A method for the identification of material parameters related to orthotropic yielding

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A Method for the Identification of Material Parameters Related to Orthotropic Yielding

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Abstract

The main focus of this study is the implementation of an identification technique used for parameter estimation in mathematical models of material behaviour. The identification technique is a numerical-experimental procedure whereby the results of a numerical analysis are compared with experimental results. The parameters in the numerical model can then be adjusted until a good correlation is achieved. Previously, this technique had been used to find parameters relating to inhomogeneous materials [1], [2]. The results of previous work allowed a better understanding of what was required for the implementation of the technique.

The work covered finite element modelling of orthotropic materials displaying orthotropic yielding behaviour as described in [3]. The intention was to check the validity of the technique for future numerical-experimental parameter estimations. 'Perfect' observations from the finite element models were used, neglecting modelling or observation errors which may be significant during estimation. Noise was then introduced to the 'perfect' observations to simulate measurement error. The performance of the models was studied. It was possible to find conditions for optimum operation of the technique.
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Chapter 1

Introduction

In certain cases it can be difficult to determine the properties of a material with traditional testing techniques. For example, inhomogeneous materials are extremely difficult to characterize. It may also not be possible to take a sample without affecting the material properties. For instance woven fabric composites can be subject to the effects of fibre pull-out near the edges of the specimen. Also in the determination of parameters relating to the yield criterion for orthotropic yielding proposed by Hill [3], some of these parameters are difficult to determine by traditional testing methods. In such cases an identification technique is useful.

The identification technique is a mixed numerical-experimental approach which requires some mathematical model of material behaviour. The parameters relating to this model can then be found by an iterative process whereby results of the numerical model are compared to experimental observations, taking into account possible modelling and observation errors. Parameters are automatically adjusted in the model until a good correlation is achieved between the numerical simulation and the experimental observations.

The purpose of the study that is presented here is to evaluate the feasibility of such a technique for orthotropic yielding parameter estimation. In the current investigation, experimental data are not used. Instead a simulation of measured data is created by the finite element model. This simulates perfect observations which can then be disturbed by a random noise factor.
For the experimental measurements, there exist methods to measure displacement fields without contact or interference with the sample. Such methods are discussed in [4]. It is envisaged that electronic speckle pattern interferometry will be used for the measurement of the displacement fields.
Chapter 2

Theoretical Background

2.1 The Identification Method

2.1.1 Outline of the Method

The identification method is a mixed numerical-experimental technique. In this case strain data from experimental work and numerical analysis are compared. The parameters are then adjusted in the numerical analysis until a good correlation is achieved.

The numerical part consists of a mathematical model for the material behaviour, containing the parameters which are to be determined. A finite element model of the sample to be tested is generated. Some trial estimations for the parameters are used for the generation of the finite element model. The results of the model (in this case strains) are compared to experimental data. The parameters are then adjusted through a numerical procedure, described in section 2.3. The new parameter estimates can then be used for another iteration of finite element analysis. These results are also compared to experimental data and the process is repeated until convergence to a suitable tolerance is achieved. The flow chart in Figure 2.1 describes the operation of the technique.
2.1.2 Scope of Applicability

The method outlined has certain advantages over traditional testing methods. In particular, there is no condition that the strain field be homogeneous. The advantages of this are:

1. A homogeneous strain field can be difficult to achieve in some cases and the traditional techniques will fail to describe material behaviour properly in such a case.

2. More information is available from an inhomogeneous strain field. It is therefore possible to determine material parameters more effectively. A large number of parameters may be determined from one test.

There are also disadvantages:

1. It may prove difficult to obtain a strain field that is sufficiently inhomogeneous with standard testing equipment. Multi-axial testing could be necessary in some cases.
2. If there is a high degree of inhomogeneity, accurate measurement of the strain field may be difficult.

3. A computer model for accurate numerical analysis is more complex and hence time-consuming.

2.2 Yield Criterion Theory

It was shown by Hill [3] that the yield criterion for a material with orthotropic yielding could be described by:

\[ 2f = F(\sigma_{yy} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2 + 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 = 1 \] (2.1)

If \( X, Y, \) and \( Z \) are the tensile yield stresses in the principal anisotropic directions and \( R, S \) and \( T \) are the yield stresses in shear with respect to these axes, then:

\[ \frac{1}{X^2} = G + H, \quad \frac{1}{Y^2} = H + F, \quad \frac{1}{Z^2} = F + G; \]

\[ 2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}; \]

\[ 2G = \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}; \]

\[ 2F = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \] (2.2)

and:

\[ 2L = \frac{1}{R^2}, \quad 2M = \frac{1}{M^2}, \quad 2N = \frac{1}{T^2} \] (2.3)

2.3 Identification of Material Parameters

A brief outline of the numerical technique used for parameter estimation of material properties is presented here. The technique uses the sequential minimum variance approach. Detailed descriptions can be found in [1] and [2].
The identification technique used for the estimation of the material parameters can be represented in its simplest form by the following equation:

\[ y_k = h_k(x) + v_k \]  

(2.4)

\( y_k \) is observational data where \( k = 1, \ldots, N \) is the number of the load case or time step. \( x \) is a column of unknown parameters, in this case the material parameters of the model used. Function \( h_k(x) \) describes the \( k \)-th observation. The observation errors are accounted for by \( v_k \). The procedure is to alter the value of the estimate for \( x, \hat{x}_k \) until a good correlation is found between \( y_k \) and the numerical simulation data. This can be achieved by minimising the expression from [5]:

\[
S_k = (\hat{x}_{k-1} - x)^T (P_{k-1} + Q_k)^{-1} (\hat{x}_{k-1} - x) + (y_k - h_k(x))^T R_k^{-1}(y_k - h_k(x))
\]

(2.5)

Matrix \( R \) is the covariance of the measurement error \( v_k \). Matrix \( P \) is the covariance of the estimation error based on the most recent parameter estimation. Matrix \( Q \) is a non-negative symmetric weighting matrix which is used to prevent \( P \) from becoming too small. It gives more weight to the observational data and less weight to the initial parameter estimates. It was found [1] that matrix \( Q \) was needed to assist the convergence of some parameter estimations.

It is shown in [1] how this leads to an iterative scheme described by the following equations:

\[
\dot{x}_{k+1} = \dot{x}_k + K_{k+1}(y_{k+1} - h_{k+1}(\hat{x}_k))
\]

(2.6)

\[
K_{k+1} = (P_k + Q_k) H_{k+1}^T (R_{k+1} + H_{k+1}(P_k + Q_k) H_{k+1}^T)^{-1}
\]

(2.7)

\[
P_{k+1} = (I - K_{k+1} H_{k+1}) (P_k + Q_k) (I - K_{k+1} H_{k+1})^T + K_{k+1} R_{k+1} K_{k+1}^T
\]

(2.8)

\( H_{k+1}\dot{x}_k \) is a linear approximation of \( h_{k+1}(\hat{x}_k) \). It is defined by:

\[
H_{k+1} = \left( \frac{\delta h_{k+1}(x)}{\delta x} \right)_{x=\hat{x}_k}
\]

(2.9)
If there are $n$ parameters to be determined, then there must be $n + 1$ finite element calculations to determine the linear approximation matrix $H_{k+1}$. When new observational data becomes available, equations 2.6 to 2.8 can be used to find the new parameter estimates.

It is also possible to use the same measurement data to update the parameter estimates. Several iterations may be performed per load or time case. The estimator for the $k$-th load or time step is then:

$$\hat{x}_{k+1}^i = \hat{x}_k^i + \left( H_k^T R_k^{-1} H_k + (P_k + Q_k)^{-1} + R_k^{-1} \right)^{-1}$$

$$\left\{ H_k^T R_k^{-1} (y_k - h_k(\hat{x}_k^i)) + R_k^{-1} (\hat{x}_{k-1}^i - \hat{x}_k^i) \right\}$$

(2.10)

The number of the iteration is indicated by the counter $i$. The update given in equation 2.10 is the one implemented for the work that is presented here. The same measurement data are input into the estimation algorithm applied to a nonlinear analysis. Each parameter estimation iteration is performed for the final load step only.
Chapter 3

Implementation of the Identification Method

3.1 Results from previous work and suggestions

The identification method came about from the need for a general method capable of determining the material parameters of an inhomogeneous material where the boundary conditions are not known and the geometry of the sample is ill defined. There are two ways in which the technique may be implemented:

1. A local approach using kinematic boundary conditions (displacements). This can be used to determine dimensionless parameters such as material orientation angle and the ratios between the stiffness parameters.

2. A global approach using both kinematic and dynamic (forces) boundary conditions. Exact material parameters can be determined in this way.

It was shown by Ratingen [2] that both methods were possible with the theory presented, each with its advantages and disadvantages. For the application of the method to the characterisation of biological materials, the first approach is preferable because it allows the possibility of in situ testing of materials, where boundary conditions and geometry are difficult to define. Kinematic boundary conditions (displacements) are defined in the model.
These displacements are obtained from measured data. This is the method used by Ratingen [2] for his work on characterization of dog skin.

The focus of the study in this case was the feasibility of the application of the method to orthotropic yielding parameter estimation. Only kinematic boundary conditions were defined. Hence this is strictly speaking a local approach. Since we have defined the stiffness in the model, the values of the dimensionless parameters in each case can be compared to the expected numerical value of the parameter.

In the comparison of numerical to experimental observations, it must be remembered that clamping effects are often difficult to model properly. For this reason a small section of the material is compared with experimental observations. This small section must be well clear of any boundary effects.

Hendriks [1] and Ratingen [2] found that there were certain conditions under which the parameter estimation performed best.

1. An inhomogeneous strain field in the sample with a large domain of strains. Ratingen [2] compared the performances of various geometries of a woven fabric material. It was found that the best approximation of the material parameters occurred when the strain domain was large. Generally, the more inhomogeneous the strain field, the better the approximation of the material parameters due to the greater volume of available information.

2. A mesh which is fine enough to give a good description of the displacement field. If the mesh is too coarse, the output from the finite element analysis will not be a good approximation of the real situation. Modelling errors will have a large influence, affecting the parameter estimation.

3. The mesh also cannot be too fine, since this will cause the computations to be extremely time-consuming. Some peculiarities with regard to the number of elements were shown by Hendriks [1]. He showed that there can be a marked effect introduced by the number of elements. Increasing the number of elements can improve the estimation of some
parameters while others are worse. This characteristic is, however, due to the behaviour of the particular material investigated.

Each parameter estimation case has certain conditions under which it operates best. Therefore a careful study of the method and samples to be used must be made in each case before attempting parameter estimation. A sample geometry with a large strain domain is advantageous. The performance of the model must then be assessed, simulating actual test conditions as closely as possible.

3.2 Selection of a Sample Geometry

A finite element model of the experimental sample which is used for the characterisation of the material is needed. Four finite element models of various samples were generated initially. For the purposes of this investigation, the material in each sample was assumed to be isotropic in behaviour and to exhibit strain hardening. The material properties were as follows:

- Young’s modulus: 67 GPa
- Yield stress (0%) extension: 200 MPa
- Yield stress (100%) extension: 257 MPa
- Poisson’s ratio: 0.35

Plots of the four samples can be seen in Figures 4.1a-d. The outer dimensions are (0.06 m x 0.1 m). The details of other dimensions are not given since they are not significant here. The thickness of the plate in each case is $1 \times 10^{-3}$ m. The lower edge of the plate is clamped in each case. The upper edge has a translation perpendicular to the lower edge and in a direction away from it. This translation is $2.5 \times 10^{-4}$ m. Linear (4-node) plane stress elements were used for the analysis.

3.3 Application of Measurement Points

In order to perform parameter estimation, points were needed where displacements could be measured. These points had to be placed in such a way that there was a large amount of information available from the regions
of the model obeying the conditions of the yield criterion. A plot of the plasticity status for sample was generated. Some points were placed in the plastic region and some outside it, since the transition from elastic to plastic behaviour is important information. These points were then used to simulate measurement points as they would be used in the experimental case.

3.4 Generation of Measurement Data

In this feasibility study, experimental observations were simulated by a numerical analysis and are not measured experimentally. The displacements of each measurement point in the x and y directions were determined by a trial run in which one iteration of parameter estimation was carried out where the initial guess for the estimation parameter is set at the 'correct' value. The displacements of the measurement points are calculated for this case. These displacements are then used in the parameter estimation with initial guesses that differ from the 'correct' values. Of course this approach simulates a case where there are no modelling errors due to the finite element modelling of the experimental situation. It also assumes that there is no measurement error of the displacement points. In order to simulate the effects of measurement error, the 'measured' data were disturbed with a noise factor. This noise was random in nature with a normal distribution and a standard deviation of 1% that of the average magnitude of the displacements.

3.5 Parameter Estimation

Perfect observational data, as outlined in section 3.4, were used for parameter estimation. The estimated parameters can then be compared with the original input data used to generate the displacements. In this way the feasibility of the technique can be assessed for future experimental work.

The following input data was used to generate the displacements:
Parameter | Value
---|---
\( X \) | \( 2.00 \times 10^8 \ Pa \)
\( Y \) | \( 1.80 \times 10^8 \ Pa \)
\( Z \) | \( 1.60 \times 10^8 \ Pa \)
\( T \) | \( 1.40 \times 10^8 \ Pa \)

There should be no difference between 'measured' values and calculated values of displacements when parameter estimates converge to their correct values. This situation will almost never be found in practice. Modelling and experimental errors will always affect the performance of the method. The performance of the models with measurement noise (error) is extremely important.

Three parameter estimation cases were considered, where the angles of orthotropy were 0°, 45° and 60° respectively. This angle is measured from the positive x-axis on the model (Figure 4.1c) towards the y-axis.

Parameter estimation was performed in the Diana finite element package according to the iteration procedure outlined in the previous section. Four parameters were estimated in each case, so for each iteration, five finite element analyses were required to determine the matrix \( \mathbf{H} \).
Chapter 4

Results

4.1 Selection of a Sample Geometry

The Von Mises stress distributions and the strain domains were compared for each preliminary finite element model. These results are shown in Figures 4.1 and 4.2. It was found that there was not much significant difference between the stress distributions of the three models. The patterns and peak values of the Von Mises stress did not change much. The strain domains were, however, rather different, as shown in Figures 4.2a-c. The strain domain of model 3 displayed the most promising distribution for parameter estimation and was thus selected for this purpose.

4.2 Allocation of Measurement Points

Measurement points had to be positioned so that they gave a good spread of data covering both elastic and plastic regions in the model. Plots of the plasticity status of the material for each of the orthotropy angles 0°, 45° and 60° are shown in Figures 4.3a-c. In each case, the highest degree of plasticity is in the region surrounding the cutout with the small fillet radius. There is also a marked transition in this region from plastic to elastic behaviour. It was decided to locate measurement points in this region for parameter estimation. The location of the points used for all three orthotropy angles is shown in Figure 4.3d. Although the section surrounding the larger fillet radius also has some degree of plasticity, it is neither as severe nor does
Table 4.1: Average displacements and standard deviation.

<table>
<thead>
<tr>
<th>Sample</th>
<th>0° Orthotropy</th>
<th>45° Orthotropy</th>
<th>60° Orthotropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Displacement (m)</td>
<td>$5.97 \times 10^{-5}$</td>
<td>$6.26 \times 10^{-5}$</td>
<td>$6.21 \times 10^{-5}$</td>
</tr>
<tr>
<td>Standard Deviation (m)</td>
<td>$5.34 \times 10^{-7}$</td>
<td>$5.60 \times 10^{-7}$</td>
<td>$5.55 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

it have the rapid transition from plastic to elastic displayed by the smaller radius.

The highest degree of plasticity is displayed by the sample with the 0° angle of orthotropy. The maximum plastic strain in this sample is $7,917 \times 10^{-2}$. The maximum plastic strains for the 45° and 60° cases were $5,230 \times 10^{-2}$ and $4,566 \times 10^{-2}$ respectively. The reasons for this variation are discussed in section 5.1.

4.3 Displacements and Noise

The average displacements of the measurement points and the standard deviation for each sample are summarised in Table 4.1. A small routine was written to disturb the measurement data. This routine was always the first run in MATLAB. The random values generated for each case were thus identical. This gives a noise distribution which is the same for each of the samples considered. The performance of the samples can thus be established without having to consider that some input data might be better than others. The results in Table 4.1 confirm this, where it can be seen that the standard deviation is 0.9% of the mean displacement in each case.

4.4 Parameter estimation results

Initially a parameter estimation was performed for the 0° orthotropy case with 'perfect' displacements displaying no noise. The results of this parameter estimation can be seen in Figure 4.4a. All of the parameters converge to almost their correct values within 6 iterations. The worst estimation was that of parameter T, which displayed a 1.4% error. For the same 0° sample disturbed with the random 1% noise, the parameter estimation is very poor.
Again the worst parameter estimation is that of the shear, $T$. The best estimation is that for $Y$.

The estimation for the sample with an angle of $45^\circ$ is shown in Figure 4.4c. Note that the estimation is far quicker than that for the $0^\circ$ cases (5 iterations). Parameters converge very close to their correct values. Parameters $X$ and $Y$ converge very fast, but their values are incorrect.

The estimation for the $60^\circ$ angle of orthotropy is shown in Figure 4.4d. Convergence is slightly slower than the $45^\circ$ case (6-7 iterations), but the accuracy of the final estimates is greatly improved.
Figure 4.1a: Model 1 Von Mises stress distribution. Maximum stress $2.15 \times 10^8$ Pa.

Figure 4.1b: Model 2 Von Mises stress distribution. Maximum stress $2.15 \times 10^8$ Pa.

Figure 4.1c: Model 3 Von Mises stress distribution. Maximum stress $2.15 \times 10^8$ Pa.

Figure 4.1d: Model 4 Von Mises stress distribution. Maximum stress $2.15 \times 10^8$ Pa.
Figure 4.2a: Model 1 strain domain.  
Figure 4.2b: Model 2 strain domain.  
Figure 4.2c: Model 3 strain domain.  
Figure 4.2d: Model 4 strain domain.
Figure 4.3a: Sample 1 plasticity status. Angle of orthotropy=0°. Maximum plastic strain $7.917 \times 10^{-2}$.

Figure 4.3b: Sample 2 plasticity status. Angle of orthotropy=45°. Maximum plastic strain $5.230 \times 10^{-2}$.

Figure 4.3c: Sample 3 plasticity status. Angle of orthotropy=60°. Maximum plastic strain $4.566 \times 10^{-2}$.

Figure 4.3d: Selected measurement points for parameter estimation. Number of points = 95.
**LEGEND**

- Parameter $X$. Correct value $= 2.0 \times 10^8$
- Parameter $Y$. Correct value $= 1.8 \times 10^8$
- Parameter $Z$. Correct value $= 1.6 \times 10^8$
- Parameter $T$. Correct value $= 1.4 \times 10^8$

**Figure 4.4a:** Sample 1 parameter estimation. Angle of orthotropy $= 0^\circ$. 0% noise.

**Figure 4.4d:** Sample 1 parameter estimation. Angle of orthotropy $= 0^\circ$. 1% noise.

**Figure 4.4c:** Sample 2 parameter estimation. Angle of orthotropy $= 45^\circ$. 1% noise.

**Figure 4.4d:** Sample 3 parameter estimation. Angle of orthotropy $= 60^\circ$. 1% noise.
Chapter 5

Discussion and Recommendations

5.1 Plasticity and Allocation of Points

The highest degree of plasticity occurred when the orthotropy angle was $0^\circ$ and lowest when it was $60^\circ$. This can be explained, at a first glance, by the fact parameter $Y$ was chosen smaller than parameter $X$. Hence as the loading direction corresponds more with the lower yield stress, the plasticity status must increase. Studying in more detail, we see from [3] that the directions of the maximum tensile stress can be determined from the following expression:

\[
\frac{2(FG + GH + HF)}{N(F + G)}
\]  

(5.1)

If the expression 5.1 has a value greater than unity, then the tensile yield stress has a maximum at an angle of $45^\circ$ and minimums at $0^\circ$ and $90^\circ$. If it is smaller than unity, the opposite applies. Hence one would expect the plasticity to be lower than for any of the other orthotropy angles. This is not the case.

The number of measurement points chosen was 95. Increasing the number of points should improve the accuracy but will also increase the computation time for parameter adjustment in each iteration. Since this is not significant compared to the time taken for the finite element analyses, the number of
points may be increased to check if the accuracy improves. However the measuring system currently in use can only measure 126 points, giving an upper limit for this check.

5.2 Parameter Estimation

For the 0\degree angle of orthotropy with no noise, the quickest convergence is observed for the parameter $Y$. This is easily explained by the fact that the maximum stress is in the y-direction. Hence there is more available information on the yielding in this direction than in any other direction. The parameter with the poorest accuracy of convergence is the shear yield stress, $T$. This is the case particularly when measurement errors or 'noise' are introduced. If one studies equations 2.1 to 2.3, it can be seen that the yield stresses in the x, y and z-directions are very closely linked. With a good knowledge of one of these parameters (in this case $\sigma_{yy}$) and a reasonable knowledge of $\sigma_{xx}$, it is possible to determine $F$, $G$ and $H$. If we know $F$, $G$ and $H$ then from equations 2.1 and 2.2, it is possible to determine the parameters $X$, $Y$ and $Z$ with reasonable accuracy. However, in the case of the yield stress in shear, parameter $T$, the information is not as closely linked to that required for the other parameters. Hence there is little assistance from the other information to determine this parameter and the information on the shear stresses is poor.

From the results of the 0\degree case, it appeared that in order to determine $T$, a good knowledge of $\sigma_{zy}$ is essential. To test this hypothesis, the model with the direction of orthotropy as 45\degree was implemented. As seen in Figure 4.4c, there was a significant improvement in the estimation of parameter $T$. The speed of convergence was faster than the 0\degree case with no noise. However, the accuracy of the parameter estimates $X$ and $Y$ was substantially reduced. It appeared that for this case the information was sufficient to determine the shear, but at a cost of losing information for the estimation of $X$ and $Y$. The accuracy of the convergence of $Z$ is purely coincidental. If one studies equations 2.1 and 2.2, it is possible to see that if $X$ and $Y$ have roughly the same error, including the sign (which they do in this case), then $Z$ will not be affected significantly.
The performance of the 60° angle of orthotropy sample appears to be consistent with the hypothesis outlined above. For this case, it appears that there is enough information to determine all the parameters under investigation even when the 'measured' data is disturbed with a noise factor. It may be possible to determine some optimum angle for which all the parameters have good information. This has not been done, however, because it is believed that this optimum angle would be a function of the various material yield parameters. These are the values that are to be estimated, so at best the 'optimum' angle would only be an educated guess.

It is clear that the material should not be tested with the angle of orthotropy at 0° or 45°. It should be tested at some intermediate angle which is not known and most likely cannot be determined. A way of checking whether the parameters have converged to their correct values is to compare the results of estimation from two or more samples with different angles of orthotropy. This would give an increased confidence in the obtained results.

Parameter Z has been estimated quite well with the identification method. It would prove difficult to determine this parameter with traditional testing techniques and several samples may have to be used. The identification technique could determine the parameter with one sample in the ideal case, although it is been recommended here that several samples be examined to increase the confidence in the obtained results.
Chapter 6

Conclusions

It has been demonstrated here that the identification technique used can successfully identify the parameters of orthotropic yielding described in [3]. The feasibility of the technique has therefore been established for future experimental work. There are some limitations with regard to conditions under which the technique will converge firstly and converge to the correct value secondly. It has been demonstrated how the angle of orthotropy can have a marked affect on the performance of parameter estimation.

It is therefore wise to be extremely careful in implementing the technique. The method is certainly not general enough to be applied to just any similar parameter estimation case. It is first necessary to carry out trial runs of the experiment to be performed, as has been done here. In this way conditions for efficient and correct operation of the model can be established.

It is clear that the technique has marked advantages over other traditional testing techniques when it is applied to a problem such as determining orthotropic yield stresses. Some of the parameters that can be determined by this method would be extremely difficult to determine with traditional testing methods. More than one sample should be analysed with different angles of orthotropy in order to check the obtained results. The identification method is a time-consuming approach which works well for the application demonstrated here.
References


