Geometric error modelling of machine tools

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Geometric Error Modelling of Machine Tools

Authors: H.A.M. Spaan
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            BCR-91/018

CONFIDENTIAL
GEOMETRIC ERROR MODELLING OF MACHINE TOOLS

First milestone report of the BCR-project:
Development of Methods for Numerical Error Correction of Machine Tools

September 1991

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1. Introduction

1.1 General introduction

In the manufacturing of complex products machine tools play an important role. Due to several reasons one can observe a tendency towards higher accuracy at all levels of production. As the driving forces for higher accuracy [1] could be listed the requirement for:

- better performance and reliability of the products;
- miniaturization and integration of product-parts for weight and space saves purposes;
- automatic assembly;
- active noise reduction of accurate parts in gearboxes etc;
- response to increased accuracy in other fields, for example electronics.

Forced by this requirement, the performance of machine tools is continuously being improved. The classical way to increase the performance of a machine tool is to enhance the behaviour of the mechanical structure. In terms of geometric behaviour this implies the aim at faultless movements of the carriages, exact squareness between the guides and no finite stiffness effects of the elements. For the improvement of the thermal behaviour various experiments with thermal control [2], thermal invariant structures [3] and compensating heat sources [4, 5, 6] are carried out. However these methods of improvement are costly and the physical limits will soon be achieved. For this reason, new techniques to improve the overall behaviour of machine tools, are being developed. With the aid of computer technology it is possible to compensate for the errors existing in machine tools instead of avoiding them. However, this error correction method requires a very thorough investigation of the machine tool's behaviour and the factors influencing it. This is the fundamental reason for the initiation of a research project, supported by BCR, to develop methods for the numerical error correction of machine tools.
1.2 Description of the project
When the basic function of a machine tool is reviewed, it can be expressed as the transformation of rough material into a usable product. If the product has specified dimensions and tolerances, the function of the machine tool is to generate the product with accompanying specifications within a given time and at given costs. A number of influences on this transformation are responsible for errors in the dimensions of the final product. An overview of the main influences that may disturb the final accuracy of the product, is depicted in figure 1.1 [7].

Fig. 1.1 Sources of errors in machine tools.
In this BCR-project we will concentrate on the most influential error sources. Together these errors contribute to more than seventy percent of the resulting error of a machine tool [8, 9]. These error sources can be described as:

- **Geometric errors due to imperfect movements of the carriages.**
  In the production of the guides the manufacturer tries to avoid unwanted motions of the carriages, for example rotations. But even with the present production techniques the limits of achievable accuracy are restricted and therefore the carriage will show certain deviations from the perfect behaviour. These deviations will manifest themselves as position and orientation errors of the tool with respect to the workpiece.

- **Geometric errors due to finite stiffness of the elements of the machine.**
  One should make a distinction between the static and dynamic stiffness of a machine tool. The static stiffness of the machine is important when the machine is loaded with heavy workpieces, when heavy machine parts are moving while machining or when large cutting forces are to be expected. In this case the bending of machine parts will result in a deviation in the position of the tool with respect to the workpiece, and as a result this will introduce an error in the dimensions of the product. The dynamic stiffness contributes merely to the surface roughness of the product and less to the dimensional properties. Therefore, in this research project, it will not be taken into account.

- **Thermal behaviour of the machine's structure.**
  Under the influence of several internal and external heat sources a machine tool will demonstrate a certain thermal behaviour with, due to expansion of the individual elements of the machine, an accompanying distortion. The resulting errors in the position and orientation of the tool are at least of the same order of magnitude as the errors caused by geometric errors [10].
The main goal of this project is the improvement of the accuracy of a commercially available machine tool by the implementation of a correction algorithm into the control system. The error correction should comprise the most significant geometric errors, errors due to the finite stiffness of the machine and also reduce the effect of the thermal behaviour of the machine's structure. This goal requires a systematic investigation of the main error sources in multi axis machine tools. Therefore the available experience in research on accuracy of coordinate measuring machines will be adapted and extended to machine tools.

In order to achieve the aims of the project the following contractors participate in the project:

- Eindhoven University of Technology, Metrology Laboratory (TUE);
- Physikalisch Technische Bundesanstalt (PTB);
- Philips Industrial Electronics, Machine Tool Controls (Philips);
- Maho Aktiengesellschaft (Maho).

Together these partners will develop methods to model and describe the influence of a certain error source on the accuracy of a machine tool. Based on this research a correction algorithm will be developed. This error correction will be implemented on a five axis milling machine, supplied by Maho, to validate the developed methods.

In order to achieve the aims of the project, three workpackages have been defined:

- First, a geometric error model has to be developed and validated;
- Secondly, methods have to be developed to describe and analyse the thermally induced errors;
- Finally, an error correction has to be developed and implemented in order to validate both the geometric and thermomechanical models.
1.3 First milestone report
The project has been initiated on November 1st 1989 and will last until the end of December 1992. During the first 18 months of the project, a classification of machine tools has been developed and a literature study has been carried out. In chapter 2 these subjects will be discussed.
The other activities of the partners will also be summarized in this chapter.
Preliminary results of a conceptual description of the thermomechanical behaviour will be presented for both a statistical and an analytical approach. Also the first developed concept for an error correction will be described.

In the next chapters the geometric error model and its validation will be discussed extensively. In chapter 3 the mathematical model will be presented, capable of determining the resulting error of the tool with respect to the workpiece, as a function of the individual geometric errors. The individual errors will be described with functions, using piecewise polynomials. This technique will be explained.
The model will be applied to the Maho milling machine under research.
Two different methods to determine the geometric error components are used. First the error components are measured directly, using laser interferometer, level meters and displacement sensors. Secondly, the holeplate method, as developed by PTB, is used. Both techniques with accompanying measurement results will be presented in chapter 4.
Based on the direct measurements, the geometric error model will be used to simulate the holeplate measurements. In order to validate the model, this simulation will be compared with the actual measurements of the holeplate. The results of this comparison will be presented in chapter 5.
Finally, in the last chapter, conclusions will be drawn with respect to the used modelling technique and its possibilities and limitations.
2. State of the project

2.1 Introduction
The main goal of this BCR-project is the improvement of the accuracy of a commercially available machine tool, by adapting and extending correction methods used in the field of coordinate measuring machines. Therefore, the most influential error sources will be investigated: the basic geometric errors, thermal behaviour of the machine's structure and finite stiffness effects. Based on this research a correction algorithm will be developed and implemented. This algorithm will be validated on the milling machine, supplied by Maho for this research.

In order to achieve the aims of the project the following tasks will be carried out by the partners involved in the project:

- **Classification of multi-axis machine tools**
  A survey of the different types of five axis CNC milling machines will be made in order to establish the origins and characteristics of their related errors.

- **Bibliographical studies**
  A bibliographical study will be made to establish the characteristics of currently available error models, methods of temperature correction, related measurements and methods of parameter extraction from measurements of workpieces.

- **Agreements and standardizations for the exchange of data**
  In order to make the exchange of information possible a standard data format will be defined, measurement programs and interfaces will be developed and an error model will be chosen.

- **Geometric error modelling including finite stiffness**
  A general error model capable of modelling the geometric error structure of an arbitrary five-axis milling machine will be developed. This model will be evaluated by determining all geometric error components with direct measurements or by using artifacts. Simulations will be carried out to verify this error model. Also the finite stiffness effects of the machine tool caused by moving parts, cutting force and the weight of workpieces will be considered.
The development of thermomechanical models
In order to investigate the thermomechanical behaviour of a machine tool, both a statistical and an analytical approach will be investigated. For both approaches measurement techniques will be developed to determine the thermally induced errors. Based on measurements, both methods will be evaluated, verified and compared. The possibility of a combination of both models will be investigated.

Correction
A concept for a real-time error correction will be developed. This concept will be implemented in the hard- and software of the controller of the milling machine. The concept will be verified by milling workpieces with and without error correction. These workpieces will be evaluated to demonstrate the improved accuracy obtained with the real-time correction. Finally the uncertainties associated with the error correction method will be evaluated.

Not all partners will contribute the same amount of work to a certain workpackage, especially while the experience of the partners differs. Both PTB and TUE have experience in the research on the accuracy of coordinate measuring machines and machine tools. Philips has knowledge in the field of controlling and error correction of machine tools. Maho has experience in the field of machine tools and related errors. Therefore a distinction has to be made between the four partners, concerning the research to be carried out. A complete summary of these workpackages is given in table 2.1. Within this table, also a time-schedule is given. According to the time-schedule the first four tasks should be finished now, as the project is running for 18 months. In the next paragraphs a summary will be given of these completed workpackages. Also the state of the workpackages, which are not finished yet, will be presented in the following paragraphs.
Table 2.1 Workplan of the project, including a time-schedule.

2.2 Classification

During a project-meeting, all partners agreed that a classification should be developed to simplify the identification of significant errors. Therefore a separation is made between machine types with the same kind of errors (type dependant errors). A first approach, made by Maho and Philips [11], identifies the most important type dependant geometric and thermally induced errors of machine tools. Using this approach and some relevant literature [12], TUE made a proposal for a final version. During the last BCR-project meeting, which took place on December 20th, 1990, all partners accepted this version as a final version.

In this classification [13] a distinction is made between the basic machine and the additional rotary axes. Because rotary axes are available as an option, the three linear axes constitute the basic machine. In all cases the rotary axes will be be added to this basic machine and therefore they will not interfere with the kinematic chain of linear axes. Also the spindle of a milling machine can be defined as a rotary axis. However, the spindle will not be included in this classification, since the errors introduced by the spindle are not machine type dependent.
Basic machine
The basic machine has three perpendicular linear axes. These linear axes represent the kinematic chain of the basic machine. One end of this kinematic chain supports the tool, at the other end the workpiece can be positioned. The base of the machine is situated within this kinematic chain. As measurements mostly are referred to the base of a machine, a distinction between two chains can be made:

- A - chain: this chain supports the tool;
- B - chain: this chain supports the workpiece.

In figure 2.1 an example of a milling machine with its kinematic chain representation is depicted.

![Example of A- and B-chain in a knee-type milling machine.](image)

Fig. 2.1 Example of A- and B-chain in a knee-type milling machine.

As a machine consists of three axes, four classes can be discriminated with a different number of axes present in a particular chain. Within these classes two groups are distinguished, where the construction of the machine causes another type dependent error structure. In table 2.2 the resulting classification is presented.
<table>
<thead>
<tr>
<th>Number of axes in:</th>
<th>Classification</th>
<th>Some of the class dependent errors during traverse of the axis without process forces</th>
<th>Examples of machine types</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-Chain</td>
<td>B-Chain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>Class IA</td>
<td>Bending of: Table guide = F(Table, Weight/Load); Ram = F(Ram)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Chain: V → H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A-Chain: H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Class IB</td>
<td>Bending of: Table guide = F(Table, Weight/Load)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Chain: H → H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A-Chain: V</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Class IIA</td>
<td>Bending of: Table guide = F(Table, Weight/Load); Column = F(Ram, Vertical slide); Ram = F(Ram)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Chain: H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A-Chain: V → H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Class IIB</td>
<td>Bending of: Table guide = F(Table, Weight/Load); Column/bridge = F(Ram guide)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Chain: H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A-Chain: H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>V → H</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>Class IIIA</td>
<td>Bending of: Bridge guide = F(Bridge); Bridge = F(Ram guide)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A-Chain: H → H → V</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Class IIIB</td>
<td>Bending of: Column guide = F(Column); Column = F(Ram guide); Ram = F(Ram)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A-Chain: H</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>V → H</td>
<td></td>
</tr>
</tbody>
</table>

→: Sequence in kinematic chain
H: Horizontal axis of motion
V: Vertical axis of motion

F(E): Function of position of element

Table 2.2 Classification of basic machine types.
**Rotary axes**

Apart from three linear axes, a five axis milling machine consists of two rotary axes with perpendicular axes of rotation. Taking two rotary axes it is possible to distinguish three classes with a different number of rotary axes present in a chain. In the next table 2.3 this classification is shown.

<table>
<thead>
<tr>
<th>Number of rotary elements in:</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-Chain</td>
<td>B-Chain</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 2.3 Classification of the additional rotary tables*

With the described classification a distinction can be made between different types of five axis milling machines that manifest the same kind of errors (i.e. type dependent errors). With the knowledge of the type dependent errors the definition of a calibration setup and the error structure of the machine are significantly simplified. As the classification distinguishes between different number of axes present in the A- and B-chain, it becomes less difficult to model the geometric error structure of the milling machine by using this classification.
2.3 Bibliographical studies
A bibliographical study has been carried out to establish the characteristics of currently available error models, methods of temperature correction, related measurements and methods of parameter extraction from measurements of workpieces. In order to simplify the traceability of a certain article, TUE developed a database-system, capable of containing all relevant literature [14]. Within this database-system the following subjects can be filed:

- Name of the author(s);
- Title of publication;
- Name of the magazine or University;
- The numbers of the pages, where the article is published;
- Date of publication;
- Keywords;
- Rated (importance indication);
- Summary (maximum 5 lines);
- Place where the publication is stored.

This database-system is located at TUE. Therefore all partners have send their literature to TUE, who will take care of the maintenance of the database-system. The literature study gives a good indication on the state of the research activities, carried out at other institutes. As for the research topics of this BCR-project, the results of the literature study will be presented in the paragraphs, dealing with a corresponding subject.

In paragraph 3.1 the research on modelling the geometric errors is presented, carried out at other institutes. In paragraph 4.2 the results of the research on the finite stiffness effects is discussed. The research activities on thermal behaviour of machines is presented in paragraph 2.6.

According to the time-schedule, the literature study should be finished now. But, as the amount of relevant publications always increases, this database-system will be updated till the end of the project.
2.4 Agreements and standardizations for the exchange of data

During the first year of the project PTB extended the VDA-data format, as defined in DIN 66301 [15], for the exchange of temperature and geometric data. PTB supplied all partners with a program (written in Turbo Pascal) and a manual [16] to support this data-format.

In order to exchange the information on geometric errors, all partners agreed to use the German standard, laid down in VDI 2617, Blatt 3 [17] for the identification of a certain error. To identify the individual geometric errors three characters are used. The first, lower case, character represents the axis of movement, for instance "x" for the X-axis. The second, lower case, character represent the type of geometric error, i.e. translation, rotation or squareness. The last, lower case, character represents the axis along which, or rotation about which, the geometric error is acting.

The agreed measurement programs are described in the next paragraphs, including the necessary interfaces.

2.5 Geometric error model including finite stiffness

Both TUE and PTB have been developing mathematical models to describe the geometric error structure of the milling machine. TUE has developed a general error model which can be applied to multi-axis machine of arbitrary configuration. PTB has developed a similar, less complicated, model, whose use is restricted currently to machines consisting of three linear orthogonal axes. Since similar results can be obtained using the TUE model and as the TUE model is capable of handling a more complex machine, all partners agreed to use the TUE general model.

To model and/or estimate the error components PTB and TUE are investigating a different approach. TUE directly measures the error components, based on conventional methods using laser interferometer, inductive displacement sensors and electronic levelmeters. PTB, on the contrary, focuses on the indirect acquisition of machine tool error components using artifacts. One of these artifacts is a holeplate, developed for coordinate measuring machines. By measuring this holeplate, it is possible to determine the error components of a machine, knowing the deviations of the used object.
Both approaches have been applied to the milling machine, supplied by Maho and installed at TUE. As the amount of measurements, to be carried out, is very large, a dedicated program has been developed. This program is capable of controlling the position of the machine tool and collecting and storing the measurement results. To control the milling machine, Philips, the manufacturer of the control system, delivered special communication software and supplied information concerning the used interfaces and communication protocols.

The holeplate method requires a milling machine equipped with measuring facilities. Therefore Maho delivered and installed an infrared measuring probe. Using this probe system, a holeplate is measured in several positions. The measurement setups and accompanying results of both approaches will be described in chapter 4.

Besides the holeplate method, which asks for a measuring facility on the milling machine, PTB is working on a method to determine the machine tool errors by manufacturing special designed workpieces. Because the workpieces will be milled on a machine tool, they will not only contain information about the geometric errors, but also about the thermally induced errors of a machine tool. As the research on the thermal behaviour of a milling machine has just been initiated and will continue during the second half of the project's duration, PTB will continue their research activities on workpieces. A short description of this method will be presented in paragraph 2.6, where the state of the research activities on the thermomechanical behaviour is described.

With the above mentioned methods, all error components can be determined, caused by imperfect movements of the guides. In addition to these errors a number of forces act on the machine's structure. These forces cause the structure to deform and consequently disturb the actual position of the tool. While machining a workpiece three basic types of forces [18] are present:

1) forces induced by the cutting process;
2) gravity forces acting on the machines components;
3) gravity forces acting on the workpiece (workpiece load).
Errors due to forces of type 1

The type 1 forces act directly between the tool and the workpiece, causing the tool holder to deflect and thereby introducing errors in the dimensions of the product.

In order to calculate the magnitude of deflections caused by cutting forces, first these cutting forces have to be estimated.

Allowing a roughness of the milled product with a maximum $R_t$-value of 10 $\mu$m, the geometry of the cutting tool yields a maximum allowable feed per tooth of the cutter of 0.2 mm. The cutting forces induced under this condition and practical process parameters, have a magnitude of 500 - 1000 N. Considering a machine tool stiffness of $10^8$ Nm$^{-1}$, a maximum deflection of 10 $\mu$m may be expected, which equals the $R_t$-value. To mill a product very accurate, not only the geometry of the product should be very accurate, but also the roughness of the milled surface should be very small. Taking into account that the stated $R_t$-value is a theoretical value, based on the tools geometry only (in practice the $R_t$-value will be larger), it can be concluded that a correction for the deflection will not significantly improve the accuracy of the milled product, which is deteriorated by the bad surface roughness.

Thus, taking into account the huge effort required to model the errors due to cutting forces and the relative low improvement of the workpiece's accuracy, the errors due to these forces will not be investigated any further during this project.

Therefore, in order to achieve a very accurate product, it is advisable to mill a workpiece in at least two steps (strongly advised by Maho):

1) Most of the material is removed during milling the workpiece with large depths of cut and a large feed;
2) To achieve the desired dimensions, the workpiece is finished by milling with small depths of cut and a small feed.

During the finishing process the static cutting force will be much smaller than the above made estimation. Combined with the high stiffness of the machine tool, the cutting force will not cause a significant deflection.
Errors due to forces of type 2
The type 2 forces are caused by movement of large masses. As a result of these gravitation forces the geometric errors of one guide are dependent of the position of one or more other guides. An example of this effect is depicted in figure 2.2. Therefore, if the machine type indicates a certain finite stiffness effect due to movement of masses, the measurement of the individual geometric errors should be organized to comprehend these effects.

Fig. 2.2 Possible effect of the movement of a carriage on the geometric error of a guide due to finite stiffness.

The finite stiffness error of a axis, dependent on the position of the axis itself, is actually included in the model. This error is determined during the geometry measurements of the milling machine, but the geometric error and the finite stiffness error are not separable. A separation, however, is not necessary as the resulting error can be modelled perfectly in the developed geometric error model. This concept is similar to the "hardware-correction" practiced by many machine tool manufacturers.
Errors due to forces of type 3

The type 3 forces are dependent on the weight of the workpiece put on the machine tool. The effect of these forces is that the machine parts will bend and thereby cause a change in the geometric errors. TUE has investigated this finite stiffness effect (see paragraph 4.2), by measuring the change in geometric errors, when a load is placed upon the table of the milling machine. Based on this change in geometric errors, simulations are carried out, using the developed simulation software. The results of these simulations are presented in paragraph 5.3.

The finite stiffness errors due to gravity forces acting on a workpiece are actually included in the error model. As workpieces can be placed upon the milling machine having different weights, the change in geometric errors will be dependent of the weight of these workpieces. The developed error model, however, is capable of modelling the change in geometric errors for different loads. In order to validate the error model, both simulations and verification measurements will be carried out for different workpieces. This validation will be carried out during next period of the projects duration.

Considering the results of the simulations, the finite stiffness effects due to a load on the workpiece table cause a significant change in the resulting error of the tool with respect to the workpiece. These finite stiffness effect have even a larger contribution to the resulting error in the XY-plane than the geometric errors sec. Therefore, an appropriate software correction will be developed to correct these errors. The results of this error correction will be presented in the final project report.
2.6 The development of thermomechanical models

Thermal influences on the machine tool account for the largest part of the inaccuracy of the product [24, 25, 26]. In order to model the thermal behaviour of a machine tool, several approaches have been investigated with varying success [27, 28]. In general there are three methods to describe the thermal behaviour. First, a complex and time consuming approach is the use of finite element analysis methods (FEA). A number of researchers have applied this methodology on (sub-)structures of machine tools [29, 30]. Apart from the difficulties in the modelling procedure due to the complex structure of a machine tool, several uncertainties in the model, such as radiation, convection and conduction, cause the contribution of this methodology to be of limited practical importance [31, 32]. Therefore this approach will not be investigated during this project.

Secondly, simple analytical methods can be applied to determine the changes in the geometric errors due to thermal influences. PTB will focus their activities on this method by a transfer and adaptation of successful methods, developed to describe the thermomechanical behaviour of coordinate measuring machines.

Thirdly, an empirical method will be applied to determine the actual relationship between the thermal distribution and the displacement of the tool holder. The first results of this statistical approach, investigated by TUE, will be presented in this paragraph.

Analytical approach

Thermomechanical errors of coordinate measuring machines have been successfully described using simple analytical methods. The transfer and adaptation of this technique to machine tools and its further development is one of the tasks of PTB within this BCR-project.

While in most practical cases the temperature distributions in coordinate measuring machines are of a relatively uncomplicated kind; those in machine tools are not. This is mainly due to the high amount of heat generated within those machines. Heat generation, which is in many cases locally and timely confined. Besides the empirical modelling of quick transient effects (spindles) and a "history-dependent" correction using a multiple of reference states, the analytical approach, known from coordinate measuring machines, is developed. All three approaches shall together meet the needs of a thermomechanical error correction on machine tools. In the next period, this analytical error model will be developed.
In order to determine the error components of a milling machine, PTB has developed specially designed test workpieces. The main objective of these workpieces is to determine the machine tool errors under working conditions. Such a method was lacking so far, but it is indispensable if a full scale error correction, including thermomechanical effects, shall be verified. PTB will use these workpieces for the data acquisition, necessary for the development of a thermomechanical error model. Finally, the workpieces might be used for the acceptance test of machine tools.

Figure 2.3 and figure 2.4 show ways to extract the machine related error components from workpiece features. The workpiece features which are employed to determine certain machine errors are marked with double circles.

![Table of Error Components](image)

**Fig. 2.3 Machine error extraction from test workpieces**
It is possible to extract the errors of all linear axes and those of the rotary table. With some examples the principle will be explained:

- Positioning errors can be directly gained from the workpiece's linear dimensions;
- Squareness errors are directly imaged in the angles between orthogonal sides of the workpiece;
- The roll angle of the Z-axis can be determined from the difference between the squareness errors in two workpiece positions, namely two parallel positions in the YZ-plane, one with the milling head pointing in the positive X-direction and one in the negative X-direction.

Four workpiece positions are sufficient to separate all machine error components. The positions are in principle three cross sections through the working volume, parallel to two coordinate axes each. However, two parallel plate positions in the XY-plane are needed. In figure 2.5 the latest design of a test workpiece is depicted, with more than 1200 milled elements.

Besides the mentioned errors of the linear axes, which are extracted from the tracks and bores on the workpiece periphery, the errors of the rotary table show themselves in the tracks and bores in the center of the workpiece. It is also possible to detect zero point drifts with some of the features in the center.
The central part is only manufactured in the plane, parallel to the plane of the rotary axis. The periphery has to be produced in four positions. Except for the geometric elements reserved for the rotary axis, all elements exist 12 times in form of steps in the tracks and holes. The rotary table's errors are assumed to be invariant to temperature.
With this kind of workpieces, it is possible to record 12 different thermal states of the machine by means of four workpieces of three different states by means of one single workpiece. This has been done for 24 different states. Some are part of a continuous warming up of the machine, while some represent cycles with intermittent spindle operation.

To evaluate the workpiece geometry, a program has been developed to determine object features, like positions, angles, straightness etc. These features can then be transformed into machine related errors, like position errors, roll, pitch, yaw, etc. In figure 2.6 a result of such an evaluation is depicted. It shows the maximum deviation of a test workpiece during four hours of continuously warming up of the machine. The evaluation does not yet take the thermomechanical deformation of the machine tool into consideration. In figure 2.7 the axial slip of the rotary table is depicted.

**Fig. 2.6 Squareness error during continuously warming up for four hours**
Fig. 2.7 Axial slip of the rotary table

It can be concluded that the determination of all error components of a milling machine is feasible with test workpieces. Therefore, the development and verification of the thermomechanical model will be based on corresponding test workpieces and recorded temperature data. As it is not possible, at this state of the project, to separate the geometric and thermomechanical error components with sufficient accuracy, these workpieces will not be used for the evaluation and verification of the geometric error model.
Statistical approach
A number of heat sources contribute to the thermal deformation of the machine tool. Previous measurements at Maho's laboratories indicate that the friction in the spindle bearings can be regarded as the main source.
In order to verify this conclusion for the milling machine under research some experiments are carried out. These experiments include the measurement of the thermal distribution of the machine tool. During these measurements the machine tool was loaded with a spindle speed of 6000 rpm and simultaneously the carriages were moved back and forth over the range of the axes. The results of the preliminary temperature measurements are depicted in figure 2.8.

Fig. 2.8 Temperatures on a machine tool under continuous load.
In order to determine the relationship between the thermal distribution and the deformation of the machine tool, a measurement setup is built, based on a commonly applied principle [33, 34, 35]. For correction purposes, the principal interest is not the deformation of each machine component, but the displacement of the tool with respect to the workpiece. Therefore, a measurement setup is designed to obtain the displacement of the tool holder in three orthogonal directions, and the two relevant rotations (picture 2.1). Five contactless eddy current displacement transducers are used to measure this displacement.

![Measurement setup for drift measurements](image)

*Pict. 2.1 Measurement setup for drift measurements*

The measurement of the thermal distribution, which is continuously changing, is carried out by extensive temperature measurement equipment. In figure 2.9 the position of the temperature sensors on the machine tool is depicted. The choice of the position of each sensor is determined by:

- the capability of obtaining the thermal gradients in all directions. This is known as the so-called box-model, which means the sensors are located at the corners of an imaginary cube;
- a sensor density in relation to the importance of the heat sources;
- a minimum of three sensors on each measuring scale.
Fig. 2.9 Position of the temperature sensors on the machine tool

For a practical measurement the surface of the cylinder is positioned to 0.5 mm of the transducers. In this position the machine tool is loaded with a spindle speed over a specified time. The programming of the machine tool and the collection of the measurement data are carried out automatically by a computer.

With this measurement setup described, measurement cycles can be carried out, applying a spindle speed as load to the machine tool. During these measurements the temperature correction of the manufacturer of the machine tool shall be put non-active. In figure 2.10 the results of a first measurement with a spindle speed of 5000 rpm for 6 hours, followed by a spindle stop of 8 hours, is presented.

As the standard available temperature compensation of the machine tool operates on the information of the temperature of the spindle head, the temperature variation of the spindle head is marked out on the abscissa. This choice enables us to draw conclusions with respect to the suitability of one temperature sensor. On the ordinate the measured displacement of the tool holder is plotted. The solid part of the graph represents the warming up time, whereas the dashed part represents the cool down period. During this measurement the spindle was positioned in the center of the working space.
Drift of the tool holder with a load of 5000 rpm for 6 hours

Temperature rise of the spindle head [degree centigrade]

Displacement of the tool holder [mm]

-27-

Fig. 2.10 Results of a drift measurement in X-, Y- and Z-direction

From the results depicted in figure 2.10 a remarkable conclusion can be deduced. If a temperature rise on the spindle head of approximately 25 °C is measured, the displacement in Y-direction can be 40 as well as 52 μm, depending on warming up or cooling down of the machine tool.

In the Z-direction this effect is even larger and the predicted displacement at a temperature rise of 25 °C can even range as much as 40 μm. From these observations we can conclude that one temperature sensor for correction of thermal behaviour is insufficient.

The presented results are obtained in the center of the working space. The same series of measurements are repeated for different Z-coordinates of the machine i.e. with the Z-carriage extracted and retracted. These measurements yield a different behaviour than those obtained in the center. An example the results in Y-direction, measured on different positions of the Z-carriage, is depicted in figure 2.11. Here a load sequence is applied of 5000 rpm for 6 hours and no load for 8 hours. The solid line represents the warm up period, whereas the dashed line depicts the cool down period.
Fig. 2.11 Drift of the tool holder on different Z-positions

The goal of this part of the study is to determine a relationship between the relevant temperature sensors and the measured displacement of the tool holder, based on empirical obtained measurement results. The temperature sensors must be chosen such that the predictive value of the model is maximal, whereas the number of sensors is minimal.

Several approaches will be carried out during next period in order to obtain the relation between relevant temperature sensors and the displacement. First, preliminary calculations using least squares fitting procedures will be performed. However, as the theoretical importance of a particular temperature sensor is unknown, a methodology for relevance detection is desired. A possible approach is the use of statistical criteria for elimination and calculation of the optimal model. Therefore the data sets will be implemented in a software package for statistical analysis.
2.7 Correction

An error correction will be developed and implemented in the hard- and software of the controller of the milling machine, which enables milling of test workpieces with and without error correction. These workpieces will be evaluated to demonstrate the improved accuracy obtained with the real-time correction.

First, an error correction will be developed to correct the most significant errors. Therefore, the error correction workpackage is split into two parts. The first part deals with the correction of thermally induced errors, the second part with geometric errors including the finite stiffness effects. The state of the research on this error correction is presented in this paragraph.

**Correction for thermally induced errors**

The thermal displacement of the tool tip position due to temperature variations at various points in the machine tool, can be compensated. Based on the research carried out at TUE, the relationship between the thermal distribution and the deformation of the machine tool will be determined, using a statistical approach. This approach will take into account the temperature of several parts of the machine, the position along the Z-axis and the expansions of the X-, Y- and Z-measuring scales. In order to verify this method, an experimental setup of the control system will be made by Philips that supports the manufacturing of workpieces.

In figure 2.12 the links of the experimental temperature correction system with it environment is depicted.

![Data context diagram of the experimental temperature correction system](image-url)
The system interfaces to:

1) a number of temperature sensors on the milling machine;
2) the data that represent the positions along the machine axes;
3) the motion controller task, which handles the correction data;
4) the VDU update procedure, which makes the compensations available to the VDU (VDU: Video Display Unit).

In figure 2.13 the decomposition of the experimental temperature compensation system is depicted. Processes 1, 2 and 3 are implemented in an IBM-compatible computer, which handles the temperature dependent part of the temperature compensation. These processes will be implemented by TUE. The other processes will be implemented into the control system by Philips. This part handles the dependence of the temperature compensation on the position of the machine's carriages.
The compensation table is the interface between the processes running on the PC and the NC. It contains the coefficients for the computation by interpolation of the position dependent part of the temperature compensation. This table is updated by process 3 any 60 seconds to account for machine temperature changes.

Fig. 2.13 Decomposition of the experimental temperature correction system
The compensation table is read and split into a part that contains the dependence of the X, Y and Z compensation on the Z position, a part that contains the dependence of the X compensation on the X position, and a part that contains the dependence of the Y compensation on the Y position. The latter two parts represent the contributions of the expansion of the X and Y measuring rulers to the X and Y compensations, respectively. Both the X and the Y rulers are provided with three temperature sensors; one on both edges and one in the middle. It is assumed that there is a linear gradient between two of these three sensors; this results in a compensation which is a quadratic function of the position between the two sensors (figure 2.14). Since the measurements of the temperature and Z dependence were made at fixed positions for X and Y, the expansion with respect to these positions will be superimposed to the compensations, calculated dependent from the temperature and the Z position, at the X and Y measurement positions.

Fig 2.14 Compensation due to temperature gradient along the Y measuring scale
Correction for geometric errors

For the correction of the geometric errors the mathematical model, as developed by TUE, will be applied. With the known error sources, the resulting error, of the machine's tool with respect to the workpiece, can be calculated at any position of the machine's carriages. However, as the position of the carriages can change rapidly, the calculation of the correction terms must be performed every cycle of the control systems set point generation. This implies that every 15 msec a new correction vector must be available to the motion controller. The hardware of the controller is not capable to evaluate the total model, within the given time conditions, despite the fast hardware, delivered by Philips. In fact, this hardware provides some limitations on the possible error correction methodology. These limitations can be summarized as:

1) No evaluation of sine and cosine terms is possible due to lack of time;
2) The orientation of the tool can not be corrected due to the first reason and hardware limitation;
3) A maximum of three correction functions, other than the correction for scale errors, can be applied;
4) The correction functions are allowed to be dependent of only one axis;
5) The correction values must be available to the control system in a look-up table for speed purposes;
6) The correction of the scale errors is treated separately in look-up tables.

The last two conditions necessitate a description of the resulting error function in a look-up table with fixed grid points. The density and distribution of the grid points can be chosen freely so there is no large loss of information by this transformation.

Restricted by these limitations, the most significant contributions of an error component to the resulting error of the tool, with respect to the workpiece, will be determined and corrected. In the next period, Philips will implement this preliminary error correction together with the first correction for the thermally induced errors.

As the error correction for the geometric errors is not satisfactory, Philips has started a new development, in close cooperation with TUE. This concept will enable the error correction of all significant geometric errors and the thermally induced errors. During the next period this concept will be elaborated by the partners involved.
3. Geometric error model

3.1 Introduction
One of the major influences that determine the accuracy of multi axis machines are systematic errors in the movements of the carriages i.e. geometric errors. The main problem in this field is to determine a relationship that describes the systematic error in the location (i.e. position and orientation) of the tool, in dependence of the position of the machine's carriages. Many studies have assessed the problem of describing this relationship [36 - 42]. The applied methods range from correlation models, to trigonometric analysis, to 'error matrix' representations. However, in recent reports a tendency towards the use of rigid body kinematics can be observed. This method yields, in case of a three axis machine, a description of the location error of the tool as a linear combination of 21 measurable geometric errors.

Before one can develop a geometric error model, a definition for the geometric errors has to be chosen. In the next paragraph the chosen definition will be presented. Then a general methodology will be described for the construction of a model, which relates the various geometric errors to the location error of the tool. This methodology, developed by TUE [43, 44], will be applied to the five axis milling machine under research. With the elaboration of the general model to a machine specific model, a useful tool is obtained for software error compensation. Therefore, this model will be used for the future development and implementation of a software correction.

3.2 Definition for the geometric errors
In general machine tools, in particular milling machines, possess three perpendicular linear axes of movement. In addition to these three axes, rotary axes can be mounted on the machine tool. The three linear axes form a cartesian coordinate system which allows the tool to be positioned at any place within the range of the axes. Normally the linear movements along an axis of this system are performed by a carriage-guide system. In the ideal situation the spatial position of the tool can be determined by the positions of the carriages, presented by the regarding measuring systems. However, due to the imperfect geometrical shape of the guides of a machine tool, the carriages will display erroneous movements. These erroneous movements will result in an error of the location of the tool with respect to the workpiece. In order to classify these erroneous movements we first present some kinematic principles.
Basically a body possesses six degrees of freedom that determine its location (i.e., position and orientation) in space [45, 46]. These degrees of freedom are built up out of three translations and three rotations. Consequently a body can reveal six sources of error which result in another position and orientation of the body than expected. As a carriage of a machine tool is basically a body in space with five degrees of freedom suppressed, this theory does also apply to these elements. Application of this theory to a linear carriage-guide system implicates that, due to imperfections in the shape of the guide, the carriage will display straightness errors, rotations about all three axes and an error in the position along the guide.

In figure 3.1 a linear carriage-guide system is depicted with its possible erroneous movements.

![Carriage-guide system with possible geometric errors.](image)

In order to avoid wrong interpretation of each geometric error a definition is required of the nomenclature of the erroneous movements.
In principle any definition suits the purpose but for uniformity reasons all partners agreed to use the German standard, laid down in VDI 2617, Blatt 3 [47]. This standard uses three characters to identify the individual geometric errors. Hereby the first, lower-case, character represents the axis of movement, for instance "x" for the guide in figure 3.1. The second, lower-case, character represents the type of geometric error i.e. translation or rotation. The last, lower-case, character represents the axis along which, or rotation about which, the geometric error is acting. For example, if this notation is applied to the rotation of a carriage of the X-guide about the Y-axis, the geometric error source is denoted as "xry".

It must be noted that the geometric error of the carriage in the direction of movement (xtx, yty or ztz) does not find its cause in the guide but in the measuring system attached to the guide. Thereby, in principle, it is not a geometric error. However, in this milestone report it will be treated as an error in the geometry for simplicity reasons.

Also rotary elements are liable to erroneous movements. Similar to linear axes a definition of the individual errors is necessary. In figure 3.2 a rotary element is depicted with its geometric errors. This definition is again according to the German standard VDI 2617, Blatt 3. The identification of rotary axes of machine tools is defined in DIN 66217 and ISO 841 [48, 49]. According to this definition the rotary axis around the X-axis is denoted as the A-axis, around the Y-axis as the B-axis and around the Z-axis as the C-axis. In conjunction with the above presented definition of geometric errors for linear axes, the geometric errors of rotary axes are also denoted by three characters. Hereby the first, lower case, character represents the rotary axis of movement ("a", "b" or "c"), the second, lower-case, character represents the nature of the error (i.e. "t" or "r"). The last, lower-case, character represents the axis along which, or rotation about which, the geometric error is acting (i.e. "x", "y" or "z"). As an example, the scale error of a rotary element, which provides the rotation around the Z-axis, is denoted as "crz".
As the coordinate frame rotates along with the rotary element, additional definitions of the directions of the X-, Y- and Z-axis are necessary. In this milestone report is chosen to let the direction of the X-axis of the coordinate frame attached to the rotary element, coincide with the X-direction of the machine coordinate frame when the rotary element is at its zero position. If the rotary element starts its movement, the coordinate frame XYZ rotates along with the rotary element. This definition is necessary to avoid problems in the definition of direction if two serial rotary elements are applied.

Fig. 3.2 Rotary axis with its geometric errors.

3.3 The modelling system
In the modelling of the error structure of multi axis machine tools, several levels can be specified. At the top of the modelling system (figure 3.3) stands the general model. This general model relates errors in the location of the tool, with respect to the workpiece, to errors in the location of coordinate frames attached to succeeding components of the machine (i.e. the geometric errors).

Elaboration of the general model for a machine tool type yields the type dependent model. This model contains the common properties of the error structures belonging to machine tools of the same type. Thereby the type dependent model must be placed below the general. In the optimal situation the type dependent model can be also used to store type dependent errors, for example finite stiffness effects and thermal behaviour. However, as the division into type dependent and individual errors is a complex problem, the practical significance of this error classification is still limited.
The result of the modelling methodology is the so-called individual model. This model describes the error structure of an individual machine tool at a certain time and place. With this model the machine's accuracy can be unambiguously assessed and, if desired, improved by software error correction.

![Diagram of the modelling system of the geometric error structure](image)

**Fig. 3.3 Modelling system of the geometric error structure**

### 3.4 The general model
The general model can be applied to multi axis machine tools, composed of rotary and linear elements in an arbitrary serial configuration. It relates errors between the actual and nominal location of the tool (with respect to the workpiece), to errors in the location of coordinate frames attached to succeeding components of the machine. Such errors describe the difference between the nominal and actual geometry of machine parts enclosed by two frames. The number and position of the coordinate frames is chosen such that there is one kinematic element, i.e. carriage-guide system, between each two frames. This choice is adequate for application of this methodology to machine tools, coordinate measuring machine and robots.
Starting with the global coordinate system 0 attached to the machine tool's foundation, the orthogonal frames are successively numbered. As depicted in figure 3.4, a prefix is added to this number. This identifies the corresponding frame as being part of kinematic chain 'a' from foundation to tool, or chain 'b' from foundation to workpiece. This differentiation of the kinematic chain into two separate chains is made for convenient assessment of the geometric errors [51, 52]. Two additional frames 'wp' and 'tl' are introduced, which are attached to workpiece and tool respectively.

**Fig. 3.4** Nomenclature of the coordinate frames attached to a multi axis machine with \(n + m\) kinematic elements
Mathematical Derivation of the General Model

The nominal relation between the homogeneous coordinates \( k^p \) and \( l^p \) of a point \( p \) in frames \( k \) and \( l \) respectively, can be described by a \( 4 \times 4 \) transformation matrix \( k^T_l \) [53]:

\[
k^p = k^T_l l^p
\]  

where:

\[
k^T_l = \begin{bmatrix}
R_l & t_l \\
0 & 1
\end{bmatrix}
\]  

\[
k^p = \begin{bmatrix}
k_Px & k_py & k_pz & 1
\end{bmatrix}^T
\]

In this transformation the \( 3 \times 3 \) matrix \( R_l \) describes the orientation of frame \( l \) with respect to frame \( k \). The \( 3 \times 1 \) vector \( t_l \) contains the coordinates of the origin of frame \( l \) in frame \( k \).

The inverse transformation \( l^T_k = (k^T_l)^{-1} \) can be expressed as:

\[
l^p = l^T_k k^p
\]  

where:

\[
l^T_k = \begin{bmatrix}
R_l^T & (-R_l^T t_l) \\
0 & 1
\end{bmatrix}
\]

For a multi axis machine, composed of \( n \) kinematic elements in chain 'a' and \( m \) elements in chain 'b', successive application of these transformations yields the following expression for the nominal location \( wp^T_{tl} \) of the tool coordinate system 'tl' in the workpiece coordinate system 'wp'.

\[
wp^T_{tl} = \begin{bmatrix}
w_{p^T_{0}} & 0
\end{bmatrix}^T
\]

\[
= wp^T_{bm} \prod_{k=m}^{1} \begin{bmatrix}
1 & \prod_{k=k}^{b_k} \begin{bmatrix}
1 & \prod_{k=1}^{a_{k-1}} \begin{bmatrix}
ak_{k-1} & T_{ak}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\]

\[
t_{tl}
\]
Contrary to this ideal or nominal situation, an actual machine tool possesses errors in the relative location of subsequent frames, as well as in the location of the tool with respect to the last frame 'an' of the kinematic chain. Because none of the contemporary multi axis machines show an absence of Abbe offsets, the relevant errors in the relative location of two subsequent frames are not limited to those in the moving direction of the enclosed kinematic element (i.e. scale errors).

Consequently all possible errors of a guide as defined in paragraph 3.2 have to be taken into account. For the location of frame k with respect to frame k-1 this implies:

- translational errors \( k-1_\text{x} \), \( k-1_\text{y} \), \( k-1_\text{z} \) along the x, y and z axes of frame k respectively.
- angular errors \( k-1_\text{cx} \), \( k-1_\text{cy} \), \( k-1_\text{cz} \) about the x, y and z axes of frame k respectively.

In figure 3.5 an example of a two-dimensional carriage-guide system is depicted with the two coordinate frames and the related errors.

*Fig. 3.5 Carriage-guide system and possible errors*
The nomenclature of the individual errors in the general model differs from the definition as presented in paragraph 3.2. The reason for this departure are the lengthy formulas that arise in the process of developing a general description of an error structure. With the above presented notation, addition and multiplication of a variable number of terms becomes relatively easy to summarize. However, once the general model is worked out for a particular machine tool, the individual geometric errors will be nominated accordingly to the definitions of paragraph 3.2.

In the analysis of the effect of angular errors on the machine tool's accuracy, linearisation is applied, i.e. \( \cos(e) = 1 \) and \( \sin(e) = e \). Since the absolute values of these errors are relatively small for the target group of machine tools, this approximation is valid. Application of this approximation yields additive and commutative properties for the various errors.

This condition results in the following relationship between the actual transformation \( T_{ak} \) and its nominal \( T_k \):

\[
k_{-1} T_{ak} = k_{-1} T_k \cdot \text{Trans}[x, k_{-1} e_{kx}] \cdot \text{Trans}[y, k_{-1} e_{ky}] \cdot \text{Trans}[z, k_{-1} e_{kz}] \cdot \\
\text{Rot}[x, k_{-1} e_{kx}] \cdot \text{Rot}[y, k_{-1} e_{ky}] \cdot \text{Rot}[z, k_{-1} e_{kz}]
\]

\[ [3.8] \]

\[
k_{-1} T_{ak} = k_{-1} T_k (1 + k_{-1} \delta T_k)
\]

\[ [3.9] \]

where:

\[
k_{-1} \delta T_k = \begin{bmatrix}
0 & -k_{-1} e_{kz} & k_{-1} e_{ky} & k_{-1} e_{kx} \\
-k_{-1} e_{kz} & 0 & -k_{-1} e_{kx} & k_{-1} e_{ky} \\
k_{-1} e_{ky} & k_{-1} e_{kx} & 0 & -k_{-1} e_{kz} \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[ [3.10] \]

\[ I : 4 \times 4 \text{ identity matrix.} \]

\( \text{Trans}[x, k_{-1} e_{kx}] \): translation along the local x-axis by a distance \( k_{-1} e_{kx} \)

\( \text{Rot}[x, k_{-1} e_{kx}] \): rotation about the local x-axis by an angle \( k_{-1} e_{kx} \).
Similarly, the actual location \( \mathbf{T}_{a_{tl}} \) of the tool coordinate system with respect to the workpiece coordinate system can be expressed as:

\[
\mathbf{T}_{a_{tl}} = \mathbf{T}_{tl} (I + \mathbf{\delta}T_{tl})
\]  

[3.11]

where:

\[
\mathbf{\delta}T_{tl} = \begin{bmatrix}
0 & -\mathbf{e}_{tlz} & \mathbf{e}_{tly} & \mathbf{e}_{tlx} \\
\mathbf{e}_{tlz} & 0 & -\mathbf{e}_{tlx} & \mathbf{e}_{tly} \\
-\mathbf{e}_{tly} & \mathbf{e}_{tlx} & 0 & \mathbf{e}_{tlz} \\
0 & 0 & 0 & 0
\end{bmatrix}
\]  

[3.12]

Here transformation \( \mathbf{\delta}T_{tl} \) contains the errors in the location of the tool coordinate frame with respect to the workpiece. It consists of translational errors:

\[
\mathbf{e}_{tl} = [\mathbf{e}_{tlx} \mathbf{e}_{tly} \mathbf{e}_{tlz}]^T,
\]

defined along the x-, y- and z-axis of the nominal toolframe 'tl', and angular errors:

\[
\mathbf{e}_{tl} = [\mathbf{e}_{tlx} \mathbf{e}_{tly} \mathbf{e}_{tlz}]^T
\]

about the x-, y-, and z-axis.

Successive application of relation [3.9], yields the following expression for the actual location \( \mathbf{T}_{a_{tl}} \) of the tool coordinate system with respect to the workpiece coordinate system:

\[
\mathbf{T}_{a_{tl}} = \mathbf{T}_{tl} \left[ \prod_{k=m}^{1} \left( \mathbf{T}_{bk} \mathbf{T}_{ak} \right) \prod_{k=1}^{n} \left( \mathbf{T}_{ak} \right) \right] \mathbf{T}_{tl} (I + \mathbf{\delta}T_{tl})
\]

[3.13]

\[
\mathbf{T}_{a_{tl}} = \mathbf{T}_{tl} \left[ \prod_{k=m}^{1} \left( \mathbf{I} - \mathbf{\delta}T_{bk} \right) \mathbf{T}_{bk} \right] \mathbf{T}_{tl} \left[ \prod_{k=1}^{n} \left( \mathbf{I} + \mathbf{\delta}T_{ak} \right) \mathbf{T}_{ak} \right] \mathbf{T}_{tl} (I + \mathbf{\delta}T_{tl})
\]

[3.14]
Transformation \( \delta T_{an} \) contains the errors in the location of the tool with respect to the last frame 'an' of the kinematic chain. For metal cutting machine tools, errors such as spindle induced errors, tool misalignment, tool wear and thermal tool expansion can be included in this transformation [54].

Note that the transformation from frame \( bm \) to the workpiece frame \( (wp \, T_{bm}) \) is separated from the error terms. This implies that errors between the workpiece and the machine tool are not taken into account. The reason for this exclusion is that the workpiece coordinate frame is actually generated in the machine coordinate frame, in case of a fully machined workpiece. For partial machining, the errors in the location of the workpiece frame are randomly distributed. Therefore no calculation and correction of the induced errors is carried out.

In the elaboration of relation [3.14], an approximation will be made by ignoring higher order effects, consisting of the product of a matrix \( \delta T \) with one or more similar matrices. This approximation is valid, since the difference between the actual and nominal machine structure usually does not significantly change the active arm of angular errors and the direction in which the various errors act.

Combining relation [3.11] with relation [3.14] now yields the following expression for the error matrix \( wp \, \delta T_{tl} \) in the relative location between tool and workpiece:

\[
wp \, \delta T_{tl} = -tl \, T_{bm} \sum_{k=m}^{1} \left[ bm \, T_{bk} \, bk-1 \, \delta T_{bk} \, bk \, T_{an} \right] \, an \, T_{tl} + \\
\]

\[
tl \, T_{0} \sum_{k=1}^{n} \left[ 0 \, T_{ak} \, ak-1 \, \delta T_{ak} \, ak \, T_{an} \right] \, an \, T_{tl} + \, \delta T_{tl} \tag{3.15}
\]

A more convenient description, which also provides more intuitive insight in the basic error relationships, can be obtained by decomposing the error transformations of relation [3.15] into their basic errors \( e \) and \( e \). This procedure requires extensive vector algebra [55], after which the angular and translational errors between tool and workpiece, as formerly denoted in \( wp \, \delta T_{tl} \), can be summarized in the 6 \( \times \) 1 vector \( wp \, E_{t} \). This vector can be expressed as similarly denoted errors in the relative location between succeeding frames:
This vector represents the errors of the tool with respect to the workpiece, defined in the tool-frame.

\[ \text{where: } \text{wp}_\text{tl} E_k = \left[ \begin{array}{cccc} \text{wp} \epsilon t_x' & \text{wp} \epsilon t_y' & \text{wp} \epsilon t_z' & \text{wp} \epsilon t_x' & \text{wp} \epsilon t_y' & \text{wp} \epsilon t_z' \end{array} \right]^T \]

This matrix, the so-called F-matrix, denotes the effect of the errors \( k-1 E_k \), acting between the elements \( k-1 \) and \( k \), on the resulting error between tool and workpiece.

Here \( t_k^F \times t_k^R \) denotes a 3 \( \times \) 3 matrix whose columns contain the vector cross product of vector \( t_k^F \) with the respective columns of matrix \( t_k^R \).
The errors \( \mathbf{e}_{wl} \) in the relative position of the frames attached to tool and workpiece, are expressed as a linear combination of the transformation of the errors \( \mathbf{e}_{k+1} \) in the relative position of two subsequent frames, and the effect of related angular errors \( \mathbf{E}_k \). This active arm \( \mathbf{t}_k \) is determined by the structure and dimensional properties of the machine tool. In the following section this will be elaborated for a specific machine tool.

The errors \( \mathbf{e}_{wl} \) in the relative orientation between tool and workpiece, are transformed to the tool coordinate frame by a rotation \( \mathbf{R}_k \).

Finally error \( \mathbf{E}_l \) in the relative location of the tool with respect to the last frame 'an' of the kinematic chain, is added.

Note that the errors \( \mathbf{E}_l \) are defined in the nominal tool coordinate system. For correction purposes it is necessary to transform these errors to the machine coordinate system. This can be implemented in relation [3.16], by either premultiplying each of the \( 3 \times 3 \) sub-matrices of \( \mathbf{F}_k \) with the appropriate orientation transformation \( \mathbf{R}_k \), or by backtransformation of the resulting error to the direction of the machine's axes.

**Nominal location of the coordinate frames**

As already discussed, the chosen nominal location of the various frames seriously affects the efficiency of the final model. A generally useful model can be obtained by placing the frames in the centroid of the various kinematic elements, with one axis aligned with the respective axis of movement.

In the evaluation of the effect of geometric errors, we note a difference between carriage-guide systems where the carriage moves on a fixed guide, and those where the guide moves on a fixed carriage. For a system where the carriage moves on a fixed guide, the position of the carriage has no part in the active arm of the rotational errors. This in contrast with a system where the guide moves in a fixed carriage. Here the position of the guide is an integrated part of the active arm for rotational errors, induced in the carriage.
For the model this implies that a distinction has to be made between kinematic elements whose corresponding frame moves with the carriage and those where this frame is fixed relative to the guide (figure 3.6). This can be incorporated into the model by introducing so-called shape and joint transformations. The shape transformation \( k^{-1}S_k \) describes the relative nominal location between frames \( k-1 \) and \( k \), in case their respective kinematic elements are at home position. Joint transformation \( J_k \) describes the nominal angular or translational movement of kinematic element \( k \) (\( q_k \)). In accordance with the characteristics of the respective kinematic elements, application of these transformations yields the following expression for the relative nominal location \( k^{-1}T_k \) between two succeeding coordinate frames:

- Moving \( \rightarrow \) Moving:
  \[ k^{-1}T_k = k^{-1}S_k J_k \]  \hspace{1cm}[3.20]

- Moving \( \rightarrow \) Fixed:
  \[ k^{-1}T_k = k^{-1}S_k \]  \hspace{1cm}[3.21]

- Fixed \( \rightarrow \) Moving:
  \[ k^{-1}T_k = J_k k^{-1}S_k J_k \]  \hspace{1cm}[3.22]

- Fixed \( \rightarrow \) Fixed:
  \[ k^{-1}T_k = J_k k^{-1}S_k \]  \hspace{1cm}[3.23]

*Fig. 3.6* Connection between two kinematic elements with a 'fixed' and 'moving' coordinate frame
3.5 Elaboration of the general model to the type dependent model
The methodology described will be applied to the five axis milling machine under research (figure 3.7).

*Fig. 3.7 Five axis milling machine under research*

This machine tool consists of one horizontal linear element and one rotary element in chain 'a' from foundation to tool. Chain 'b' from foundation to workpiece consists of two linear elements, one vertical and one horizontal, and one rotary element with a vertical axis of rotation. In the first stage of the modelling process, coordinate frames are located in the workpiece, the tool and in the centroid of each joint. In figure 3.8 the kinematic representation of this milling machine is depicted. Note that the length of the tool is implemented in the model by a variable named 'L'.
The frames located in the various kinematic elements can be characterized as:

- Frame \( t_1 \) : fixed
- Frame \( a_2 \) : fixed
- Frame \( a_1 \) : fixed
- Frame \( b_2 \) : moving
- Frame \( b_1 \) : moving
- Frame \( b_3 \) : fixed
- Frame \( wp \) : fixed

The frame of the workpiece is thought to be at the same location as frame \( b_3 \).

**Fig. 3.8 Kinematic representation of the five axes milling machine under research**

Application of the above presented formulas, and abbreviation of 'cos(q)' and 'sin(q)' to 'cq' and 'sq' respectively, results in the expression of the nominal coordinate transformations between succeeding frames. As an example the transformation matrix from frame \( t_1 \) to frame \( a_2 \) can be calculated as:
\[ a_2 T_{ul} = \begin{bmatrix} cqa_2 & -sqa_2 & 0 & 0 \\ sqa_2 & cqa_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 200+L \\ 0 & 0 & 1 & -140 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cqa_2 & -sqa_2 & 0 & -(200+L)sqa_2 \\ sqa_2 & cqa_2 & 0 & (200+L)cqa_2 \\ 0 & 0 & 1 & -140 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

The calculation of all required transformation matrices is presented in Appendix A. These transformation matrices can be used to express the nominal position of the tool-frame relative to the workpiece-frame. As the elaboration of equation [3.16] requires the construction of the F-matrices out of \( u_k R_k \) and \( u_k t \) for all kinematic elements, the first step is to calculate the \( u_k T_k \) matrices. From these \( u_k T_k \) matrices, the required \( u_k R_k \) and \( u_k t_k \) matrices can be extracted (see equation [3.2]).

Application of expression [3.5] onto the obtained transformation matrices yields, for example [3.24], the following matrix:

\[ a_2 R_{11}^T = \begin{bmatrix} cqa_2 & sqa_2 & 0 & 0 \\ -sqa_2 & cqa_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad a_2 R_{11}^T a_2^T = \begin{bmatrix} 0 \\ -(200+L) \\ 140 \end{bmatrix} \]

\[ u_{T_{a2}} = \begin{bmatrix} cqa_2 & sqa_2 & 0 & 0 \\ -sqa_2 & cqa_2 & 0 & -(200+L) \\ 0 & 0 & 1 & 140 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

This procedure can also be carried out on all other transformation matrices (see Appendix A) and yields eventually the transformation from frame 'wp' to frame 'tl':

\[ u_{T_{wp}} = \begin{bmatrix} cqa_2 .cqb3 & sqa_2 & cqa_2 .sqa_2 \\ -sqa_2 .cqb3 & cqa_2 & -sqa_2 .sqa_2 \\ -sqa_2 & 0 & cqb3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 305 .sqa_2 + qbl .sqa_2 + (-350+qbl) .cqa_2 \\ 305 .cqa_2 + qbl .cqa_2 + (-350+qbl) .sqa_2 - (200+L) \\ 310 - qa_1 \\ 1 \end{bmatrix} \]
The next step is the determination of the F-matrices that describe the effect of the individual errors, between the coordinate frames, on the total error between tool and workpiece. From the calculated transformation matrices we can deduce that (see equation [3.2]):

\[
\begin{align*}
R_{a2}^t &= R_{a1}^t = R_0^t = R_{b1}^t = R_{b2}^t = R_{b3}^t = \\
&= \begin{bmatrix}
  cqa2 & \text{sqa}2 & 0 \\
  -\text{sqa}2 & cqa2 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\end{align*}
\] [3.27]

\( R_{wp}^t \) contains the additional transformation of the rotary element b3. However, as we state that the error between frame b3 and frame wp is zero, the obtained term for equation [3.16] will automatically yield a contribution of nil. Therefore it is not necessary to calculate the \( F_{wp}^t \)-matrix.

For the elaboration of the relevant F-matrices the vectors \( u_k^t \) need to be calculated. These calculation are fully described in Appendix A, underlying two examples are presented.

\[
\begin{align*}
\begin{bmatrix}
  u_{a2}^t \\
  u_{a1}^t
\end{bmatrix}
&= \begin{bmatrix}
  0 \\
  -(200+L)
\end{bmatrix} = \begin{bmatrix}
  ta_{2x} \\
  ta_{2y} \\
  ta_{2z}
\end{bmatrix} \quad [3.28] \\
\begin{bmatrix}
  210.\text{sqa}2 \\
  210.\text{cqa}2 - (200+L) \\
  805 - \text{qa}1
\end{bmatrix}
&= \begin{bmatrix}
  ta_{1x} \\
  ta_{1y} \\
  ta_{1z}
\end{bmatrix} \quad [3.29]
\end{align*}
\]

The matrix \( F_k^t \) is defined as (see equation [3.19]):

\[
F_k^t = \begin{bmatrix}
R_k^t \\
(u_k^t \times R_k^t) R_k^t
\end{bmatrix}
\]

The vector cross product of the vectors \( u_k^t \) and the matrix \( R_k^t \) can be summarized as:

\[
\begin{bmatrix}
  tkz.\text{sqa}2 & -tkz.\text{cqa}2 & tky \\
  tkz.\text{cqa}2 & tkz.\text{sqa}2 & -tkx \\
  -tkx.\text{sqa}2 - tky.\text{cqa}2 & tkx.\text{cqa}2 - tky.\text{sqa}2 & 0
\end{bmatrix}
\]

[3.30]

with \( k = a2, a1, b1, b2, b3 \)
Implementation of this relation into expression [3.19] yields the following general F-matrix:

\[
\begin{bmatrix}
  cqa2 & sqa2 & 0 & 0 & 0 & 0 \\
  -sqa2 & cqa2 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  tkz.sqa2 & -tkz.cqa2 & tky & cqa2 & sqa2 & 0 \\
  tkz.cqa2 & tkz.sqa2 & -tkx & -sqa2 & cqa2 & 0 \\
  (-tkx.sqa2-tky.cqa2) & (tkx.cqa2-tky.sqa2) & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

[3.31]

The index \( k \) indicates the concerning coordinate frame, i.e. \( a_2, a_1, b_1, b_2 \) or \( b_3 \).

Application of relation [3.16], yields the following expression for the errors \( \text{wp}^e_{il} \) and \( \text{wp}^e_u \) in the orientation and position of the tool coordinate frame with respect to the workpiece coordinate frame:

Orientation errors

\[
\begin{align*}
\text{wp}^e_u &= a_2^e_{il} + \begin{bmatrix}
  cqa2 & sqa2 & 0 \\
  -sqa2 & cqa2 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix} a_1^e_{a2} + \begin{bmatrix}
  cqa2 & sqa2 & 0 \\
  -sqa2 & cqa2 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix} a_1^e_{a1} - \begin{bmatrix}
  cqa2 & sqa2 & 0 \\
  -sqa2 & cqa2 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix} e_{b1}^a \\
- \begin{bmatrix}
  cqa2 & sqa2 & 0 \\
  -sqa2 & cqa2 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix} b_1^e_{b2} - \begin{bmatrix}
  cqa2 & sqa2 & 0 \\
  -sqa2 & cqa2 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix} e_{b3}^b \\
\end{align*}
\]

[3.32]
Position errors

\[ wp_{dl}^e = a_{2e}^e_{dl} \]

\[
\begin{bmatrix}
140. sa_2 & -140. c_2 & -(200+L) & c_2 & s_2 & 0 \\
140. c_2 & 140. sa_2 & 0 & -s_2 & c_2 & 0 \\
-(200+L).sa_2 & (200+L).sa_2 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
(805-q_1)sa_2 & -(805-q_1)c_2 \\
(805-q_1)c_2 & (805-q_1)sa_2 \\
-(210sa_2)c_2 & -(210c_2-(200+L))c_2 & -(210sa_2)c_2 & -(210c_2-(200+L))sa_2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
210c_2-(200+L) & c_2 & s_2 & 0 \\
-210sa_2 & -s_2 & c_2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
(825-q_1)sa_2 \\
(825-q_1)c_2 \\
-(632.5sa_2 + qb1sa_2)c_2 & -(632.5c_2 + qb1c_2 - (200+L))c_2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-(825-q_1)c_2 \\
(825-q_1)sa_2 \\
(632.5sa_2 + qb1sa_2)c_2 & -(632.5c_2 + qb1c_2 - (200+L))sa_2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
632.5c_2 + qb1c_2 & -(200+L) & c_2 & s_2 & 0 \\
-(632.5sa_2 + qb1sa_2) & -s_2 & c_2 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
\[
\begin{pmatrix}
(685-q_1)s_{q_2} \\
(685-q_1)c_{q_2} \\
-(700s_{q_2} + q_1s_{q_2} + (-350 + q_2)c_{q_2})s_{q_2} - \\
(700c_{q_2} + q_1c_{q_2} - (-350 + q_2)s_{q_2} - (200+L))c_{q_2} \\
-(685-q_1)c_{q_2} \\
(685-q_1)s_{q_2} \\
(700s_{q_2} + q_1s_{q_2} + (-350 + q_2)c_{q_2})c_{q_2} - \\
(700c_{q_2} + q_1c_{q_2} - (-350 + q_2)s_{q_2} - (200+L))s_{q_2} \\
700c_{q_2} + q_1c_{q_2} - (-350 + q_2)s_{q_2} - (200+L) \\
-c_{q_2} s_{q_2} 0 \\
-(700s_{q_2} + q_1s_{q_2} + (-350 + q_2)c_{q_2}) - s_{q_2} c_{q_2} 0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
(310-q_1)s_{q_2} \\
(310-q_1)c_{q_2} \\
-(305s_{q_2} + q_1s_{q_2} + (-350 + q_2)c_{q_2})s_{q_2} - \\
(305c_{q_2} + q_1c_{q_2} - (-350 + q_2)s_{q_2} - (200+L))c_{q_2} \\
-(310-q_1)c_{q_2} \\
(310-q_1)s_{q_2} \\
(305s_{q_2} + q_1s_{q_2} + (-350 + q_2)c_{q_2})c_{q_2} - \\
(305c_{q_2} + q_1c_{q_2} - (-350 + q_2)s_{q_2} - (200+L))s_{q_2} \\
305c_{q_2} + q_1c_{q_2} - (-350 + q_2)s_{q_2} - (200+L) \\
-c_{q_2} s_{q_2} 0 \\
-(305s_{q_2} + q_1s_{q_2} + (-350 + q_2)c_{q_2}) - s_{q_2} c_{q_2} 0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}
\]

\[
[3.33]
\]
4. Determination of the errors

4.1 Introduction

In the preceding chapter, a general model has been postulated which relates errors in the actual location of frames attached to tool and workpiece, to errors in the location of frames attached to succeeding components of a multi axis machine. Furthermore, this general model has been elaborated to construct the type dependent model, which describes this relationship for a certain machine. As a final step in the development of the individual model, the relation between the geometric errors and the position of the carriages has to be obtained. This requires an extensive performance evaluation of the machine tool.

In this project, two different approaches will be investigated to determine the errors of a machine tool. First, a more or less conventional method is used, to measure the error components using laserinterferometer, levelmeters and displacement sensors. The results of this method, as investigated by TUE, will be presented in paragraph 4.2.

Secondly, the error components are determined with artifacts. One of these artifacts is a holeplate, developed by PTB. This holeplate method has been applied by TUE and PTB to the Maho milling machine under research. The measurement setups and the accompanying results will be presented in paragraph 4.3.

4.2 Acquisition of the error components with direct measurements

In order to complete the individual model, which describes the error structure of an individual machine tool at a certain time and place, the error components of the machine tool have to be identified. These error components will be measured directly, using instruments as laserinterferometer levelmeters and displacement sensors. With these measurements not only the geometric errors will be measured, but also the change in geometric errors due to finite stiffness effects of the milling machine under research. The measurement setups and the obtained results, from the TUE experiments, will be discussed in this section.
One of the problems in the assessment of the geometric errors is the physical impossibility to measure in the center of the elements, as accordingly to the definition of the position of the coordinate frames is required. This implies for the obtained translational errors that a correction to the center of the elements is necessary. The correction value is determined by the position of the measurement i.e. the influence of rotational errors on the measured displacement. In figure 4.1 a two-dimensional example is depicted for the measurement of $tx$. In this case, the bare measurement data have to be corrected with the influence of the rotation $e$ (i.e. $eD$) in order to obtain the true error $tx$, defined in the center of the carriage.

![Diagram](image.png)

**Fig. 4.1 Example of a measurement of $tx$ and the effect of rotations**

Underlying the assessment of the bare measurement data will be discussed. As the machine tool under research is placed in a temperature stabilized room with a temperature of $23 \pm 0.5 \, ^\circ C$, this temperature is accepted as the reference temperature for all measurements.

**Software for automation of the measurements**

As the number of measurements, required for the purpose defined in this project, is very large, dedicated software is developed. This software has basically two main tasks:

1) Control of the position of the machine tool;
2) Collecting and storing of measurement results.
In order to perform task 1, an interface has been accomplished with the milling machine under research. This interface uses the RS232 port of an IBM-compatible computer for data transport. The communication with the milling machine runs under a specified protocol which is developed by Philips. With this interface the computer can control almost any feature of the machine tool. This allows convenient programming of, for instance, the positioning of the axis for measurement purposes.

The second task of the program is to gather the results from the installed measurement devices. As most modern instruments are available with an IEEE-488 interface, the communication and installation facilities have been developed for all necessary measurement devices as a laser interferometer, electronic level meters, temperature measurement equipment and inductive displacement transducers. With this interface facility, the computer aided setup and read out of these instruments becomes relatively simple. Besides the IEEE interfacing, the program is developed to have communication facilities through RS232 and keyboard input to accommodate any measurement situation that might occur.

The program is completely menu-driven, which makes it convenient to use. A typical measurement definition consists of the following steps:

- definition of the error to be measured, the instrument to be used, the interface type and interface address (IEEE);
- software installation of the measurement instrument (if necessary);
- definition of the initial position of the machine tool;
- definition of the measurement sequence:
  - definition of the start-position of the measurement;
  - definition of the axis and range of the measurement, the required positions (linear steps back and forth, pilgrimstep or a number of random positions);
  - definition of the number of repeats of the measurement sequence
- definition of the file for data storage and the number of repeats of the measurement.
After the above described steps have been completed, a measurement will be carried out automatically, yielding a datafile and an accompanying measurement information file. During the measurements all relevant information is displayed on the screen to allow checking of incoming data. The program scans the reading of the measurement instrument until it remains within a specified range before taking the measurement reading. This procedure enables us to eliminate dynamic effects induced by positioning the machine tool.

In figure 4.2 a schematic overview is presented of the applied interfaces and the used measurement instruments. All the underlying described measurements are carried out by application of this software package.

![Fig. 4.2 Scheme with measurement instruments and interfaces](image-url)
Measurements of the linear axes

With the aid of the above presented program all 21 geometric errors present in the system of the three linear axes have been determined. The measurement sequence of these measurements is defined over the entire range of the respective axis of movement with a measurement step of 10 mm. All measurements are carried out back and forth over the range of the axis.

Underlying some measurement results are presented. First, the measurement of \( \text{xrz} \) is a typical example of a measurement setup and obtained results. Secondly, a couple of measurements are discussed that give cause to further investigation. An overview of all measurement setups and accompanying results is presented in Appendix B.

In the underlying graphs the bare measurement data are depicted. This implies that no correction is carried out for thermal expansion or, in case of translational errors, for the influence of rotations, as indicated in figure 4.1.

All measurement equipment, used to determine the geometric error components, is calibrated at Metrology Laboratory of Eindhoven University, which is certified by the Dutch Calibration Organisation [56]. The used instruments are:

- HP 5528 laser interferometer with accompanying optics, air-sensor and temperature sensors;
- Wyler electronic levelling meters;
- Hilger Watts autocollimator;
- Hilger Watts polygon;
- Ceramic square;
- Inductive displacement transducers;
- Sipp rotary table.

For the measurement of the rotation error \( \text{xrz} \) a laser interferometer is used. The measurement uncertainty associated with this instrument is less than 0.2 arcsec. The measurement setup for the rotation error \( \text{xrz} \) is depicted in figure 4.3. The reference interferometer is mounted to the ram of the milling machine while the retroreflector is connected to the workpiece table, which performs the movement in X-direction.
The results of this measurement are presented in figure 4.4. These results directly reflect the rotation error between the coordinate frames attached to the X- and Y-axis respectively. This rotation error is not only composed of the geometric error of the X-guide, caused by the imperfect guide, but also of the finite stiffness effect of the guide. When the workpiece table is moving from $X=0$ to $X=700$ mm, the X-axis will bend (rotation around the Z-axis). This yields a rotation error, almost linear dependent on the position of the X-axis (figure 4.4).
For the measurement of the rotation error xrx a set of electronic levelmeters is applied. The measurement uncertainty of these electronic levelmeters is less than 0.5 arcsec.

The measurement setup for xrx is depicted in figure 4.5. The reference levelmeter is mounted on the ram, thereby eliminating the effect of rotation of the overall machine structure, while the measurement levelmeter is placed on the workpiece table, which performs the movement in X-direction. The rotation error does not depend on the position of measurement, so the obtained results directly reflect the rotation error between the coordinate frames attached to the X- and Y-axis respectively.

![Fig. 4.5 Measurement setup for xrx](image)

Execution of the described measurement yields the results that are graphically depicted in figure 4.6. In this graph all bare measurement data are depicted that are obtained by repeating the measurement 20 times. The rotation error of the machine is defined in arcsec (1 arcsec = 4.8e-6 rad).

In the results of xrx, individual peakvalues of the measurand can be observed with magnitudes of the same order as the measured error. Plotting the measurement results sequentially reveals that the peak error repeats equidistant. In figure 4.7 the measurement results of xrx are depicted together with a quasi timescale, which yields the conclusion that the peakvalues repeat every 30 minutes. From an inspection of the machine constants it appeared that the periodic peaks are induced by the lubrication pump of the machine tool, that is activated every 30 minutes.
**Fig. 4.6** Error xrx versus position of the X-carriage

**Fig. 4.7** Results of measurement of xrx sequentially and time indicator of 30 minutes
A laser interferometer is also used to measure the scale errors of the milling machine. The measurement uncertainty of this instrument is less than $0.05 + 0.5\times L \, \mu m$ ($L$ in m).

The measurement setup, to determine the scale error of the Z-axis ($ztz$), is schematically depicted in figure 4.8. The interferometer is mounted on the workpiece table, while the retroreflector is connected to the ram of the machine tool. The influences of the rotation $zrx$ and $zry$ have to be eliminated from the obtained measurement results. This yields the error $ztz$ of the coordinate frame positioned in the centroid of the Z-carriage.

![Measurement setup for $ztz$](image)

**Fig. 4.8 Measurement setup for $ztz$**

Execution of the described measurement yields the bare, uncorrected results that are graphically depicted in the first part of figure 4.9. In the results of $ztz$ a clear form of hysteresis can be observed. This is not caused by the hardware of the machine, but purely by a reproducing temperature field over the Z-scale. In figure 4.10 the temperature on three positions of the Z-scale is depicted. These temperatures were obtained during the measurement of $ztz$. The second graph in figure 4.9 represents the error $ztz$ corrected for the effect of the changes in the temperature of the Z-scale. Clearly the hysteresis has disappeared.
Fig. 4.9 Error ztz versus position of the Z-carriage

Fig. 4.10 Temperature of the Z-scale during measurement of ztz
Analysis of the cause of the thermal problem yields the conclusion that the hydraulic installation warms up the Z-scale by radiation. In order to avoid this problem it is therefore advisable to isolate the scale of the Z-axis.

The straightness measurements are carried out with a ceramic square and inductive displacement sensors. The results of these measurements can be used for both the analysis of the straightness errors and the analysis of the squareness errors of the milling machine. The straightness error is defined as the difference between the actual data and its least-square. In figure 4.11 the result of a straightness measurement is depicted, after the measurement data are corrected for the error induced by the square. This result show that the machine has a very small straightness error. The other results are shown in appendix B, together with the squareness results.

![Fig. 4.11 Straightness error ztx](image)
Measurements of the rotary axes

The Maho milling machine contains not only three linear axes, but also two rotary axes. In order to complete the individual model, also the geometric errors induced by these two rotary table have to be determined.

For the measurement of the linearity error of the B-axis, an optical polygon and an autocollimator is used. The polygon has a measurement uncertainty less than 1 arcsec. The autocollimator introduces a measurement uncertainty less than 0.5 arcsec. The measurement is carried out by positioning the B-axis with a step that equals the angle between two succeeding planes of the polygon. With the autocollimator the difference between the orientation of the two planes is measured. Using a polygon, which has 12 sides, together with the autocollimator, it is possible to measure the position error of the B-axis over the whole range (360°) with steps of 30°.

With the above presented measurement method it is possible to obtain the position error with steps as large as the angle between two succeeding planes of the polygon. In order to determine the position error between these steps another method is used. A very accurate rotary table is placed upon the B-axis of the milling machine. this rotary table introduces an uncertainty less than 0.5 arcsec. In the center of this table a plane mirror is placed. The measurement sequence is carried out by rotating the B-axis of the milling machine a randomly chosen angle. Using the rotary table, the plane mirror is rotated back to the zero position, which is measured by the autocollimator.

Both measurement methods are applied to the B-axis of the Maho milling machine. To measure the position error with the first method the polygon is place in the center of the B-axis, as depicted in figure 4.12. The autocollimator is placed outside the milling machine on a stable tripod. In the same figure the measurement setup for the second method is depicted. Here a SIPP rotary table is placed in the center of the B-axis. A plane mirror is placed upon this table. The autocollimator is also placed outside the milling machine.
Fig. 4.12 Both the measurement setups for brγ.

The results of both measurements are depicted in figure 4.13. As these results are corrected for the systematic errors of the used instruments, they directly reflect the position error of the B-axis. The results show a randomly distributed position error smaller than 4 arcsec. As the resolution of the B-axis equals 3.6 arcsec, it can be concluded that the B-axis has an position error smaller than its resolution.

Fig. 4.13 Error brγ versus position of the B-axis
Also the position error of the C-axis of the Maho milling machine is measured. As the C-axis rotates around the Z-axis of the milling machine, which is a horizontal axis, it was not possible to connect the SIPP rotary table to this axis. Therefore this axis is only measured with the first method. In order to get more information on the position error of the intermediate positions the measurement was carried out with different start positions.

The measurement setup for the position error crz is depicted in figure 4.14. The polygon is placed upon a rotation table, which is connected to the C-axis of the milling machine. This rotation table is used for the different start positions. In order to create a suitable measurement setup, an optical square is placed on the table of the milling machine. The autocollimator is placed outside the milling machine on a stable tripod.

![Fig. 4.14 Measurement setup for crz](image)

The result of this measurement is presented in figure 4.15. This result directly reflects the position error of the C-axis. From this result, it can be concluded that the position error of the C-axis, which has a resolution of 3.6 arcsec, is very small, concerning the range of this axis.
Fig 4.15 Error \( crz \) versus position of the C-axis

Based on the found, small position errors of rotation tables and the results from research carried out at other institutes \[57\] it can be concluded that rotation tables have a very high accuracy, compared to the other error sources in general and the geometric errors of the linear axes in particular.

**Measurements of the finite stiffness effects**

With the above mentioned methods, all error components can be determined, caused by imperfect movements of the guides. In addition to these errors a number of forces act on the machine's structure. These forces cause the structure to deform and consequently disturb the actual position of the tool. While machining a workpiece three basic types of forces \[58\] are present:

1) forces induced by the cutting process;
2) gravity forces acting on the machines components;
3) gravity forces acting on the workpiece (workpiece load).
Errors due to forces of type 1

The type 1 forces act directly between the tool and the workpiece, causing the tool holder to deflect and thereby introducing errors in the dimensions of the product.

In order to calculate the magnitude of deflections caused by cutting forces, first these cutting forces have to be estimated. Allowing a roughness of the milled product with a maximum $R_t$-value of 10 $\mu$m, the geometry of the cutting tool yields a maximum allowable feed per tooth of the cutter of 0.2 mm. The cutting forces induced under this condition and practical process parameters, have a magnitude of 500 - 1000 N. Considering a machine tool stiffness of $10^8$ Nm$^{-1}$, a maximum deflection of 10 $\mu$m may be expected, which equals the $R_t$-value. To mill a product very accurate, not only the geometry of the product should be very accurate, but also the roughness of the milled surface should be very small. Taking into account that the stated $R_t$-value is a theoretical value, based on the tools geometry only (in practice the $R_t$-value will be larger), it can be concluded that a correction for the deflection will not significantly improve the accuracy of the milled product, which is deteriorated by the bad surface roughness.

Thus, taking into account the huge effort required to model the errors due to cutting forces and the relative low improvement of the workpiece's accuracy, the errors due to these forces will not be investigated any further during this project.

Therefore, in order to achieve a very accurate product, it is advisable to mill a workpiece in at least two steps (strongly advised by Maho):

1) Most of the material is removed during milling the workpiece with large depths of cut and a large feed;

2) To achieve the desired dimensions, the workpiece is finished by milling with small depths of cut and a small feed.

During the finishing process the static cutting force will be much smaller than the above made estimation. Combined with the high stiffness of the machine tool, the cutting force will not cause a significant deflection.
Errors due to forces of type 2

The type 2 forces are caused by movement of the large masses, e.g. of the workpiece table. As a result of the gravity forces acting on these masses, the geometric errors of one guide are dependent on the position of one or more other guides. An example of this effect is depicted in figure 4.16. Therefore, if the machine type indicates a certain finite stiffness effect due to movement of masses, the measurement and model of the individual geometric errors should be organized to comprehend these effects.

Fig. 4.16 Possible effect of the movement of a carriage on the geometric error of a guide due to finite stiffness.

The finite stiffness error of a axis, dependent on the position of the axis itself, is actually included in the model. This error is determined during the geometry measurements of the milling machine, but the geometric error and the finite stiffness error are not separable. A separation, however, is not necessary as the resulting error can be modelled perfectly in the developed geometric error model. This concept is similar to the "hardware-correction" practiced by many machine tool manufacturers.
The finite stiffness error, as depicted in figure 4.16, can be determined separate as this error is dependent on the position of another axis. The error $yrz$ is measured, using the measurement setup as depicted in figures B.16 & B.17, with the workpiece table on the positions $X=0$, $X=350$ and $X=700$. The measurement results showed no significant change in the geometric error $yrz$ due to finite stiffness. Although the presented model and estimation techniques can cope with such errors, it was not necessary to include this error $yrz$ for the studied machine.

*Errors due to forces of type 3*

The type 3 forces are dependent of the weight of the workpiece put on the machine tool. The effect of these forces is that the machine parts will bend and thereby cause a change in the geometric errors. In order to investigate this change in geometric errors a load is placed upon the table of the milling machine. During movement of the X-axis three rotations are measured, using laserinterferometer ($xry$) and levelmeters ($xrx$, $xrz$) (figure 4.17). The results of this measurement are depicted in figure 4.18.

![Figure 4.17 Measurement setup for $xrx$, $xry$ and $xrz$](image-url)
In order to determine the effect on the rotation errors of the Y-axis, caused by a load of a workpiece, two rotations are measured with levelmeters (yrx, yrz) (figure 4.19). The results of these measurements are depicted in figure 4.20. This measurement is carried out on three different X-positions, to determine the dependence of the position of the X-guide on the error components of the Y-guide (see also point 2).
Fig. 4.19 Measurement setup for \( y_{rx} \) and \( y_{rz} \)

Fig. 4.20 Rotation errors \( y_{rx} \) and \( y_{rz} \) versus position of the Y-carriage, on different X-positions
In order to analyze the effect of these additional errors on the resulting inaccuracy of the machine tool, simulations are carried out based on the finite stiffness errors. The results of these simulations are presented in paragraph 5.3.

4.3 Acquisition of the error components using the holeplate method

PTB is developing several methods to determine the geometric components using artifacts, e.g. the workpiece method, presented in chapter 2. This research is based on the experience gained with another method: the holeplate. This holeplate is developed to calibrate coordinate measuring machines. In order to verify the geometric error model, based on the direct measurements, this holeplate method will be applied to the Maho milling machine under research, by using the milling machine as a measuring machine. The method and accompanying results will be presented in this paragraph.

**The holeplate method**

In order to determine the geometric errors of a coordinate measuring machine, a 2-dimensional reference object is developed. This artifact consists of a grid of holes, with an equal mutual distance. By measuring these holes, and knowing the deviation from the nominal mutual distance, it is possible to evaluate the performance of a coordinate measuring machine, with respect to the geometric errors. When this artifact is placed upon the machine, so that the different locations of the holeplate constitute the sides of a cube, and the holeplate is measured with different probe-styli, it is possible to determine all 21 geometric error components of a machine consisting of three linear axes [64].

As stated before, the holeplate has to be calibrated to be suitable for calibration of coordinate measuring machines. This calibration is carried out by PTB with laserinterferometer measurements in two directions (linear) and a subsequent multiorientation measurement in at least three orientations.
These multiorientation measurements are carried out to eliminate virtually all systematic errors of the measuring instrument including (stationary) temperature effects and reduces drifts and random errors by at least a factor of 2.5 [64]. By using a laser interferometer, the holeplate is directly traceable to the national standard of length (figure 4.21).

With both methods it is possible to determine all 21 geometric error components of a milling machine, consisting of three linear axes. But as the PTB method refers to another coordinate frame, direct comparison of the individual error components is not possible. Therefore it has been decided to compare the actual holeplate measurements with simulations of these measurements, based on the individual model, developed by TUE.
In the next part, the holeplate measurements and accompanying results will be presented. This method, developed by PTB, is carried out by TUE and PTB on the Maho milling machine under research.

**Measurements and results of the holeplate method**

PTB supplied a holeplate with a grid of 9 x 9 holes. The nominal distance between two holes is 50 mm. The actual distance is determined during a calibration at PTB. This holeplate is measured on the Maho milling machine, using an infrared probesystem, delivered and installed by Maho.

Using the probesystem, the milling machine will determine the location of each hole referred to the reference hole. As the actual location of these holes is known, it is possible to determine the geometric error of the milling machine within a certain plane. This error will not only consist of the geometric error of the milling machine but also the repeatability of the used probe system will be present in the measured error. In order to determine this repeatability of the probe system, a repeatability test is carried out with all probe configurations, as depicted in figure 4.22.

![Fig. 4.22 Used probe configurations, to measure the holeplate](image)

The following repeatability is found for each probe configuration:

<table>
<thead>
<tr>
<th>Probe</th>
<th>Repeatability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2S_{xc} = 0.63 , \mu m$; $2S_{zc} = 0.84 , \mu m$</td>
</tr>
<tr>
<td>2</td>
<td>$2S_{xc} = 1.1 , \mu m$; $2S_{yc} = 0.80 , \mu m$</td>
</tr>
<tr>
<td>3</td>
<td>$2S_{xc} = 1.8 , \mu m$; $2S_{zc} = 1.3 , \mu m$</td>
</tr>
<tr>
<td>4</td>
<td>$2S_{yc} = 1.8 , \mu m$; $2S_{zc} = 7.5 , \mu m$</td>
</tr>
<tr>
<td>5</td>
<td>$2S_{yc} = 1.4 , \mu m$; $2S_{zc} = 7.5 , \mu m$</td>
</tr>
</tbody>
</table>
These results show a large repeatability in Z-direction for probes 4 and 5. This is caused by the position of the probetip: not in the center beneath the switching probe system.

It is possible to determine the error components within a range of 400 mm with the available holeplate. As the machine has a range of X=700 mm, Y=500 mm and Z=600 mm, the holeplate has to be positioned on different locations within a certain plane. Therefore a total of 24 holeplate measurements have to be carried out to determine all error components within the whole range of the milling machine. As the measuring scales and the holeplate will expand due to temperature changes, the temperature of these parts will be measured during the experiments, using the software as described in paragraph 4.2.

Underlying the holeplate measurement in the YZ-plane is presented. An overview of all measurement setups with accompanying results is presented in Appendix C.

The different holeplate positions in the YZ-plane are depicted in figure 4.23. For this measurement setup probe 4 and probe 5 are used. The milling machine is programmed to measure the outmost holes.

![Fig. 4.23 Measurement setup for holeplate measurements in the YZ-plane](image)
Before the measurement results can be compared with the simulations, the actual measurements have to be corrected for the expansion of the measuring scales of the milling machine and the expansion of the holeplate itself due to temperature changes. As it is not possible to perfectly align the holeplate along one of the axes of the milling machine, the measurement results will show an alignment error. This alignment error is eliminated by a mathematically transformation of the measurement results, as the holeplate would be aligned along the axis, which is the lowest in the kinematic chain.

In figure 4.24 the result of the holeplate measurement on position 5 in the YZ-plane is depicted, after correction for the above mentioned effects.

![Graph showing the result of the holeplate measurement in the YZ-plane on position 5, after correction.](image)

**Fig. 4.24**  *Result of the holeplate measurement in the YZ-plane on position 5, after correction*

The location of each hole is determined during a measurement for and back. Figure 4.24 shows a large difference between those two measurements. This is mainly caused by the bad repeatability of the probe system in Z-direction.
During the experiments, the holeplate method turned out to be a very fast method for determination of the geometric performance of a milling machine, when equipped with a measuring facility. This method even turned out to be accurate, but this accuracy can be deteriorated extremely by the necessary locations of the probe tip.

The corrected results of the actual holeplate measurements, as depicted in figure 4.24, can now be used for direct comparison with simulations of the holeplate measurements. These simulations and accompanying results will be presented in the next chapter.
5. Simulation and validation of the error model

5.1 Introduction
Before the general error model can be accepted as a tool for software error correction, this model has to be verified. Therefore, the general model has been elaborated to the type dependent model (Maho milling machine type). Based on this type dependent model the individual model can be determined, by modelling the geometric error components. The used modelling technique and the resulting functions will be presented in this chapter.

The individual model will be used to simulate the holeplate measurements, which are described in paragraph 4.3 and appendix C. The results of these simulations will be compared with the actual measurements of the holeplate. Based on this comparison, conclusions will be drawn with respect to the developed model and the applied holeplate method.

A simulation of holeplate measurements will also be carried out to determine the significance of the change in the geometric errors caused by finite stiffness effects due to the load of a workpiece. Further investigations will be defined using the results of the simulations.

5.2 Modelling the individual geometric errors
The measurements, described in chapter 4 and appendix B, yield datasets consisting of discrete measured errors. However, in the application of the individual model as a basis for software error compensation, it is preferable to have a function based description of these error components. This implies that fitting techniques are applied to the datasets.

Although most mathematical functions can be applied to describe the general trend of an error, their use is limited for modelling the, often irregular, shape of the remainder. That is, the behaviour of an error in one region of the domain may be totally unrelated to the behaviour in another region. Polynomials, along with most other mathematical functions, have just the opposite property: their behaviour in a small region determines their behaviour everywhere. A group of functions that possesses this property to a lesser extent, are the so-called piecewise polynomials [65].
Piecewise polynomials can be described as a set of polynomials defined upon limited continuous parts of the domain. The pieces join in the so-called knots, obeying continuity conditions with respect to the function value itself and an arbitrary number of derivatives. The number and degrees of the polynomial pieces, the nature of the continuity restrictions and the number and positions of the knots may vary in different situations, which gives piecewise polynomials the desired flexibility.

**Definition of piece-wise polynomials**

A straightforward mathematical implementation of the continuity restrictions can be obtained by the use of truncated polynomials, or "+"- functions, as basic elements in the piecewise polynomial models. The "+"- function is defined as:

\[
\begin{align*}
  & u_+ = u & \text{if } u > 0 \\
  & u_+ = 0 & \text{if } u \leq 0
\end{align*}
\]

In general, with \( k \) knots \( t_1, \ldots, t_k \) and \( k+1 \) polynomial pieces each of degree \( n \), the truncated power representation of a piecewise polynomial \( p(x) \) with no continuity restrictions can be written as:

\[
p(x) = \sum_{j=0}^{n} \beta_{0j} x^j + \sum_{i=1}^{k} \sum_{j=0}^{n} \beta_{ij} (x - t_i)_+
\]

Note that the presence of a term \( \beta_{ij} (x - t_i)_+^j \), allows a discontinuity at \( t_i \) in the \( j \)-th derivative of \( p(x) \). Thus different continuity restrictions can be imposed at different knots simply by deleting the appropriate terms. Normally, it is sufficient to ensure that each model is continuous with respect to the function value and its first derivative.
An inherent problem in constructing the individual model is the unknown nature of the errors to be described. The model's potential to accommodate irregular errors, is to an extensive degree determined by the number and position of the knots. If the position of these knots are considered variable, that is, parameters to be estimated, they enter into the regression problem in a nonlinear fashion, and all the problems arising in nonlinear regression are present [65]. The use of variable knot positions also carries the practical danger of overfitting the data, and makes testing of hypotheses, considering areas of structural change, virtually impossible. Unless prior information is available, we use a basic model which contains enough polynomial pieces with a fixed length and a maximum allowable degree of two, to accommodate the most complex error expected.

In the parameter estimation process, a stepwise regression procedure is implemented to remove statistically insignificant parameters from the model [65]. The reason for this removal is twofold:

- including insignificant parameters hardly improves the model's quality of fit, but increases the variance of the estimated parameters and response;
- identification of structural parameters enhances the diagnostic properties of the individual model.

*Fitting the measurement results with piece-wise polynomials*

The data obtained from the measurements, as described in Appendix B, are averaged. A least squares fitting procedure has been applied to these data sets, using piece-wise polynomials. The piece-wise polynomials are defined by the position of the knots and the coefficients of the polynomial for each interval. An example of a result is graphically depicted in figure 5.1. An overview of all fitted data, and the accompanying coefficients of the polynomials, is presented in Appendix D.
Fig. 5.1 Example of fitting with piece-wise polynomials on error zry

In the graph depicted in figure 5.1, the dashed lines represent the averaged results of a number of back and forth measurements. The solid line represents the fit through these averaged measurement data using piece-wise polynomials.

This method yields a continuous description of each geometric error, based on discrete measurements, and will be used for the simulations of the holeplate measurements. When the comparison of these simulations with the actual measurements yields satisfactory results, this information will also be used for the development of software error compensation.
5.3 Verification of the error model with simulations of the holeplate method

TUE developed a dedicated software package, which enables the calculation of the error vector on any position within the range of the modelled machine, using the individual model. This error vector is calculated with respect to the workpiece coordinate frame, taking into account the dimensions of the tool. In order to facilitate simulations, the error vector can be determined as a function of the position of all axes. Thus it is possible to evaluate the geometric performance of the modelled machine, within its range.

One of the possibilities of the software package is to evaluate the geometric performance of the modelled machine in a plane. In figure 5.2 the error vector in the XZ-plane on Y=250 is depicted, using a tool with a length of 150 mm. This error vector is calculated with respect to (X,Z) = (0,0) and as a function of the X- and Z-coordinates (the X- and Z-coordinates correspond with the X and Z positions of the axes of the milling machine). In figure 5.3 the Y-component of the error vector is depicted as a function of the X- and Z-coordinates.

This software package is used for the simulations of the holeplate measurements. Based on the location of the holeplate and the used probe the actual positions of the axes of the milling machine are calculated. Knowing these positions and the dimensions of the probe (defined as tool dimensions), it is possible to calculate the error vector, with the above mentioned software package.
Fig. 5.2 Result of a simulation of the error vector in the XZ-plane on Y=250, using a tool with a length of 150 mm.

Fig. 5.3 Y-component of the error vector in the XZ-plane.
To simulate the measurements of the holeplate, the difference between the error vector with respect to the reference hole and the error vector with respect to the evaluated hole is determined. The program can calculate this error vector for each hole, as it would be measured by the milling machine.

These simulations enable direct comparison with the actual measurements carried out on the Maho milling machine under research. But, before comparing the results, the actual measurements have to be corrected for the expansion of the scales and the expansion of the holeplate. Also the alignment error of the holeplate along the axes of the milling machine has to be corrected.

After these corrections, direct comparison with the simulations is possible. This comparison has been carried out for all holeplate measurements. Underlying a couple of comparison results are discussed that gave cause to further investigations.

In figure 5.4 both the corrected measurement results and the simulations are depicted, for the holeplate measurement on position 1 in the XY-plane. In order to simplify the comparison, the residual between the measured result and the simulation is depicted in figure 5.5, together with the corrected measurement.

![Figure 5.4](image)

**Fig. 5.4** Result of the actual holeplate measurement and the simulation on position 1 (Z=292) in the XY-plane
Before drawing conclusions with respect to the made comparison, the uncertainty of the residuals has to be calculated.

First, the uncertainty associated with the simulations is estimated. These simulations are based on the individual model, which is determined by direct measurements. Taking into account the uncertainty of the used instruments, an upper limit estimation of the simulated holeplate measurements yields:

$$2S_{sim} = 6.3 \, \mu m.$$  \[5.1\]

This uncertainty is determined by calculating the positive square root of the variance of the result, multiplied by a factor $k = 2$. The variance of the result is given by adding the variances corresponding to the different uncertainty components, multiplied by the squares of relevant partial derivatives \[66\]. In order to get an upper limit estimation, the maximum value for the partial derivatives is substituted, considering the application.
Secondly, an estimation is made of the uncertainty introduced by the holeplate measurements.

As presented in paragraph 4.3, the location of the holes is determined during a calibration of holeplate, with an uncertainty of:

$$2S_{\text{plate}} = 0.7 \mu m.$$ \[5.2\]

The holes are measured on the Maho milling machine under research. The probe system, together with the used probe configurations yield the following uncertainty:

- Probe 1: $$2S_{xc1} = 0.63 \mu m; \quad 2S_{zc1} = 0.84 \mu m$$ \[5.3\]
- Probe 2: $$2S_{xc2} = 1.1 \mu m; \quad 2S_{yc2} = 0.80 \mu m$$ \[5.4\]
- Probe 3: $$2S_{xc3} = 1.8 \mu m; \quad 2S_{zc3} = 1.3 \mu m$$ \[5.5\]
- Probe 4: $$2S_{yc4} = 1.8 \mu m; \quad 2S_{zc4} = 7.5 \mu m$$ \[5.6\]
- Probe 5: $$2S_{yc5} = 1.4 \mu m; \quad 2S_{zc5} = 7.5 \mu m$$ \[5.7\]

The measurements are corrected for the expansion of the scales of the milling machine and the expansion of the holeplate. The uncertainty of the used temperature sensors yield an upper limit estimation for the uncertainty of this correction of:

$$2S_{\text{scale}} = 0.46 \mu m \quad 2S_{\text{plate}} = 0.96 \mu m$$ \[5.8\] \[5.9\]

The comparison of the simulations of the holeplate with the corrected measurement results, as depicted in figure 5.5, yield an upper limit estimation for the uncertainty of the residuals of:

$$2S_{\text{Residual}} = 6.8 \mu m$$ \[5.10\]

Taking into account this upper limit estimation, it can be concluded that for the comparison, depicted in figure 5.5, there is no significant difference between the model and the actual error structure of the milling machine. However, the uncertainty associated with the squareness measurements is 1.5 arcsec, which yields a large contribution to the uncertainty of the residuals.
The comparison of the holeplate measurements with the simulations show a systematic effect in the residuals, which can be caused by a squareness error. Therefore, it can be concluded that more accurate squareness measurements probably will yield a better model.

The results of another comparison between the simulation and the actual holeplate measurement in the XY-plane are depicted in figures 5.6 and 5.7. The residuals show also no significant difference between the model and the actual error structure of the milling machine, taking into account their uncertainty. However, at the end of the X-axis' range, the residuals show a systematic difference. Though within the range of the uncertainty, these residuals are not randomly distributed, which was to be expected. Therefore, the cause of this systematic difference is still subject of a study.

As the residuals, at the remaining positions, are much smaller than the measured geometric errors, a significant improvement of the accuracy of the milling machine is possible, using the developed model for software error compensation.

![Fig. 5.6](image)

Fig. 5.6 Result of the actual holeplate measurement and the simulation on position 8 (Z=292) in the XY-plane
The comparison of the holeplate measurements with corresponding simulations is depicted in figures 5.8 and 5.9, for position 3 in the XZ-plane. Also for this comparison an upper limit estimation for the uncertainty associated with the residuals can be calculated:

\[
2S_{\text{Residual}}^{xy} = 6.7 \, \mu m
\]  

[5.11]

Taking into account this uncertainty, a significant improvement of the milling machine's accuracy would be possible for the geometric error component in Z-direction. The difference between the simulations and the actual results in X-direction, which was also present in the simulation of the holeplate measurement on position 8 in the XY-plane, is also found here. This effect is still subject of a study.
Fig. 5.8 Result of the actual holeplate measurement and the simulation on position 3 (Y=157) in the XZ-plane

Fig. 5.9 Result of the holeplate measurement, together with the residual between measured result and simulation
Finally, the results of the comparison for position 4 in the YZ-plane are depicted in the figures 5.10 and 5.11. Also for this comparison an upper limit estimation of the uncertainty of the residuals can be calculated, yielding:

$$2S_{\text{Residual}}^{yz} = 12.7 \, \mu m$$

This uncertainty is mainly caused by the bad repeatability of the used probe configuration (see paragraph 4.3).

Considering this uncertainty, the results of the comparison yield a much better similarity than to be expected from the estimated uncertainty. Therefore, it can be concluded that the residuals probably have a lower uncertainty as indicated by the upper limit estimation. But, therefore, the upper limit estimation is a very useful tool to prove the significance of the used modelling technique.

![Graph](image)

**Fig. 5.10** Result of the actual holeplate measurement and the simulation on position 4 (X=550) in the YZ-plane
The residuals between measured result and simulation (figure 5.11) are randomly distributed. Therefore, it can be concluded that no systematic effect is missing in the individual model. This result among the other results, is very promising when the developed individual model will be used as a tool for the final version of a software error compensation.

5.4 Simulations of holeplate measurements based on the finite stiffness errors
In addition to the geometric errors, the actual position of the tool can be disturbed by a number of forces acting on the machine's structure. In the chapters 2 and 4, three different basic types of forces are distinguished. However, it is only necessary to evaluate the significance of the forces of type 3 (workpiece load), as forces of type 1 will not cause a significant deflection during finishing (see chapter 2) and forces of type 2 are not separable from the basic geometric errors.
In order to analyze the effect of the change in geometric error caused by the finite stiffness errors due to a workpiece load on the resulting inaccuracy of the machine tool, simulations are carried out of a holeplate measurement. These simulations coincide with the simulations presented in paragraph 5.3, except for the used error model, which is only based on the finite stiffness errors.

In figure 5.12 the results of a holeplate measurement simulation is depicted on the same position as the simulations depicted in figures 5.4 and 5.5. It is obvious that the predicted deviation based on the finite stiffness errors (max. ± 20 μm) is larger than the predicted deviation based on the geometric errors sec (max. ± 12 μm).

![Graph showing deviation due to table load]

**Fig. 5.12** Result of the holeplate measurement simulation on position 1 in the *XY-plane*
The same comparison is carried out for position 8 in the XY-plane (figures 5.6 & 5.7). The predicted deviation (max. $\pm 25 \, \mu m$), depicted in figure 5.13, is of the same magnitude as the predicted deviation based on the geometric errors (max. $\pm 30 \, \mu m$). Comparing the simulations on position 3 in the XZ-plane and position 4 in the YZ-plane (figures 5.14 & 5.15), yields the conclusion that in these planes the finite stiffness errors have a smaller contribution to the resulting error of the machine tool than the geometric errors sec.

![Graph showing deviation due to table load](image)

**Fig. 5.13** Result of the holeplate measurement simulation on position 8 in the XY-plane
Fig. 5.14 Result of the holeplate measurement simulation on position 3 in the XZ-plane

Fig. 5.15 Result of the holeplate measurement simulation on position 4 in the YZ-plane
Considering above presented results, the finite stiffness errors due to a load on the workpiece table are significant, especially in the XY-plane, where they are larger than the geometric errors sec. As workpieces can be placed upon the milling machine having different weights, the change in geometric errors will be dependent of the weight of these workpieces. The developed error model, however, is capable of modelling the change in geometric errors for different loads. In order to validate the error model, both simulations and verification measurements will be carried out for different workpieces. This validation will be carried out during next period of the projects duration.

It turned out that the developed error model is very well capable of modelling errors due to the finite stiffness of the milling machine. Also the simulation program can evaluate the effect of the finite stiffness errors. Thus a software error correction will be developed for these kind of errors. The results of this error correction will be presented in the final project report.
6. Conclusions

The main goal of this BCR-project is the improvement of the accuracy of a commercially available machine tool. Therefore, the most influential error sources will be investigated: basic geometric errors, thermal behaviour of the machine's structure and finite stiffness effects. Based on this research a correction algorithm will be developed and implemented. In order to achieve these aims of the project several workpackages have been defined, including a time-schedule. Four workpackages are finished now (April 1991), which corresponds with this time-schedule.

First, bibliographical studies are carried out, to determine the state of the research at other institutes. Based on this research, the main goal of this BCR-project has been defined: to improve the accuracy of a commercially available machine tool. This will mainly be achieved by correction for thermally induced errors, which contribute, together with the geometric errors, to more than seventy percent of the resulting error of a machine tool.

Secondly, a classification of multi-axis machine tools has been developed. With this classification it is possible to distinguish different types of five axis milling machines that manifest the same kind of errors. With the knowledge of the type dependent errors the definition of a calibration setup and the error structure of the machine are significantly simplified.

Thirdly, to make the exchange of information possible, a standard data format has been defined, measurement programs and interfaces have been developed.

The last workpackage that has been finished is the development and validation of the geometric error model. This general model, developed by TUE, relates the errors in the location of the tool, with respect to the workpiece, to errors in the location of coordinate frames attached to succeeding components of the machine. With this general model it is not only possible to model a specific machine tool, like the Maho milling machine under research, but it can be applied to any multi axis machine, composed of rotary and linear elements in an arbitrary serial configuration.
The general model has been elaborated to the individual model, which describes the geometric error structure of the Maho milling machine. Therefore all geometric error components have been measured, using the developed direct measuring techniques. These measuring techniques turned out to be a very accurate and flexible method to determine all geometric error components, but time consuming.

Also the change in geometric errors has been investigated, when a load (workpiece) is placed upon the table of the milling machine. Based on this change in geometric errors, simulations are carried out, using the developed simulation software.

Considering the results of the simulations, the finite stiffness effects due to a load on the workpiece table cause a significant change in the resulting error of the tool with respect to the workpiece.

In order to verify the developed model, a totally different approach to determine the geometric errors of the milling machine is applied.

PTB developed a method to determine the geometric performance of coordinate measuring machines, using a holeplate. This holeplate is applied to the Maho milling machine under research, using this machine as a measuring machine. As the actual location of each hole is known, it is possible to calculate the geometric error induced by the milling machine. This method turned out to be a very fast method to determine the geometric performance of a machine, which is able to measure the difference in the location of two holes. However, this method is very dependent on the repeatability of the used probe system and is limited in its use to machines with a relatively small range.

The above mentioned holeplate method is simulated, using the individual model. During comparison of this simulation with the actual measurements, it appeared that the simulations correspond very well with the measurements. Therefore, it can be concluded that the used modelling technique, together with the applied measurement techniques, prove to describe the error structure of the milling machine in a correct way. As the results are promising, the developed model will be used as a basic tool for software error compensation.
The final version of the software error correction will not only correct the geometric errors of the milling machine, but also the thermally induced errors. Therefore, the research activities on this subject are already initiated. The first results of the statistical approach, investigated at TUE are very promising for the final version of a software error compensation and therefore used by Philips for a first version of an error correction. PTB has started the development of an analytical model to describe the thermomechanical behaviour of a milling machine. The first concepts of these models are already discussed with the partners. Contrary to TUE, who uses direct measurement techniques, PTB will continue their investigations on the data acquisition using workpieces. These workpieces turned out to be a very suitable method to determine the error components of a milling machine. Thereby, they will be used for the validation of the final software error compensation and for the development of the analytical thermomechanical model. During the first period, Maho supported all partners with practical information concerning the improvement of the accuracy of milling machines. They also delivered hard- and software to utilise the milling machine for dedicated research tasks, such as measuring the holeplate. They will continue this support during the next period. Using the available results of the research carried out by the partners, Philips has started the development of a first version of a software error compensation. This error correction will correct the most significant contributions of the geometric errors and the thermomechanical behaviour, using the first results of the statistical approach. Based on the experience gathered with this research, the final version of the software compensation will be developed.
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A. Elaboration of the general model to the type dependent model

In paragraph 3.4 the following general equation is derived for the expression of the location error of the tool as a function of the location errors of subsequent coordinate frames:

\[
wpE_{tl} = - \sum_{k=1}^{m} (tl^{F_{bk}} bk^{-1} E_{bk}) + \sum_{k=1}^{n} (tl^{F_{ak}} ak^{-1} E_{ak}) + anE_{tl}
\]  \[A.1\]

where:

\[
wpE_{tl} = \begin{bmatrix}
wp^e_{tx} & wp^e_{ty} & wp^e_{tz} & wp^e_{tx}' & wp^e_{ty}' & wp^e_{tz}'
\end{bmatrix}^T
\]  \[A.2\]

This vector represents the errors of the tool with respect to the workpiece, defined in the tool-frame.

\[
k-1E_k = \begin{bmatrix}
k-1^e_{xx} & k-1^e_{xy} & k-1^e_{xz} & k-1^e_{xx}' & k-1^e_{xy}' & k-1^e_{xz}'
k-1^e_{xy} & k-1^e_{yy} & k-1^e_{yz} & k-1^e_{xy}' & k-1^e_{yy}' & k-1^e_{yz}'
k-1^e_{xz} & k-1^e_{yz} & k-1^e_{zz} & k-1^e_{xz}' & k-1^e_{yz}' & k-1^e_{zz}'
\end{bmatrix}^T
\]  \[A.3\]

This vector represents the errors in the location of frame \(k\) with respect to frame \(k-1\), i.e. the geometric errors of the kinematic element \(k\).

\[
tl^{F_k} = \begin{bmatrix}
tl^{R_k} & 0 \\
(tl^{t_k} \times tl^{R_k}) & tl^{R_k}
\end{bmatrix} \quad (6 \times 6) \text{ matrix}
\]  \[A.4\]

This matrix denotes the effect of the errors \(k-1E_k\), acting between the elements \(k-1\) and \(k\), on the resulting error between tool and workpiece. Here \(tl^{t_k} \times tl^{R_k}\) denotes a \(3 \times 3\) matrix whose columns contain the vector product of vector \(tl^{t_k}\) with the respective columns of matrix \(tl^{R_k}\).

Underlying this formula will be elaborated for the milling machine under research. Thereby the explanation of the different terms, as presented in paragraph 3.4, is supposed to be known.
The five axis milling machine, under research (figure A.1) consists of one horizontal linear element and one rotary element in chain 'a' from foundation to tool. Chain 'b' from foundation to workpiece consists of two linear elements, one vertical and one horizontal, and one rotary element with a vertical axis of rotation. In the first stage of the modelling process, coordinate frames are located in the workpiece, the tool and in the centroid of each joint. In figure A.2 the kinematic representation of this milling machine is depicted. Note that the length of the tool is characterized by introducing a variable 'L'.

The frames located in the various kinematic elements can be characterized as:

- Frame tl : fixed
- Frame a2 : fixed
- Frame a1 : fixed
- Frame wp : fixed
- Frame b3 : fixed
- Frame b2 : moving
- Frame b1 : moving

The frame of the workpiece is thought to be at the same location as frame b3.
Fig. A.2 Kinematic representation of the five axes milling machine under research

Application of the above presented formulas, and abbreviation of \( \cos(q) \) and \( \sin(q) \) to 'cq' and 'sq' respectively, results in the expression of the nominal coordinate transformations between succeeding frames as:

\[
T_{a_2t_1} = \text{Fixed} \rightarrow \text{Fixed} = J_{a_2a_2}S_{t_1} = \begin{bmatrix} cqa_2 & -sqa_2 & 0 & 0 \\ sqa_2 & cqa_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 200+L \\ 0 & 0 & 1 & -140 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
= \begin{bmatrix} cqa_2 & -sqa_2 & 0 & -(200+L)sqa_2 \\ sqa_2 & cqa_2 & 0 & (200+L)cqa_2 \\ 0 & 0 & 1 & -140 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[\text{[A.5]}\]
\[ a_1 T_{a2} = \text{Fixed} \rightarrow \text{Fixed} = J_{a2} a_1 S_{a2} \]
\[
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & qa_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -210 \\
0 & 0 & 1 & -665 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
\[
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -210 \\
0 & 0 & 1 & -665 + qa_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[ a_1 T_{a1} = \text{Moving} \rightarrow \text{Fixed} = S_{a1} \]
\[
= \begin{bmatrix}
1 & 0 & 0 & 350 \\
0 & 1 & 0 & -95 \\
0 & 0 & 1 & 495 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[ a_1 T_{b1} = \text{Moving} \rightarrow \text{Moving} = S_{b1} J_{b1} \]
\[
= \begin{bmatrix}
1 & 0 & 0 & 350 \\
0 & 1 & 0 & 327.5 \\
0 & 0 & 1 & 515 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & qb_1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
\[
= \begin{bmatrix}
1 & 0 & 0 & 350 \\
0 & 1 & 0 & 327.5 + qb_1 \\
0 & 0 & 1 & 515 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[ b_1 T_{b2} = \text{Moving} \rightarrow \text{Moving} = S_{b2} J_{b2} \]
\[
= \begin{bmatrix}
1 & 0 & 0 & -350 \\
0 & 1 & 0 & 67.5 \\
0 & 0 & 1 & -140 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & qb_2 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
\[
= \begin{bmatrix}
1 & 0 & 0 & -350 + qb_2 \\
0 & 1 & 0 & 67.5 \\
0 & 0 & 1 & -140 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
These transformation matrices can be used to express the nominal position of the tool-frame relative to the workpiece-frame. As the elaboration of equation [A.1] requires the construction of the F-matrices out of $\mathbf{R}_k$ and $\mathbf{t}_k$ for all kinematic elements, the first step is to calculate the $\mathbf{T}_k$ matrices. From these $\mathbf{T}_k$ matrices, the required $\mathbf{R}_k$ and $\mathbf{t}_k$ matrices can be extracted (see equation [A.12]).

Application of expression [3.5] onto expressions [A.5] to [A.11] yields the following matrices:

\[
\begin{align*}
\mathbf{b}_2^T \mathbf{b}_3 &= \text{Moving} \rightarrow \text{Fixed} = \mathbf{b}_2^T \mathbf{b}_3 \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -395 \\ 0 & 0 & 1 & -375 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{[A.10]} \\
\mathbf{b}_3^T \mathbf{w}_p &= \text{Fixed} \rightarrow \text{Fixed} = \mathbf{b}_3^T \mathbf{w}_p \\
&= \begin{bmatrix} \cos \beta_3 & 0 & \sin \beta_3 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta_3 & 0 & \cos \beta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos \beta_3 & 0 & \sin \beta_3 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta_3 & 0 & \cos \beta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{[A.11]}
\end{align*}
\]

Application of expression [3.5] onto expressions [A.5] to [A.11] yields the following matrices:

\[
\begin{align*}
a^T_{21} & = \begin{bmatrix} \cos \alpha_2 & \sin \alpha_2 & 0 & 0 \\ -\sin \alpha_2 & \cos \alpha_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & \quad -a^T_{21} \mathbf{t}_{a2} = \begin{bmatrix} 0 \\ -(200+L) \\ 140 \end{bmatrix} \\
\implies \mathbf{T}_{a2} &= \begin{bmatrix} \cos \alpha_2 & \sin \alpha_2 & 0 & 0 \\ -\sin \alpha_2 & \cos \alpha_2 & 0 & -(200+L) \\ 0 & 0 & 1 & 140 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{[A.12]} \\
a^T_{12} & = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, & \quad -a^T_{12} \mathbf{t}_{a1} = \begin{bmatrix} 0 \\ -210 \\ 665-\alpha_1 \end{bmatrix} \\
\implies \mathbf{T}_{a1} &= \begin{bmatrix} \cos \alpha_2 & \sin \alpha_2 & 0 & 210.\sin \alpha_2 \\ -\sin \alpha_2 & \cos \alpha_2 & 0 & 210.\cos \alpha_2 - (200+L) \\ 0 & 0 & 1 & 805-\alpha_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{[A.13]}
\end{align*}
\]
\[
\begin{align*}
0^R_{a_1} &= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}, \\
-0^R_{a_1''a_1} &= \begin{bmatrix}
-350 \\
95 \\
-495 \\
\end{bmatrix}
\end{align*}
\]

\[u_0^T = u_{a_1'a_1} T_0 \quad \text{[A.14]}\]

\[
\begin{bmatrix}
cqa2 & sqa2 & 0 & -350.cqa2 + 95.sqa2 + 210.sqa2 \\
-sqa2 & cqa2 & 0 & 350.sqa2 + 95.cqa2 + 210.cqa2 - (200+L) \\
0 & 0 & 1 & 310 - qa1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[u_{b1}^T = u_{b1'b1} T_{b1} \quad \text{[A.15]}\]

\[
\begin{bmatrix}
cqa2 & sqa2 & 0 & 632.5.sqa2 + qb1.sqa2 \\
-sqa2 & cqa2 & 0 & 632.5.cqa2 + qb1.cqa2 - (200+L) \\
0 & 0 & 1 & 825 - qa1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[u_{b2}^T = u_{b2'b2} T_{b2} \quad \text{[A.16]}\]

\[
\begin{bmatrix}
cqa2 & sqa2 & 0 & 700.sqa2 + qb1.sqa2 + (-350+qb2).cqa2 \\
-sqa2 & cqa2 & 0 & 700.cqa2 + qb1.cqa2 - (-350+qb2).sqa2 - (200+L) \\
0 & 0 & 1 & 685 - qa1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[u_{b3}^T = u_{b3'b3} T_{b3} \quad \text{[A.17]}\]

\[
\begin{bmatrix}
cqa2 & sqa2 & 0 & 305.sqa2 + qb1.sqa2 + (-350+qb2).cqa2 \\
-sqa2 & cqa2 & 0 & 305.cqa2 + qb1.cqa2 - (-350+qb2).sqa2 - (200+L) \\
0 & 0 & 1 & 310 - qa1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[u_{wp}^T = u_{b3'b3} T_{wp} \quad \text{[A.18]}\]

\[
\begin{bmatrix}
cqa2 & cqb3 & sqa2 & cqa2.sqb3 \\
-sqa2 & cqb3 & cqa2 & -sqa2.sqb3 \\
-sqb3 & 0 & cqb3 \\
0 & 0 & 0 & \\
305.sqa2 + qb1.sqa2 + (-350+qb2).cqa2 \\
305.cqa2 + qb1.cqa2 - (-350+qb2).sqa2 - (200+L) \\
310 - qa1 \\
1 \\
\end{bmatrix}
\]
The next step is the determination of the F-matrices that describe the effect of the individual errors, between the coordinate frames, on the total error between tool and workpiece. From the above calculated transformation matrices we can deduce that (see equation [3.2]):

\[
\begin{bmatrix}
  cqa2 & sqa2 & 0 \\
  -sqa2 & cqa2 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]

\[\text{[A.19]}\]

\(t_{\text{R}_w}^R\) contains the additional transformation of the rotary element \(b_3\). However, as we state that the error between frame \(b_3\) and frame \(wp\) is zero, the obtained term for equation [A.1] will automatically yield a contribution of nil. Therefore it is not necessary to calculate the \(F_{wp}\) matrix.

For the elaboration of the relevant F-matrices the vectors \(t_{\text{Ik}}^t\) can be calculated as:

\[
\begin{align*}
  t_{\text{a}_2}^t &= \begin{bmatrix} 0 \\ -(200+L) \\ 140 \end{bmatrix} = \begin{bmatrix} ta_{2x} \\ ta_{2y} \\ ta_{2z} \end{bmatrix} \quad [A.20] \\
  t_{\text{a}_1}^t &= \begin{bmatrix} 210.sqa2 \\ 210.cqa2 - (200+L) \\ 805 - qa1 \end{bmatrix} = \begin{bmatrix} ta_{1x} \\ ta_{1y} \\ ta_{1z} \end{bmatrix} \quad [A.21] \\
  t_{0}^t &= \begin{bmatrix} -350.cqa2 + 305.sqa2 \\ 350.sqa2 + 305.cqa2 - (200+L) \\ 310 - qa1 \end{bmatrix} = \begin{bmatrix} t0x \\ t0y \\ t0z \end{bmatrix} \quad [A.22] \\
  t_{b1}^t &= \begin{bmatrix} 632.5.sqa2 + qb1.sqa2 \\ 632.5.cqa2 + qb1.cqa2 - (200+L) \\ 825 - qa1 \end{bmatrix} = \begin{bmatrix} tb_{1x} \\ tb_{1y} \\ tb_{1z} \end{bmatrix} \quad [A.23] \\
  t_{b2}^t &= \begin{bmatrix} 700.sqa2 + qb1.sqa2 + (-350+qb2).cqa2 \\ 700.cqa2 + qb1.cqa2 - (-350+qb2).sqa2 - (200+L) \\ 685 - qa1 \end{bmatrix} = \begin{bmatrix} tb_{2x} \\ tb_{2y} \\ tb_{2z} \end{bmatrix} \quad [A.24] \\
  t_{b3}^t &= \begin{bmatrix} 305.sqa2 + qb1.sqa2 + (-350+qb2).cqa2 \\ 305.cqa2 + qb1.cqa2 - (-350+qb2).sqa2 - (200+L) \\ 310 - qa1 \end{bmatrix} = \begin{bmatrix} tb_{3x} \\ tb_{3y} \\ tb_{3z} \end{bmatrix} \quad [A.25]
\]
The matrix $F_k$ is defined as (see equation [A.3]):

$$
F_k = \begin{bmatrix}
R_k & 0 \\
(\mathbf{t}_k \times R_k) & R_k
\end{bmatrix}
$$

The outproduct of the vectors $\mathbf{t}_k$ and the matrix $R_k$ can be summarized as:

$$
\begin{bmatrix}
t_k z_2 - t_k x_2 & t_k y_1 \\
t_k x_2 & t_k z_2
\end{bmatrix}
\begin{bmatrix}
t_k x_2 \\
t_k z_2
\end{bmatrix}
$$

with $k = a2, a1, b1, b2 b3$

Implementation of this relation into expression [A.3] yields the following general $F$-matrix:

$$
\begin{bmatrix}
c_{a2} & s_{a2} & 0 & 0 & 0 & 0 \\
-s_{a2} & c_{a2} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
t_k z_2 s_{a2} & -t_k z c_{a2} & t_k y_2 c_{a2} & s_{a2} & 0 & 0 \\
t_k z_2 c_{a2} & t_k z_2 s_{a2} & -t_k x_2 & -s_{a2} & c_{a2} & 0 \\
(-t_k x_2 s_{a2} - t_k y_2 c_{a2}) (t_k c_{a2} 2 - t_k y_2 s_{a2}) & 0 & 0 & 0 & 1
\end{bmatrix}
$$

The index $k$ indicates the concerning coordinate frame, i.e. $a2, a1, b1, b2$ or $b3$

Application of relation [A.1], yields the following expression for the errors $e_{w_{p_t}}$ and $e_{w_{p_t}}$ in the orientation and position of the tool coordinate frame with respect to the workpiece coordinate frame:

### Orientation errors

$$
\begin{align*}
\mathbf{w} & = a_2^k e_{w_{p_t}} + \begin{bmatrix}
c_{a2} & s_{a2} & 0 \\
-s_{a2} & c_{a2} & 0 \\
0 & 0 & 1
\end{bmatrix} a_1 e_{a_2} + \begin{bmatrix}
c_{a2} & s_{a2} & 0 \\
-s_{a2} & c_{a2} & 0 \\
0 & 0 & 1
\end{bmatrix} e_{a_1} - \begin{bmatrix}
c_{a2} & s_{a2} & 0 \\
-s_{a2} & c_{a2} & 0 \\
0 & 0 & 1
\end{bmatrix} \mathbf{b}_1 e_{b_1} \\
- \begin{bmatrix}
c_{a2} & s_{a2} & 0 \\
-s_{a2} & c_{a2} & 0 \\
0 & 0 & 1
\end{bmatrix} b_1 e_{b_2} - \begin{bmatrix}
c_{a2} & s_{a2} & 0 \\
-s_{a2} & c_{a2} & 0 \\
0 & 0 & 1
\end{bmatrix} b_2 e_{b_3}
\end{align*}
$$
Position errors

\[ wp_{\text{ul}} = a_{\text{ul}}^e \]

\[ \begin{bmatrix}
140. \cdot \text{sqa}_2 & -140. \cdot \text{cqa}_2 & -(200+L) \cdot \text{cqa}_2 & \text{sqa}_2 & 0 \\
140. \cdot \text{cqa}_2 & 140. \cdot \text{sqa}_2 & 0 & -\text{sqa}_2 & \text{cqa}_2 & 0 \\
(200+L) \cdot \text{cqa}_2 & (200+L) \cdot \text{sqa}_2 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\text{sqa}_2 \\
\text{cqa}_2 \\
\text{sqa}_2 \\
\text{cqa}_2 \\
0 \\
0
\end{bmatrix} = a_1^E a_2^E 
\]

\[ \begin{bmatrix}
(805-\text{qa}_1) \cdot \text{sqa}_2 & -(805-\text{qa}_1) \cdot \text{cqa}_2 \\
(805-\text{qa}_1) \cdot \text{cqa}_2 & (805-\text{qa}_1) \cdot \text{sqa}_2 \\
-(210 \cdot \text{sqa}_2)-(210 \cdot \text{cqa}_2-(200+L)) \cdot \text{cqa}_2 & -(210 \cdot \text{sqa}_2)-(210 \cdot \text{cqa}_2-(200+L)) \cdot \text{sqa}_2
\end{bmatrix} \begin{bmatrix}
\text{cqa}_2 \\
\text{sqa}_2 \\
\text{sqa}_2 \\
\text{cqa}_2 \\
0 \\
0
\end{bmatrix} = E_{a1} 
\]

\[ \begin{bmatrix}
(825-\text{qa}_1) \cdot \text{sqa}_2 \\
(825-\text{qa}_1) \cdot \text{cqa}_2 \\
-(632.5 \cdot \text{sqa}_2 + \text{qb}_1 \cdot \text{sqa}_2) \cdot \text{sqa}_2 & -(632.5 \cdot \text{cqa}_2 + \text{qb}_1 \cdot \text{cqa}_2 - (200+L)) \cdot \text{cqa}_2 \\
-(825-\text{qa}_1) \cdot \text{cqa}_2 \\
(825-\text{qa}_1) \cdot \text{sqa}_2 \\
(632.5 \cdot \text{sqa}_2 + \text{qb}_1 \cdot \text{sqa}_2) \cdot \text{cqa}_2 & -(632.5 \cdot \text{cqa}_2 + \text{qb}_1 \cdot \text{cqa}_2 - (200+L)) \cdot \text{sqa}_2
\end{bmatrix} \begin{bmatrix}
\text{cqa}_2 \\
\text{sqa}_2 \\
\text{cqa}_2 \\
\text{sqa}_2 \\
0 \\
0
\end{bmatrix} = E_{a1} 
\]
\[
\begin{bmatrix}
(685-\text{qa}1)\text{sqa}2 \\
(685-\text{qa}1)\text{cqa}2 \\
-(700\text{sqa}2 + \text{qb1sqa}2 + (-350 + \text{qb2cqa}2)\text{sqa}2 - (200+L))\text{cqa}2 \\
(700\text{cqa}2 + \text{qb1cqa}2 - (-350 + \text{qb2sqa}2 - (200+L))\text{sqa}2 \\
-(685-\text{qa}1)\text{cqa}2 \\
(685-\text{qa}1)\text{sqa}2 \\
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(700\text{cqa}2 + \text{qb1cqa}2 - (-350 + \text{qb2sqa}2 - (200+L))\text{sqa}2 \\
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\[[A.29]\]
B. Acquisition of the geometric errors with direct measurements

Underlying the measurement setups for the geometric errors and the obtained results are depicted. These measurement data are gathered using the software package described in paragraph 4.2. All data is available on floppy and therefore easy to assess for the software, used for fitting the measurement data.

The results display the bare measurement data which implies that no correction for thermal expansion, nor for influences of rotations, is carried out.

All measurement equipment, used to determine the geometric error components, is calibrated at Metrology Laboratory of Eindhoven University, which is certified by the Dutch Calibration Organisation [NKO-014]. The used instruments are:

- HP 5528 laserinterferometer with accompanying optics, air-sensor and temperature sensors;
- Wyler electronic levelmeters;
- Hilger Watts autocollimator;
- Hilger Watts polygon;
- Ceramic square;
- Inductive displacement transducers;
- Sipp rotary table.

B.1 Measurements of the X-axis

For the measurement of the scale error $x_{tx}$, a laserinterferometer is used with accompanying linear optics. The measurement uncertainty associated with this option of the laserinterferometer is less than $0.05 + 0.5 \times L \text{ } \mu m$ ($L$ in m).

The measurement setup is depicted in figure B.1. The interferometer is mounted to the ram of the machine tool, while the retroreflector is connected to the workpiece table, which performs the movement in X-direction. The influences of the rotation $x_{ry}$ and $x_{rz}$ have to be eliminated from the obtained measurement results. This yields the error $x_{tx}$ of the coordinate frame positioned in the centroid of the X-carriage.

Execution of the described measurement yields the results as graphically depicted in figure B.2. The error in the position of the machine, in this case $x_{tx}$, is defined as:

$$\text{Error} = \text{True displacement (laser)} - \text{Assigned displacement (machine)}$$  \hspace{1cm} [B.1]
Fig. B.1 Measurement setup for xtx

Fig. B.2 Error xtx versus position of the X-carriage

For the measurements of the rotation of the X-axis, the laser interferometer, with accompanying angular optics, is used for determination of xry and xrx, while the error xrx is determined by a set of electronic levelmeters. This option of the laser interferometer introduces an uncertainty less than 0.2 arcsec (1 arcsec = 4.8e-6 rad). The electronic levelmeters has an uncertainty less than 0.5 arcsec.
The measurement setup for $x_{rx}$ is depicted in figure B.3. The reference levelmeter is mounted on the ram of the machine tool, thereby eliminating the effect of rotation of the overall machine structure, while the measurement levelmeter is placed on the workpiece table, which performs the movement in X-direction. The rotation error does not depend on the position of measurement, so the obtained results directly reflect the rotation error between the coordinate frames attached to the X- and Y-axis respectively.

![Fig. B.3 Measurement setup for $x_{rx}$](image)

Execution of the described measurement yields the results that are graphically depicted in figure B.4. The rotation error of the machine is defined in arcsec.

![Fig. B.4 Error $x_{rx}$ versus position of the X-carriage](image)
In the results of xrx, peaks can be observed with magnitudes of the same order as the measured error. Plotting the measurements sequentially yields to the conclusion that the peak error repeats every 30 minutes. In figure B.5 the measurement results of xrx are depicted together with a quasi timescale. From an inspection of the machine constants it appeared that the periodic peaks are induced by the lubrication pump of the machine tool, that is activated every 30 minutes.

Fig. B.5 Results of measurement of xrx sequentially and time indicator of 30 minutes

The measurement setups for xry and xrz are depicted in figure B.6 and B.7. The reference interferometer is mounted to the ram of the machine tool while the retroreflector is connected to the workpiece table, which performs the movement in X-direction. The results of these measurements are presented in figure B.8 and B.9. Also these results directly reflect the rotation errors between the coordinate frames of the X- and Y-axis respectively.
Fig. B.6 Measurement setup for xry

Fig. B.7 Measurement setup for xrz
Measurement of $xry$

20 measurements

Position of the x-axis [mm]

Fig. B.8 Error $xry$ versus position of the X-carriage

Measurement of $xrz$

20 measurements

Position of the x-axis [mm]

Fig. B.9 Error $xrz$ versus position of the X-carriage
B.2 Measurements of the Y-axis
For the measurement of the scale error \( y_{ty} \) the laser interferometer is applied with accompanying linear optics. The measurement setup is depicted in figure B.10. The interferometer is mounted to the ram of the machine tool, while the retroreflector is placed on the workpiece table, which performs the movement in Y-direction. The influences of the rotation \( y_{rx} \) and \( y_{rz} \) have to be eliminated from the obtained measurement results. This yields the error \( y_{ty} \) of the coordinate frame positioned in the centroid of the Y-carriage.

![Diagram of measurement setup for \( y_{ty} \)](image)

*Fig. B.10 Measurement setup for \( y_{ty} \)*

Execution of the described measurement yields the results that are graphically depicted in figure B.11.
Fig. B.11 Error yty versus position of the Y-carriage

Several instruments are applied to measure the rotation errors of the Y-axis. First, for the determination of the error yrx, the laser interferometer is applied. The setup for this measurement is depicted in figure B.12. The interferometer is mounted to the ram of the machine tool, while the retroreflector is placed on the workpiece table, which performs the movement in Y-direction. The obtained results directly reflect the rotation error between the frame attached to the Y-axis and the machine coordinate frame.

Fig. B.12 Measurement setup for yrx
Execution of the described measurement yields the results that are graphically depicted in figure B.13. The rotation error of the machine is given in arcsec.

![Measurement of \( \gamma_{rx} \)](image)

**Fig. B.13 Error \( \gamma_{rx} \) versus position of the Y-carriage**

The measurement results of \( \gamma_{rx} \) show identical peaks as the results of \( \gamma_{rx} \). This is also caused by the lubrication pump that operates every 30 minutes.

The measurement setup for \( \gamma_{ry} \) is depicted in figure B.14. While this measurement cannot be carried out by a laser interferometer nor a set of levelmeters, two straightness measurements are performed. The straightness measurement is performed by movement of a straight-edge, in this case a calibrated surface of a squareness reference block, along a displacement transducer. The reading of the transducer is a measure of the error \( \gamma_{tz} \) with a contribution of \( \gamma_{ry} \).
By choosing the setup as depicted in figure B.14 the effect of the error yry in the measurement result will reverse sign between measurement 1 and 2. With the necessary displacement in X-direction between the measurements the change in active arm is known. The results of these measurements have to be corrected for the effect of the error xrx on the orientation of the straight-edge. This error results in a displacement Y*rx which is not caused by the Y-axis.

Taking the above procedure the error yry can be calculated as:

\[
yry = \frac{ytz(2) - ytz(1) - Y*rx}{x(1) - x(2)} \quad \text{[Rad]}
\]

with:
- ytz(2) and ytz(1) are the uncorrected results of the straightness measurement;
- x(1) and x(2) are the positions of the X-carriage during the respective straightness measurements;
- Y is the position of the Y-carriage during the straightness measurements.
Execution of the described measurement yields the bare measurement results that are graphically depicted in figure B.15a. In figure B.15b the calculated error $y_{ry}$ is presented.

![Graph showing two measurements of $y_{tz}$ and calculated $y_{ry}$](image)

**Fig. B.15a** Measured errors $y_{tz}$ on two positions of the X-carriage  
**B.15b** Calculated $y_{ry}$ from straightness measurements

The measurement setup for $y_{rz}$ is depicted in figure B.16. The reference interferometer is mounted to the ram of the machine tool, while the retroreflector is connected to the workpiece table, which performs the movement in Y-direction. Also these results directly reflect the rotation errors between the coordinate frame attached to the Y-axis and the machine coordinate frame.
Fig. B.16 Measurement setup for \( \text{yrz} \)

Fig. B.17 Error \( \text{yrz} \) versus position of the Y-carriage
B.3 Measurements of the Z-axis
For the measurement of the scale error ztz, the laser interferometer is applied with accompanying linear optics.
The measurement setup is depicted in figure B.18. The interferometer is mounted on the workpiece table, while the retroreflector is connected to the ram of the machine tool. The influences of the rotation zrx and zry have to be eliminated from the obtained measurement results. This yields the error ztz of the coordinate frame positioned in the centroid of the Z-carriage.

![Fig. B.18 Measurement setup for ztz](image)

Execution of the described measurement yields the results that are graphically depicted in figure B.19. In the results of ztz a clear form of hysteresis can be observed. This is not caused by the hardware of the machine, but purely by a reproducing temperature field over the Z-scale. In figure B.20 the temperature on three positions of the Z-scale is depicted. These temperatures were obtained during the measurement of ztz. The second graph in figure B.19 represents the error corrected for these thermal effects. Clearly the hysteresis has disappeared.
Fig. B.19 Error zTz versus position of the Z-carriage

Fig. B.20 Temperature of the Z-scale during measurement of zTz
Several instruments are applied to measure the rotation errors of the Z-axis. First, for the determination of the error $z_{rx}$, we applied the laser interferometer with the angular optics. The setup for this measurement is depicted in figure B.21. The interferometer is placed on the workpiece table while the angular retroreflector mounted on the ram of the machine tool. The obtained results directly reflect the rotation error between the machine coordinate frame and the frame attached to the Z-axis.

![Measurement setup for $z_{rx}$](image)

**Fig. B.21 Measurement setup for $z_{rx}$**

Execution of the described measurement yields the results that are graphically depicted in figure B.22. The rotation error of the machine is given in arcsec.
Fig. B.22 Error zrx versus position of the Z-carriage

The measurement setup for zry is depicted in figure B.23. This measurement is also carried out by a laser interferometer and angular optics. The same setup is used for the determination of zrx, except the optics are rotated about the Z-axis over ninety degrees.

Fig. B.23 Measurement setup for zry
By choosing this measurement setup the results directly reflect the error \( z_{ry} \). These results are graphically depicted in figure B.24. The rotation error of the machine is given in arcsec.

![Measurement of \( z_{ry} \)](image)

**Fig. B.24 Error \( z_{ry} \) versus position of the Z-carriage**

The measurement setup for \( z_{rz} \) is depicted in figure B.25. The reference levelmeter is mounted on the workpiece table, while the measurement levelmeter is connected to the ram of the machine tool. Also these results (figure B.26) directly reflect the rotation errors between the coordinate frame attached to the Z-axis and the machine coordinate frame.
Fig. B.25 Measurement setup for zrz

Fig. B.26 Error zrz versus position of the Z-carriage
B.4 Straightness measurements

The straightness measurements are carried out with a ceramic square and inductive displacement sensors. The results of these measurements can be used for both the analysis of the straightness errors and the analysis of the squareness errors of the milling machine. The straightness error is defined as the difference between the actual data and its least-square. In figure B.27 the result of a straightness measurement is depicted, after the measurement data are corrected for the error induced by the square. This result show that the machine has a very small straightness error.

Fig. B.27 Straightness error \( ztx \)

As the straightness measurements are carried out during the determination of the squareness errors, the results of these measurements will be presented in the next paragraph, together with the results of the squareness measurements.
B.5 Determination of the Squareness Errors

The squareness error between the carriages is determined by measurement of a ceramic reference block. This block is calibrated on a 3D measuring machine and possesses a squareness error of +1.7 arcsec. The measurements are carried out using inductive displacement transducers.

To measure the squareness error two straightness measurements are carried out. To calculate the squareness error, the pure straightness error will be eliminated by Least-Square fitting.

The squareness errors are included in the model as offsets of rotation errors i.e. they will not be treated as separate geometric errors in the model.

Underlying the three different squareness measurements are described and the measurement results presented.

**Squareness error between the X- and Z-guide**

For this measurement the reference block is placed on the machine table. The block is supported on three points. The block is aligned along the X-axis to secure that the displacement transducers remain in their calibrated range (0-2 mm). First, the displacement transducer is mounted on the ram of the machine tool and the reference side of the block is measured, yielding the error xtz and the alignment error of the block (figure B.28 and B.29).

![Fig. B.28 Measurement setup of xtz (top view of machine tool)](image-url)
Secondly, the other side of the block is measured yielding the error \( z_{tx} \) and an error that includes both the alignment error of the block, the squareness error in the block and the squareness error of the machine tool (figures B.30 and B.31).

**Fig. B.29 Measurement results of \( z_{tx} \)**

**Fig. B.30 Measurement setup of \( z_{tx} \) (top view of machine tool)**
Out of these measurement results the following conclusions can be drawn. The positive angle between the best fitted line through of the measurement results and the X-axis is explained by an alignment error of the block, whereas the block is rotated about the Y-axis.

The measurements indicate an alignment error ($\alpha$) of 0.7 arcsec. The second measurement yields an alignment error ($\beta$) of 2.1 arcsec. The total uncorrected out of squareness, defined as the actual angle included by the guides minus 90 degrees, is in this case calculated by: $-\beta - \alpha$, thus -2.8 arcsec. However, the squareness error of the block, i.e. 1.7 arcsec, must be added to this result to achieve the total squareness error of the Z-axis with respect to the X-axis. Applying this correction yields a squareness error between the Z- and the X-axis of -1.1 arcsec.

**Squareness error between the X- and Y-guide**
Again the reference block is supported by the machine table and aligned along the X-axis. First, the displacement transducer is mounted on the ram of the machine tool and the reference side of the block is measured, yielding the error $xy$ and the alignment error of the block (figure B.32 and B.33).
Fig. B.32 Measurement setup of xty (front view of machine tool)

Fig. B.33 Measurement results of xty

Secondly, the other side of the block is measured yielding the error ytx and an error that includes both the alignment error of the block, the squareness error in the block and the squareness error of the machine tool (figure B.34 and B.35).
Fig. B.34 Measurement setup of ytx (front view of machine tool)

Fig. B.35 Measurement results of ytx

From these measurement results the following conclusions can be drawn. The positive angle between the best fitted line through the measurement results and the X-axis is explained by an alignment error of the block whereas the block is rotated around the Z-axis.

The measurements indicate an alignment error ($\alpha$) of $-0.1$ arcsec.

The second measurement yields an alignment error ($\beta$) of $+12.6$ arcsec. The total uncorrected out of squareness is in this case calculated by: $-\beta - \alpha$, thus yielding $-12.5$ arcsec. However, the squareness error of the block, i.e. 1.7 arcsec, must be added to this result, so the total squareness error of the Y-axis with respect to the X-axis is $-10.8$ arcsec.
Squareness error between the Y- and Z-guide
Again the reference block is supported by the machine table. The block is now aligned along the Z-axis. First, the reference side of the block is measured, yielding the error $z_{xy}$ and the alignment error of the block (figure B.36 and B.37).

![Figure B.36 Measurement setup of zty (side view of machine tool)](image)

**Fig. B.36 Measurement setup of zty (side view of machine tool)**

![Figure B.37 Measurement results of zty](image)

**Fig. B.37 Measurement results of zty**
Secondly, the other side of the block is measured yielding the error $ytz$ and an error that includes both the alignment error of the block, the squareness error in the block and the squareness error of the machine tool (figure B.38 and B.39).

Fig. B.38 Measurement setup of $ytz$ (side view of machine tool)

Fig. B.39 Measurement results of $ytz$

Out of these measurement results the following conclusions can be drawn. The positive angle between the best fitted line through the measurement results and the Z-axis is explained by an alignment error of the block. The measurements indicate an alignment error ($\alpha$) of -5.4 arcsec.
The second measurement yields an alignment error ($\beta$) of -3.1 arcsec. The total uncorrected out of squareness is in this case calculated by: $\alpha - \beta$, thus yielding 8.5 arcsec. However, the squareness error of the block, i.e. 1.7 arcsec, must be subtracted from this result, so the total squareness error of the Y-axis with respect to the Z-axis is -4.0 arcsec.

B.6 Measurements of the rotary axes

The Maho milling machine contains not only three linear axes, but also two rotary axes. In order to complete the individual model, also the geometric errors induced by these two rotary table have to be determined.

For the measurement of the linearity error of the B-axis, a optical polygon and an autocollimator is used. The polygon has a measurement uncertainty less than 1 arcsec. The autocollimator introduces a measurement uncertainty less than 0.5 arcsec. The measurement is carried out by positioning the B-axis with a step that equals the angle between two succeeding planes of the polygon. With the autocollimator the difference between the orientation of the two planes is measured. Using a polygon, which has 12 sides, together with the autocollimator, it is possible to measure the position error of the B-axis over the whole range (360°) with steps of 30°.

With the above presented measurement method it is possible to obtain the position error with steps as large as the angle between two succeeding planes of the polygon. In order to determine the position error between these steps another method is used. A very accurate rotary table is placed upon the B-axis of the milling machine. this rotary table introduces an uncertainty less than 0.5 arcsec. In the center of this table a plane mirror is placed. The measurement sequence is carried out by rotating the B-axis of the milling machine a randomly chosen angle. Using the rotary table, the plane mirror is rotated back to the zero position, which is measured by the autocollimator.

Both measurement methods are applied to the B-axis of the Maho milling machine. To measure the position error with the first method the polygon is place in the center of the B-axis, as depicted in figure B.40. The autocollimator is placed outside the milling machine on a stable tripod. In the same figure the measurement setup for the second method is depicted. Here a SIPP rotary table is placed in the center of the B-axis. A plane mirror is placed upon this table. The autocollimator is also placed outside the milling machine.
Fig. B.40 Both the measurement setups for bry.

Fig. B.41 Error bry versus position of the B-axis
The results of both measurements are depicted in figure B.41. As these results are corrected for the systematic errors of the used instruments, they directly reflect the position error of the B-axis. The results show a randomly distributed position error smaller than 4 arcsec. As the resolution of the B-axis equals 3.6 arcsec, it can be concluded that the B-axis has an position error smaller than its resolution.

Also the position error of the C-axis of the Maho milling machine is measured. As the C-axis rotates around the Z-axis of the milling machine, which is a horizontal axis, it was not possible to connect the SIPP rotation table to this axis. Therefore this axis is only measured with the first method. In order to get more information on the position error of the intermediate positions the measurement was carried out with different start positions.

The measurement setup for the position error crz is depicted in figure B.42. The polygon is placed upon a rotation table, which is connected to the C-axis of the milling machine. This rotation table is used for the different start positions. In order to create a suitable measurement setup, an optical square is placed on the table of the milling machine. The autocollimator is placed outside the milling machine on a stable tripod.
The result of this measurement is presented in figure B.43. This result directly reflects the position error of the C-axis. From this result, it can be concluded that the position error of the C-axis is very small, concerning the range of this axis (120°), which has a resolution of 3.6 arcsec.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{measurement_of_crz.png}
\caption{Error \( crz \) versus position of the C-axis}
\end{figure}
C. Measurements and results of the holeplate method

C.1 Introduction
In this appendix the measurements and accompanying results of the holeplate method will be presented. This method, developed by PTB, is applied by TUE and PTB on the Maho milling machine under research. PTB supplied a holeplate with a grid of 9 x 9 holes. The nominal distance between two holes is 50 mm. The actual distance is determined during a calibration at PTB. This holeplate is measured on the Maho milling machine, using an infrared probesystem, delivered and installed by Maho. Using this probesystem the milling machine will determine the location of each hole referred to the reference hole. As the actual location of these holes is known, it is possible to calculate the geometric error of the milling machine within a certain plane. This error will not only consist of the geometric errors of the milling machine, but also the repeatability of the used probe system will be present in the measured error. In order to determine this repeatability of the probe system, a repeatability test is carried out with all probe configurations, as depicted in figure C.1.

![Fig. C.1 Used probe configurations](image)

The following repeatability is found for each probe configuration:

- **Probe 1**: $2S_{xc} = 0.63 \, \mu m; \quad 2S_{zc} = 0.84 \, \mu m$
- **Probe 2**: $2S_{xc} = 1.1 \, \mu m; \quad 2S_{yc} = 0.80 \, \mu m$
- **Probe 3**: $2S_{xc} = 1.8 \, \mu m; \quad 2S_{zc} = 1.3 \, \mu m$
- **Probe 4**: $2S_{yc} = 1.8 \, \mu m; \quad 2S_{zc} = 7.5 \, \mu m$
- **Probe 5**: $2S_{yc} = 1.4 \, \mu m; \quad 2S_{zc} = 7.5 \, \mu m$
These results show a large repeatability in Z-direction for probes 4 and 5. This is caused by the position of the probe tip: not in the center beneath the switching probe system.

It is possible to determine the error components within a range of 400 mm with the available holeplate. As the machine has a range of X=700 mm, Y=500 mm and Z=600 mm, the holeplate has to be positioned on different locations within a certain plane. Therefore a total of 24 holeplate measurements have to be carried out in order to determine all error components within the whole range of the milling machine. As the measuring scales and the holeplate will expand due to temperature changes, the temperature of these parts will be measured during the experiments, using the software as described in paragraph 4.2. Underlying the measurement setups with accompanying results are presented.
C.2 Holeplate measurement in the XY-plane

The different holeplate positions in the XY-plane are depicted in figure C.2. For this measurement setup probe 2 is used. The milling machine is programmed to measure the outmost holes. During this measurement the temperature is measured from the holeplate and the scales of the milling machine.

![Measurement setup for holeplate measurements in the XY-plane](image)

Before the measurement results can be compared with the simulations, the actual measurements have to be corrected for the expansion of the measuring scales of the milling machine and the expansion of the holeplate itself due to temperature changes. As it is not possible to perfectly align the holeplate along one of the axes of the milling machine, the measurement results will show an alignment error. This alignment error is eliminated by a mathematically transformation of the measurement results, as the holeplate would be aligned along the axis, which is the lowest in the kinematic chain.

In figure C.3 the result of the holeplate measurement on position 1 in the XY-plane is depicted, after correction for the above mentioned effects. As the location of each hole is determined during a measurement back and forth, the results of both measurement sequences are depicted.
Fig. C.3 Result of the holeplate measurement in the XY-plane on position 1

In figure C.4 the result of the holeplate measurements in the XY-plane on position 8 is depicted.

Fig. C.4 Result of the holeplate measurement in the XY-plane on position 8
C.3 Holeplate measurement in the XZ-plane

The different holeplate positions in the XZ-plane are depicted in figure C.5. For this measurement setup the probes 1 and 3 are used. Again the milling machine is programmed to measure the outmost holes. During this measurement the temperature is measured from the holeplate and the scales of the milling machine.

![Diagram of holeplate measurement setup in XZ-plane](image)

*Fig. C.5 Measurement setup for holeplate measurements in the XZ-plane*

In figure C.6 the results of the holeplate measurement on position 3 in the XZ-plane is depicted, after correction for temperature and alignment errors. As the location of each hole is determined during a measurement back and forth, the results of both measurement sequences are depicted.
Fig. C.6 Result of the holeplate measurement in the XZ-plane on position 3

In figure C.7 the result of the holeplate measurements in the XZ-plane on position 8 is depicted.

Fig. C.7 Result of the holeplate measurement in the XZ-plane on position 8
C.4 Holeplate measurement in the YZ-plane

The different holeplate positions in the YZ-plane are depicted in figure C.8. For this measurement setup the probes 4 and 5 are used. The milling machine is programmed to measure the outmost holes. During this measurement the temperature from the holeplate and the scales of the milling machine is measured.

In figure C.9 the result of the holeplate measurement on position 4 in the YZ-plane is depicted, after correction for temperature and alignment errors. As the location of each hole is determined during a measurement back and forth, the results of both measurement sequences are depicted.
Fig. C.9  Result of the holeplate measurement in the YZ-plane on position 4

In figure C.10 the result of the holeplate measurements in the YZ-plane on position 5 is depicted.

Fig. C.10  Result of the holeplate measurement in the YZ-plane on position 5
D. Fitting measurement data with piece-wise polynomials

D.1 Introduction
The data obtained from the measurements, as described in chapter 4 and appendix B, are averaged. A least squares fitting procedure has been applied to these data sets, using piece-wise polynomials. The piece-wise polynomials are defined by the position of the knots and the coefficients of the polynomial for each interval. Underlying an overview of all fitted data, and the accompanying coefficients of the polynomials, is presented. The dashed lines in the graphs represent the averaged results of a number of measurements, moving the carriage back and forth. The solid line represents the fit of the error expressed as a set of polynomials. All fitting procedures are performed with continuity restrictions in the knots for the functional value and its first derivative, using a maximum degree of two for the polynomials.
This method yields a continuous description of each geometric error, based on discrete measurements.
The shown coefficients are valid as polynomial coefficients for the specified interval. Thus, it is not necessary to evaluate the polynomial form the beginning of the axis, to calculate the error.
D.2 Fitting rotation errors of the X-axis

**Fit on xrx results**

**Fig. D.1 Fitting with piece-wise polynomials of error xrx**

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<th>Linear coeff. $\beta_{11}$</th>
<th>Quadratic coeff. $\beta_{12}$</th>
</tr>
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<tr>
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<tr>
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<tr>
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<tr>
<td>7</td>
<td>600-700</td>
<td>$1.8482189e+01$</td>
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</tr>
</tbody>
</table>
Fit on xry results

Average xry and fitted polynomials

Position of the X-carriage [mm]

Error [mm]

Fig. D.2 Fitting with piece-wise polynomials of error xry

<table>
<thead>
<tr>
<th>i</th>
<th>Interval</th>
<th>Intercept ((\beta_{10}))</th>
<th>Linear coeff. ((\beta_{11}))</th>
<th>Quadratic coeff. ((\beta_{12}))</th>
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<tbody>
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Fit on \( xrz \) results

Average \( xrz \) and fitted polynomials

Fig. D.3 Fitting with piece-wise polynomials of error \( xrz \)

<table>
<thead>
<tr>
<th>( i )</th>
<th>Interval</th>
<th>Intercept ( (\beta_{i0}) )</th>
<th>Linear coeff. ( (\beta_{i1}) )</th>
<th>Quadratic coeff. ( (\beta_{i2}) )</th>
</tr>
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<tbody>
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D.3 Fitting the rotation errors of the Y-axis

*Fit on yrz results*

![Graph showing fitting with piece-wise polynomials of error yrz](image)

**Fig. D.4 Fitting with piece-wise polynomials of error yrz**

<table>
<thead>
<tr>
<th>i</th>
<th>Interval</th>
<th>Intercep $(\beta_{10})$</th>
<th>Linear coeff. $(\beta_{11})$</th>
<th>Quadratic coeff. $(\beta_{12})$</th>
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Fit on yry results

Fig. D.5 Fitting with piece-wise polynomials of error yry

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Fit on yrz results

![Graph showing average yrz and fitted polynomials.]

**Fig. D.6 Fitting with piece-wise polynomials of error yrz**

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D.4 Fitting the rotation errors of the Z-axis

Fit on zrx results

![Graph showing fitting results](image)

**Fig. D.7 Fitting with piece-wise polynomials of error zrx**

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<th>Interval</th>
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<th>Quadratic coeff. ($\beta_{i2}$)</th>
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Fit on zry results

Fig. D.8 Fitting with piece-wise polynomials of error zry

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<th>Intercept ( \beta_{10} )</th>
<th>Linear coeff. ( \beta_{11} )</th>
<th>Quadratic coeff. ( \beta_{12} )</th>
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Fit on zrz results

Fig. D.9 Fitting with piece-wise polynomials of error zrz

<table>
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<th>Intercept ((\beta_{i0}))</th>
<th>Linear coeff. ((\beta_{i1}))</th>
<th>Quadratic coeff. ((\beta_{i2}))</th>
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<td>-1.8508369e+00</td>
<td>7.0422548e-03</td>
<td>-2.5341112e-06</td>
</tr>
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</table>
D.5 Fitting the scale errors

The scale errors are defined in the centroid of the carriage. In practical situations it is impossible to obtain these errors directly. But, with a carefully administrated measurement position, it is possible to eliminate the influence of rotational errors on the measured scale error. Underlying the bare measurement data, the determined influence of rotations and the calculated fit of the true scale error, is depicted.

Fit on xtx results
Note that these results reveal a clear periodic term in the error. In order to obtain a acceptable fit through the data, the general trend of the error is handled by piece-wise polynomials, whereas the periodic term is fitted separately and superimposed on the general trend.
Fig. D.10 Fitting with piece-wise polynomials of error xtx

Results of the determination periodic term ($\lambda = 39.6$ mm):

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<th>Quadratic coeff.</th>
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</tr>
<tr>
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<td>3.9792780e-03</td>
<td>2.1953863e-05</td>
<td>8.057136e-08</td>
</tr>
<tr>
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<tr>
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<tr>
<td>7 400-500</td>
<td>-1.0791816e-02</td>
<td>5.5991125e-05</td>
<td>-3.9114952e-08</td>
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<td>3.1258289e-02</td>
<td>-1.1220929e-04</td>
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<td>-5.7886128e-02</td>
<td>1.8493876e-04</td>
<td>-1.1853791e-07</td>
</tr>
</tbody>
</table>
Fit on yty results

**Average yty and fitted polynomials**

- Bare measurement data
- Fitted polynomials on corrected data
- Influence of yrz and yrx

**Fig. D.11 Fitting with piece-wise polynomials of error yty**

<table>
<thead>
<tr>
<th>i</th>
<th>Interval</th>
<th>Intercept $(\beta_{10})$</th>
<th>Linear coeff. $(\beta_{11})$</th>
<th>Quadratic coeff. $(\beta_{12})$</th>
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</tbody>
</table>
Fit on ztz results

![Graph](image)

**Fig. D.12** Fitting with piece-wise polynomials of error ztz

<table>
<thead>
<tr>
<th>i</th>
<th>Interval</th>
<th>Intercept $(\beta_{i0})$</th>
<th>Linear coeff. $(\beta_{i1})$</th>
<th>Quadratic coeff. $(\beta_{i2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0-200</td>
<td>-7.1077194e-04</td>
<td>3.1978343e-05</td>
<td>4.4584063e-08</td>
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<tr>
<td>1</td>
<td>200-400</td>
<td>-1.8833918e-03</td>
<td>4.3704542e-05</td>
<td>1.5268566e-08</td>
</tr>
<tr>
<td>2</td>
<td>400-600</td>
<td>1.4866087e-03</td>
<td>2.6854539e-05</td>
<td>3.6331069e-08</td>
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</tbody>
</table>