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ON THE HEAT-FLOW DISTRIBUTION FROM AND THE TEMPERATURE PROFILE ON AN EQUALLY HEATED RECTANGLE, CALCULATED BY SERIES DEVELOPMENT

J. K. NIEUWENHUIZEN

Technische Hogeschool, Eindhoven, The Netherlands

SUMMARY

A method is proposed to approximate the solutions of a certain class of differential equations, linear or nonlinear, in two or three dimensions, provided that the boundary conditions are given on a rectangle or a parallelepiped, respectively. For other boundary shapes the co-ordinate system must be transformed to meet that requirement. As in illustration, an example is given for the solution of Poisson's equation in two dimensions with a constant heat source, giving the temperatures on the rectangle together with the heat-flow distribution along its edges. The basis of the method is a Taylor-series development around one point; the result is given in terms of as many partial derivatives in that point as is desired. A similar method has already been described by Small² for the 'heat equation' $\partial\theta/\partial t = \partial^2\theta/\partial x^2$. Compared with finite difference and finite element solutions, these methods have the advantage that the solution is continuous, whereas first and second derivatives such as heat fluxes are available at hardly any effort. The results are compared with those of the exact solution. Even if the size of the determinant is limited to 4×4 , the accuracy is already better than 98.98 per cent. When more effort is spent to solve a system of 10 equations, the accuracy is better than 99.95 per cent.

METHOD

Consider a function $\theta_{[x,y,z]}$ which is continuous in all its derivatives. Then

$$\theta_{[x+\Delta x, y+\Delta y, z+\Delta z]} = \sum_{p_x=0}^{\infty} \sum_{p_y=0}^{\infty} \sum_{p_z=0}^{\infty} \frac{\partial^{p_x+p_y+p_z}}{\partial x^{p_x} \partial y^{p_y} \partial z^{p_z}} \theta_{[x,y,z]} \cdot \frac{\Delta x^{p_x}}{p_x!} \cdot \frac{\Delta y^{p_y}}{p_y!} \cdot \frac{\Delta z^{p_z}}{p_z!}$$

Writing for short

$$\{p_x, p_y, p_z, \theta\} \quad \text{for} \quad \frac{\partial^{p_x+p_y+p_z}}{\partial x^{p_x} \partial y^{p_y} \partial z^{p_z}} \theta_{[x,y,z]}$$

one has

$$\theta_{[x+\Delta x, y+\Delta y, z+\Delta z]} = \sum_{p_x=0}^{\infty} \sum_{p_y=0}^{\infty} \sum_{p_z=0}^{\infty} \{p_x, p_y, p_z, \theta\} \cdot \frac{\Delta x^{p_x}}{p_x!} \cdot \frac{\Delta y^{p_y}}{p_y!} \cdot \frac{\Delta z^{p_z}}{p_z!}$$

If the boundaries coincide with planes for x , y , z is constant respectively, they put relatively simple restrictions on the possible functions $\{p_x, p_y, p_z, \dots\}$; the differential equation $L(\theta, x, y, z) = 0$ can be worked out to relations between those functions.

As soon as sufficient equations are developed for the desired number of functions, they may—under the usual conditions—be solved. As soon as $\{p_x, p_y, p_z, \theta\}$ are known for all $p_x + p_y + p_z < r$, the value of θ at any x , y , z can be approximated by a Taylor series.

As a simple example, we consider a rectangle which extends from $x = -1$ to $x = +1$ and

from $y = -b$ to $y = +b$. Poisson's equation reads

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = f_{[x,y]} \quad \left[\frac{K}{m^2} \right]$$

where

$$f_{[x,y]} = -\frac{1}{\lambda} \cdot q$$

or

$$\frac{1}{l^2} \cdot \frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{b^2} \cdot \frac{\partial^2 \theta}{\partial \eta^2} = f_{[\xi,\eta]}, \quad \xi := \frac{x}{l}, \eta := \frac{y}{b}$$

whereas the boundary conditions are

$$\theta = \theta_r \text{ for } \xi = \pm 1 \text{ and for } \eta = \pm 1$$

Consequently

$$\begin{aligned} \sum_{p_\xi=0}^{\infty} \sum_{p_\eta=0}^{\infty} \frac{1}{l^2} \cdot \{p_\xi + 2, p_\eta, \theta\} \cdot \frac{\xi^{p_\xi}}{p_\xi!} \cdot \frac{\eta^{p_\eta}}{p_\eta!} + \sum_{p_\xi=0}^{\infty} \sum_{p_\eta=0}^{\infty} \frac{1}{b^2} \cdot \{p_\xi, p_\eta + 2, \theta\} \cdot \frac{\xi^{p_\xi}}{p_\xi!} \cdot \frac{\eta^{p_\eta}}{p_\eta!} \\ = \sum_{p_\xi=0}^{\infty} \sum_{p_\eta=0}^{\infty} \{p_\xi, p_\eta, f\} \cdot \frac{\xi^{p_\xi}}{p_\xi!} \cdot \frac{\eta^{p_\eta}}{p_\eta!} \end{aligned}$$

Since this should be valid for all $-1 < \xi < +1$ and $-1 < \eta < +1$, Poisson's equation is equivalent to

$$\frac{1}{l^2} \cdot \{p_\xi + 2, p_\eta, \theta\} + \frac{1}{b^2} \cdot \{p_\xi, p_\eta + 2, \theta\} = \{p_\xi, p_\eta, f\}; p_\xi, p_\eta > 0 \quad \text{while } p_\xi, p_\eta > 0$$

THE BOUNDARY CONDITIONS

Since $\theta_{[1,\eta]}$ is available for $-1 < \eta < +1$, $\langle 0, p_\eta, \theta \rangle_{[1,0]}$ is known for every $p_\eta > 0$. Together with similar conditions at $[-1, 0]$, $[0, 1]$ and $[0, -1]$, boundary conditions of type 1 (i.e. θ given on boundary) lead to

$$\begin{aligned} \sum_{m=0}^{\infty} \frac{1}{(2m)!} \{2m, 2n, \theta\} &= \theta_r \quad \text{for } n=0; \\ &= 0 \quad \text{for } n < 0; \\ \sum_{m=0}^{\infty} \frac{1}{(2m)!} \{2m, 2n+1, \theta\} &= 0; \\ \sum_{m=0}^{\infty} \frac{1}{(2m+1)!} \{2m+1, n, \theta\} &= 0; \end{aligned}$$

Similarly for n :

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{1}{(2n)!} \{2m, 2n, \theta\} &= \theta_r \quad \text{for } m=0; \\ &= 0 \quad \text{for } m < 0; \\ \sum_{n=0}^{\infty} \frac{1}{(2n)!} \{2m+1, 2n, \theta\} &= 0; \\ \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} \{m, 2n+1, \theta\} &= 0; \end{aligned}$$

whereas Poisson reads

$$\frac{1}{l^2} \cdot \{m+2, n, \theta\} + \frac{1}{b^2} \cdot \{m, n+2, \theta\} = \{m, n, f\}$$

or, with $c := b/l$

$$\boxed{\{m, n+2, \theta\} = b^2 \cdot \{m, n, f\} - c^2 \cdot \{m+2, n, \theta\}}$$

is $\{m, n, f\} = 0$ for $m > 0$ or $n > 0$ and then

$n = 0$, odd series:

$$\frac{1}{1!} \{1, 0, \theta\} + \frac{1}{3!} \{3, 0, \theta\} + \frac{1}{5!} \{5, 0, \theta\} + \frac{1}{7!} \{7, 0, \theta\} + \cdots = 0$$

$n = 2$, odd series:

$$\frac{1}{1!} \{1, 2, \theta\} + \frac{1}{3!} \{3, 2, \theta\} + \frac{1}{5!} \{5, 2, \theta\} + \frac{1}{7!} \{7, 2, \theta\} + \cdots = 0$$

or

$$-\frac{c^2}{1!} \{3, 0, \theta\} - \frac{c^2}{3!} \{5, 0, \theta\} - \frac{c^2}{5!} \{7, 0, \theta\} + \cdots = 0$$

.....

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so that

$$\frac{1}{1!} \{1, 0, \theta\} + \frac{1}{3!} \{3, 0, \theta\} + \frac{1}{5!} \{5, 0, \theta\} + \frac{1}{7!} \{7, 0, \theta\} + \cdots = 0$$

$$\frac{1}{1!} \{3, 0, \theta\} + \frac{1}{3!} \{5, 0, \theta\} + \frac{1}{5!} \{7, 0, \theta\} + \cdots = 0$$

$$\frac{1}{1!} \{5, 0, \theta\} + \frac{1}{3!} \{7, 0, \theta\} + \cdots = 0$$

from which $\{2m+1, n, \theta\} = 0$

$$\{m, 2n+1, \theta\} = 0$$

and also $\{2m+1, 2n+1, \theta\} = 0$

$n = 0$, even series:

$$\frac{1}{0!} \{0, 0, \theta\} + \frac{1}{2!} \{2, 0, \theta\} + \frac{1}{4!} \{4, 0, \theta\} + \frac{1}{6!} \{6, 0, \theta\} + \cdots = \theta_r$$

$m = 0$, even series

$$\frac{1}{0!} \{0, 0, \theta\} + \frac{1}{2!} \{0, 2, \theta\} + \frac{1}{4!} \{0, 4, \theta\} + \frac{1}{6!} \{0, 6, \theta\} + \cdots = \theta_r$$

or

$$\frac{1}{0!} \{0, 0, \theta\} - \frac{c^2}{2!} \{2, 0, \theta\} + \frac{c^4}{4!} \{4, 0, \theta\} - \frac{c^6}{6!} \{6, 0, \theta\} + \cdots = \theta_r - \frac{b^2}{2!} \{0, 0, f\}$$

$n = 2$, even series:

$$\frac{1}{0!} \{0, 2, \theta\} + \frac{1}{2!} \{2, 2, \theta\} + \frac{1}{4!} \{4, 2, \theta\} + \frac{1}{6!} \{6, 2, \theta\} + \cdots = 0$$

or

$$-c^2 \frac{1}{0!} \{2, 0, \theta\} - \frac{c^2}{2!} \{4, 0, \theta\} - \frac{c^2}{4!} \{6, 0, \theta\} - \frac{c^2}{6!} \{8, 0, \theta\} + \cdots = -b^2 \{0, 0, f\}$$

$$\frac{1}{0!} \{2, 0, \theta\} + \frac{1}{2!} \{4, 0, \theta\} + \frac{1}{4!} \{6, 0, \theta\} + \frac{1}{6!} \{8, 0, \theta\} + \cdots = l^2 \{0, 0, f\}$$

 $m = 2$, even series:

$$\frac{1}{0!} \{2, 0, \theta\} + \frac{1}{2!} \{2, 2, \theta\} + \frac{1}{4!} \{2, 4, \theta\} + \frac{1}{6!} \{2, 6, \theta\} + \cdots = 0$$

or

$$\frac{1}{0!} \{2, 0, \theta\} - \frac{c^2}{2!} \{4, 0, \theta\} + \frac{c^4}{4!} \{6, 0, \theta\} - \frac{c^6}{6!} \{8, 0, \theta\} + \cdots = 0$$

.....

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This gives the following table of coefficients for

 $\{0, 0, \theta\} - \theta_r, \{2, 0, \theta\}, \{4, 0, \theta\}, \{6, 0, \theta\}, \dots, \{18, 0, \theta\}$:

$\frac{1}{0!}$	$\frac{1}{2!}$	$\frac{1}{4!}$	$\frac{1}{6!}$	$\frac{1}{8!}$	$\frac{1}{10!}$	$\frac{1}{12!}$	$\frac{1}{14!}$	$\frac{1}{16!}$	$\frac{1}{18!}$	0
$\frac{1}{0!}$	$-\frac{c^2}{2!}$	$+\frac{c^4}{4!}$	$-\frac{c^6}{6!}$	$+\frac{c^8}{8!}$	$-\frac{c^{10}}{10!}$	$+\frac{c^{12}}{12!}$	$-\frac{c^{14}}{14!}$	$+\frac{c^{16}}{16!}$	$-\frac{c^{18}}{18!}$	$-\frac{c^2}{2!} \cdot bl\{0, 0, f\}$
	$\frac{1}{0!}$	$\frac{1}{2!}$	$\frac{1}{4!}$	$\frac{1}{6!}$	$\frac{1}{8!}$	$\frac{1}{10!}$	$\frac{1}{12!}$	$\frac{1}{14!}$	$\frac{1}{16!}$	$+\frac{1}{c} \cdot bl\{0, 0, f\}$
	$\frac{1}{0!}$	$-\frac{c^2}{2!}$	$+\frac{c^4}{4!}$	$-\frac{c^6}{6!}$	$+\frac{c^8}{8!}$	$-\frac{c^{10}}{10!}$	$+\frac{c^{12}}{12!}$	$-\frac{c^{14}}{14!}$	$+\frac{c^{16}}{16!}$	0
		$\frac{1}{0!}$	$\frac{1}{2!}$	$\frac{1}{4!}$	$\frac{1}{6!}$	$\frac{1}{8!}$	$\frac{1}{10!}$	$\frac{1}{12!}$	$\frac{1}{14!}$	0
		$\frac{1}{0!}$	$-\frac{c^2}{2!}$	$+\frac{c^4}{4!}$	$-\frac{c^6}{6!}$	$+\frac{c^8}{8!}$	$-\frac{c^{10}}{10!}$	$+\frac{c^{12}}{12!}$	$-\frac{c^{14}}{14!}$	0
			$\frac{1}{0!}$	$\frac{1}{2!}$	$\frac{1}{4!}$	$\frac{1}{6!}$	$\frac{1}{8!}$	$\frac{1}{10!}$	$\frac{1}{12!}$	0
			$\frac{1}{0!}$	$-\frac{c^2}{2!}$	$+\frac{c^4}{4!}$	$-\frac{c^6}{6!}$	$+\frac{c^8}{8!}$	$-\frac{c^{10}}{10!}$	$+\frac{c^{12}}{12!}$	0
				$\frac{1}{0!}$	$\frac{1}{2!}$	$\frac{1}{4!}$	$\frac{1}{6!}$	$\frac{1}{8!}$	$\frac{1}{10!}$	0
				$\frac{1}{0!}$	$-\frac{c^2}{2!}$	$+\frac{c^4}{4!}$	$-\frac{c^6}{6!}$	$+\frac{c^8}{8!}$	$-\frac{c^{10}}{10!}$	0

In a first manual approximation, we restrict ourselves to the four unknowns $\{0, 0, \theta\} - \theta_r$, $\{2, 0, \theta\}$, $\{4, 0, \theta\}$, $\{6, 0, \theta\}$:

$$\begin{array}{ccccc} \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{6!} & 0 \\ \frac{1}{0!} & -\frac{c^2}{2!} & +\frac{c^4}{4!} & -\frac{c^6}{6!} & -\frac{c^2}{2!} bl\{0, 0, f\} \\ \frac{1}{0!} & \frac{1}{2!} & \frac{1}{4!} & \frac{1}{c} & bl\{0, 0, f\} \\ \frac{1}{0!} & -\frac{c^2}{2!} & +\frac{c^4}{4!} & 0 & \end{array}$$

With

$$\text{Det} = \frac{1}{6! \cdot 2!} \cdot \left(\frac{1}{c} + c\right) \cdot \left(\frac{1}{c^3} + c^3\right) - \frac{1}{4! \cdot 4!} \cdot \left(\frac{1}{c^2} - c^2\right)^2 - \frac{1}{4! \cdot 2! \cdot 2!} \cdot \left(\frac{1}{c} + c\right)^2$$

One finds

$$\begin{aligned} \{0, 0, \theta\} - \theta_r &= \frac{bl\{0, 0, f\}}{\text{Det}} * \left[\left(\frac{1}{c^3} + c^3\right) \cdot \left(\frac{1}{4! \cdot 4! \cdot 2!} - \frac{1}{6! \cdot 2! \cdot 2!}\right) + \left(\frac{1}{c} + c\right) \cdot \left(\frac{1}{4! \cdot 2! \cdot 2! \cdot 2!} - \frac{1}{6! \cdot 4!}\right) \right] \\ \{2, 0, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} * \left[\left(\frac{1}{c^3} + c^3\right) \cdot \frac{1}{6! \cdot 2!} + \left(\frac{1}{c^2} - c^2\right) \cdot \frac{c}{4! \cdot 4!} - \left(\frac{1}{c} + c\right) \cdot \frac{1}{4! \cdot 2! \cdot 2!} \right] \\ \{0, 2, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} * \left[\left(\frac{1}{c^3} + c^3\right) \cdot \frac{1}{6! \cdot 2!} - \left(\frac{1}{c^2} - c^2\right) \cdot \frac{1}{4! \cdot 4! \cdot c} - \left(\frac{1}{c} + c\right) \cdot \frac{1}{4! \cdot 2! \cdot 2!} \right] \\ \{4, 0, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} \cdot \frac{1}{c^2} * \left[\left(\frac{1}{c^3} + c^3\right) \cdot \frac{1}{6!} - \left(\frac{1}{c} + c\right) \cdot \frac{1}{4! \cdot 2!} \right] \\ \{2, 2, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} * \left[-\left(\frac{1}{c^3} + c^3\right) \cdot \frac{1}{6!} + \left(\frac{1}{c} + c\right) \cdot \frac{1}{4! \cdot 2!} \right] \\ \{0, 4, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} \cdot c^2 * \left[\left(\frac{1}{c^3} + c^3\right) \cdot \frac{1}{6!} - \left(\frac{1}{c} + c\right) \cdot \frac{1}{4! \cdot 2!} \right] \\ \{6, 0, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} \cdot \frac{1}{c^3} * \left[-\left(\frac{1}{c^2} - c^2\right) \cdot \frac{1}{4!} \right] \\ \{4, 2, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} \cdot \frac{1}{c} * \left[+\left(\frac{1}{c^2} - c^2\right) \cdot \frac{1}{4!} \right] \\ \{2, 4, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} \cdot c * \left[-\left(\frac{1}{c^2} - c^2\right) \cdot \frac{1}{4!} \right] \\ \{0, 6, \theta\} &= \frac{bl\{0, 0, f\}}{\text{Det}} \cdot c^3 * \left[+\left(\frac{1}{c^2} - c^2\right) \cdot \frac{1}{4!} \right] \end{aligned}$$

The values of $\{m, n, \theta\}$; $m + n \leq 6$ are tabulated in Appendix I. Attention is asked for the perfect symmetry in values for c in relation to those of $1/c$.

An analytical solution is given in Reference 1, which in our variables reads

$$\frac{\theta - \theta_r}{b l f} = -\frac{1 - \xi^2}{2c} + \frac{16}{\pi^3 c} \sum_{n=0}^{\infty} (-1)^n \frac{\cos - [(2n+1)(\pi\xi/2)] \cdot \cosh - [(2n+1)(\pi\eta c/2)]}{(2n+1)^3 \cdot \cosh - [(2n+1)(\pi c/2)]}$$

Since the symmetry in ξ and η is difficult to notice, comparison with our results is complicated; however, heat fluxes are also somewhat hidden. What we can do is calculate $\{0, 0, \theta\}$ by substituting $\xi = 0$ and $\eta = 0$ for various values of c and $n < 3$. This gives the following results:

c	Our result	Exact solution	Discr. (%)
1.00000	-0.29167	-0.29469	-1.02
0.90909	-0.29019	-0.29315	-1.01
0.82645	-0.28581	-0.28863	-0.976
0.75131	-0.27875	-0.28132	-0.912
0.68030	-0.26930	-0.27155	-0.829
0.62092	-0.25787	-0.25974	-0.718

so that our approximation, although only four terms were evaluated, is already accurate up to 1 per cent or better.

The temperature gradients as reported in Appendix I are given in terms of $\partial\theta/\partial\xi$ and $\partial\theta/\partial\eta$. Heat flux densities are calculated as

$$q_x = -\lambda \frac{\partial\theta}{\partial x} = -\frac{\lambda}{l} \frac{\partial\theta}{\partial\xi}; \quad q_y = -\lambda \frac{\partial\theta}{\partial y} = -\frac{\lambda}{l} \frac{\partial\theta}{\partial\eta}$$

This ratios of the heat flux densities at the centre of the edges are given as q_y/q_x in Appendix I.

Obviously, even better approximations might be realized through calculation of higher derivatives. However, the development of the eleven 10×10 determinants into an exact formulation is looked upon as almost a Gordian knot. Numerical development is restricted, due to the poor condition of the determinants; the coefficients of the 4×4 determinants differ by three orders of magnitude, and those of the 10×10 determinants by as much as 15. Therefore, the values are successively approximated, i.e. in step 1 a first approximation for, e.g., $\{0, 0, \theta\}$ through $\{6, 0, \theta\}$ is developed from the 4×4 determinants; in a second step, corrections of and first approximations for $\{0, 0, \theta\}$ through $\{6, 0, \theta\}$ and first approximations of $\{8, 0, \theta\}$ through $\{10, 0, \theta\}$ are calculated from the 6×6 determinants with, on the right-hand side, four zero and two non-zero, values, calculated from the first approximations of $\{0, 0, \theta\}$ through $\{6, 0, \theta\}$.

The procedure may be repeated as many times as desired, leading to the following result (Appendixes II and III):

c	Our result	Exact solution	Discr. (%)
1.00000	-0.29457	-0.29469	-0.0407
0.90909	-0.29304	-0.29315	-0.0375
0.82645	-0.28853	-0.28863	-0.0346
0.75131	-0.28125	-0.28132	-0.0249
0.68030	-0.27152	-0.27155	-0.0110
0.62092	-0.25974	-0.25974	-0.0000

CONCLUSIONS

The method described is not limited to two dimensions, nor does it require linearity of the differential equation. Dependent upon the effort that one wishes to spend, one may approximate the solution as closely as is desired.

APPENDIX I

Results of 4 terms approximation (prog 300-5)

b/l : 0.62

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.25787	-0.24800	-0.21815	-0.16765	-0.09541	-0.00000
Ksi 0.20:	-0.25108	-0.24152	-0.21256	-0.16350	-0.09315	0.00004
Ksi 0.40:	-0.22912	-0.22051	-0.19438	-0.14984	-0.08546	0.00065
Ksi 0.60:	-0.18695	-0.18008	-0.15909	-0.12283	-0.06945	0.00355
Ksi 0.80:	-0.11539	-0.11127	-0.09840	-0.07529	-0.03948	0.01237
Ksi 1.00:	-0.00000	0.00005	0.00088	0.00443	0.01393	0.03380

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.71873	0.69448	0.62257	0.50559	0.34779	0.15515
Eta = 1	0.53769	0.52598	0.48862	0.41897	0.30596	0.13405
qy / qx	1.20485	1.21976	1.26400	1.33462	1.41681	1.39146

b/l : 0.68

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.26930	-0.25913	-0.22830	-0.17589	-0.10045	-0.00000
Ksi 0.20:	-0.26165	-0.25181	-0.22198	-0.17117	-0.09786	0.00005
Ksi 0.40:	-0.23725	-0.22845	-0.20169	-0.15586	-0.08917	0.00080
Ksi 0.60:	-0.19156	-0.18462	-0.16333	-0.12636	-0.07151	0.00419
Ksi 0.80:	-0.11657	-0.11246	-0.09955	-0.07618	-0.03959	0.01398
Ksi 1.00:	-0.00000	0.00006	0.00095	0.00477	0.01502	0.03643

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.71350	0.68970	0.61910	0.50409	0.34869	0.15848
Eta = 1	0.56852	0.55498	0.51266	0.43641	0.31767	0.14443
qy / qx	1.16661	1.17814	1.21240	1.26754	1.33385	1.33428

b/l : 0.75

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.27875	-0.26839	-0.23690	-0.18309	-0.10500	-0.00000
Ksi 0.20:	-0.27031	-0.26031	-0.22990	-0.17784	-0.10209	0.00006

Ksi	0.40:	-0.24371	-0.23483	-0.20771	-0.16100	-0.09247	0.00091
Ksi	0.60:	-0.19497	-0.18802	-0.16665	-0.12928	-0.07333	0.00469
Ksi	0.80:	-0.11716	-0.11310	-0.10028	-0.07684	-0.03971	0.01527
Ksi	1.00:	-0.00000	0.00006	0.00100	0.00506	0.01592	0.03863

Gradient / blf :

Eta (Ksi) 0.0 0.2 0.4 0.6 0.8 1.0

Ksi = 1	0.70616	0.68301	0.61426	0.50198	0.34963	0.16206
Eta = 1	0.59734	0.58202	0.53485	0.45215	0.32778	0.15316

qy / qx	1.12590	1.13421	1.15895	1.19889	1.24783	1.25791
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b/l : 0.83

Temperatures / blf :

Eta: 0.0 0.2 0.4 0.6 0.8 1.0

Ksi	0.00:	-0.28581	-0.27540	-0.24364	-0.18900	-0.10893	-0.00000
Ksi	0.20:	-0.27670	-0.26667	-0.23605	-0.18327	-0.10574	0.00006
Ksi	0.40:	-0.24823	-0.23937	-0.21221	-0.16509	-0.09527	0.00099
Ksi	0.60:	-0.19698	-0.19011	-0.16888	-0.13146	-0.07485	0.00506
Ksi	0.80:	-0.11708	-0.11310	-0.10049	-0.07719	-0.03983	0.01621
Ksi	1.00:	-0.00000	0.00007	0.00104	0.00527	0.01659	0.04029

Gradient / blf :

Eta (Ksi) 0.0 0.2 0.4 0.6 0.8 1.0

Ksi = 1	0.69617	0.67388	0.60751	0.49866	0.34995	0.16505
Eta = 1	0.62358	0.60657	0.55478	0.46589	0.33606	0.15990

qy / qx	1.08384	1.08916	1.10497	1.13048	1.16196	1.17220
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b/l : 0.91

Temperatures / blf :

Eta: 0.0 0.2 0.4 0.6 0.8 1.0

Ksi	0.00:	-0.29019	-0.27987	-0.24827	-0.19343	-0.11215	-0.00000
Ksi	0.20:	-0.28052	-0.27060	-0.24018	-0.18730	-0.10870	0.00006
Ksi	0.40:	-0.25057	-0.24185	-0.21499	-0.16799	-0.09751	0.00104
Ksi	0.60:	-0.19744	-0.19073	-0.16989	-0.13281	-0.07602	0.00529
Ksi	0.80:	-0.11624	-0.11240	-0.10012	-0.07719	-0.03991	0.01681
Ksi	1.00:	-0.00000	0.00007	0.00107	0.00539	0.01699	0.04132

Gradient / blf :

Eta (Ksi) 0.0 0.2 0.4 0.6 0.8 1.0

Ksi = 1	0.68310	0.66185	0.59842	0.49368	0.34911	0.16677
Eta = 1	0.64679	0.62821	0.57211	0.47742	0.34236	0.16442

qy / qx	1.04154	1.04409	1.05165	1.06378	1.07875	1.08453
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b/l : 1.00

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.29167	-0.28160	-0.25060	-0.19627	-0.11460	-0.00000
Ksi 0.20:	-0.28160	-0.27193	-0.24213	-0.18980	-0.11093	0.00007
Ksi 0.40:	-0.25060	-0.24213	-0.21593	-0.16960	-0.09913	0.00107
Ksi 0.60:	-0.19627	-0.18980	-0.16960	-0.13327	-0.07680	0.00540
Ksi 0.80:	-0.11460	-0.11093	-0.09913	-0.07680	-0.03993	0.01707
Ksi 1.00:	-0.00000	0.00007	0.00107	0.00540	0.01707	0.04167

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.66667	0.64667	0.58667	0.48667	0.34667	0.16667
Eta = 1	0.66666	0.64666	0.58667	0.48667	0.34667	0.16667
qy / qx	1.00000	1.00000	1.00000	1.00001	1.00001	1.00001

8

b/l : 1.10

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.29019	-0.28052	-0.25057	-0.19744	-0.11624	-0.00000
Ksi 0.20:	-0.27987	-0.27060	-0.24185	-0.19073	-0.11240	0.00007
Ksi 0.40:	-0.24827	-0.24018	-0.21499	-0.16989	-0.10012	0.00107
Ksi 0.60:	-0.19343	-0.18730	-0.16799	-0.13281	-0.07719	0.00539
Ksi 0.80:	-0.11215	-0.10870	-0.09751	-0.07602	-0.03991	0.01699
Ksi 1.00:	-0.00000	0.00006	0.00104	0.00529	0.01681	0.04132

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.64679	0.62821	0.57211	0.47742	0.34236	0.16442
Eta = 1	0.68310	0.66185	0.59842	0.49368	0.34911	0.16677
qy / qx	0.96012	0.95778	0.95090	0.94005	0.92701	0.92207

b/l : 1.21

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.28581	-0.27670	-0.24823	-0.19698	-0.11708	-0.00000
Ksi 0.20:	-0.27540	-0.26667	-0.23937	-0.19011	-0.11310	0.00007
Ksi 0.40:	-0.24364	-0.23605	-0.21221	-0.16888	-0.10049	0.00104
Ksi 0.60:	-0.18900	-0.18327	-0.16509	-0.13146	-0.07719	0.00527
Ksi 0.80:	-0.10893	-0.10574	-0.09527	-0.07485	-0.03983	0.01659
Ksi 1.00:	-0.00000	0.00006	0.00099	0.00506	0.01621	0.04029

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.62359	0.60658	0.55478	0.46589	0.33606	0.15990
Eta = 1	0.69617	0.67387	0.60751	0.49866	0.34995	0.16505
qy / qx	0.92265	0.91814	0.90501	0.88459	0.86062	0.85310

b/l : 1.33

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.27875	-0.27031	-0.24371	-0.19497	-0.11716	-0.00000
Ksi 0.20:	-0.26839	-0.26032	-0.23483	-0.18802	-0.11310	0.00006
Ksi 0.40:	-0.23690	-0.22991	-0.20772	-0.16665	-0.10028	0.00100
Ksi 0.60:	-0.18309	-0.17784	-0.16100	-0.12928	-0.07684	0.00506
Ksi 0.80:	-0.10500	-0.10209	-0.09247	-0.07333	-0.03971	0.01592
Ksi 1.00:	-0.00000	0.00006	0.00091	0.00469	0.01527	0.03863

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.59734	0.58203	0.53486	0.45215	0.32778	0.15316
Eta = 1	0.70616	0.68301	0.61426	0.50198	0.34963	0.16206
qy / qx	0.88818	0.88167	0.86285	0.83411	0.80140	0.79498

b/l : 1.46

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.26930	-0.26165	-0.23725	-0.19156	-0.11657	-0.00000
Ksi 0.20:	-0.25913	-0.25182	-0.22845	-0.18462	-0.11246	0.00006
Ksi 0.40:	-0.22830	-0.22198	-0.20169	-0.16333	-0.09955	0.00095
Ksi 0.60:	-0.17590	-0.17117	-0.15586	-0.12636	-0.07618	0.00477
Ksi 0.80:	-0.10045	-0.09786	-0.08917	-0.07151	-0.03959	0.01502
Ksi 1.00:	-0.00000	0.00005	0.00080	0.00419	0.01398	0.03643

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.56852	0.55499	0.51267	0.43642	0.31767	0.14443
Eta = 1	0.71349	0.68969	0.61909	0.50409	0.34869	0.15848
qy / qx	0.85719	0.84880	0.82481	0.78894	0.74971	0.74947

b/l : 1.61

Temperatures / blf :

Eta:	0.0	0.2	0.4	0.6	0.8	1.0
Ksi 0.00:	-0.25787	-0.25109	-0.22913	-0.18695	-0.11539	-0.00000
Ksi 0.20:	-0.24800	-0.24152	-0.22051	-0.18008	-0.11127	0.00005
Ksi 0.40:	-0.21815	-0.21257	-0.19438	-0.15909	-0.09840	0.00088
Ksi 0.60:	-0.16765	-0.16350	-0.14984	-0.12283	-0.07529	0.00443
Ksi 0.80:	-0.09541	-0.09315	-0.08547	-0.06945	-0.03948	0.01393
Ksi 1.00:	-0.00000	0.00004	0.00065	0.00355	0.01237	0.03380

Gradient / blf :

Eta (Ksi)	0.0	0.2	0.4	0.6	0.8	1.0
Ksi = 1	0.53769	0.52598	0.48863	0.41898	0.30596	0.13405
Eta = 1	0.71873	0.69447	0.62257	0.50559	0.34779	0.15516
qy / qx	0.82998	0.81983	0.79114	0.74928	0.70582	0.71867

APPENDIX II

Results of 10 terms approximation (prog 300-6)

b/l 0.62

Temperatures / blf :							
Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.25974	-0.24980	-0.21974	-0.16887	-0.09608	-0.00000
Ksi	0.20:	-0.25313	-0.24351	-0.21437	-0.16495	-0.09402	-0.00000
Ksi	0.40:	-0.23159	-0.22299	-0.19684	-0.15215	-0.08726	-0.00000
Ksi	0.60:	-0.18972	-0.18302	-0.16250	-0.12688	-0.07386	-0.00007
Ksi	0.80:	-0.11756	-0.11380	-0.10214	-0.08140	-0.04907	-0.00034
Ksi	1.00:	-0.00000	0.00002	0.00024	0.00074	0.00146	0.00484
Gradient / blf :							
Ksi = 1 :		0.73221	0.71280	0.65469	0.55088	0.37496	0.08741
Eta = 1 :		0.54119	0.53069	0.49632	0.42765	0.29660	0.03690
qy / qx :		1.19035	1.19906	1.22091	1.25025	1.27394	0.67983

b/l 0.68

Temperatures / blf :							
Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.27152	-0.26127	-0.23021	-0.17738	-0.10128	-0.00000
Ksi	0.20:	-0.26401	-0.25412	-0.22411	-0.17293	-0.09893	-0.00000
Ksi	0.40:	-0.23993	-0.23116	-0.20445	-0.15853	-0.09131	-0.00000
Ksi	0.60:	-0.19438	-0.18764	-0.16694	-0.13079	-0.07650	-0.00003
Ksi	0.80:	-0.11866	-0.11492	-0.10332	-0.08257	-0.04991	0.00006
Ksi	1.00:	-0.00000	0.00001	0.00013	0.00039	0.00099	0.00524
Gradient / blf :							
Ksi = 1 :		0.72609	0.70682	0.64795	0.54079	0.36049	0.07370
Eta = 1 :		0.57290	0.56092	0.52210	0.44615	0.30657	0.04561
qy / qx :		1.15521	1.16189	1.17973	1.20790	1.24509	0.90612

b/l 0.75

Temperatures / blf :							
Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.28125	-0.27082	-0.23911	-0.18484	-0.10600	-0.00000
Ksi	0.20:	-0.27292	-0.26289	-0.23232	-0.17987	-0.10337	-0.00000
Ksi	0.40:	-0.24654	-0.23771	-0.21071	-0.16398	-0.09493	-0.00000
Ksi	0.60:	-0.19777	-0.19106	-0.17037	-0.13400	-0.07882	0.00000
Ksi	0.80:	-0.11913	-0.11545	-0.10400	-0.08339	-0.05060	0.00036
Ksi	1.00:	-0.00000	0.00001	0.00007	0.00019	0.00083	0.00582
Gradient / blf :							
Ksi = 1 :		0.71774	0.69877	0.64005	0.53214	0.35182	0.06839
Eta = 1 :		0.60265	0.58924	0.54618	0.46336	0.31549	0.05210
qy / qx :		1.11759	1.12238	1.13581	1.15896	1.19355	1.01392

b/l 0.83

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.28853	-0.27806	-0.24610	-0.19100	-0.11010	-0.00000
Ksi	0.20:	-0.27948	-0.26943	-0.23870	-0.18556	-0.10721	-0.00000
Ksi	0.40:	-0.25112	-0.24234	-0.21537	-0.16834	-0.09803	0.00000
Ksi	0.60:	-0.19970	-0.19309	-0.17263	-0.13639	-0.08074	0.00003
Ksi	0.80:	-0.11891	-0.11531	-0.10410	-0.08380	-0.05112	0.00057
Ksi	1.00:	-0.00000	0.00000	0.00003	0.00010	0.00082	0.00628

Gradient / blf :

Ksi = 1 :	0.70667	0.68818	0.63046	0.52385	0.34612	0.06600
Eta = 1 :	0.62986	0.61513	0.56815	0.47898	0.32340	0.05630
qy / qx :	1.07849	1.08155	1.09041	1.10637	1.13055	1.03217

b/l 0.91

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.29304	-0.28269	-0.25093	-0.19566	-0.11349	-0.00000
Ksi	0.20:	-0.28340	-0.27349	-0.24301	-0.18981	-0.11038	-0.00000
Ksi	0.40:	-0.25346	-0.24484	-0.21825	-0.17144	-0.10053	0.00000
Ksi	0.60:	-0.20004	-0.19360	-0.17360	-0.13786	-0.08222	0.00004
Ksi	0.80:	-0.11791	-0.11444	-0.10358	-0.08375	-0.05145	0.00072
Ksi	1.00:	-0.00000	0.00000	0.00001	0.00006	0.00083	0.00653

Gradient / blf :

Ksi = 1 :	0.69250	0.67468	0.61873	0.51494	0.34132	0.06386
Eta = 1 :	0.65408	0.63815	0.58766	0.49281	0.33032	0.05914
qy / qx :	1.03896	1.04043	1.04477	1.05272	1.06455	1.01861

b/l 1.00

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.29457	-0.28449	-0.25340	-0.19869	-0.11611	-0.00000
Ksi	0.20:	-0.28449	-0.27486	-0.24507	-0.19251	-0.11280	-0.00000
Ksi	0.40:	-0.25340	-0.24507	-0.21923	-0.17321	-0.10239	0.00000
Ksi	0.60:	-0.19869	-0.19251	-0.17321	-0.13836	-0.08323	0.00005
Ksi	0.80:	-0.11611	-0.11280	-0.10239	-0.08323	-0.05156	0.00080
Ksi	1.00:	-0.00000	-0.00000	0.00000	0.00005	0.00080	0.00660

Gradient / blf :

Ksi = 1 :	0.67500	0.65803	0.60451	0.50471	0.33618	0.06144
Eta = 1 :	0.67500	0.65803	0.60451	0.50471	0.33618	0.06144
qy / qx :	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999

b/l 1.10

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.29304	-0.28340	-0.25346	-0.20004	-0.11791	0.00000
Ksi	0.20:	-0.28269	-0.27349	-0.24484	-0.19360	-0.11444	-0.00000
Ksi	0.40:	-0.25093	-0.24301	-0.21825	-0.17360	-0.10358	0.00000
Ksi	0.60:	-0.19566	-0.18981	-0.17144	-0.13786	-0.08376	0.00005
Ksi	0.80:	-0.11349	-0.11038	-0.10053	-0.08222	-0.05145	0.00080
Ksi	1.00:	-0.00000	-0.00000	-0.00000	0.00004	0.00071	0.00648

Gradient / blf :

Ksi = 1 :		0.65408	0.63814	0.58763	0.49275	0.33020	0.05887
Eta = 1 :		0.69251	0.67466	0.61863	0.51470	0.34088	0.06316
qy / qx :		0.96250	0.96112	0.95704	0.94960	0.93849	0.97527

b/l 1.21

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.28853	-0.27948	-0.25112	-0.19970	-0.11891	0.00000
Ksi	0.20:	-0.27806	-0.26943	-0.24234	-0.19309	-0.11531	-0.00000
Ksi	0.40:	-0.24610	-0.23870	-0.21537	-0.17263	-0.10410	0.00000
Ksi	0.60:	-0.19100	-0.18556	-0.16834	-0.13639	-0.08380	0.00004
Ksi	0.80:	-0.11010	-0.10721	-0.09803	-0.08074	-0.05112	0.00072
Ksi	1.00:	-0.00000	-0.00000	-0.00000	0.00002	0.00056	0.00611

Gradient / blf :

Ksi = 1 :		0.62986	0.61512	0.56812	0.47892	0.32328	0.05581
Eta = 1 :		0.70669	0.68811	0.63009	0.52297	0.34455	0.06350
qy / qx :		0.92726	0.92451	0.91659	0.90246	0.88082	0.94025

b/l 1.33

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.28125	-0.27292	-0.24654	-0.19777	-0.11913	0.00000
Ksi	0.20:	-0.27082	-0.26289	-0.23771	-0.19106	-0.11545	-0.00001
Ksi	0.40:	-0.23911	-0.23232	-0.21071	-0.17037	-0.10400	0.00001
Ksi	0.60:	-0.18484	-0.17987	-0.16398	-0.13400	-0.08339	0.00005
Ksi	0.80:	-0.10600	-0.10337	-0.09493	-0.07882	-0.05061	0.00058
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	0.00034	0.00541

Gradient / blf :

Ksi = 1 :		0.60265	0.58924	0.54617	0.46331	0.31539	0.05116
Eta = 1 :		0.71782	0.69859	0.63909	0.52987	0.34772	0.06193
qy / qx :		0.89489	0.89075	0.87914	0.85925	0.82834	0.90941

b/l 1.46

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.27152	-0.26401	-0.23993	-0.19438	-0.11866	0.00001
Ksi	0.20:	-0.26127	-0.25412	-0.23116	-0.18764	-0.11492	-0.00001
Ksi	0.40:	-0.23021	-0.22411	-0.20445	-0.16694	-0.10332	0.00000
Ksi	0.60:	-0.17738	-0.17293	-0.15853	-0.13079	-0.08258	0.00007
Ksi	0.80:	-0.10128	-0.09893	-0.09131	-0.07650	-0.04992	0.00042
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00003	0.00003	0.00434

Gradient / blf :

Ksi = 1 :	0.57290	0.56092	0.52209	0.44613	0.30647	0.04373
Eta = 1 :	0.72632	0.70647	0.64586	0.53580	0.35144	0.05942
qy / qx :	0.86592	0.86025	0.84493	0.82029	0.78324	0.92810

b/l 1.61

Temperatures / blf :

Eta :		0.0	0.2	0.4	0.6	0.8	1.0
Ksi	0.00:	-0.25974	-0.25313	-0.23159	-0.18972	-0.11756	0.00003
Ksi	0.20:	-0.24980	-0.24351	-0.22299	-0.18302	-0.11380	-0.00002
Ksi	0.40:	-0.21974	-0.21437	-0.19684	-0.16250	-0.10215	-0.00002
Ksi	0.60:	-0.16887	-0.16495	-0.15215	-0.12688	-0.08142	0.00012
Ksi	0.80:	-0.09608	-0.09402	-0.08726	-0.07386	-0.04910	0.00033
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00007	-0.00039	0.00306

Gradient / blf :

Ksi = 1 :	0.54119	0.53069	0.49631	0.42764	0.29648	0.03325
Eta = 1 :	0.73276	0.71219	0.65062	0.54103	0.35702	0.05906
qy / qx :	0.84072	0.83328	0.81397	0.78556	0.74772	1.10297

APPENDIX III

Results of exact formula (prog 300-10)

Prog. Calslaw, versie 840802

C&Jaeger b/l 0.62

Temperatures / blf :

Eta:		0.00	0.20	0.40	0.60	0.80	1.00
Ksi	0.00:	-0.25974	-0.24980	-0.21974	-0.16886	-0.09608	-0.00000
Ksi	0.20:	-0.25312	-0.24350	-0.21438	-0.16501	-0.09424	-0.00075
Ksi	0.40:	-0.23158	-0.22298	-0.19683	-0.15214	-0.08725	-0.00000
Ksi	0.60:	-0.18971	-0.18301	-0.16249	-0.12685	-0.07380	0.00015
Ksi	0.80:	-0.11756	-0.11380	-0.10216	-0.08144	-0.04908	-0.00000
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000

C&Jaeger b/l 0.68

Temperatures / blf :							
Eta:		0.00	0.20	0.40	0.60	0.80	1.00
Ksi	0.00:	-0.27155	-0.26130	-0.23024	-0.17740	-0.10130	-0.00000
Ksi	0.20:	-0.26405	-0.25416	-0.22415	-0.17300	-0.09913	-0.00068
Ksi	0.40:	-0.23997	-0.23121	-0.20449	-0.15856	-0.09133	-0.00000
Ksi	0.60:	-0.19443	-0.18769	-0.16700	-0.13086	-0.07654	0.00014
Ksi	0.80:	-0.11869	-0.11496	-0.10340	-0.08273	-0.05018	-0.00000
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000

C&Jaeger b/l 0.75

Temperatures / blf :							
Eta:		0.00	0.20	0.40	0.60	0.80	1.00
Ksi	0.00:	-0.28132	-0.27089	-0.23917	-0.18489	-0.10602	-0.00000
Ksi	0.20:	-0.27299	-0.26295	-0.23239	-0.17995	-0.10354	-0.00062
Ksi	0.40:	-0.24661	-0.23779	-0.21079	-0.16406	-0.09498	-0.00000
Ksi	0.60:	-0.19785	-0.19114	-0.17048	-0.13414	-0.07895	0.00012
Ksi	0.80:	-0.11919	-0.11552	-0.10411	-0.08363	-0.05107	-0.00000
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000

C&Jaeger b/l 0.83

Temperatures / blf :							
Eta:		0.00	0.20	0.40	0.60	0.80	1.00
Ksi	0.00:	-0.28863	-0.27816	-0.24619	-0.19106	-0.11014	-0.00000
Ksi	0.20:	-0.27958	-0.26953	-0.23879	-0.18565	-0.10737	-0.00056
Ksi	0.40:	-0.25122	-0.24244	-0.21548	-0.16845	-0.09811	-0.00000
Ksi	0.60:	-0.19980	-0.19320	-0.17277	-0.13658	-0.08094	0.00011
Ksi	0.80:	-0.11897	-0.11539	-0.10423	-0.08408	-0.05174	-0.00000
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.00000

C&Jaeger b/l 0.91

Temperatures / blf :							
Eta:		0.00	0.20	0.40	0.60	0.80	1.00
Ksi	0.00:	-0.29315	-0.28280	-0.25103	-0.19575	-0.11354	-0.00000
Ksi	0.20:	-0.28352	-0.27360	-0.24312	-0.18992	-0.11053	-0.00051
Ksi	0.40:	-0.25357	-0.24496	-0.21838	-0.17158	-0.10064	-0.00000
Ksi	0.60:	-0.20014	-0.19371	-0.17376	-0.13808	-0.08248	0.00010
Ksi	0.80:	-0.11798	-0.11452	-0.10371	-0.08406	-0.05215	-0.00000
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.00000

C&Jaeger b/l 1.00

Temperatures / blf :

Eta:		0.00	0.20	0.40	0.60	0.80	1.00
Ksi	0.00:	-0.29469	-0.28461	-0.25352	-0.19879	-0.11617	-0.00000
Ksi	0.20:	-0.28461	-0.27498	-0.24520	-0.19263	-0.11294	-0.00047
Ksi	0.40:	-0.25352	-0.24519	-0.21936	-0.17336	-0.10251	-0.00000
Ksi	0.60:	-0.19879	-0.19262	-0.17337	-0.13859	-0.08352	0.00009
Ksi	0.80:	-0.11617	-0.11287	-0.10251	-0.08352	-0.05229	-0.00000
Ksi	1.00:	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.00000

APPENDIX IV: NOTATION

	Units	Dimension
θ = Temperature	[K]	$[\theta]$
p = Integer		
x, y, z = Coordinates	[m]	$[L]$
ξ, η = Dimensionless coordinates		
l = Length of rectangle	[m]	$[L]$
b = Width of rectangle	[m]	$[L]$
f = Function		
q = Heat generated per unit volume	$[W \cdot m^{-3}]$	$[M \cdot L^{-1} \cdot T^{-3}]$
λ = Heat conductivity	$[W \cdot m^{-1} \cdot K^{-1}]$	$[M \cdot L^{-1} \cdot T^{-3} \cdot \theta^{-1}]$

General:

$$\theta_{[x,y,z]} = \text{value of } \theta \text{ at } x, y, z$$

$$\{p_x, p_y, p_z, \theta\} = \frac{\partial^{p_x + p_y + p_z}}{\partial x^{p_x} \cdot \partial y^{p_y} \cdot \partial z^{p_z}} \theta_{[x,y,z]}$$

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