Highest-Order Coupler Points of Watt-1 Linkages, Tracing Symmetrical 6-Bar Curves

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Abstract
Certain non-symmetrical Watt-1 linkage mechanisms may produce symmetrical curves. The dimensions of such mechanisms then have to meet five precise conditions. They have been derived from the two possibilities that symmetrize the Assembly Configuration containing the four 6-bar curve cognates. Each possibility leads to a different mechanism. The one that can be driven without additional conditions, contains a kite 4-bar, carrying rigid triangles that are similar and otherwise related. Then, a 'kite-cell' is involved, transforming the input-circle into a symmetrical Watt-1 curve of the 8th or of the 12th degree, depending on the choice of coupler-plane. The second possibility leads to a mechanism in which not only a point, but also a straight-line reaches the image positions. This type, however, must move through a 'stretched position' with a two-fold coupler motion. Then, gear-pairs or the like are necessary to overcome such a bifurcated position.

1. Introduction
Six-bar curves may be produced by coupler points of 6-bar linkage mechanisms of the Watt-type or of the Stephenson-type. Each type contains a 4-bar loop to which a linkage-dyad is adjoined in order to form a kinematic chain with six links. For Watt’s type the adjoined dyad connects two points that are attached to adjacent sides of the 4-bar, whereas for Stephenson’s type, points attached to the opposite sides are connected. A further distinction is made by appointing the frame in the linkage. For the Watt-1 mechanism, for instance, this is done by appointing a binary link as frame. We further have the Watt-2 mechanism, and also the Stephenson-1, -2 and -3 mechanisms.

Conditions for symmetry of the curves for the Stephenson-3 mechanism have been investigated by Levitskii[1] and Antuma[2], whereas the symmetry-conditions for the Stephenson-1 mechanism have been treated by the author[3] of this paper.

The author[4] has also investigated the symmetry-conditions for the Watt-1 mechanism in which the tracing point was attached to a floating link of ‘lower order’ implying that the floating link contained at least one point tracing a circle. For particular mechanisms such as the focal linkages of the Watt-1 type, for which the tracing point is of ‘higher order’, the author[5] similarly investigated the conditions of symmetry, in order to cause them to produce so-called ‘half-symmetrical’ curves.

Here, in this paper, however, a more general case will be considered, for which the coupler points E are of higher order and are, therefore, attached to the dyad-link KD which is directly hinged to the coupler point K of the 4-bar AABB₀. (See Fig. 1.)

Primrose et al.[6], who among others studied the Watt-1 mechanism, found that such coupler points generally produce 6-bar curves of degree 16 and genus 5. If the curves are symmetrical, they may be of lower degree. We intend to show that this is the case when kite 4-bars are involved. The main issue of this paper, however, is to find the conditions that have to be imposed on the mechanism for a higher order coupler point to produce symmetrical curves of the most general form.

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‡The coupler of a 4-bar mechanism is of lower order again, as then the coupler contains even two points tracing a circle.
§Of higher order in the sense that the floating link, containing such a point, does not include points tracing a circle.
We propose to use *cognate theory*\[7\] to derive the symmetry-conditions to be met by the Watt-1 mechanism. This may be carried out by combining the mechanism with one of its curve-cognates to form a symmetrical configuration. The symmetry-conditions that have to be imposed on the (initial) Watt-1 mechanism are then derived from this configuration.

2. The Assembly Configuration made Symmetrical

It is known from cognate theory\[7\], that a *four-fold* generation exists for the curve produced by a tracing-point $E$ attached to the dyad-link $KD$. Thus, including the initial mechanism, four *curve* cognates exist, producing the same 6-bar curve. They are (a) the initial Watt-1 mechanism, (b) its double Roberts’ *coupler* cognate, producing identical motion for the bar $KD$, (c) the curve cognate of the initial mechanism, obtained by an exchange of the bars of the adjoined dyad, and finally, (d) the curve cognate of the double Roberts’ coupler cognate, obtained in a similar way.

The four curve cognates together, form the so-called ‘Assembly Configuration’. Part of this configuration is demonstrated in Fig. 2, showing the cognates (a) and (d). It is possible to make them *reflected identical* with respect to the common axis $BoE$. As a consequence, not only (a) and (d) but also (b) and (c) then are images of each other with respect to this axis. To carry this out, it is necessary to arrange the triads $AoAKE$ and $A^*oA^*K'E$ symmetrically about the axis $BoE$. In addition, the dyads $BCD$ and $B'C'D'$ will each have to be the image of the other with respect to this axis of symmetry. With due observance of the rules of cognition, this then leads to a result which is also obtainable directly through the introduction of an auxiliary point $B^*$ on the axis of symmetry. For the sake of brevity, however, only this direct way of derivation will be explained.

**Derivation of half the Symmetric Assembly Configuration**

In order to make the curve cognates (a) and (d) images with respect to the axis of symmetry, we proceed as follows: (See Fig. 3) A 4-bar $A_0BB_0$ with coupler point $B^*$, and in which $AB = B_0B = BB^*$, is drawn in its *design* position (also called the *symmetry* position). In this position the crank $A_0A$ falls along the fixed link $A_0B_0$. The point $B^*$ attached to the coupler $AB$
Figure 2. Half the Assembly Configuration, containing the initial Watt-I mechanism as well as the curve cognate of the double Roberts coupler cognate.

Figure 3. Symmetrical linkage and derivation of Watt-I 6-bar producing a symmetrical curve, traced by $E$. 
traces a symmetrical 4-bar coupler curve. The line connecting $B_0$ to the design position of $B^*$ is the axis of symmetry of the curve.

Next, we draw the image 4-bar $A_0^*A'' - B^* - B^*B_0$ of the original mechanism with respect to the axis of symmetry. Then, according to Roberts’ Law, the common coupler point $B^*$ of these two 4-bars produces the same coupler curve. The image and the initial 4-bars are, therefore, 4-bar curve cognates of one another. Their common coupler point $B^*$, when moved, causes the cranks $A_0A$ and $A''_0A''$ to rotate at the same angular speed. It can also be seen that the configuration, being part of the Roberts’ Configuration, contains a parallelogram linkage which in this case is the rhombus $B_0B^*B^*B$.

A linkage dyad $KEK'$ is now added. This dyad has equal arms and connects two symmetrically located 4-bar coupler points ($K$ and $K'$) in the design position of the configuration. Thus, the dyad-joint $E$ lies on the axis of symmetry in that position. As it is symmetrically attached to a symmetric configuration, joint $E$ clearly moves along a symmetric curve.

4. Derivation and properties of the Watt-1 mechanism

The mechanism assembled in section 3, contains part of a Roberts’ Configuration. It is therefore an overconstrained mechanism which, because of its symmetry, guides points $B^*$ and $E$ along symmetrical paths. The non-symmetrical Watt-1 mechanism can now be derived from this symmetrical mechanism. To carry this out, we stretch-rotate the kite $KB^*K'E$ about $K$ so that $B^*$ moves to $B$. We so obtain the 4-bar kite $KBCD$ which is similar to the original kite $KB^*K'E$. The factor $f_K$ of stretch-rotation, being equal to the vector ratio $KB/KB^*$, is independent of time. The two kites therefore may remain similar, causing corresponding links to rotate at the same angular speed. (In notation: $KBCD = f_KKB^*K'E$, appointing $K$ as its center of stretch-rotation.) Thus, $KD$ may be attached to $KE$ and, similarly, $BC$ to $BB_0$, the latter having the same angular speed as $B^*B''$ or $B^*K'$. If we now obliterate the image mechanism and $B''$, we are left with the Watt-1 mechanism $A_0ABB_0-CDK$, in which the (higher) coupler point $E$ clearly traces a symmetrical 6-bar curve. The properties to be met by this mechanism are derived as follows: From the stretch-rotation applied about the point $K$, we find that

$$\Delta KBD = f_K \cdot KB^*E,$$

hence

$$KB/KB^* = KD/KE,$$

so that

$$\Delta KB^* - \Delta DKE.$$

Further, if we define the point $B'$ through the relationship

$$\Delta B_0B'B = \Delta B''K'B',$$

we may write that

$$BC = f_K \cdot B^*K' = f_K \cdot BB''.$$

hence

$$BC/BB'' = KB/KB^*.$$

Thus, with $BC = KB$, it follows that

$$\Delta CBB'' = \Delta KB^*.$$
Therefore, as the triangles $B'B^K$ and $BKB^*$ are reflected congruent so are the triangles $CBB'$ and $BB'B$. Thus, the rigid quadrilateral $CBB'B_0$ is an isosceles trapezium and so a cyclic quadrilateral.

From this, it follows that

$$\angle CB_0B = (\angle BCB_0 = \angle CB'B = \angle BB'K = ) \angle DEK.$$ 

Also,

$$\angle BCB_0 = \angle DEK + \angle KDE.$$

As further the corresponding kite diagonals $B'E$ and $BD$ are stretch-rotated about the angle $\angle BKB^* = \angle DKE = \pi - \angle B_0CB$, they meet in a point $S$ of the trapezium. Therefore, the axis of symmetry, the diagonal $DB$ and the circumcircle of $\triangle B_0BC$ are concurrent at $S$.

(For random positions, for which $\pi \neq \angle AA_0B_0 \neq 0$, the line $B_0E$, the diagonal $DB$ and the circumcircle of $\triangle B_0BC$ equally have a common point $S$.) Thus,

$$\angle A_0B_0E = \angle A_0B_0B^* = \pi - \angle SB_0A_0$$

$$= \pi - \frac{1}{2} \angle A_0B_0A_0 = \pi - \frac{1}{2} \angle BBA$$

$$= \pi - \frac{1}{2} (\angle KBA - \angle KBB^*)$$

$$= \pi - \frac{1}{2} (\angle KBA - \angle CB_0B - \angle BCB_0)$$

which is an expression for the direction of the axis of symmetry.

The properties derived will be used for a direct design of our Watt-I mechanism. Clearly, the Watt-I mechanism, drawn in full lines in the figure, will produce the same symmetric curve as did the point $E$ of the symmetric mechanism, from which we started. In conclusion, we have found altogether 5 conditions for our Watt-I mechanism to produce a symmetrical 6-bar curve. These conditions are respectively

1. $AB = B_0B$
2. $BC = BK$
3. $CD = DK$
4. $\angle DEK = \angle CB_0B$
5. $\angle EKD = \pi - \angle BCB_0$.

Instead of the conditions (4) and (5), it is possible to use the set of conditions

$$\begin{align*}
\angle DEK &= \pi - \angle B_0B_0C \\
\angle EKD &= \angle B_0CB.
\end{align*}$$

(4A)  
(5A)

In Fig. 4, for instance, the conditions (4) and (5) are applicable, whereas Fig. 5 demonstrates the application of the conditions (4A) and (5A).

In order to avoid this ambiguity, it is possible to relate the triangles $EKD$ and $B_0CB$ otherwise (See again Fig. 4):

Let then the circumcircle of $\triangle B_0BC$ intersect the circle about $B$, having the radius $BC$, at the points $C$ and $G$. then,

$$\angle EKD = \pi - \angle BDB_0 = \angle B_0GB.$$
Figure 4. Simple design of a Watt-1 linkage, producing 4 symmetrical branches of a 12th degree 6-bar curve.

Also, because $GB = CB$,

$$\angle BB_0G = -\angle BB_0C = \angle DEK.$$ 

Hence,

$$\triangle BB_0G \sim \triangle DEK.$$

This similarity replaces the conditions (4) and (5) as well as the conditions (4A) and (5A), unambiguously.

Two unambiguous relations between the sides of the triangles $EKD$ and $B_0CB$ are derived as follows: For this, we intersect the ray $B_0G$ and the circle about $B$, at the points $G$ and $H$. Then, for the (geometric) power of point $B_0$ with respect to this circle, we may write

$$B_0H \cdot B_0G = B_0B^2 - BC^2.$$ 

As further the triangles $B_0CB$ and $B_0HB$ are reflected identical, we have that $B_0H = B_0C$, so that with the similarity condition (4B), (5B), this power turns into

$$B_0C \cdot (EK/DK) \cdot BC = B_0B^2 - BC^2.$$
5. Design of the Watt-1 Mechanism

The design of the 6-bar mechanism to produce a symmetrical curve, may be carried out as follows: (See Fig. 5)

(a) Draw the 4-bar linkage $A_0ABB_0$, for which $AB = B_0B$, with point $A$ lying on the frame-link $A_0B_0$.
(b) Attach a random point $C$ to the rocker, or secondary crank, $B_0B$
(c) Attach a point $K$ to the coupler $AB$ such that $BK = BC$
(d) Adjoin an isosceles linkage dyad $CDK$
(e) Intersect the circle circumscribed about the triangle $B_0BC$ and the circle about $B$ with radius $BC$, at the points $G$ and $C$.
(f) Make the floating $\triangle DKE \sim \triangle BGB_0$.
(g) Verify that the diagonal $BD$, the circumcircle of $\triangle B_0BC$ and the symmetry-axis $B_0E$ all intersect at the one point $S$. (Note that the points $S, K, E$ and $D$ always lie on a circle.)

For obvious reasons we then say, that the triangles $EKD$ and $B_0CB$ are 'quasi-similar' to one another.

If one studies the complete Assembly Configuration, one finds that the symmetry-conditions, here derived, remain the same for all 4 curve-cognates. Thus, through cognation, no other sets of conditions are to be obtained.

Thus,

$$\frac{EK}{DK} = \frac{(B_0B^2 - BC^2)}{(B_0C \cdot BC)} \quad (4C)$$

and

$$\frac{DK}{DE} = \frac{BC}{BB_0} \quad (5C)$$

The orientation of $\triangle EKD$ being decided by the sign of the expression for $EK/DK$. 

Figure 5. Design of the Watt-1 mechanism, producing 4 symmetrical branches of a 12th degree and complete Watt-1 curve.
Generally, a Watt-I mechanism involves 15 degrees of freedom in design, not counting the motion variable of the mechanism. They consist of the 4 coordinates that determine the fixed link and a further 11 link-lengths. Thus, 10 parameters remain to be chosen by the designer in this case. They are the 4 coordinates for the frame-link, one parameter for the crank-length \( A_0A \), one for the link-length \( AB \), two for the sides \( BC \) and \( B_0C \), and, finally, two parameters for the sides \( AK \) and \( DK \).

A computer program, set up by Mr. A. T. J. M. Smals, has been used on a Hewlett and Packard table-top calculator (type no. 9825A) in combination with a Hewlett and Packard plotter (type no. 9872A) in order to demonstrate the symmetry of the curves that are traced by the Watt-I mechanisms of this design.

Figure 5, for instance, shows an example, in which the basic 4-bar linkage is a double crank. The complete curve is a \textit{quadruncursal} one. Each branch belongs to a chosen orientation of the isosceles dyads \( B_0BA \) and \( CDK \). (The orientations of the three rigid triangles of the 6-bar, however, remain the same for all branches that are traced by the (higher) coupler point.)

We further note that all branches have the same axis of symmetry through the fixed center \( B_0 \). Finally, a line perpendicular to axis of symmetry intersects the curve at up to 12 real points of intersection.

In Fig. 6, the basic 4-bar linkage is a non-Grashof one, with the result that twice two branches merge so that only two branches actually appear.

In Fig. 7, finally, a special case has been considered in which all the rigid triangles are stretched, so that each branch is traced twice.

6. The Degree of the Curve, Traced by Point \( E \)

Figure 8 shows a particular 8-bar mechanism that has one degree of freedom in motion. It consists of the 4-bar \( A_0ABB_0 \), the kite \( KEK'B' \), the rhombus \( B''BB'B'' \), and the rigid and reflected similar triangles \( B''K'B'' \) and \( BKB'' \), the latter being rigidly attached to the isosceles triangle \( ABB'' \). This mechanism really is half the Assembly Configuration without the redundant bar \( A''A'' \). It therefore produces the same Watt-I curve as before. The points \( B'', E \) and \( B_0 \) of this mechanism remain collinear. So, we can regard the mechanism as a transformer, that transforms the symmetrical 4-bar coupler curve, traced by \( B'' \), into a symmetrical 6-bar Watt-I curve, traced by point \( E \) of the mechanism.

![Figure 6. The complete Watt-1 curve that consists of two symmetrical branches.](image-url)
Figure 7. When two symmetric branches are traced twice.

In order to derive the degree of this Watt-1 curve, we note that for each point \( B^* \) on the axis of symmetry two points \( E \) exist. These are obtained by reflection of the isosceles dyad \( KEB' \) in the diagonal \( KK' \) of the kite. Further, as \( B^* \) moves along a coupler curve of degree 6, a total of 6 points \( B^* \) lies on the axis of symmetry. Two of them, however, are located at \( B_0 \), which is a double point of the (4-bar) coupler curve. The corresponding points \( E \) (to this double point) lie somewhere on the tangents to the (4-bar) coupler curve at \( B_0 \), not on the axis of symmetry. (As point \( B^* \) approaches \( B_0 \) along a tangent of the curve, this tangent, also being the connecting line of \( B^* \) and \( E \), joins \( E \) at a certain distance from \( B_0 \), but not on the axis of symmetry.) Thus, this axis intersects the symmetrical Watt-1 curve at \((6-2)*2 = 8\) points outside \( B_0 \). This number, however, does not account for the possibility in which \( E \) lies on the axis, and \( B^* \) does not. Such a possibility, namely, may arise when point \( E \) coincides with \( B_0 \). If \( E = B_0 \), a triangle \( BKE \)

Figure 8. Symmetrical driven cell transforming a 4-bar coupler curve into a symmetrical Watt-1 curve of degree 12.
exists, that is rigidly attached to the coupler triangle $ABK$. Thus, two different distances $EA$ exist, that correspond to the two possible orientations of $\Delta BKE$. A circle about $B_0$ with radius $EA$ intersects the input-circle at two real or at two imaginary points $A$. This occurs twice. So, a total of four positions $A$ may exist, that correspond to four positions $E$ at $B_0$. Hence, $B_0$ is a quadruple point $E$ of the Watt-I curve. The Watt-I curve, therefore, intersects its symmetry-axis at $(6-2) \times 2 + 4 = 12$ points. Hence, the curve that is traced by the higher order coupler point $E$ is of degree 12.

The lower order coupler point $E^x$, Fig. 9, which attached to the lower order coupler plane $CD$ of the mechanism, also traces a symmetrical Watt-I curve[4]. The latter, however, is only of degree 8.

7. The Kite-Cell, Transforming a Circle into a Symmetrical Watt-I Curve of Degree 12

The Watt-I mechanism, assembled in section 5, may be separated into two parts, namely the input-crank $A_0A$ and a remaining two-degree-of-freedom mechanism, which is to be named the kite-cell. (See Fig. 9)

In doing so, we may regard the curve traced by point $A$ as the input curve and the curve traced by $E$ (or $E^x$) as the output curve. The particulars of the kite-cell are easily extracted from the symmetry conditions of the Watt-I mechanism, in which the cell forms a part. The dimensions of the kite-cell, are therefore subjected to the following conditions

1. $AB = B_0B$
2. $BC = BK$
3. $CD = DK$
4. $EK/DK = (B_0B^2 - BC^2)/(B_0B \cdot BC) = (E^x E^2 - DC^2)/(E^x E \cdot DC)$
5. $DK/DE = BC/B_0B = DC/DE$.

The tracing point $E^x$, apparently, is attached to the bar $CD$ of the cell, such that $\Delta E^x DC \sim \Delta B_0 BC$. In case the cell contains point $E^x$ as well as point $E$, the cell is named a complete one.

From section 5 we deduce, that the kite-cell defined above transforms any circle traced by $A$ into a symmetrical Watt-I curve, traced by $E$. As further $\angle AB_0E = \pi - \frac{1}{2} \angle B^x BA = constant$, we find that $E$ traces a symmetrical curve if $A$ follows any curve that is symmetrical about a

![Figure 9. Complete kite-cell transforming a circle into symmetrical Watt-I curves of degree 8 and 12.](image)
If, for instance, point A traces a 4-bar coupler curve that is symmetrical about an axis through $B_0$, then point $E$ will equally trace a symmetrical curve. This may be realized by forcing another point $A$ of plane $BK$, that lies on the same distance from $B$, to move along a circle.

From Ref. [4] we derive, that if $A$ traces a circle or any other curve that is symmetrical about an axis through the fixed center $B_0$, then also point $E'$ will trace a symmetrical curve. We further derive, (See again Ref. [4]), that the tracing points $E$ and $E'$ of the cell, always remain aligned with point $B'$ and with the fixed center $B_0$. Therefore, the curves traced by the points $E$ and $E'$ have a common axis of symmetry.

As further

$$DC[DE^*] = BC[BB_0] = DK[DE] = DC[DE],$$

it follows that

$$DE^* = DE.$$ 

We finally note that the circumcircles of the triangles $B_0BC$, $DKE$ and $CDE'$ of the cell all have a common point $S$ at any point of time.

8. Symmetrical Arrangement of a Source Mechanism in Conjunction with its Double Roberts' Coupler Cognate

Another way to symmetrize the Assembly Configuration is to make the first cognate (a) and the second cognate (b) reflected images of each other with respect to the axis of symmetry. This means, that the double Roberts' coupler cognate must be obtained from the source mechanism through reflection. As before, such a reflection has to comply with the normal rules for cognation; see Ref. [7] and under. Consequently the common joints $B_0$, $K$ and $E$, the latter being the actual tracing point, all have to lie on the axis of reflection when the two cognates are in the symmetrical position. The remaining joints, such as $A'$, $B'$, $C'$ and $D'$ then are the reflected images of the initial joints $A_0$, $A$, $B$, $C$ and $D$. (See Fig. 10.)

**Figure 10.** Source mechanism and its double Roberts coupler cognate, generating symmetrical positions for their common coupler plane $DKD'$. 

[Diagram of the source mechanism and its double Roberts coupler cognate, showing the symmetrical positions for their common coupler plane $DKD'$.]
Thus, all the cognate dimensions are the same as for the source mechanism. The cognate conditions, extracted from p. 185 of Ref. [7] are

1. $\triangle BB_0C \sim \triangle B'C'B_0 \sim \triangle KD'D$
2. $\triangle ABK \sim \triangle KB'A' \sim \triangle A_0B_0A'_0$
3. $\triangle KDC \sim \triangle KD'C'$ (2 similar dyads)
4. $\triangle B_0A_0A \sim \triangle B_0A'_0A'$ (2 similar dyads)
5. $\Box B_0BKB'$ is a parallelogram linkage.

The conditions 1 and 5, in conjunction with the fact that $KD' = KD$ as well as $KB' = KB$, then lead to the equalities:

$$BC = BB_0 = KB.$$  

As further $A_0B_0 = A_0'B_0$, condition 2 additionality leads to the fact that $AB = KB$. So, in total, we have

$$AB = KB = B_0B = CB,$$

which we will call the star-conditions of the mechanism. From the cognate-condition 3, we next derive that

$$\frac{KC'}{KC} = \frac{K'D}{KD},$$

so that

$$\angle CKC' = \angle DKD'.$$

As, also, $K$ lies on the axis of symmetry, which is the axis of reflection between the bars $CD$ and $C'D'$, these angles are only equal if $K = CD \times C'D'$. Therefore, $K$, $C$ and $D$ are collinear. We may similarly prove that the joints $A_0$, $A$ and $B_0$ are aligned in the symmetry position of the mechanism.

Figure 11. Two symmetrical positions of a 6-bar mechanism, producing two identically reflected branches of a 6-bar curve.
Figure 12. Two identical reflected branches of a Watt-1 curve.

So, in total, there are five conditions for symmetry of the mechanism. They are successively:

1. \( AB = B_0B = KB = CB \)
2. Points \( C, D \) and \( K \) are collinear when \( A_0, A \) and \( B_0 \) are aligned.
3. In the symmetrical design-position for which \( A \) lies on \( A_0B_0 \), the tracing-point \( E \) must lie on the axis of symmetry.

The source mechanism, demonstrating these conditions, actually produces symmetrical positions of the (common) coupler-triangle \( DKD' \). In particular, the altitude \( KE \) of this triangle obtains symmetrical positions.

In fact, any point of this altitude, produced or not, traces a symmetrical Watt-1 curve of highest order. Point \( K \) is an exception, for it traces merely a symmetrical 4-bar coupler curve.

We deduce from the Assembly Configuration that symmetrical positions of \( \triangle DKD' \) are obtained from each other by successive reflection in the altitude \( KE \) and in the axis of symmetry. (See also Fig. 11.) They may also be obtained by a singular rotation about a point that is common to the altitude as well as to the axis of symmetry. Another way to arrive at the 'image position' is first to rotate \( \triangle DKE \) about \( K \) until dyad \( CDK \) has reached its image position with respect to the diagonal \( CK \). We then rotate the result about the fixed center \( B_0 \) until the 'image position' of the dyad \( C-DKE \) has been reached.

Two corresponding 'image positions' of this dyad blend only in the stretched position of the dyad, which is in fact the design position of the mechanism. Therefore, in its design position, point \( E \) lies at a blending point—or switch point—for two branches of the curve, each being the other's reflection with respect to the axis of symmetry. Thus, the two branches intersect at two points on the axis of symmetry of the curve, one of them being an ordinary double point, the other a switch point of the curve with coincident tangents. Figure 12 finally, demonstrates an example in which the basic 4-bar is a non-Grashof one. Also the two triangles \( ABK \) and \( B_0BC \) are stretched. Here too, the two branches are each other's image with respect to the axis of symmetry of the (complete) curve.

References


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**Symmetrische Kurven erzeugende Koppelpunkte höchster Ordnung eines Watt-I-Gelenkgetriebes**

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