Tuning and performance of a CFT master-slave robot system

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Tuning and Performance of a CFT Master-Slave Robot System

Report number: DCT.2002.70
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November 15, 2002

Eindhoven University of Technology, the Netherlands
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1 Introduction

Instead of using one robot, sometimes it is desired that two or more robots execute a certain task together. This is either because of the complexity of the task or the limitations of a single robot. The robots can be either identical or different and working together gives rise to maneuverability that cannot be achieved by a single robot. As soon as two or more robots have to cooperate synchronization is of great importance. Such synchronization is a control problem where, at least for one of the robots, a suitable controller has to be designed such that this robot (slave) follows the other robot (master) properly. Most controllers also assume the availability of all state variables, implying the presence of extra sensors in each joint. This is usually not the case because of the complexity in the implementation and due to the savings in cost, weight and volume that can be obtained. Frequently only position measurements of both master and slave robots are available. To overcome this problem numerical differentiation and the use of observers for example can be considered.

In this paper two identical CFT robots are treated. The CFT robot is a Cartesian robot with a basic elbow configuration designed and built by Philips Centre for Manufacturing Technology (CFT). It consists of a two links arm that is placed on a rotating and translational base, and has a passively actuated tool connected at the end of the outer link. The CFT robot is a pick and place robot and is actuated by 4 servomotors. It has 4 degrees of freedom in the Cartesian space and 7 degrees of freedom in the joint space. The robot has only 4 encoders for position measurements. The tool connected at the end of the outer link is a kinematically constrained planar support which is designed to keep a horizontal plane at all time.

In a 4 years Ph.D. program carried out at the Eindhoven University of Technology, synchronization of robots with only position measurements was studied and analytically proved, see [3]. For experimental validation of the synchronization controllers a pair of two CFT robots was used. The same setup is considered in this study. The synchronization goal was achieved by using model-based controllers and observers. Since the goal in the synchronization study was to show that synchronization based only on position measurements is viable in complex systems, it is likely that for the specific multi-robot system some performance improvement can still be accomplished.

The goal of this study is to improve the performance of the master-slave synchronization of the two CFT robots as well as to give guidelines for the tuning of the proportional and derivative action on the model-based synchronization controller. The performance is expressed in a relation with an error index, which relates the synchronization to the positions of the master and slave robot. First, a comparison study is carried where the performance of a controller in joint space is compared with the performance in Cartesian space. Also a PD, PID and a model-based controller are compared to prove that a model-based controller indeed has the best performance for this system. Second, an alternative for tuning the proportional and derivative action of the model-based synchronization controller is presented. The proposed alternative is based on feedback linearization and makes use of transfer functions and design methods for linear systems. Finally, the performance of the synchronization controller is improved and compared to the arbitrarily chosen gains, which were used for the synchronization goal in the Ph.D. program as mentioned above.

Section 2 presents a short description of the dynamic model and kinematic relations of the CFT transposer robots. In section 3 some terms are defined and the methodology for achieving the goal is given. Section 4 shows a comparison study between the Cartesian and joint space. A comparison between a PD, PID and a model-based controller is carried out in section 5. In section 6 several options to tune the proportional and derivative gains of a model-based controller are discussed. Some conclusions and recommendations are presented in section 7.

For any one who would like to do the experiments a short manual for the experimental setup is added to this report in appendix D.

2 The CFT transposer robot

In this section a short description of the CFT transposer robots is given. The dynamic model and kinematic relations are presented. Because it is beyond the scope of this report only a summary of the models necessary for the understanding of this report, are presented. For further details about the dynamic model the interested reader is referred to [3].
The multi-robot system, which is treated in this study, consists of two robots. The robots are industrial transposer robots that are designed by the Centre for Manufacturing Technology (CFT). The robots are identical in their structure and design, which means that they have the same dynamical model. The difference of the robots lies in the physical parameters like inertias and friction coefficients for example.

The CFT robot is a pick and place robot that is used for assembling. The CFT robot is a Cartesian based elbow configuration robot. It consists of a two links arm, which is placed on a rotating and translational base, and it has a passively actuated tool connected at the end of the outer link. It has four degrees of freedom in the Cartesian space, denoted by $x_i (i=1,\ldots,4)$ and seven degrees of freedom in the joint space, denoted by $q_j (j=1,\ldots,7)$, and is actuated by 4 DC brushless servomotors. In the joint space, the set of joint coordinates $\{q_3, q_6, q_7\}$ is kinematically constrained, which means that the robot can be represented in the joint space by 4 degrees of freedom namely $\{q_1, q_2, q_4, q_5\}$. The servomotor link pair is stiff enough to be considered as a rigid joint.

### 2.1 Kinematics in Cartesian space

The four Cartesian degrees of freedom $x_i (i=1,\ldots,4)$ are respectively up and down, forward and backward movement of the arm, rotation and translation of the base in which the arm is mounted. The robot is equipped with encoders attached to the shaft of the motors. The tool connected at the end of the outer link is kinematically constrained and is designed to stay horizontal at all time. Figure 2.1 shows a schematic diagram of the robot, the physical dimensions of the CFT robot are listed in table 2.1. $d_{i,i+1}$ denotes the distance between the origin of frame $\{i\}$ and $\{i+1\}$. The coordinates $x_{c3}$ and $x_{c4}$ are absolute coordinates and are referred with respect to an inertial frame $\{0\}$. The coordinates $x_{cl}$ and $x_{c2}$ are relative coordinates and are referred with respect to a frame at the edge of the translational platform, frame $\{e\}$.

The coordinates of the tip of the tool $(x_T, y_T, z_T)$ that is connected at the end of the outer link are given with respect to frame $\{0\}$ by

$$
\begin{align*}
     x_T &= (x_{c2} - d_z + L_q) \cos(x_{c3} - 0.8292) \\
     y_T &= x_{c4} + d_z + (x_{c2} - d_z + L_q) \sin(x_{c3} - 0.8292) \\
     z_T &= L_2 + 0.25 - x_{cl} - L_7
\end{align*}
$$

Equation (2.1) describes the direct kinematics of the robot in Cartesian space and determines the position of the tip of the tool in the robot as function of the coordinates $x_{c1}, x_{c2}, x_{c3}$ and $x_{c4}$.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value [m]</th>
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<th>Value [m]</th>
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<tr>
<td>$L_2, d_1, L_3$</td>
<td>0.25</td>
<td>$L_8$</td>
<td>0.48</td>
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<tr>
<td>$L_4$</td>
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<td>$d_4, d_5$</td>
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<tr>
<td>$L_5$</td>
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<td>$d_6$</td>
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<td>$L_6$</td>
<td>0.30</td>
<td>$d_7$</td>
<td>0.185</td>
</tr>
<tr>
<td>$L_7$</td>
<td>0.08</td>
<td>$d_2, d_5$</td>
<td>0.0916</td>
</tr>
</tbody>
</table>

Table 2.1: Dimensions of the robot
2.2 Kinematics in joint space

For the definition of the kinematics in joint space, Denavit-Hartenberg parameters are used. The coordinate frames are assigned as shown in figure 2.2. The correspondent set of Denavit-Hartenberg parameters is listed in table 2.2. The joint coordinates $q_1, q_2$ are the translations along $z_1, z_2$ respectively. For $i=2,4,5,6,7$ the joint coordinate $q_i$ is the rotation about the $z_i$-axis. As mentioned before, the joint coordinates $q_3, q_6, q_7$ are kinematically constrained. Thus, it follows that the kinematics between the tip of the tool and the actuated joints is given by

\begin{align}
    x_7 &= -(L_6 + d_{2,v} + L_5 - L_2 - L_4 - L_6) \sin(q_2) + \frac{1}{2} \left[ \cos(-q_2 + q_4) - \cos(q_2 + q_4) \right] L_6 \\
    &+ \frac{1}{2} \left[ \cos(q_5 - q_2 + q_4) - \cos(q_5 + q_2 + q_4) \right] (L_5 - L_4) \\
    y_7 &= q_1 + (L_6 + d_{2,v}) \cos(q_2) - \frac{1}{2} \left[ \sin(q_2 + q_4) - \sin(q_4 - q_2) \right] L_6 \\
    &- \frac{1}{2} \left[ \sin(q_5 - q_2 + q_4) + \sin(q_5 + q_2 + q_4) \right] (L_5 - L_4) \\
    z_7 &= L_2 - L_7 + (L_6 + L_4) \cos(q_4 + q_6) + L_5 \cos(q_4 + q_5)
\end{align}
Since the kinematics in equation (2.1) and (2.2) represent the same point in space with respect to frame {0}, it follows that there exists a unique relationship between the Cartesian coordinates $x_4$, $x_5$ and $x_6$ and the joint coordinates $q_1$, $q_2$, $q_4$ and $q_5$, which is given by
2.3 Joint space dynamics

From the Euler-Lagrange approach and the Denavit-Hartenberg parameters given in Table 2.2, the dynamics of the CFT robots in the multi-composed system are given by

\[ M(q_i)\dot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + g(q_i) + f(q_i) = \tau_i \quad \text{with } i = m, s \]

where as in [3] the friction forces \( f(q_i) \) are modeled as

\[ f(q_i) = B_{v,i}\dot{q}_i + B_{f,i}(1 - \frac{2}{1 + e^{2\alpha_i}}) + B_{f,i}(1 - \frac{2}{1 + e^{2\beta_i}}) \]

The notation \( q_i \), for \( i = m, s \) refers to the master and slave robot rather than to the \( j \)-th joint, \( j = 1, 2, 4, 5 \), in the \( i \)-th robot. Here, \( q_i = [q_{i,1}, q_{i,2}, q_{i,4}, q_{i,5}]^T \) is the vector of generalized coordinates of robot \( i \), \( M(q_i) \in \mathbb{R}^{4 \times 4} \) is the symmetric, positive definite inertia matrix, \( g(q_i) \in \mathbb{R}^4 \) denotes the gravity forces, \( C(q_i, \dot{q}_i) \in \mathbb{R}^{4 \times 4} \) represents the Coriolis and centrifugal forces, \( f(q_i) \in \mathbb{R}^4 \) are the forces due to friction effects, and \( \tau_i = [\tau_{i,1}, \tau_{i,2}, \tau_{i,4}, \tau_{i,5}]^T \) is the vector of external torques. The parameters in the matrices \( M(q_i), C(q_i, \dot{q}_i), g(q_i) \) and the friction forces \( f(q_i) \) are listed in Appendix A.

3 Problem formulation

In this report a multi-robot system formed by two (or more) robots is considered, such that the motion of one of the robots is independent of the others. The dominant robot is referred to as the master robot. The master robot is driven by a controller already designed and not relevant for the synchronization goal. The non-dominant robot for which interconnections and feedback controllers have to be designed is referred to as the slave robot. From now on a master and a slave robot are considered. There is no loss of generality since the controller is decentralized. So the same structure of controller can be applied to one or more robots indistinctly.

This report addresses the performance of the master-slave system. Even though, it is not clear how the word “performance” should be interpreted. In master-slave synchronization the goal is to design a control law for the slave robot, such that its position and velocity synchronize with the master robot. Thus the goal is to follow the trajectory of the master robot. Because only positions can be measured, we can define the performance in a kind of index that relates the synchronization to the positions of the slave and master robot. We define this index by

\[ \text{error index} = \int_0^t e(\tau)^T e(\tau) d\tau \]

with \( e = q_s - q_m \), i.e. the position coordinate of the slave robot minus the position coordinate of the master robot. To have a good performance, this index should be as small as possible. Therefore we can relate the performance to this index as

\[ \text{performance} \sim [\text{error index}]^{-1} \]
An alternative for the error index as defined by (3.1) is

\[
\text{error index} = \frac{1}{t} \int_0^t e(\tau)^T e(\tau) d\tau \tag{3.3}
\]

The performance can be related to this error index in the same way as in (3.2). For this study, the definition of the error index in (3.1) is used.

The main goal of this study is to improve the performance of a mechanical master-slave system in synchronization in case of partial knowledge of the states, i.e. only positions are measured. For this purpose a comparison study is carried out first. A PD controller in Cartesian and joint space is compared to know if there is any difference in performance. Because of the complexity of a model-based controller, it is not sensible to work with it if there is not a significant difference in performance with the simpler PD and PID controllers. Therefore, a PD, PID and model-based controller are compared too. From this comparison study it is clear which controller yields the best performance. Nevertheless, the gains of the model-based synchronization controller are still arbitrarily chosen. Therefore a better performance can be achieved if the gains of the controller are tuned in a systematic way. Several options are considered to tune the gains of the controller and eventually they are implemented in the master-slave system to validate the results.

4 Cartesian and joint space

There are different control techniques which can be applied to the CFT robot, e.g. PID controllers, model-based controllers, adaptive controllers. Before an advanced controller can be used, first the workspace in which the controller will be designed must be determined. In section 2, two workspaces are defined, namely the Cartesian space and the joint space. If there is no difference in performance of the robots in Cartesian and joint space, then it is much easier to choose for the Cartesian space. The algorithms are much simpler and the coordinates and forces do not need a transformation to the joint space. Therefore, in this section a simple PD controller is used to compare the performance of the control action in Cartesian and in joint space.

4.1 PD controller for tracking

Before considering the robots in synchronization, we first look at the controllers for tracking. The PD controller can be described as

\[
\tau = -K_p e - K_d \dot{e}
\]

with the tracking error \( e = q_s - q_d, \dot{e} = \dot{q}_s - \dot{q}_d \), and \( q_s, q_d \) the position coordinate of the slave robot and the desired trajectory. \( K_p \) and \( K_d \) correspond to the proportional and derivative gain respectively. As described in [4] the force and torque which generate equivalent displacements in the robot, are related by the transpose of the Jacobian of the kinematic relation between the Cartesian and joint coordinates.

\[
\tau = J(q)^T F
\]

Relation (4.2) allows converting any Cartesian quantity into a joint space quantity without calculating any inverse kinematic function. A more detailed description of the relation between forces and torques is given in Appendix B. To be able to compare the performance of the controller in Cartesian and joint space, the gains for the PD controller must be equivalent. Because it is not clear yet which values to choose for the gains, a set of gains is chosen that does not saturate the voltages but is able to do the control action. To have equivalent gains in the Cartesian space, the gains in the joint space are divided by the corresponding elements of \( K^T \), the proportional gains which relate the forces \( F_{x_4} \) to the voltages applied at the servoamplifiers in the motor (see Appendix B). This is sufficient to relate \( \tau_{q_1} \) and \( \tau_{q_2} \) to \( F_{x_4} \) and \( F'_{x_4} \) respectively. For \( \tau_{q_4} \) and \( \tau_{q_5} \) it is a bit more difficult because there is a kind of coupling between \( x_{c_1}, x_{c_2} \) and \( q_4, q_5 \). Therefore the worst case, in which the difference between the two spaces is
the largest, is considered. This means that the torques should be multiplied with the maximum value of $\frac{1}{\Delta_J}$.

Here $\frac{1}{\Delta_J}$ is a coupling factor between the Cartesian and joint space. $\Delta_J$ represents the determinant of the Jacobian matrix which is given in appendix B. By some experiments this parameter is measured to have a maximum value of 25. For the experiments the gains as listed in table 4.1 are used. The results of the experiments are shown in figure 4.1 and 4.2.

<table>
<thead>
<tr>
<th>Joint space</th>
<th>Cartesian space</th>
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</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>$K_d$</td>
</tr>
<tr>
<td>$\tau_{q1}$</td>
<td>15000 20</td>
</tr>
<tr>
<td>$\tau_{q2}$</td>
<td>2000 50</td>
</tr>
<tr>
<td>$\tau_{q4}$</td>
<td>12000 50</td>
</tr>
<tr>
<td>$\tau_{q5}$</td>
<td>12000 50</td>
</tr>
</tbody>
</table>

Table 4.1: Gains that are used for comparison of Cartesian and joint PD controllers

Figure 4.1: Error index in Cartesian and in joint space for $x_{c1}$ and $x_{c2}$

Figure 4.2: Error index in Cartesian and in joint space for $x_{c3}$ and $x_{c4}$

Figure 4.2 shows that for the rotational ($x_{c3}$) and translation ($x_{c4}$) movement the difference in the error index for the Cartesian and joint space is very small. This is in agreement with the fact that respectively $q_1$, $q_2$ are related to $x_{c4}$, $x_{c3}$ by a proportional constant relation, so similar performance is expected when
considering equivalent gains. Figure 4.1 shows the error index for $x_{c1}$ and $x_{c2}$, after some time the PD controller in joint space has a smaller error index. As the only difference between the two spaces is caused by the Jacobian, the better performance in joint space can only be caused by this Jacobian. The Jacobian introduces time varying scaling and coupling effects. The time varying scaling is caused by the determinant $\Delta_f$ which is a function of $q_4, q_5$. In Cartesian space the coupling between the two links of the arm is not considered in the forces, while in the joint space the coupling between the joints $q_4, q_5$ is considered in the forces $F_{x_{c1}}$ and $F_{x_{c2}}$, see (B.2). By considering this coupling in the arms and the time varying scaling, the calculated forces in the joint space are more accurate and therefore give a better performance.

Therefore we can conclude that for this system with a single robot a tracking controller in joint space has a better performance than a tracking controller in Cartesian space. This does not automatically mean that for a master-slave system the same conclusion can be drawn. In the master-slave synchronization the slave robot uses the measured positions of the master robot. Because in measured variables there is some noise present, this can influence the performance. Also in the joint space there are more conversions that could result in amplification of the noise and therefore in a larger error in the position. Therefore it is necessary to do the same experiment in master-slave synchronization.

### 4.2 PD controller for synchronization

For the slave robot the same gains as listed in table 4.1 are used. The controller for the master robot is designed in Cartesian space since it has no influence on the performance of the controller of the slave robot. Figure 4.3 shows the results of the experiment with the master-slave system. Again, the results show that the joint space has a better performance than the Cartesian space. From these results the conclusion can be drawn that also in master-slave synchronization the performance of controllers in joint space is better than in Cartesian space. Therefore the controllers will be evaluated and studied only in the joint space.

**Figure 4.3:** Error index for synchronization in Cartesian and in joint space for $x_{c1}$ and $x_{c2}$

**Figure 4.4:** Error index for synchronization in Cartesian and in joint space for $x_{c3}$ and $x_{c4}$
5 Synchronization controllers

In this section a PD, PID and a model-based controller are compared. Because of its complexity it is not sensible to use a model-based controller if the same performance can be achieved with a PD or PID controller. This comparison is carried out to show the difference in performance of the three controllers. In the first subsection the model-based synchronization controller is presented. The second subsection shows the comparison between a PD and the model-based controller. The last subsection presents the influence on the performance of the integral action in a PID controller.

5.1 Model-based synchronization controller

Most controllers assume the availability of all state variables, implying the presence of extra sensors in each joint. If the master and slave positions, velocities and accelerations are available for measurement, then master-slave synchronization can be obtained by using some of the tracking controllers proposed in literature. Since this is not the case, these tracking controllers cannot be implemented. In this master-slave system only joint positions \( q_m \) and \( \hat{q}_s \) can be measured, it means that there is neither access to \( \hat{q}_m \) nor \( \hat{q}_s \).

In [3] a model-based synchronization controller is proposed which is formed by a feedback control law and two non-linear model-based observers. In this way the synchronization controller depends on position measurements and estimated values for the velocities and accelerations. The proposed controller is described as

\[
\tau_s = M_s(q_s)\hat{q}_m + C_s(q_s,\hat{q}_s)\hat{q}_m + g_s(q_s) + f(q_s) - K_p e - K_d \dot{e}
\]  

(5.1)

where \( \hat{q}_s, \dot{e}, \hat{q}_m, \dot{q}_m \in \mathbb{R}^n \) represent the estimates of \( q_s, \dot{q}_s, \hat{q}_m, \dot{q}_m \) and \( e = q_s - \hat{q}_m, \dot{e} = \dot{q}_s - \dot{q}_m \) represent the synchronization errors. \( K_p \) and \( K_d \) are the proportional and derivative gains respectively.

From the two observers proposed in [3] one is used for the estimation of the synchronization error and the other is used for the estimation of the slave joint variables. The observer for the estimation of the synchronization error is given by

\[
\frac{d}{dt} \hat{e} = \hat{\dot{e}} + \Lambda_1 \tilde{e}
\]

\[
\frac{d}{dt} \tilde{e} = -M_s(q_s)^{-1}[C_s(q_s,\hat{q}_s)\dot{\hat{e}} + K_p \hat{\dot{e}} + K_d \hat{\dot{e}}] + \Lambda_2 \tilde{e}
\]

(5.2)

where the estimated position and velocity synchronization errors \( \tilde{e}, \tilde{\dot{e}} \) are defined by

\( \tilde{e} = e - \dot{e} \) and \( \tilde{\dot{e}} = \dot{\dot{e}} - \dot{\dot{e}} \) and \( \Lambda_1, \Lambda_2 \in \mathbb{R}^{n\times n} \) are positive definite gain matrices. The observer for the estimation of the slave joint variables is given by

\[
\frac{d}{dt} \hat{\dot{q}}_s = \hat{\dot{q}}_s + L_p_1 \hat{\ddot{q}}_s
\]

\[
\frac{d}{dt} \hat{\ddot{q}}_s = -M_s(q_s)^{-1}[C_s(q_s,\hat{q}_s)\dot{\hat{\dot{e}}} + K_p \hat{\dot{e}} + K_d \hat{\dot{e}}] + L_p_2 \hat{\ddot{q}}_s
\]

(5.3)

where the estimated position and velocity errors \( \hat{\dot{q}}_s, \hat{\ddot{q}}_s \) are defined by

\( \hat{\dot{q}}_s = \dot{q}_s - \dot{q}_s \) and \( \hat{\ddot{q}}_s = \ddot{q}_s - \ddot{q}_s \) and \( L_p_1, L_p_2 \in \mathbb{R}^{n\times n} \) are positive definite gain matrices.

Because the master robot variables \( \dot{q}_m \) and \( \ddot{q}_m \) are not available, estimated values for \( \dot{q}_m \) and \( \ddot{q}_m \) are used in (5.1). However, the master robot dynamic model is assumed to be unknown, then the variables \( \dot{q}_m \) and \( \ddot{q}_m \) must be reconstructed based on information of the slave robot and the synchronization closed loop.
system. From (5.1) and the definition of the estimated variables \( \hat{e}, \hat{\dot{e}}, \hat{\dot{q}}, \hat{\dot{q}} \), we can consider that estimated values for \( \hat{q}_m, \hat{\dot{q}}_m, \hat{\ddot{q}}_m \) are given by

\[
\begin{align*}
\hat{q}_m &= \hat{\dot{q}}_s - \hat{\dot{e}} \\
\hat{\dot{q}}_m &= \hat{\ddot{q}}_s - \hat{\ddot{e}} \\
\hat{\ddot{q}}_m &= \frac{d}{dt}(\hat{\dot{q}}_s - \hat{\dot{e}})
\end{align*}
\] (5.4)

From the definition of the observers (5.2), (5.3) it follows that

\[
\hat{\dot{q}}_m = -(M_s(q_s)^{-1}K_p + L_{p2})\tilde{e} + L_{p2}\tilde{q}_s
\] (5.5)

which gives clear insight into how the master joint acceleration \( \hat{\dot{q}}_m \) is reconstructed.

### 5.2 Comparison of different controllers

The proposed model-based synchronization controller (5.1-5.3) is complicated and it takes a lot of time to develop it and tune the gains in the right way. This is one of the main reasons why such controllers are not used very often in industry. Instead, PD and PID controllers are used, which have the advantage of simplicity in implementation. For the sake of comparison a PD, PID, and model-based controller are implemented on the slave system. The synchronization errors are measured and the performance of the controllers is compared by means of the error index (3.1). To be sure that the results do not depend on the values of the gains several experiments are carried out in which the proportional and derivative gains are varied. The different values for \( K_p \) and \( K_d \) are listed in the following tables

<table>
<thead>
<tr>
<th></th>
<th>( K_p ) (100%)</th>
<th>( K_p ) (120%)</th>
<th>( K_p ) (140%)</th>
<th>( K_p ) (150%)</th>
<th>( K_p ) (160%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1 )</td>
<td>10000</td>
<td>12000</td>
<td>14000</td>
<td>15000</td>
<td>16000</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>1333</td>
<td>1600</td>
<td>1867</td>
<td>2000</td>
<td>2133</td>
</tr>
<tr>
<td>( q_4 )</td>
<td>8000</td>
<td>9600</td>
<td>11200</td>
<td>12000</td>
<td>12800</td>
</tr>
<tr>
<td>( q_5 )</td>
<td>8000</td>
<td>9600</td>
<td>11200</td>
<td>12000</td>
<td>12800</td>
</tr>
</tbody>
</table>

Table 5.1: Five experiments in which \( K_p \) was varied and \( K_d \) was kept constant at

\[
K_d = [20 \ 50 \ 50 \ 50]^T
\]

<table>
<thead>
<tr>
<th></th>
<th>( K_d ) (100%)</th>
<th>( K_d ) (200%)</th>
<th>( K_d ) (300%)</th>
<th>( K_d ) (400%)</th>
<th>( K_d ) (500%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1 )</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
</tr>
<tr>
<td>( q_4 )</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
</tr>
<tr>
<td>( q_5 )</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
</tr>
</tbody>
</table>

Table 5.2: Five experiments in which \( K_d \) was varied and \( K_p \) was kept constant at

\[
K_p = [15000 \ 2000 \ 12000 \ 12000]^T
\]

Tables 5.1 and 5.2 show two sets of experiments in which the gains are systematically increased. The observer gains are kept constant at \( \Lambda_1 = L_{p1} = [500 \ 500 \ 500 \ 500]^T \) and

\[
\Lambda_2 = L_{p2} = [50000 \ 50000 \ 50000 \ 50000]^T
\].
Figure 5.1: Error index of the model based controller (solid) and a PD controller (dotted) for different proportional gains for $q_1$, $q_2$, $q_4$ and $q_5$.

Figure 5.2: Error index of the model based controller (solid) and a PD controller (dotted) for different derivative gains for $q_1$, $q_2$, $q_4$ and $q_5$. 

Tuning and Performance of a CFT Master-Slave Robot System
The results show that there is a significant difference in the error index between the PD controller and the model-based controller. Figures 5.1 and 5.2 show that in some cases, the model-based controller has a higher error index in the transient than the PD controller. This is due to an overcompensation of the model-based controller in the transition phase. In all cases the slope of the error index is smaller for the model-based controller and after some time the error index for the model-based controller is smaller in all cases.

Figure 5.3 shows the error index at 15 seconds when keeping $K_d$ constant and varying $K_p$ and Figure 5.4 shows the error index for a constant $K_p$ and a varying $K_d$. From these figures it is very clear that the model-based controller has a better performance than the PD controller.

![Figure 5.3: Error index for PD and model-based controllers at t=15 seconds for varying $K_p$ values. The gray bars represent the PD controller and the black bars represent the model-based controller.](image-url)
5.3 Introducing the Integral action

A PID controller is implemented to see if it can reduce the difference in performance between the PD and model-based controller. For the PID controller two experiments with two different integral gains $K_I$ are carried out as listed in table 5.3. For the proportional and derivative gains $K_p$ (150%) and $K_d$ (200%) respectively are used.

<table>
<thead>
<tr>
<th></th>
<th>$K_I$ (100%)</th>
<th>$K_I$ (300%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>$q_2$</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>$q_4$</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>$q_5$</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 5.3: Integral gains used for comparison of PID with PD and model-based controllers.
Figure 5.4: Error index for model-based (black solid), PD (gray solid), PID (K_p=100%) (black dotted) and PID (K_p=300%) (gray dotted) controller.

Figure 5.4 shows the results of the comparison of the PID controller with the PD and model-based controller. It can be seen that the slope of the error index of the PID controller is not changing with respect to the slope of the error index of the PD controller. This means that the integral action in the PID controller adds an offset in the error index but does not change the slope. For this experiment two different values for the integral gain K_I are used. From the results the conclusion can be drawn that depending on the value of K_I certain improvement of the PD controller can be achieved, however it can also result in overcompensation. The PID controller shows some improvement of the PD controller, especially for the coordinates that are influenced by gravity, but the PID controller still has lower performance than the model-based controller.

6 Improving performance of the model-based controller

From the comparison study in the previous section it is clear that the model-based synchronization controller yields the best performance. Nevertheless the gains in the proportional action (K_p) and derivative action (K_d) have been arbitrarily chosen. Therefore it can be expected better performance of the model-based synchronization controller if the K_p and K_d gains are tuned in a systematic way. In this section we consider several options to tune the K_p and K_d gains.

6.1 1st method: Neglected couplings

In this first method, for simplicity we neglect the couplings between q_1, q_2, q_4 and q_5 and consider the system as a linear system. The transfer functions of the system can be estimated by means of measurements of the frequency response data.

For each of the degrees of freedom q_1, q_2, q_4 and q_5, one is excited while the other three are kept fixed. On the input signal of the exciting coordinate of the system white noise is added while the sensitivity of the corresponding excited coordinate is measured. The sensitivity function can be written as
S = \frac{1}{1 + CH}, \text{ where } S \text{ is the sensitivity, } C \text{ is the transfer function of the controller and } H \text{ is the transfer function of the system.}

This means that when the sensitivity is measured and the controller is known, the transfer function \( H \) of the system can be determined.

In section 5, for each set of gains \( K_p \) and \( K_d \) that were used for the PD controller and the model-based controller, the model-based controller always showed a better performance. Therefore it can be expected that the gains that are tuned for the PD controller in order to have a better performance will also show a better performance for the model-based controller. Therefore, the gains of a PD controller will be tuned and will be used as the \( K_p \) and \( K_d \) gains for the model-based controller.

To be able to tune the gains, first the bandwidth of each degree of freedom of the slave system must be determined. The frequency response data is obtained for the four coordinates of the system. By fitting a model to the experimentally determined data, the transfer function for each of the coordinates is obtained. The bandwidth is determined by taking the magnitude of the smallest right-half plane zero as explained in [2]. All the bandwidths are in the region of 10 Hz. For simplicity the bandwidths are chosen all equal to 10 Hz. With this information, the gains are tuned to have a phase margin of 45 degrees and a gain margin of 6 dB, to assure stability. The PD controller is tuned by means of Bode diagrams with the software program DIET, see [5].

The gains are obtained for the slave robot with a PD controller. They are used for a model-based synchronization controller with two observers (see equation 5.1-5.3) which both use the same gains as the model-based controller. This means that the obtained gains will also affect the behavior of the observers and therefore it is not sure that the system will be stable at the considered bandwidth. Therefore the gains are tuned for a set of bandwidths.

After implementation, all the considered combinations of gains for the different set of bandwidths seemed to give an unstable system. This is probably because the tuned system is too different from the real system because of the neglects of the couplings, the model-based controller and the observers.

6.2 2nd method: Feedback Linearization

In this method, we take the couplings between the four coordinates into account, so we do not neglect the influences of the four coordinates on each other. This is done by considering a controller that 'cancels' the nonlinear dynamics of the model. The model for the slave robot can be written as

\[
M(q_s)\ddot{q}_s + C(q, \dot{q}_s, \dot{q}_s)\dot{q}_s + g(q_s) + f(\dot{q}_s) = \tau_s
\]  

(6.1)

where \( q_s \) refers to the position of the slave robot and the friction forces are modeled as in (2.5) for \( i=s \).

To linearize the system the controller has to be designed, such that the non-linear terms can be cancelled in the closed loop equation. In the nominal case with state feedback, the controller that cancels the non-linear terms completely can be described as

\[
\tau_s = M(q_s)\ddot{q}_r + C(q, \dot{q}_s, \dot{q}_s)\dot{q}_s + g(q_s) + f(\dot{q}_s) \\
\text{with } \dot{q}_r = \dot{q}_d - K_p e - K_d \dot{e}
\]  

(6.2)

Substitution of equation (6.2) in equation (6.1) leaves us with the closed loop equation

\[
\ddot{q}_r - \ddot{q}_s = 0
\]  

(6.3)
which is equivalent to

$$\ddot{e} + K_p e + K_d \dot{e} = 0$$

(6.4)

As stated earlier in this report, only the link positions $q_{u}, q$, are measured, therefore (6.2) cannot be implemented. With observers we can estimate the unknown states and use them to modify the controller in (6.2). This gives the controller

$$\tau_s = M(q_s)\dot{q}_s + C(q_s, \dot{q}_s)\ddot{q}_s + g(q_s) + f(\dot{q}_s)$$

(6.5)

with $\ddot{q}_s = \ddot{q}_d - K_p e - K_d \dot{e}$. Here the PD action is designed as a PD block in Simulink (a design tool in Matlab) where $\dot{e}$ is estimated automatically with a numerical differentiation method that could not be determined. Therefore and for simplicity we write $\dot{e}$ not as an estimated value, this yields

$$\ddot{e} + K_p e + K_d \dot{e} = \Delta$$

(6.6)

$$\Delta = M(q_s)^{-1}((C(q_s, \dot{q}_s)\ddot{q}_s - C(q_s, \dot{q}_s)\ddot{q}_s) + (f(\dot{q}_s) - f(\dot{q}_s)))$$

(6.7)

$\Delta$ is the uncertainty due to the estimation error. For simplicity, we assume the ideal case in which the observers are designed optimal so that we can assume $\Delta = 0$. This results in a linear closed loop equation which is equal to (6.4)

$$\ddot{e} + K_p e + K_d \dot{e} = 0$$

(6.8)

The closed loop equation (6.8) is linear, so this means that we have a linearized system that is decoupled. For this system, observers can be designed that are linear as well.

$$\frac{d}{dt} \dot{e} = \dot{\dot{e}} + \Lambda_1 \ddot{e}$$

(6.9)

$$\frac{d}{dt} \dot{\dot{e}} = -K_p e - K_d \dot{e} + \Lambda_2 \ddot{e}$$

$$\frac{d}{dt} \ddot{q}_s = \ddot{q}_s + L_p \ddot{q}_s$$

(6.10)

$$\frac{d}{dt} \ddot{q}_s = \ddot{q}_d - K_p e - K_d \dot{e} + L_p \ddot{q}_s$$

The observers in equation (6.9) and (6.10) are used for the estimation of the errors and the slave joint variables respectively. The detailed calculation of the observers can be found in Appendix B. Now we have a feedback linearized system, where for the frequency response measurements of the slave system, the computed torque part of the controller and the observers can be considered as the linear plant with input $\dot{q}_s$ and output $\ddot{q}_s$.

To be able to tune the gains, first the bandwidth of the closed loop system must be determined. The frequency response data of the system is obtained for the four coordinates. By fitting a model to the experimentally determined data, the transfer function for each of the coordinates is obtained. The bandwidth is determined by taking the magnitude of the smallest right-half plane zero as described in [2].
Tuning and Performance of a CFT Master-Slave Robot System

The gains are tuned in the open loop system with the software program DIET (see [5]) in order to have a stable closed loop system. The proportional gains are increased until the system crosses the 0 dB line at the desired bandwidth. Sometimes it is not possible with a PD controller to cross the 0 dB line at the desired bandwidth because there is a resonance in that region. In that case, the bandwidths for which the gains are tuned are chosen lower to be sure that the gains yield a stable system.

Now that the $K_p$ gains are fixed we can concentrate on the $K_d$ gains. In the real system it seemed that the observer is very sensitive for the $K_d$ gains, high $K_d$ gains make the system unstable. The derivative gains $K_d$ in the observers are multiplied with $\hat{e}$, which is an estimation of $\dot{e}$. Since there is some noise present in the estimations, $K_d$ acts also as an amplifier of noise. This noise amplification can cause saturation of the voltages that can lead to instability of the system. Therefore we should be careful with choosing high $K_d$ gains. Therefore we choose $K_d$ gains that give a phase margin of approximately 20 degrees instead of 45 degrees that is used for linear systems. With this method, the gains as listed in table 6.2 are obtained.

<table>
<thead>
<tr>
<th>Bandwidth [Hz]</th>
<th>Phase margin [deg]</th>
<th>$K_p$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>11</td>
<td>5000</td>
<td>20</td>
</tr>
<tr>
<td>$q_2$</td>
<td>5</td>
<td>7000</td>
<td>80</td>
</tr>
<tr>
<td>$q_4$</td>
<td>4</td>
<td>9000</td>
<td>60</td>
</tr>
<tr>
<td>$q_5$</td>
<td>3</td>
<td>10000</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 6.2: Initial tuned gains

We have now tuned the gains for a feedback-linearized system to be implemented in a highly nonlinear system with a model-based controller and two model-based observers. The gains lead to a stable nonlinear system, but because the nonlinear system is not tuned directly but uses another system to generate tuned gains, it does not guarantee the best performance of the system. Therefore the obtained gains should be considered as starting values that at least give a stable nonlinear system that could be improved further.

Since the system is highly nonlinear and coupled in a complex way, further improvement by online tuning, i.e. while the system is running, is necessary. First the proportional gains are increased such that the synchronization error decreases. Then the proportional gains are varied to decrease the synchronization error even more. While tuning online, the following points should be taken into account.

- The $K_p$ and $K_d$ values of each of the coordinates $q_1$, $q_2$, $q_4$ and $q_5$ have influence on the other three coordinates.
- There are some sets of $K_p$ and $K_d$ values that are not stable for the fixed sets of $L_{p1}$ and $L_{p2}$.
- Increasing the gains for the controller, especially $K_d$, can have a negative effect on the estimation errors in the observers.
- High $K_p$ and $K_d$ values can saturate the voltages of the system, which can lead to high errors or even instability.
With online tuning we get the improved values for the gains as listed in table 6.4. Here the comparison is made with the previous chosen gains, listed in table 6.3, which were used for the synchronization goal in the Ph.D. program as mentioned in the introduction. The results are shown in figures 6.1 and 6.2.

Table 6.3: Previous chosen gains

<table>
<thead>
<tr>
<th></th>
<th>$K_p$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>15000</td>
<td>20</td>
</tr>
<tr>
<td>$q_2$</td>
<td>2000</td>
<td>50</td>
</tr>
<tr>
<td>$q_4$</td>
<td>12000</td>
<td>50</td>
</tr>
<tr>
<td>$q_5$</td>
<td>12000</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 6.4: Gains obtained by online tuning

<table>
<thead>
<tr>
<th></th>
<th>$K_p$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>60000</td>
<td>50</td>
</tr>
<tr>
<td>$q_2$</td>
<td>10000</td>
<td>50</td>
</tr>
<tr>
<td>$q_4$</td>
<td>15000</td>
<td>30</td>
</tr>
<tr>
<td>$q_5$</td>
<td>15000</td>
<td>30</td>
</tr>
</tbody>
</table>

Figure 6.1: Error index for the previous chosen gains (solid) and the new tuned gains (dotted)

Figure 6.1 shows that for some coordinates of the system with the new tuned gains the transient shows a jump in the error index. This is due to the high proportional and derivative gains that cause overcompensation in the transition phase. For all coordinates it is clear from the slopes of the error indices that the new tuned gains show a smaller error after the initial effects have died out. This means that the performance has improved for all the four degrees of freedom.
Figure 6.2: Synchronization errors $e_s = q_s - q_m$ for the previous chosen gains (black) and the new tuned gains (gray)

Figure 6.2 presents the synchronization errors for the new tuned gains and the previous chosen gains. From this figure it is even clearer that the performance improves. This improvement is relatively big for $q_1$ and $q_2$ and smaller for $q_4$ and $q_5$.

To give the performance improvement in a percentage the average error $\bar{e}$ of the new tuned gains and the previous chosen gains is determined as

$$\bar{e} = \frac{e_{\text{max}} - e_{\text{min}}}{2}$$  \hspace{1cm} (6.11)

with $e_{\text{max}}$ the maximum synchronization error and $e_{\text{min}}$ the minimum synchronization error. Then the performance improvement can be defined as

$$\text{performance improvement} = \frac{\bar{e}_{\text{previous gains}} - \bar{e}_{\text{new gains}}}{\bar{e}_{\text{previous gains}}} \cdot 100\%$$  \hspace{1cm} (6.12)

For the four coordinates this improvement has been calculated and listed in table 6.5.

<table>
<thead>
<tr>
<th>Performance improvement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>69%</td>
</tr>
<tr>
<td>$q_2$</td>
<td>74%</td>
</tr>
<tr>
<td>$q_4$</td>
<td>18%</td>
</tr>
<tr>
<td>$q_5$</td>
<td>35%</td>
</tr>
</tbody>
</table>

Table 6.5: Performance improvement
These results show a big improvement for the performance of $q_1$ and $q_2$. For $q_4$ and $q_5$ the improvement is smaller. This is probably because the previous chosen gains for $q_4$ and $q_5$ are by coincidence close to the gains with the best performance, so that the improvement can not be so big any more. This is also clear when we look at the small difference in the previous and new gains for $q_4$ and $q_5$ in table 6.3 and 6.4.

6.3 3rd method: Nichols-Ziegler

In the third method the linearized system of method 2 is used. Instead of a trajectory for the coordinates, a step function is used where for each coordinate three setpoints are used. With the first method of Nichols-Ziegler as described in [1], the $K_p$ and $K_d$ gains are estimated for the setpoints. For each coordinate, the mean value is calculated. The values obtained for the gains give an unstable system because they are too small to control the system properly. Therefore this method is not useful for this system.

7 Conclusions and recommendations

7.1 Conclusions

- A controller in joint space can present a better performance than in Cartesian space. This is due to time varying scaling caused by the determinant of the Jacobian of the kinematic relation between the Cartesian and joint coordinates and coupling effects introduced by this Jacobian. In Cartesian space the coupling between the two links of the arm is not considered in the forces, while in the joint space the coupling between the joints $q_4$ and $q_5$ is considered in the forces $F_{x_1}$ and $F_{x_2}$.

- A model-based controller has better performance than a PD controller. In the transient the error of the model-based controller can be bigger because of an overcompensation of the dynamics of the computed torque part in the model-based controller, but when the initial effects die out the model-based controller always shows a better performance.

- Depending on the value of the integral gain certain improvement in performance with respect to a PD controller can be achieved by a PID controller, specifically for the coordinates that are influenced by gravity $(x_{11}, x_{22})$. However it can also result in overcompensation. Nevertheless, the PID controller still has a lower performance than the model-based controller.

- It is not (yet) possible to tune the gains of a nonlinear system directly. Therefore, a linearized system should be considered which neglects as less elements as possible from the nonlinear system. Too many simplifications lead to gains that are tuned for a system that is too different from the system in which they are implemented.

- The proposed alternative for tuning the proportional and derivative gains of the PD action in the model-based synchronization controller is based on feedback linearization. This method proposes a controller that compensates for the nonlinear terms in the model, such that the closed loop system becomes linear. For this linearized system observers can be designed that are linear as well. For the four coordinates in the linearized system transfer functions are obtained and are used for tuning the PD action with techniques for linear systems. The tuned gains for this linear system are used as gains for the nonlinear system.

- The derivative gains $K_d$ in the observers are multiplied with $\hat{e}$, which is an estimation of $\dot{e}$. Since there is some noise present in the estimations, $K_d$ acts also as an amplifier of noise. This noise amplification can cause saturation of the voltages that can lead to instability of the system. Therefore the derivative gains can not be chosen relatively big, so that the performance improvement is mainly caused by the proportional gains $K_p$. 

_________________________________________________________________________________
The gains obtained by tuning a feedback-linearized system are used as gains for the original nonlinear system with model-based observers and controllers. They may lead to a stable nonlinear system, but do not guarantee the best performance of the system. Therefore, those gains should be considered as starting values that at least give a stable nonlinear system that can be improved further.

Because the nonlinear system is not tuned directly but uses another system to generate tuned gains, it is necessary to improve the performance further by means of online tuning i.e. tuning while the system is running. The gains obtained then are close to the best performance if the error is not changing obviously.

The previous chosen gains which were used for the synchronization goal in the Ph.D. program, as mentioned in the introduction, are compared with the new tuned gains which are obtained by the proposed feedback linearization method. The performance improvement obtained by the proposed method is for $q_1$, $q_2$, $q_4$ and $q_5$ respectively 69%, 74%, 18% and 35%. Because the previous chosen gains of $q_4$ and $q_5$ are by coincidence close to the new tuned gains the performance improvement cannot be relatively big.

7.2 Recommendations

Performance improvement can be achieved by tuning also the gains of the observers in a systematic way. By increasing $L_{p1}$ and $L_{p2}$ which are chosen equal to $\Lambda_1$ and $\Lambda_2$ respectively, the part multiplied with these gains has a higher weighting factor than before. The influence of $K_p$ and $K_d$ in the observer is therefore reduced, so that these gains can be increased more without making the system unstable.

To reduce the noise in the estimations of the observers, a low pass filter can be used to filter the high frequency noise. This leads to estimations with less noise, so that saturation of the voltages is not reached so fast. This gives a better performance of the synchronization.

The physical parameters for this system are based on estimations and are kept fixed. This means that there is an uncertainty in the estimated parameters which contributes to a lower performance. By considering adaptive control, where the values of the physical parameters are converging to a constant value, the parameters can be estimated better which can lead to a better performance of synchronization.
Appendix A: Dynamic model and estimated parameters of the CFT-robot

Here the entries of the dynamic model of the CFT transposer robot are presented. The dynamics of the CFT transposer robot is given by

\[
M(q_i)\ddot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + g(q_i) + f(\dot{q}_i) = \tau_i \quad \text{with} \quad i = m, s
\]  

(A.1)

where as in [3] the friction forces \( f(\dot{q}_i) \) are modeled as

\[
f(\dot{q}_i) = B_{v,i}\dot{q}_i + B_{f,i}(1 - \frac{2}{1 + e^{2\omega_{i,cl}\dot{q}_i}}) + B_{f2,i}(1 - \frac{2}{1 + e^{2\omega_{i,cl}\dot{q}_i}})
\]  

(A.2)

The estimated physical parameters are listed in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>description</th>
<th>value</th>
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<tbody>
<tr>
<td>( \theta_1 )</td>
<td>( m_1 + m_2 )</td>
<td>121.3049</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>( m_2l_c d )</td>
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</tr>
<tr>
<td>( \theta_3 )</td>
<td>( m_2l_{bc} )</td>
<td>4.1955</td>
</tr>
<tr>
<td>( \theta_4 )</td>
<td>( m_2(l_{ca}^2 + l_{cb}^2) + m_3(l_{ca}^2 + l_{cd}^2) + m_4l_{ca}^2 + m_4l_{cd}^2 + m_5l_{bc}^2 + m_7(l_{ca}^2 + l_{cd}^2) + m_8(l_{ca}^2 + l_{cd}^2) + I_{xx1} + I_{xx2} + I_{xy1} + I_{xy2} + I_{yy1} + I_{yy2} + I_{xx3} + I_{xx4} + I_{yx1} + I_{yx2} + I_{xx5} + I_{xx6} + I_{xx7} + I_{xx8} + I_{xx8} + I_{xx8} + I_{xx8} + I_{xx8} )</td>
<td>1.7453</td>
</tr>
<tr>
<td>( \theta_5 )</td>
<td>( m_4l_{cd} )</td>
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<tr>
<td>( \theta_6 )</td>
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<td>( \theta_7 )</td>
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<td>( \theta_{11} )</td>
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<td>( \theta_{16} )</td>
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<tr>
<td>( \theta_{32} )</td>
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</table>

Table A.1: Estimated parameters for the CFT transposer robot
Entries of the inertia matrix $M(q_i)$

The entries of the inertia matrix $M(q_i) \in \mathbb{R}^{4 \times 4}$, as function of the generalized coordinates $q_i = [q_{i,1} \quad q_{i,2} \quad q_{i,3} \quad q_{i,4}]^T$ are given by

$$M_{1,1} = \theta_1 + \theta_{11} + \theta_{12}$$

$$M_{1,2} = (-\theta_{12}d_{2,0'} - \theta_{11}d_{2,0'} - \theta_3) \sin(q_2) + (\theta_2 + d_0 \theta_{11}) \cos(q_2) + \frac{1}{2}((L_4 - L_6) (\theta_{12} + \theta_{11}) - \theta_5) (\cos(q_5 + q_2 + q_4) - \cos(q_5 - q_2 + q_4)) + \frac{1}{2}(\theta_4 + \theta_6) \sin(q_2 + q_4) - \sin(-q_2 + q_4)) + \frac{1}{2}(\theta_9 + \theta_{12}L_6)(\cos(-q_2 + q_4) - \cos(q_2 + q_4))$$

$$M_{1,3} = \frac{1}{2}(-\theta_5 - \theta_7 - \theta_{12}L_6)(\cos(q_2 + q_4) + \cos(-q_2 + q_4)) + \frac{1}{2}((L_4 - L_6)(\theta_{12} + \theta_{11}) - \theta_5) (\cos(q_5 - q_2 + q_4) - \cos(q_5 - q_2 + q_4)) + \frac{1}{2}(\theta_8 + \theta_6) \sin(q_2 + q_4) + \sin(-q_2 + q_4) + \frac{1}{2}(\sin(q_5 - q_2 + q_4) + \sin(q_5 + q_2 - q_4)) \theta_{10}$$

$$M_{1,4} = \frac{1}{2}((L_4 - L_6)(\theta_{12} + \theta_{11}) - \theta_5) (\cos(q_5 + q_2 + q_4) + \cos(q_5 - q_2 + q_4)) + \frac{1}{2}(\sin(q_5 + q_2 + q_4) + \sin(q_5 - q_2 + q_4)) \theta_{10}$$

$$M_{2,2} = \frac{1}{2}(\cos(q_5 + q_2 + q_4) - 2 \cos(q_4)d_{2,0'}) \theta_9 + \frac{1}{2}((L_4 - L_6)(\sin(q_5 + q_2 + q_4)) - 2 \cos(q_4)d_{2,0'}) \theta_9 + \frac{1}{2}(\theta_2 + d_0 \theta_{11}) \cos(q_2) + \frac{1}{2}((L_4 - L_6) (\theta_{12} + \theta_{11}) - \theta_5) (\cos(q_5 + q_2 + q_4) - \cos(q_5 - q_2 + q_4)) + \frac{1}{2}(\cos(q_4) - \cos(q_5) - 1 + \cos(q_5 + q_4)) \theta_{12} + \frac{1}{2}(\cos(q_5 + q_2 + q_4) - \cos(q_5)) (L_4 - L_5) - 2d_{2,0'} \sin(q_4)) \theta_6 - 2 \sin(q_4 + q_5) L_5 + \sin(q_4) L_5 d_{2,0'} \theta_{12} + ((\cos(q_5 - q_2 + q_4) - \cos(q_5)) (L_4 - L_5) - 2d_{2,0'} \sin(q_4)) \theta_7 + \frac{1}{2}(\theta_1 + (\cos(q_4) - 1) L_5 - \frac{1}{2} (\cos(q_5 + q_2 + q_4) - 1) (L_5 + L_5) \theta_{12} + \frac{1}{2}(\cos(q_5 + q_2 + q_4) - 1) L_4 + \cos(q_5 - q_2 + q_4)) L_5 \theta_{12} + \frac{1}{2}(\cos(q_5 - q_2 + q_4) - 1) L_4 - 2d_{2,0'} \sin(q_4 + q_5) \theta_9 + (2 \sin(q_4 + q_5)d_{2,0'} - (\cos(q_5) + \cos(q_5 + 2q_4)) L_2) L_4 \theta_{12}
\[ M_{2,3} = -\theta_7 d_6 \cos(q_4) + \theta_8 d_6 \sin(q_4) + \theta_{11} d_6 (L_4 - L_5) \cos(q_4 + q_5) \]
\[ M_{2,4} = \theta_{11} d_6 (L_4 - L_5) \cos(q_4 + q_5) \]

\[ M_{3,3} = ((L_4 - L_5)L_4 + 2L_3 L_2)\theta_{12} + L_3 L_2 \theta_{11} + (\theta_9 - \theta_5 - \theta_7)L_4 + 2\theta_7 L_5) \cos(q_5) + L_4 (\theta_6 + \theta_8) \sin(2q_4) 
\]
\[ -L_4 \left( \frac{1}{2} L_4 + L_6 \right) \theta_{12} + \frac{1}{2} L_4 \theta_{11} + \theta_5 + \theta_7 \cos(2q_4) \]
\[ -L_4 \left( L_5 + \frac{1}{2} L_4 \theta_{11} + \theta_5 + \theta_7 \right) \cos(q_5 + 2q_4) \]
\[ + (\theta_8 + \theta_{10} + \theta_6) L_4 \sin(q_5 + 2q_4) + (2L_2 - L_4) \theta_8 \]
\[ - (\theta_6 + \theta_{10} + \theta_4) L_4 \sin(q_5) + (L_4^2 + L_6 - L_4) L_4 + L_2^2 \theta_{12} \]
\[ + (\frac{1}{2} L_4 - L_5) (\theta_{12} + \theta_{11}) - \theta_9) L_4 \cos(2q_5 + 2q_4) \]
\[ + (L_3^2 - L_5 L_4 + L_2^2) \theta_{11} + (\theta_7 - \theta_9 + \sin(2q_5 + 2q_4) \theta_{10} - \theta_5) L_4 \]

\[ M_{3,4} = \frac{1}{2} \left( \sin(q_5 + 2q_4) - \sin(q_5) \right) L_4 \theta_{12} + \frac{1}{2} \theta_{12} \cos(q_5 + 2q_4) + 1) L_4^2 \]
\[ + \frac{1}{2} \cos(q_5) - \cos(2q_5 + 2q_4) - \frac{1}{2} \cos(q_5 + 2q_4) - 1 \right) L_4 \theta_9 \]
\[ + \frac{1}{2} \sin(q_5 + 2q_4) + \sin(2q_5 + 2q_4) - \frac{1}{2} \sin(q_5) \right) L_4 \theta_{10} \]
\[ + (L_5 \cos(q_5) - \frac{1}{2} \left( \cos(q_5) + \cos(q_5 + 2q_4) \right) L_4 \theta_7 \]
\[ + (L_5 - \frac{1}{2} L_4) \sin(q_5) + \frac{1}{2} L_4 \sin(q_5 + 2q_4) \theta_9 + \theta_{12} L_5^2 \]
\[ + \left( L_2^2 + \frac{1}{2} \cos(q_5) - \cos(q_5 + 2q_4) \right) L_4 L_5 + \frac{1}{2} \left( 1 + \cos(2q_5 + 2q_4) \right) \]
\[ \times (L_4^2 - 2L_4 L_5) \right) \theta_{11} - \frac{1}{2} \left( \cos(q_5) + \cos(q_5 + 2q_4) \right) L_4 \theta_9 \]
\[ + (\cos(q_5) L_6 - \frac{1}{2} (2 + 2 \cos(2q_5 + 2q_4) + \cos(q_5 + 2q_4) \]
\[ - \cos(q_5) \right) L_4 L_5 \theta_{12} - \frac{1}{2} \theta_{12} \cos(q_5) + \cos(q_5 + 2q_4) \right) \]
\[ L_6 L_4 \]

\[ M_{4,4} = \left( \frac{1}{2} L_4^2 - L_5 L_4 \right) (\theta_{12} + \theta_{11}) - L_4 \theta_9 \cos(2q_5 + 2q_4) \]
\[ + \left( \frac{1}{2} L_4^2 - L_5 L_4 + L_2^2 \right) \left( \theta_{12} + \theta_{11} \right) + \sin(2q_5 + 2q_4) \theta_{10} - \theta_9 \right) L_4 \]

**Entries of the Coriolis matrix \( C(q_i, \dot{q}_i) \)**

The entries of the Coriolis matrix \( C(q_i, \dot{q}_i) \in \mathbb{R}^{4 \times 4} \) as function of the generalized coordinates \( q_i = [q_{i,1} \quad q_{i,2} \quad q_{i,4} \quad q_{i,5}]^T \) are given by

\[ C_{1,1} = C_{2,1} = C_{3,1} = C_{4,1} = 0 \]

\[ C_{1,2} = \frac{1}{2} \left( \theta_7 + \theta_5 + L_6 \theta_{12} \right) \left( \left( \dot{q}_2 - \dot{q}_4 \right) \sin(q_4 - q_2) + \left( \dot{q}_2 + \dot{q}_4 \right) \right) \]
\[ \times \left( \sin(q_2 + q_4) \right) - \left( \left( \theta_{12} + \theta_{11} \right) d_2 \dot{\theta}_5 + \theta_3 \right) \dot{q}_2 \cos(q_2) \]
\[ + \frac{1}{2} \left( \theta_8 + \left( L_5 - L_4 \right) \left( \theta_{12} + \theta_{11} \right) \right) \left( \left( \dot{q}_4 + \dot{q}_2 + q_5 \right) \dot{q}_4 + \sin(q_5 + q_2 + q_4) \right) \]
\[ - \left( \theta_2 + d_5 \theta_{11} \right) \dot{q}_2 \sin(q_2) + \frac{1}{2} \left( \theta_8 + \theta_6 \right) \left( \left( \dot{q}_2 - \dot{q}_4 \right) \cos(q_4 - q_2) \right) \]
\[ + \left( \dot{q}_2 + \dot{q}_4 \right) \cos(q_2 + q_4) + \frac{1}{2} \left( \left( \dot{q}_2 - \dot{q}_4 \right) \right) \]
\[ \times \cos(q_4 - q_2 + q_4) + \left( \dot{q}_4 + \dot{q}_2 + \dot{q}_5 \right) \cos(q_5 + q_2 + q_4) \theta_{10} \]
\[ C_{1,3} = \frac{1}{2}(\theta_9 + \theta_6)((q_4 - q_2) \cos(q_4 - q_2) + (q_4 + q_2) \cos(q_4 + q_2)) + \frac{1}{2}(\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11}))((q_4 - q_2 + q_4) + (q_4 - q_2 + q_4) \sin(q_5 - q_2 + q_4)) + \frac{1}{2}((q_4 + \theta_9 + L_5 \theta_{12}((q_4 - q_2) \sin(q_4 - q_2) + (q_4 + q_4))) + \frac{1}{2}((q_4 + q_2 + q_4) \cos(q_5 - q_2 + q_4))) \theta_{10} \]

\[ C_{1,4} = \frac{1}{2}((q_4 + q_2 + q_5) \cos(q_5 + q_2 + q_4) + (q_4 - q_2 + q_5) \cos(q_5 - q_2 + q_4)) \]

\[ \times \cos(q_5 - q_2 + q_4)) \theta_{10} + \frac{1}{2}((\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11}) \sin(q_5 + q_2 + q_4)) \]

\[ + (q_4 + q_2 - q_5) \sin(q_5 - q_2 + q_4)) \]

\[ C_{2,2} = -\frac{1}{2}(L_4 \theta_6 - L_5 \theta_6 + \theta_6 L_4)((2q_4 + q_5) \cos(2q_4 + q_5)) \]

\[ + q_5 \cos(q_5) - q_4 d_{2,0} \cos(2q_4 + 2q_5) \]

\[ - (q_4 + q_5) \cos(q_5 + q_2 + q_4) \]

\[ - \frac{1}{2}(q_4 + q_5)(2L_4 \theta_9 - (L_5 - L_4)^2(\theta_{12} + \theta_{11})) \sin(2q_5 + 2q_4) \]

\[ - \frac{1}{2}q_4(2L_4 \theta_9 - L_5 \theta_{12}) \sin(2q_4) - q_4 L_4 \theta_6 \cos(2q_4) \]

\[ + \frac{1}{2}(L_4 \theta_5 - L_5 \theta_6 \theta_9 + L_6 \theta_4)((2q_4 + q_5) \sin(2q_4 + q_5)) \]

\[ + q_5 + q_5)(2q_4 + q_5) \sin(2q_4 + q_5)) \]

\[ - (q_4 + q_5) \sin(2q_4 + q_5)) \]

\[ + (q_4 + q_5) \sin(q_4 + q_5) \sin(q_5 + q_5) \sin(q_5 + q_5) \sin(q_5 + q_5) \]

\[ + (q_4 + q_5) \cos(2q_4 + 2q_5) \]

\[ + d_{2,0} \theta_{10} \sin(q_4) \]

\[ + q_4 d_{2,0} \theta_9 - q_4 \theta_{12} + \theta_9 \theta_{12} + \theta_9 d_{2,0} \theta_9 \sin(q_4) \]

\[ + (q_4 + q_5) \sin(q_4 + q_5) \sin(q_4 + q_5) \]

\[ + q_4 d_{2,0} \theta_9 \sin(q_4) \]

\[ + q_4 d_{2,0} \theta_9 \sin(q_4) \]

\[ - q_2 L_4 \theta_{10} \cos(2q_4 + 2q_5) \]

\[ + \frac{1}{2} q_2 (L_4 \theta_9 + \theta_9 (L_4 - L_5)(\cos(q_5) \cos(2q_4 + q_5)) \]

\[ - \frac{1}{2} q_2 (L_4 \theta_9 + \theta_9 (L_4 - L_5)(\cos(q_5) \cos(2q_4 + q_5)) \]

\[ + \frac{1}{2} q_2 ((L_5 - L_4)(L_5 \theta_{12} + \theta_7 - L_4 \theta_5) \sin(2q_4 + q_5) - \sin(q_5)) \]

\[ C_{2,3} = \frac{1}{2} q_2 (L_4 \theta_9 - L_5 \theta_{12} + 2L_4 \theta_9 \sin(2q_4) - q_2 \cos(2q_5 + 2q_4) L_4 \theta_{10} \]

\[ + \frac{1}{2} q_2 ((L_5 - L_4)^2(\theta_{12} + \theta_{11}) - 2L_4 \theta_9 \sin(2q_5 + 2q_4) \]

\[ + (q_4 + q_5)(2L_4 \theta_9 - L_5 \theta_{12} + \theta_{11} - d_{2,0} \theta_9 \sin(q_4 + q_5) \]

\[ - (L_5 - L_4)(\theta_{12} + \theta_{11}) \sin(q_4 + q_5) \]

\[ + q_4 d_{2,0} \theta_9 - q_4 \theta_{12} + \theta_{12} + \theta_9 d_{2,0} \theta_9 \sin(q_4) \]

\[ + (q_4 + q_5) \sin(q_4 + q_5) \sin(q_4 + q_5) \]

\[ + q_4 d_{2,0} \theta_9 \sin(q_4) \]

\[ + q_4 d_{2,0} \theta_9 \sin(q_4) \]

\[ - q_2 L_4 \theta_{10} \cos(2q_4 + 2q_5) \]

\[ + \frac{1}{2} q_2 (L_4 \theta_9 + \theta_9 (L_4 - L_5)(\cos(q_5) \cos(2q_4 + q_5)) \]

\[ + \frac{1}{2} q_2 ((L_5 - L_4)(L_5 \theta_{12} + \theta_7 - L_4 \theta_5) \sin(2q_4 + q_5) - \sin(q_5)) \]

\[ C_{2,4} = ((q_4 + q_5)(L_5 - L_4)d_{2,0} \theta_9 + q_2 d_{2,0} \theta_{10} \sin(q_4 + q_5) \]

\[ + \frac{1}{2} q_2 ((L_5 - L_4)^2(\theta_{12} + \theta_{11}) - 2L_4 \theta_9 \sin(2q_5 + 2q_4) \]

\[ - ((L_5 - L_4)(\theta_{12} + \theta_{11}) + \theta_9 d_{2,0} \theta_9 \cos(q_4 + q_5) \]

\[ - q_2 L_4 \theta_{10} \cos(2q_5 + 2q_4) \]

\[ + \frac{1}{2} q_2 (L_4 \theta_9 + \theta_9 (L_4 - L_5)(\cos(q_5) \cos(2q_4 + q_5)) \]

\[ + \frac{1}{2} q_2 ((L_5 - L_4)(L_5 \theta_{12} + \theta_7 - L_4 \theta_5) \sin(2q_4 + q_5) - \sin(q_5)) \]
\[ C_{3,2} = \dot{q}_2 L_4 \theta_{10} \cos(2q_8 + 2q_4) + \dot{q}_2 L_4 \theta_6 \cos(2q_4) \\
+ \dot{q}_2 \dot{q}_2 \theta_{20} (\theta_9 + (L_5 - L_4)(\theta_{12} + \theta_{11})) \cos(q_4 + q_5) \\
- \dot{q}_2 \dot{q}_2 \theta_{20} \sin(q_4 + q_5) \theta_{10} + \frac{1}{2} \dot{q}_4 (2L_4 \theta_3 - L_0 \theta_{12}) \sin(2q_4) \\
+ \frac{1}{2} \dot{q}_4 (L_4 \theta_5 - (L_5 - L_4)(L_0 \theta_{12} + \theta_7)) \sin(2q_4 + q_5) \\
\frac{1}{2} \dot{q}_4 (-(L_5 - L_4)^2(\theta_{12} + \theta_{11}) + 2L_4 \theta_5) \sin(2q_4 + 2q_8) \\
+ \frac{1}{2} \dot{q}_4 \theta_5 + \theta_7 + L_0 \theta_{12}) \cos(q_4) - \dot{q}_2 \dot{q}_2 \theta_{20} \sin(q_4) \\
+ \dot{q}_2 \dot{q}_4 \theta_6 + \theta_5 (L_4 - L_3) \cos(2q_4 + q_5) \]

\[ C_{3,3} = \frac{1}{2} L_4 (\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\theta_{12} + \theta_{11}) + 2\theta_6) \sin(2q_4 + 2q_8) \\
- \frac{1}{2} \dot{q}_4 ((2L_5 - L_4) \theta_3 - L_5 \theta_4 + L_5 \theta_4) \theta_{12} + (2L_5 - L_4) \theta_7 + \theta_{11} L_5 \theta_4 \\
+ L_4 (\theta_6 - \theta_5) \sin(q_3) + L_4 (\dot{q}_4 + \dot{q}_5) \theta_{10} \cos(2q_4 + 2q_5) \\
- \frac{1}{2} L_4 (\dot{q}_4 + \dot{q}_5)(L_5 + L_3) \theta_{12} + \theta_7 + L_5 \theta_{11} + \theta_5 + \theta_6 \\
\times \sin(2q_4 + q_5) + L_4 \dot{q}_4 (\dot{q}_6 + \theta_6) \cos(2q_4) \\
\frac{1}{2} L_4 \dot{q}_4 ((L_4 + 2L_6) \theta_{12} + 2\theta_7 + L_4 \theta_{11} + 2\theta_5) \sin(2q_4) \\
+ \frac{1}{2} L_4 (\dot{q}_4 + \dot{q}_5)(\theta_6 + \dot{q}_6 + \theta_10) \cos(q_4 + q_5) \\
\frac{1}{2} \dot{q}_4 (L_4 \theta_6 + \theta_{10}) + \theta_8 (L_4 - 2L_5) \cos(q_5) \]

\[ C_{3,4} = \frac{1}{2} (\dot{q}_5 + \dot{q}_{10} + \dot{q}_6) \cos(2q_4 + q_5) + \theta_{10} \cos(2q_4 + 2q_8))L_4 \\
\times (\dot{q}_4 + \dot{q}_5) - \frac{1}{2} (\dot{q}_4 + \dot{q}_5)(L_5 L_4 \theta_{12} + (2L_5 - L_4)(\theta_7 + L_5 \theta_{12}) \\
+ \theta_{11} L_5 L_4 + L_4 (\theta_9 - \theta_5)) \sin(q_3) + \theta_{11} L_5 L_4 + L_4 (\theta_9 - \theta_5) \sin(q_3) \\
+ \frac{1}{2} L_4 (\dot{q}_4 + \dot{q}_5)((2L_5 - L_4)(\theta_{12} + \theta_{11}) + 2\theta_5) \sin(2q_4 + 2q_8) \\
+ \frac{1}{2} (\dot{q}_4 + \dot{q}_5)((2L_5 - L_4) \theta_8 - L_4 (\theta_{10} + \theta_6)) \cos(q_3) \\
\]

\[ C_{4,2} = \dot{q}_2 \dot{q}_2 \theta_{20} ((L_5 - L_4) (\theta_{12} + \theta_{11}) + \theta_9) \cos(q_4 + q_5) \\
+ \frac{1}{2} \dot{q}_2 ((L_5 - L_4)(L_0 \theta_{12} + \theta_7) - L_5 \theta_5) (\sin(q_3) - \sin(2q_4 + q_5)) \\
+ \frac{1}{2} \dot{q}_2 (2L_4 \theta_6 - (\theta_{12} + \theta_{11})(L_5 - L_4)^2 \sin(2q_5 + 2q_4) \\
+ \dot{q}_2 L_4 \theta_{10} \cos(2q_5 + 2q_4) - \dot{q}_2 \dot{q}_2 \theta_{20} \sin(q_4 + q_5) \\
\frac{1}{2} \dot{q}_2 (L_4 \theta_6 - (L_5 - L_4) \theta_6) (\cos(q_5) + \cos(2q_4 + q_5)) \\
\]
Tuning and Performance of a CFT Master-Slave Robot System

The entries of the gravity vector $g(q_i)$ as function of the generalized joint coordinates $q_i = [q_{i,1}, q_{i,2}, q_{i,4}, q_{i,5}]^T$ and the acceleration due to gravity $g = 9.81 \, \text{m/s}^2$ are given by

$$
g_1 = g_2 = 0$$

$$
g_3 = -g(q_4 + q_5)(2L_5 - L_4)(\theta_{12} + \theta_{11}) \sin(q_4 + q_5) - g(\theta_8 + \theta_7) \cos(q_4) - g(\theta_8 + \theta_7) \sin(q_4 + q_5)$$

$$
g_4 = -g(\theta_8 + \theta_7) \sin(q_4 + q_5) - g\theta_{10} \cos(q_4 + q_5)$$

The entries of the vector of friction forces $f(q_i)$ in the transposer robot are modeled by (2.5), such that the entries of $f(\dot{q}_i)$ can be written as function of the generalized velocities $\dot{q}_i = [\dot{q}_{i,1}, \dot{q}_{i,2}, \dot{q}_{i,4}, \dot{q}_{i,5}]^T$ as follows

$$
f_1(\dot{q}_1) = \theta_{13} \dot{q}_1 + \theta_{17} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_1}}\right) + \theta_{21} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_1}}\right)
$$

$$
f_2(\dot{q}_2) = \theta_{14} \dot{q}_2 + \theta_{18} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_2}}\right) + \theta_{22} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_2}}\right)
$$

$$
f_3(\dot{q}_3) = \theta_{15} \dot{q}_3 + \theta_{19} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_3}}\right) + \theta_{23} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_3}}\right)
$$

$$
f_4(\dot{q}_4) = \theta_{16} \dot{q}_4 + \theta_{20} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_4}}\right) + \theta_{24} \left(1 - \frac{2}{1 + e^{2\theta_{20}\dot{q}_4}}\right)
$$
Appendix B: Technical information of the CFT-robot

The force and torque, which generate equivalent displacements in the robot, are related by the transpose of the Jacobian of the kinematic relation between the Cartesian and joint coordinates.

\[
\tau = J(q)^T F
\]  

Relation (B.1), which is the same as relation (4.2), allows converting any Cartesian quantity into a joint space quantity without calculating any inverse kinematic function.

\[
\begin{bmatrix}
F_{x_1} \\
F_{x_2} \\
F_{x_3} \\
F_{x_4}
\end{bmatrix}
= 
\begin{bmatrix}
\tau_{q_1} \\
\tau_{q_2} \\
\tau_{q_3} \\
\tau_{q_4}
\end{bmatrix}
= 
\frac{1}{\Delta_J}
\begin{bmatrix}
-\tau_{q_4} \sin(q_4 + q_5) + \tau_{q_4} (\sin(q_4 + q_5) + \sin(q_4)) \\
\tau_{q_4} \cos(q_4 + q_5) - \tau_{q_5} (\cos(q_4 + q_5) + \cos(q_4))
\end{bmatrix}
\]  

(B.2)

where \( \Delta_J = L_4 (\sin(q_4 + q_5) \cos(q_4) - \cos(q_4 + q_5) \sin(q_4)) \)

\[
\begin{bmatrix}
\tau_{q_1} \\
\tau_{q_2} \\
\tau_{q_3} \\
\tau_{q_4}
\end{bmatrix}
= 
\begin{bmatrix}
F_{x_4} \\
F_{x_3} \\
F_{x_2} \\
F_{x_1}
\end{bmatrix}
= 
- L_4 (F_{x_2} (\cos(q_4 + q_5) + \cos(q_4)) - F_{x_1} (\sin(q_4 + q_5) + \sin(q_4)))
- L_4 (F_{x_3} \cos(q_4 + q_5) - F_{x_4} \sin(q_4 + q_5))
\]  

(B.3)

Equations (B.2) and (B.3) relate effective forces \( F_{x_i} \) applied by the servomotors to torques in the joint space \( \tau_{q_i} \), for \( i = 1,2,3,4 \) and \( j = 1,2,4,5 \). The forces \( F_{x_i} \) are proportional to the voltage applied at the servoamplifiers in the motors, with proportional gain given by \( K_T \) in Table B.1.

<table>
<thead>
<tr>
<th>Motor</th>
<th>Coordinate</th>
<th>Total gain</th>
<th>( K_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>( x_{e1} )</td>
<td>107.5681 N/V</td>
<td>[ \frac{N}{V} ]</td>
</tr>
<tr>
<td>m2</td>
<td>( x_{e2} )</td>
<td>107.5681 N/V</td>
<td>[ \frac{N}{V} ]</td>
</tr>
<tr>
<td>m3</td>
<td>( x_{e3} )</td>
<td>7.9454 N/m</td>
<td>[ \frac{Nm}{V} ]</td>
</tr>
<tr>
<td>m4</td>
<td>( x_{e4} )</td>
<td>26.8920 N/V</td>
<td>[ \frac{N}{V} ]</td>
</tr>
</tbody>
</table>

Table B.1: Proportional gains
Appendix C: Observers for the feedback linearized system

By considering $\Delta = 0$ in the feedback linearized system (6.6, 6.7) the closed loop equation becomes

$$\ddot{e} + K_r e + K_d \dot{e} = 0$$

(C.1)

To design the observer for the estimation of the error, we use a state space representation with states $\begin{bmatrix} \hat{e} & \hat{\dot{e}} \end{bmatrix}$. This brings us to the notation

$$\frac{d}{dt} \hat{e} = \hat{\dot{e}}$$
$$\frac{d}{dt} \hat{\dot{e}} = -K_r e - K_d \dot{e}$$

(C.2)

with a compensation for estimation errors, this becomes

$$\frac{d}{dt} \hat{e} = \hat{\dot{e}} + \Lambda_1 \tilde{e}$$
$$\frac{d}{dt} \hat{\dot{e}} = K_r e + K_d \dot{e} + \Lambda_2 \tilde{e}$$

(C.3)

where the estimated position and velocity synchronization errors $\tilde{e}, \tilde{\dot{e}}$ are defined by

$\tilde{e} = e - \hat{e}$ and $\tilde{\dot{e}} = \dot{e} - \hat{\dot{e}}$ and $\Lambda_1, \Lambda_2 \in \mathbb{R}^{n \times n}$ are positive definite gain matrices.

To design the observer for the estimation of the slave joint variables, we use a state space representation with states $\begin{bmatrix} \hat{q}_s & \hat{\dot{q}}_s \end{bmatrix}$, which brings us to

$$\frac{d}{dt} \hat{q}_s = \dot{\hat{q}}_s$$
$$\frac{d}{dt} \dot{\hat{q}}_s = \ddot{\hat{q}}_s - K_r e - K_d \dot{e}$$

(C.4)

with a compensation for unknown errors, this becomes

$$\frac{d}{dt} \hat{q}_s = \dot{\hat{q}}_s + L_{p1} \tilde{q}_s$$
$$\frac{d}{dt} \dot{\hat{q}}_s = \ddot{\hat{q}}_s - K_r e - K_d \dot{e} + L_{p2} \tilde{\dot{q}}_s$$

(C.5)

where the estimated position and velocity errors $\tilde{q}_s, \tilde{\dot{q}}_s$ are defined by

$\tilde{q}_s = q_s - \hat{q}_s$ and $\tilde{\dot{q}}_s = \dot{q}_s - \hat{\dot{q}}_s$ and $L_{p1}, L_{p2} \in \mathbb{R}^{n \times n}$ are positive definite gain matrices.
Appendix D: Short manual for the experimental setup

For the experiments a multi-composed robot system formed by two robot manipulators is considered. The robots in the experimental setup are industrial transposer robots designed by the Centre for Manufacturing Technology (CFT) Philips laboratory. These robots are installed at the Dynamics and Control Technology Laboratory at the Eindhoven University of Technology. The robots are equipped with encoders attached to the shaft of the motors. Although the shaft of the motors and the corresponding links are connected by belts, the pair servomotor-link prove to be stiff enough to be considered as a rigid joint. The robots are identical in their structure and design, therefore they are represented by the same dynamic model. However the physical parameters such as inertias, friction coefficients, etc., are different for each robot.

For implementation of the controllers and communication with the robots, the experimental setup is equipped with a DS1005 dSPACE system. Throughout the experiments the frequency of the DS1005 dSPACE system is set to 2 kHz.

The files that are necessary to do the experiments for master-slave synchronization can be found in the computer that is connected to the dSPACE system of the experimental setup. The files are located in the folder: D:\users\master-slave and are listed in table D.1.

<table>
<thead>
<tr>
<th>Interface</th>
<th>Synchronization control</th>
<th>Controldesk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pizza_interface.c</td>
<td>coor_both_a.mdl</td>
<td>coor_full_both.lay</td>
</tr>
<tr>
<td>Pizza.h</td>
<td>coor_both.mdl</td>
<td></td>
</tr>
<tr>
<td>Pizza_work.c</td>
<td>coor_feed.c</td>
<td></td>
</tr>
<tr>
<td>Pizza_jog.c</td>
<td>observers.c</td>
<td></td>
</tr>
<tr>
<td>Pizza_pid.c</td>
<td>coor_routines.c</td>
<td></td>
</tr>
<tr>
<td>Pizza_papi.c</td>
<td>observers_routines.c</td>
<td></td>
</tr>
<tr>
<td>Pizza_airbag.c</td>
<td>coor_work.c</td>
<td></td>
</tr>
<tr>
<td>Pizza_motor.c</td>
<td>observers_work.c</td>
<td></td>
</tr>
<tr>
<td>Pizza_interface.dll</td>
<td>coor_feed.dll</td>
<td></td>
</tr>
<tr>
<td>Pizza_IOports.h</td>
<td>observers.dll</td>
<td></td>
</tr>
</tbody>
</table>

Table D.1: Files for synchronization experiment

For the experiments the following software programs are necessary:

• Matlab 6 (R12)
• Simulink
• DSPACE Controldesk

In Matlab also some toolboxes are needed. All the necessary programs are installed on the computer that is connected to the dSPACE system of the experimental setup.

To start an experiment, the Simulink model needs to be compiled first with RTW (Real-Time Workshop) to make it a Real-time application. The Simulink model can be compiled in the following way: In Simulink go to Tools→Real-Time Workshop→Build Model or by pressing the buttons [Ctrl+B]. During the compilation a c-code of the Simulink model is generated. This c-code is used for the Real-time application.

When the compilation is finished, the Real-time application needs to be connected to the dSPACE system. This is done by loading the file with the extension .sdf in the dSPACE Controldesk, which is the software program of dSPACE. This file can be loaded in the following way: Platform→Application→Load Application.

To start the experiment a layout is necessary. The layout Coord_full_both.lay can be used but an own layout can be created as well with Controldesk. In the layout buttons can be created to turn on the servos, the power of the robots and to start the experiments. Also, plots of important data can be created in the layout. The layout can be opened in the following way: File→open and the layout can be selected and opened.

There are three modes in dSPACE, Edit Mode, Test Mode and Animation Mode. To start the experiments we have to switch to Animation Mode. This is done by pressing on the button F5 or by Instrumentation→Animation Mode.
To save some variables we first need to click on the button \[ 
\]
in the lower right corner in Controldesk. This button opens the Hostservice were the desired variables can be selected and saved. All the plotted variables are automatically selected.

Now the experiment can be started by turning on the power, the servos and by clicking on the start button.

The whole procedure can be ordered in the following steps:

1. Check whether everything is connected properly.
2. Put the DS1005 dSPACE system on.
3. Put the power of the robots on.
4. Turn the computer on.
5. Open the Simulink model coor_both_a.mdl.
6. Compile the model with [Ctrl+B].
7. Load the model in Controldesk.
8. Open the layout.
9. Switch to Animation Mode.
10. Select the variables that need to be saved.
11. Put the power and the servos on in Controldesk.
12. Press the start button.

There are two Simulink models in the folder D:\users\master-slave which are called coor_both_a.mdl and coor_both.mdl. The first model is the model with the improved gains that is used for the experiments in this report. The second model is the model that is used for the synchronization goal in the Ph.D. program as mentioned in the introduction. Both models can be used for the experiments.
Bibliography


[5] Steinbuch, M., 2001. DIET (Do It Easy Tool), a program which facilitates the design and tuning of controllers.