table>
<thead>
<tr>
<th>Index</th>
<th>Modification-No.</th>
<th>Modification</th>
<th>Date</th>
<th>Modified by</th>
<th>Checked</th>
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<tr>
<td>a</td>
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<td>23.09.1992</td>
<td>A. Gillmann</td>
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<td>b</td>
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<td>10.11.1994/ hb</td>
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<td>P. Piesch</td>
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<td>23.02.1996/ hb</td>
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<td>Kooy</td>
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<tr>
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<td>27.08.1998</td>
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<tr>
<td>e</td>
<td>converted-into-WORD</td>
<td></td>
<td>12.10.1998/ cg</td>
<td></td>
<td>C. Ziche</td>
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<tr>
<td>f</td>
<td>modified</td>
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<td>03.09.1999/ cg</td>
<td></td>
<td>Kooy</td>
</tr>
</tbody>
</table>
8 Berechnungsgleichungen

8.1 Federarbeits
\[ W = \frac{1}{2} F \cdot s \]  
\[ (2) \]

8.2 Federkraft
\[ F = \frac{G \cdot d^4 \cdot s}{3 \cdot D^3 \cdot n} \]  
\[ (3) \]

8.3 Federweg
\[ s = \frac{G}{d^2} \cdot \frac{B^3 \cdot n}{F} \]  
\[ (4) \]

8.4 Fedarate
\[ R = \frac{G}{B^2} \cdot \frac{d^4}{D^3 \cdot n} \]  
\[ (5) \]

8.5 Schubspannungen
\[ \tau = \frac{G}{\pi} \cdot \frac{d^2}{D^3 \cdot n} \cdot F \]  
\[ \tau = \frac{G}{\pi} \cdot \frac{d}{n} \cdot \frac{1}{D^3} \cdot F \]  
\[ (6) \]
\[ (7) \]

\[ \frac{1}{D^3} \cdot \frac{1}{\pi} \]

Während die Schubspannung \( \tau \) für die Auslegung statisch bzw. quasistatisch beanspruchter Federn heranzuziehen ist, gilt die korrigierte Schubspannung \( \tau_{\text{corr}} \) für dynamisch beanspruchte Federn.

8.6 Draht- oder Stabdurchmesser
\[ d = \sqrt{\frac{F}{\pi \cdot \tau_{\text{corr}}}} \]  
\[ (8) \]

Die zulässige Schubspannung \( \tau_{\text{zul}} \) ist entsprechend dem vorliegenden Konstruktionsfall festzulegen (siehe auch Abschnitt 9).

8.7 Anzahl der wirksamen Windungen
\[ n = \frac{G \cdot d^4 \cdot s}{3 \cdot D^3 \cdot F} \]  
\[ (9) \]

8.8 Gesamtanzahl der Windungen

Die Anzahl der erforderlichen, nicht wirksamen Endwindungen hängt von der Ausführung der Federenden und dem Herstellungsverfahren ab. Man benötigt nach DIN 2095 für kaltgeformte Federn 2 und nach DIN 2096 Teil 1 und Teil 2 für warmgeformte Federn 1,8 Windungen.

Die Gesamtanzahl der Windungen beträgt für
- kaltgeformte Federn: \( n_k = n + 2 \)  
  \[ (10) \]
- warmgeformte Federn: \( n_w = n + 1,6 \)  
  \[ (11) \]

8.9 Mindestabstand zwischen den wirksamen Windungen

Bei der kleinsten, zulässigen Federlänge \( L_n = L_n + S_n \) soll die Summe der lichten Mindestabstände zwischen den einzelnen wirksamen Windungen für
- kaltgeformte Federn nach DIN 2095: \( S_n = 0,005 \cdot \frac{d}{d} + 0,1 \cdot d \)  
  \[ (12) \]
- warmgeformte Federn nach DIN 2098: \( S_n = 0,02 \cdot \left( \frac{d}{d} \right) + n \)  
  \[ (13) \]

betragen.

Bei dynamischer Beanspruchung der Federn ist der \( S_n \)-Wert bei warmgeformten Federn zu verdoppeln, bei kaltgeformten Federn muß er das 1,5fache betragen.
Appendix 8 Geometry analysis