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BRIDGMAN GROWTH OF CRYSTALS
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Introduction

Crystal growth by means of the Bridgman-oven technique is widely accepted and frequently applied nowadays. However, the understanding of the physical processes in the oven is far from complete yet. A lot of numerical models and considerations have been introduced but these approaches nearly always treat idealised situations, so that the results are not directly applicable in practice.

In this report the results of detailed calculations on the Bridgman-oven geometry are presented. This work has been carried out in support of a crystal growth project at the Faculty of Science, University of Nijmegen. The calculations have been performed in direct connection with the growth of Bismuth Gallium Oxide (BGO) crystals but are nearly independent of the crystal under consideration and thus useful for all kinds of Bridgman-oven applications.

The general features of the Bridgman-oven are as follows. An ampoule containing BGO moves down through a heater on top of a cooler. Both heater and cooler are cylindrically shaped. At the interface a diaphragm is sometimes mounted to reduce the cylinder radius locally. The wall temperature in heater and cooler are chosen to be higher respectively lower than the melting temperature of BGO, so that solidification will take place somewhere at the interface.

Important parameters during the solidification process are the vertical temperature gradient in the BGO and the form of the liquid/solid interface, determined by the isothermal contours in the ampoule.

The techniques to calculate temperature profiles in realistic ovens are presented in the following 2 sections. The speed of the ampoule through the oven is such low that dynamic effects are neglected. Only stationary states are studied. For this purpose use is made of the package ESACAP, developed at E.S.A., Noordwijk, in which the system is treated as the analogue of an electrical network. In the last section we present results and analyze in detail how temperature gradient and form of the interface are affected by the geometrical parameters of the oven and the thermal properties of the used materials. From these results a lot of insight is gained in the selection of the relevant parameters in order to obtain a specific temperature distribution in the ampoule during the solidification process. Specification of the most desirable conditions requires a deep knowledge of metallurgy and falls therefore outside the scope of this project.
The model

We shall use the following model to study Bridgman growth. The oven and the ampoule are assumed to be concentric cylinders. Owing to the cylinder symmetry the problem is essentially 2-dimensional.

The ampoule is 20 cm long with a diameter of 3 cm. The thickness of the ampoule wall is 0.025 cm. The ampoule is divided along its length into 2 cm sections and along its diameter into 0.5 cm sections. The BGO in the ampoule is also divided along its length into 2 cm sections and top and bottom are radially divided into 0.5 cm sections. The heat conduction axially through each section is described by:

\[ q = \frac{k A \Delta T}{L} \]

Here, \( q \) is the heat flow, \( k \) the thermal conductivity, \( A \) the cross-sectional area, \( \Delta T \) the temperature difference over one section and \( L \) the section length. Replacing \( q \) by current \( I \) and \( T \) by voltage \( V \) for the electrical analogue, this equation becomes:

\[ I = \frac{k A D V}{L} \]

If we interprete this equation as Ohm's law a conduction resistance \( R_c \) results, given by:

\[ R_c = \frac{L}{k A} \]

There is also heat conduction radially through the cylinder and the radial conduction resistance \( R_{cr} \) is given by: [2]

\[ R_{cr} = \frac{\log(p^2)}{2\pi r L} \]

Here \( r_1 \) is the inner radius and \( r_2 \) is the outer radius of a concentric cylinder.

Free convection in the melt is neglected because it is small for vertical growth. The movement of the ampoule takes place such slow that in the calculations the consequence of the dynamic effects can be neglected. At each moment the temperature distribution is assumed to be stationary.

The oven wall is 60 cm long and divided into 2 cm sections. It is assumed that radiation is only exchanged between oven wall and ampoule wall. The temperature at the oven wall is such high that convection and conduction in the air gap are neglected. The radiation of one section of the oven to a section of the ampoule is approximately given by the Stefan-Boltzmann relation:

\[ q = sE \alpha \left( T_o^4 - T_a^4 \right) \]
Here, $s$ is the Stefan-Boltzmann constant, $E$ the viewfactor, $A_o$ the surface area of the oven section, $T_o$ the surface temperature of the oven section and $T_a$ the surface temperature of the ampoule section. The viewfactor will depend on the emissivity of the oven wall and the ampoule, on the surface areas of both sections and on the location of both areas. For the purpose of computation the surfaces are idealised as being opaque-grey. For one surface totally enclosed by the other the viewfactor is:

$$E = \left[ \frac{1}{e_o} + \left( \frac{1}{e_a} \right)^{-1} \right]^{-1}$$

In all other cases we have:

$$E = e_o e_a F_{A_o - A_a}$$

Here, $e_o$ is the emissivity of the oven wall, $e_a$ the emissivity of the ampoule and $F_{A_o - A_a}$ does not depend on $e_o$ and $e_a$ but only on the geometry.

In the electrical analogue the equation becomes:

$$I = sE_o (V_o^4 - V_a^4)$$

Using Ohm's law, the radiation resistance $R_r$ is found to be:

$$R_r = \left[ sE_o (V_o^2 + V_a^2)(V_o + V_a) \right]^{-1}$$

In only one case reflection will be taken into account. It is when heat is emitted from a surface to another surface totally enclosed by the first one. Transmission of radiation does not take place in the model because the surfaces are assumed to be opaque. The viewfactors mentioned above are calculated with these assumptions.

Using the formulas above the radiation or conduction resistance between two nodes of the model can be calculated. Combining the nodes with each other results in a complex electrical network. In this network the temperatures of all the nodes and the heat flows between them can be calculated, if one specifies a temperature profile at the oven wall.

The values of the constants in the model are:

Stefan-Boltzmann constant: $s = 5.6693 \times 10^{-8} \ \text{W m}^{-2} \text{C}^{-4}$

Thermal conductivity of the ampoule: $k_a = 80.0 \ \text{W m}^{-1} \text{C}^{-1}$

Emissivity of the oven: $e_o = 0.8$

Length of the oven: $l_o = 0.60 \ \text{m}$

Length of the ampoule: $l_a = 0.20 \ \text{m}$

Wall thickness of the ampoule: $w_a = 0.25 \times 10^{-3} \ \text{m}$
Thermal conductivity of fluid BOO: \( k_1 = 0.45 \, \text{W/m°C} \).

Thermal conductivity of solid BOO: \( k_s = 0.45 \, \text{W/m°C} \).

Emissivity of the ampoule: \( e_a = 0.8 \).

Diameter of the oven: \( d_0 = 0.06 \, \text{m} \).

Diameter of the ampoule: \( d_a = 0.03 \, \text{m} \).

The isotherm interface has a temperature of 1055°C. To indicate the position of the ampoule with respect to the oven we need only one coordinate axis, namely the cylinder axis, with the origin at the top of the oven and pointing downwards. The bottom of the oven is at the position of 60 cm. The boundary between the heater and the cooler is then at 30 cm. The interface shall be somewhere near the boundary of the heater and the cooler.

The temperature profile at the oven wall is given in figure 1. Near the boundary between heater and cooler the temperature gradient is steepest, namely 30 °C/cm. The peaks near the boundary increase the temperature gradient.

Without those peaks the temperature gradient at the wall would be only 25 °C/cm.

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Figure 1: Temperature profile of the oven wall.
The empty Bridgman-oven

The situation in the Bridgman-oven is as follows:

1. The wall temperature of the oven is kept constant.
2. There is no radiation of heat to the environment.

Definition:

The temperature at a point on the axis of the oven is calculated by introducing an infinitesimally small test volume at that point.

The heat flow from a small cylindrical surface $A_v$ on the axis of the oven to a cylindrical section $A_i$ of the oven wall is given by:

$$q_i = sA_v E_i (T^4_v - T^4_i)$$

Subscript $v$ refers to the test volume and subscript $i$ refers to section $i$ on the oven wall. Divide the oven wall into $n$ sections of equal length. This results in $n$ radiation equations. The total sum of the heat flow has to be zero in the stationary case:

$$\sum_{i=1}^{n} q_i = 0$$

Combining these equations yields:

$$\sum_{i=1}^{n} E_i (T^4_v - T^4_i) = 0$$

or

$$T^4_v = \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} E_i}$$

The stationary temperature $T_v$ is, of course, independent of the emissivity $e_v$. This can be shown as follows.

By neglecting non-horizontal reflection, one of the $E_i$, say $E_j$, is given by:

$$E_j = \left[ \frac{1}{e_v} + \frac{A_v}{A_i} \left( \frac{1}{e_v} - 1 \right) \right]^{-1}$$

and for $i \in \{1, \ldots, n\}$, $i \neq j$:
\[ E_i = e_o e^F_{Av-Ai} \]

Because \( A_v \ll 1 \), this yields:

\[ E_j = e_v \]

Define \( c_j = 1 \) and \( c_i = e_o^F_{Av-Aj} \) for \( i \in \{1, \ldots, n\} \), \( i \neq j \), this results in:

\[ T^4_v = \frac{\sum_{i=1}^{n} c_i T^4_i}{\sum_{i=1}^{n} c_i} \]

It is obvious now that \( T_v \) is independent of \( e_v \).

Given the oven wall temperature, it is relatively easy to calculate the temperature on the axis of the oven. Comparing the temperature of the oven wall and the temperature on the axis it is seen in figure 2 that the temperature profile on the axis tends to become more smooth.

---

*Figure 2: Temperature in the empty oven compared with the oven wall temperature*
The vertical temperature profile in the ampoule will be even more smoothed out than the temperature profile on the axis in the empty oven. When the ampoule is in the oven there is through conduction a heat flow from the top to the bottom of the ampoule. This means that at the top of the ampoule the temperature is lower than the corresponding temperature in the empty oven and at the bottom of the ampoule the temperature is higher than the corresponding temperature in the empty oven. When the ampoule is in the middle of the oven the temperature gradient at the interface will be smaller than the temperature gradient in the empty oven.

The temperature gradient in the empty oven appears to be $24 \frac{C}{cm}$. 
Results and Discussion

The model above is used to calculate the temperature distribution in the oven as a function of several parameters. To study the behaviour of the temperature profile in time we place the ampoule at five different positions namely when the ampoule is 6 cm, 8 cm, 10 cm, 12 cm, 14 cm in the cooler. When the ampoule is 4 cm in the cooler all BGO is still melt. All BGO has become solid when the ampoule is 16 cm in the cooler.

The quantities of main interest are the shape or the convexity of the interface and the value of the vertical temperature gradient at the interface. We define the convexity to be negative if the shape of the interface is concave, seen from the solid. Let \( L_s \) be the position of BGO with temperature 1055 C at the axis and \( L_a \) the position of BGO with temperature 1055 C near the wall of the ampoule. The convexity is then given by:

\[
\text{convexity} = \frac{100(L_s - L_a)}{d_a}
\]

Another point of interest is the vertical position of the interface. In the following we denote therefore where the temperature at the axis equals 1055 C. Note that the top of the oven is at a position of 0 cm and after 30 cm there is the top edge of the cooler.

The default oven:

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient in cm</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.5</td>
<td>20.2</td>
<td>33.8</td>
</tr>
<tr>
<td>38</td>
<td>0.7</td>
<td>18.3</td>
<td>31.8</td>
</tr>
<tr>
<td>40</td>
<td>0.7</td>
<td>18.2</td>
<td>29.8</td>
</tr>
<tr>
<td>42</td>
<td>0.3</td>
<td>18.3</td>
<td>27.8</td>
</tr>
<tr>
<td>44</td>
<td>-0.6</td>
<td>20.4</td>
<td>25.9</td>
</tr>
</tbody>
</table>

Table 1. Results for the default oven, described in the previous section.

Comments on table 1:

Note that the movements of the ampoule and the interface are in opposite directions. This remarkable result can be explained as follows. The top and the bottom of the ampoule have large surface areas. Let us call the radiation from the oven wall to the top of the ampoule end-gain and from the bottom of the ampoule to the oven wall end-loss. When the ampoule is 6 cm in the cooler, the lower part receives a lot of heat from the upper part in the heater. That is why the position of the interface is in the cooler. In this position the end-loss is large. Thus the temperature rapidly decreases near the bottom of the ampoule and the temperature gradient at the interface is large. The
convexity of the interface profile can be explained as follows. If the ampoule is partly in the cooler the lower part of the vertical surface exchanges heat both with cooler and heater. The bottom surface, however, merely loses heat towards the cooler. Thus the temperature in the middle of the bottom is cooler than the temperature on the outside of the bottom. When the ampoule goes down the part of the ampoule in the cooler increases and the position of the interface moves towards the heater. When the ampoule has been lowered 10 cm in the cooler the position of the interface is in the heater. When the ampoule is lowered further the position of the interface shifts towards the top of the ampoule. Then the convexity will have opposite sign because of symmetry arguments.

**Influence of emissivity:**

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient ( \frac{c}{cm} )</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.3</td>
<td>17.9</td>
<td>33.9</td>
</tr>
<tr>
<td>38</td>
<td>0.7</td>
<td>17.0</td>
<td>31.8</td>
</tr>
<tr>
<td>40</td>
<td>0.7</td>
<td>17.0</td>
<td>29.8</td>
</tr>
<tr>
<td>42</td>
<td>0.7</td>
<td>17.0</td>
<td>27.8</td>
</tr>
<tr>
<td>44</td>
<td>-1.5</td>
<td>18.1</td>
<td>25.8</td>
</tr>
</tbody>
</table>

Table 2. Results for the default oven with \( e_a = 0.35 \) instead of the default value \( e_a = 0.8 \).

**Comments on table 2:**

Comparison tables 1 and 2 makes clear that changing the emissivity only effects the gradient and the convexity, whereas the position of the interface hardly changes. The effects are relatively small. When the emissivity of the ampoule becomes smaller, there is less heat absorption. The temperature deviation in the ampoule will smooth out a little because the conductivity in the ampoule and BGO remains the same. Thus the temperature gradient at the interface is smaller compared with the default oven.
Influence of conductivity of BGO:

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient ( \frac{C}{cm} )</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1.8</td>
<td>19.2</td>
<td>33.8</td>
</tr>
<tr>
<td>38</td>
<td>0.7</td>
<td>17.9</td>
<td>31.8</td>
</tr>
<tr>
<td>40</td>
<td>0.7</td>
<td>17.9</td>
<td>29.8</td>
</tr>
<tr>
<td>42</td>
<td>0.7</td>
<td>17.9</td>
<td>27.8</td>
</tr>
<tr>
<td>44</td>
<td>-0.8</td>
<td>19.3</td>
<td>25.9</td>
</tr>
</tbody>
</table>

Table 3. Results for the default oven with \( k_1 = 1.35 \) and \( k_s = 1.35 \) instead of the default values \( k_1 = 0.45 \) and \( k_s = 0.45 \).

Comments on table 3:
When the conductivities of BGO becomes larger, the temperature profile in the ampoule smoothes out and the temperature gradient at the interface is smaller compared with the default oven. The position of the interface and the convexity do not change much.

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient ( \frac{C}{cm} )</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.9</td>
<td>20.7</td>
<td>33.8</td>
</tr>
<tr>
<td>38</td>
<td>0.7</td>
<td>18.4</td>
<td>31.8</td>
</tr>
<tr>
<td>40</td>
<td>0.7</td>
<td>18.4</td>
<td>29.8</td>
</tr>
<tr>
<td>42</td>
<td>0.7</td>
<td>18.4</td>
<td>27.8</td>
</tr>
<tr>
<td>44</td>
<td>-2.0</td>
<td>20.6</td>
<td>25.9</td>
</tr>
</tbody>
</table>

Table 4. Results for the default oven with \( k_1 = 0.15 \) and \( k_s = 0.15 \) instead of the default values \( k_1 = 0.45 \) and \( k_s = 0.45 \).

Comments on table 4:
When the conductivities of BGO becomes smaller. The temperature profile in the ampoule will become steeper so the temperature gradient at the interface is a little higher.
Influence of changing both diameters:

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient ( \frac{c}{cm} )</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>4.0</td>
<td>17.3</td>
<td>33.5</td>
</tr>
<tr>
<td>38</td>
<td>0.9</td>
<td>14.2</td>
<td>31.7</td>
</tr>
<tr>
<td>40</td>
<td>0.8</td>
<td>10.3</td>
<td>29.6</td>
</tr>
<tr>
<td>42</td>
<td>0.8</td>
<td>14.3</td>
<td>27.7</td>
</tr>
<tr>
<td>44</td>
<td>-3.8</td>
<td>17.3</td>
<td>26.0</td>
</tr>
</tbody>
</table>

Table 5. Results for the default oven with \( d_a = 6 \) cm and \( d_o = 12 \) cm instead of the default values \( d_a = 3 \) cm and \( d_o = 6 \) cm.

Comments on table 5:
The temperature gradient at the interface is dramatically smoothed out when the ampoule is near the middle of the oven. Here, the heat flow can go easily down thanks to the large diameter of the ampoule. The convexities at the start and at the end of the process are larger because of the enhanced end-gain and end-loss.

Influence of diameter of the ampoule:

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient ( \frac{c}{cm} )</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1.0</td>
<td>19.1</td>
<td>33.9</td>
</tr>
<tr>
<td>38</td>
<td>0.2</td>
<td>18.7</td>
<td>31.9</td>
</tr>
<tr>
<td>40</td>
<td>0.2</td>
<td>18.7</td>
<td>29.9</td>
</tr>
<tr>
<td>42</td>
<td>0.2</td>
<td>18.7</td>
<td>27.8</td>
</tr>
<tr>
<td>44</td>
<td>-0.9</td>
<td>19.2</td>
<td>25.8</td>
</tr>
</tbody>
</table>

Table 6. Results for the default oven with \( d_a = 1 \) cm instead of the default values \( d_a = 3 \) cm.

Comments on table 6:
The small areas at the top and at the bottom of the ampoule can not receive or lose much heat. So the convexities at the top and bottom are smaller. The same argument holds for the smaller temperature gradients there.
Influence of oven wall temperature profile:

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient $\frac{C}{cm}$</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.2</td>
<td>16.6</td>
<td>33.8</td>
</tr>
<tr>
<td>38</td>
<td>0.5</td>
<td>15.6</td>
<td>31.8</td>
</tr>
<tr>
<td>40</td>
<td>0.5</td>
<td>15.6</td>
<td>29.7</td>
</tr>
<tr>
<td>42</td>
<td>0.5</td>
<td>15.6</td>
<td>27.7</td>
</tr>
<tr>
<td>44</td>
<td>-2.1</td>
<td>16.8</td>
<td>25.7</td>
</tr>
</tbody>
</table>

Table 7. Results for the default oven with a oven wall temperature profile without the peaks. The oven wall temperature changes from 1100 $^\circ$C to 1000 $^\circ$C in 4 cm, so the temperature gradient at the wall is $25 \frac{C}{cm}$. The oven wall temperature is symmetric around the boundary between the heater and the cooler.

Comments on table 7:
The temperature gradient at the interface is smaller because to the relatively small temperature gradient at the wall. Compared with the default oven the convexity and the position of the interface are nearly the same.

<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>gradient $\frac{C}{cm}$</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1.8</td>
<td>8.2</td>
<td>33.0</td>
</tr>
<tr>
<td>38</td>
<td>5.1</td>
<td>8.2</td>
<td>30.9</td>
</tr>
<tr>
<td>40</td>
<td>3.8</td>
<td>8.2</td>
<td>28.9</td>
</tr>
<tr>
<td>42</td>
<td>3.7</td>
<td>8.3</td>
<td>26.8</td>
</tr>
<tr>
<td>44</td>
<td>-2.1</td>
<td>14.57</td>
<td>25.1</td>
</tr>
</tbody>
</table>

Table 8. Results for the default oven with an oven wall temperature profile similar to the previous one except of a heater temperature of 1080 $^\circ$C. The temperature gradient at the wall is $20 \frac{C}{cm}$.

Comments on table 8:
The position of the interface shifts upwards because the temperature of the heater is lower. The temperature gradient at the interface is smaller according to the decreased temperature gradient at the wall.
position of the bottom convexity gradient position interface of the ampoule in cm c cm in cm

<table>
<thead>
<tr>
<th>position of the bottom convexity gradient position interface of the ampoule in cm</th>
<th>c cm</th>
<th>in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.7</td>
<td>14.6</td>
</tr>
<tr>
<td>38</td>
<td>-0.7</td>
<td>12.9</td>
</tr>
<tr>
<td>40</td>
<td>-0.7</td>
<td>12.9</td>
</tr>
<tr>
<td>42</td>
<td>-0.7</td>
<td>12.9</td>
</tr>
<tr>
<td>44</td>
<td>-2.4</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Table 9. Results for the default oven with a heater temperature of 1100 °C and a cooler temperature of 1020 °C. Again there are no peaks. The temperature gradient at the wall is $20\frac{C}{cm}$.

Comments on table 9:
In this case the effects are similar to the results in the preceding table. As expected, the interface shifts downwards now.

In practice the configuration of the Bridgman-oven can be changed, by mounting a diaphragm between heater and cooler. The total heat flow from heater to cooler takes then place through the ampoule. Calculation of the temperature distribution for this configuration of the Bridgman-oven yields the following results.

Influence of a diaphragm:

<table>
<thead>
<tr>
<th>position of the bottom convexity gradient position interface of the ampoule in cm</th>
<th>c cm</th>
<th>in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>2.0</td>
<td>20.8</td>
</tr>
<tr>
<td>38</td>
<td>0.3</td>
<td>20.0</td>
</tr>
<tr>
<td>40</td>
<td>0.6</td>
<td>24.4</td>
</tr>
<tr>
<td>42</td>
<td>0.8</td>
<td>19.9</td>
</tr>
<tr>
<td>44</td>
<td>-1.5</td>
<td>20.9</td>
</tr>
</tbody>
</table>

Table 10. Results for the default oven with a diaphragm.

Comments on table 10:
The presence of a diaphragm leads to enhancement of the vertical temperature gradient in the ampoule. This effect is of course most appreciable when the interface is in the vicinity of the diaphragm. Because the interface moves considerably during the solidification process, the influence of the diaphragm on the temperature gradient varies drastically in time. It may be questioned whether this is desirable.
<table>
<thead>
<tr>
<th>position of the bottom of the ampoule in cm</th>
<th>convexity</th>
<th>$\frac{C}{cm}$</th>
<th>position interface in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>3.7</td>
<td>17.5</td>
<td>33.4</td>
</tr>
<tr>
<td>38</td>
<td>0.5</td>
<td>15.5</td>
<td>31.5</td>
</tr>
<tr>
<td>40</td>
<td>1.1</td>
<td>18.0</td>
<td>29.7</td>
</tr>
<tr>
<td>42</td>
<td>1.4</td>
<td>15.4</td>
<td>27.9</td>
</tr>
<tr>
<td>44</td>
<td>-3.0</td>
<td>17.5</td>
<td>26.1</td>
</tr>
</tbody>
</table>

Table 11. Results for the default oven with diaphragm and $d_0 = 0.12$ cm and $d_a = 0.06$ cm instead of the default values $d_0 = 0.06$ cm and $d_a = 0.03$ cm.

Comments on table 11:
Also in this case the diaphragm has an influence on the temperature gradient only when the interface and diaphragm are at the same height.
Conclusions

For the growth of high quality crystals the shape of the interface and the temperature gradient at the interface are of importance. The results presented in this report give insight which factors effect these quantities.

It has been found that:
1: $e_a$, $k_1$ and $k_s$ only influence the temperature gradient at the interface, but hardly the convexity. The influence on the temperature gradient is very small.
2: Placing a diaphragm in the Bridgman-oven has only influence on the temperature gradient at the interface. The value of the temperature gradient varies a lot when the ampoule moves into the cooler. For a steady crystal growth a diaphragm might be undesirable.
3: Changing the diameter of the oven and/or of the ampoule has a great effect on the temperature gradient at the interface and on the convexity. A small diameter of the ampoule, compared with the diameter of the oven, results in small end effects.
4: The choice of the temperature profile at the oven wall is of great importance for all parameters. In the default oven this profile is symmetric with respect to the heater/cooler boundary. The interface moves then during the process in a nearly symmetric way from its lowest position in the cooler to its highest position in the heater. Introduction of an asymmetric temperature profile at the wall shifts this trajectory downwards or upwards. The consequence for the gradient and convexity can be easily deduced from the tables in the preceding section.

Recommendation

It is quite generally believed that for the growth of high quality crystals a high temperature gradient and a convex shape of the interface is favourable. The following configuration of the Bridgman-oven will match these requirements. The temperature of the heater should be much higher than the temperature of the cooler, thus the temperature profile would be very asymmetric. For example the temperature of the heater could be chosen to be 1100 C and the temperature of the cooler 800 C. With these parameters values the interface stays during the whole solidification process in the cooler with a nearly constant gradient, apart, from end effects, which depend only on the ampoule diameter.
References

(4) H.Y. Wong, Heat Transfer for Engineers (Longman London, 1977)
(6) P.Stangerup, User's Manual Esacap (Horsholm Denmark 1985)
Symbols

s  Stefan-Boltzmann constant

k_l  thermal conductivity of fluid BGO

k_s  thermal conductivity of solid BGO

k_a  thermal conductivity of the ampoule

e_o  emissivity of the oven

e_a  emissivity of the ampoule

d_o  diameter of the oven

d_a  diameter of the ampoule

l_o  length of the oven

l_a  length of the ampoule

w_a  wall thickness of the wall

T_o  temperature of the oven

T_a  temperature of the ampoule

A_o  surface area of the oven

A_a  surface area of the ampoule

F_{A_o-A_a}  viewfactor independent of e_o and e_a

E  viewfactor
Viewfactors

Two concentric circular cylinders

\[ F_{A_1-A_2} = \frac{1}{B} - \frac{1}{2B} \left( \frac{\cos^{-1} \left( \frac{Y}{X} \right) + \frac{\pi}{2}}{\cos^{-1} \left( \frac{Y}{X} \right) - \frac{\pi}{2}} \right) \]

\[ \frac{1}{\pi B} \left( \frac{Y}{X} \sin^{-1} \left( \frac{1}{B} \right) - \frac{\pi}{2} \right) \]

\[ F_{A_1-A_2} = 1 - \frac{1}{B} \left( \frac{2}{\pi B} \tan^{-1} \left( \frac{2\sqrt{B^2-1}}{C} \right) - \frac{C}{\pi B} \right) \]

\[ \frac{1}{C} \left( 1 - \frac{2}{B^2} \right) + \frac{\pi}{2} \left( \frac{4B^2+C^2}{C^2+4(B^2-1)} \right) \]

\[ B = \frac{b}{a}, \quad C = \frac{c}{a}, \quad X = B^2 + c^2 - 1 \]

\[ Y = C^2 - B^2 + 1 \]

\[ F_{A_1-A_2} = \frac{1}{B} \quad \text{and} \quad F_{A_1-A_2} = 1 \quad \text{for} \quad c \to \infty \]

\[ F_{A_1-A_2} = \frac{1}{2B^2} \left( X - \sqrt{X^2 - 4B^2C^2} \right) \]

\[ B = \frac{b}{a}, \quad C = \frac{c}{a}, \quad X = (1 + B^2 + C^2) \]

Inner surface of circular cylinder to base

\[ F_{A_1-A_2} = \frac{1}{4C} \left[ \sqrt{C^2 + 2C(1 + R^2)} + (1 - R^2)^2 \right] \]

\[ \left( 1 - R^2 + C^2 \right) \]

\[ C = \frac{c}{a}, \quad R = \frac{r}{a} \]

Inner surface of cylinder between \( C_1 \) and \( C_2 \) to base ring between \( r_2 \) and \( r_1 \)

\[ F_{A_1-A_2} = \frac{-1}{4(C_2 - C_1)} \left[ \sqrt{C_1^2 + 2C_1(1 + R_1)} + (1 - R_1)^2 \right] \]

\[ - \sqrt{C_1^2 + 2C_1(1 + R_2)} + (1 - R_2)^2 \]

\[ + \sqrt{C_1^2 + 2C_1(1 + R_2)} + (1 - R_2)^2 \]

\[ - \sqrt{C_1^2 + 2C_1(1 + R_1)} + (1 - R_1)^2 \]
Two parallel circular rings

\[ F_{1-3} = \frac{A_3}{A_3} (F_{13-34} - F_{13-4}) \]
\[ - \frac{A_1}{A_2} (F_{1-34} - F_{1-4}) \]

Between the inner wall of a circular tube \( A_3 \) and a disc \( A_1 \), \( A_2 \) and \( A_4 \) are tube end areas

\[ F_{1-3} = F_{1-2} - F_{1-4} \]
\[ F_{3-1} = \frac{c^2}{b^2} (F_{1-2} - F_{1-4}) \]