A portfolio model for foreign exchange exposure management
Soenen, L.A.

Published in:
Omega : The International Journal of Management Science

Published: 01/01/1979

Document Version
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
- A submitted manuscript is the author’s version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
A Portfolio Model for
Foreign Exchange Exposure Management

LUC A SOENEN

Eindhoven University of Technology, The Netherlands

(Received November 1978: in revised form January 1979)

This paper summarizes the results of our research into applications of decision analysis and portfolio
theory to the management of foreign exchange exposure. In contrast with much current practice
in foreign exchange management, the portfolio approach takes into explicit consideration the inher-
ent relationships among the currencies in the company's foreign currency portfolio. The hedging
model developed in this article traces out an 'efficient frontier' or trade-off curve between expected
value and variance of the foreign currency portfolio at the end of the planning period. In doing
so, the model chooses the optimal amount and method of hedging for each currency in the portfolio.

INTRODUCTION

Current exchange exposure management is
usually executed on a currency by currency
basis failing to recognize the statistical rela-
tionships among the changes in the exchange
rates of the currencies maintained by the com-
pany, not to mention exploiting the reduction
of exchange risk holding a diversified portfolio
of currencies. Although traditional exposure
analysis has been conducted for individual cur-
rencies separately, portfolio theory has taught
us that individual risks should not be measured
independently. Because of diversification, the
risk of a portfolio composed of various curren-
cies is not equal to the sum of the risks of
each currency. Just as unsystematic risk can
be reduced via diversification by the number
of stocks in an investment portfolio, so
exchange risk can be reduced in a diversified
portfolio of currencies. Consideration of the
statistical relationship among currencies allows
exposure netting as a technique to reduce
exchange risk. If one believes that exchange
rate movements of two currencies are highly
positively correlated, then a short position in
one currency will largely offset a long position
in the other currency. If the currencies are
negatively correlated, long positions in both
currencies (or short positions) will tend to
balance each other out.

Traditional techniques of foreign exchange
risk management (monetary balance, swaps,
forward markets etc.) do not explicitly recog-
nize the risk element. However, this kind of
exposure netting (i.e. balancing of foreign cur-
rency positions on the basis of their co-move-
ment) can be fully exploited by applying port-
folio theory to the management of foreign
exchange. The portfolio approach will allow
the international company to determine cor-
rectly its overall exchange exposure, to build
a well-diversified (minimum variance) currency
portfolio, and to assess the impact of major
investments on its company-wide foreign
exchange exposure.

OBJECTIVE OF FOREIGN
EXCHANGE MANAGEMENT

After an international company has consoli-
dated the foreign exchange exposure of all sub-
sidiaries and netted out assets and liabilities
per currency, its net exposure will consist of
long/short positions in various currencies. This
set of foreign currency positions constitutes the
currency portfolio of the company. The objective of the international company's foreign exchange management is then to protect the company and its subsidiaries against exchange losses, but not to seek windfall gains through currency speculation or similar hedging activities unrelated to the nature of its business. The company's profitability should be based on the return on the goods and services it produces and sells, not on the return of its currency portfolio. Protection against currency risk is the objective, not speculative profits. The multinational company can reduce its foreign exchange risk by engaging in hedging transactions.

Hedging reduces the exposure and consequently the variance of the foreign currency portfolio, but also reduces its expected end-of-period value by the hedging costs incurred. In other words, the company should minimize the variance of its foreign currency portfolio given a maximum amount of hedging costs it is willing to incur, and not maximize the expected end-of-period return on the foreign exchange portfolio. Under efficient market conditions, only the contribution of the currency to the total risk of a portfolio should influence the value of the currency. All other risks (unsystematic risk) can be eliminated by holding a diversified portfolio of currencies. Unless the price movements of two currencies are perfectly positively correlated, any variation in the exchange rate of one that is not accompanied by an equivalent and simultaneous change in the rate of the other will lead to a reduced combined variation between both currencies. In other words, the combined variation of the two currencies will be less than the sum of the fluctuations in both exchange rates individually. The total risk of the currency portfolio is thus a function of the level of correlation among all currencies, the volatility of all currencies individually, and the proportion of each currency to the base currency equivalent sum of all currencies in the portfolio (i.e. the relative importance of each currency in the portfolio). In this portfolio context, the objective of foreign exchange management is defined as the minimization of exchange risk to the company, i.e. to minimize the variance of the company's foreign currency portfolio subject to the costs incurred by hedging.

Hedging reduces variance caused by exchange rate changes but entails costs to the company. Therefore, one should concentrate on the relationship between the expected value and the variance of the company's foreign exchange portfolio at the end of the planning horizon. A portfolio model will now be developed to trace out the expected value-variance frontier for the company's foreign currency portfolio.

**STATEMENT OF THE PROBLEM**

The hedging problem can be formulated and solved as a decision problem. The data inputs required for the solution of the problem are: (a) a set of known variables (spot and forward exchange rates, domestic and foreign interest rates) and a set of variables assumed to be known (the foreign exchange exposure of the company); and (b) a set of unknown random variables (the future spot exchange rates for every currency in the portfolio at the end of the planning period). The decision variables are the amounts to be hedged in every currency in order to reduce the portfolio's variance to a selected level. These three sets of variables produce an uncertain value, $V$, i.e. the value of the company's foreign exchange portfolio at the end of the planning horizon, with expected value $V$ and variance $W$. The hedging problem is then set up as the minimization of the variance of the portfolio subject to a set of operational constraints reflecting a specified maximum level of hedging costs and bounds on the amounts of hedging transactions.

Since the risk factor is expressed in quadratic terms, the solution technique has a quadratic objective function and hence requires quadratic programming. The model then chooses the amounts to be hedged in every currency to produce a desirable combination of expected value and variance. The flowchart in Fig. 1 illustrates the inputs and outputs of the model.

**MODEL FORMULATION**

1. **General notation**

The following notation will be used in the formulation of the model:

$$V = \text{value of the firm at the end of the period}.$$
\[ X_i = \text{projected local currency denominated exposure in currency } i \text{ at the end of the period.} \]

\[ i = \text{subscript denoting the currencies in which the firm is conducting its business; i.e. } i = 1, 2, \ldots, N; \text{ with } i = 1 \text{ denoting the base currency of the company which is assumed to be the US dollar.} \]

In order to be able to make a distinction between long and short positions in different currency exposures, we add the following conventional notation:

\[ i = 1 \text{ for the base currency } = 2, 3 \ldots, n \text{ for currencies in which the company has a long exposure } = n + 1, n + 2 \ldots, N \text{ for currencies in which the company has a short exposure} \]

\[ s_{0,i} = \text{spot rate for currency } i; \]

\[ i = 2, \ldots, N. \]

\[ s_{1,i} = \text{future spot rate for currency } i; \]

\[ i = 2, \ldots, N. \]

\[ s_{0,i} = \text{forward exchange rate for currency } i; \]

\[ i = 2, \ldots, N. \]

\[ h_{1,i} = \text{amount of exposure in currency } i \text{ hedged in the forward market; } i = 2, \ldots, N. \]

\[ h_{2,i} = \text{amount of exposure in currency } i \text{ hedged in the Euro-currency market; } i = 2, \ldots, N. \]

\[ h_{3,i} = \text{amount of exposure in currency } i \text{ hedged in the local money market; } i = 2, \ldots, N. \]

\[ [1_{1,i}; u_{1,i}] = \text{lower and upper bounds of hedging transactions in the forward market for currency } i. \]

\[ [1_{2,i}; u_{2,i}] = \text{lower and upper bounds of hedging transactions in the Euro-currency market for currency } i. \]

\[ [0; u_{3,i}] = \text{lower and upper bounds of hedging transactions in the local money market for currency } i. \]

\[ TC = \text{transaction costs per unit amount of hedging (i.e. brokerage fees, information costs and administrative costs).} \]
\[ TC_1 = \text{unit } TC \text{ in the forward market, expressed as a percentage of the hedging volume.} \]
\[ TC_2 = \text{unit } TC \text{ in the Euro-currency market, expressed as a percentage of the hedging volume.} \]
\[ TC_3 = \text{unit } TC \text{ in the local money market, expressed as a percentage of the hedging volume.} \]

**Note:** We assume that all hedging transactions are taken at the beginning of the period. It is also assumed that there are no odd-day forward contracts and that all contracts are held to maturity.

\[ r_i = \text{Euro-currency interest rate for currency } i; \ i = 1, 2, \ldots, N \text{ with } r_1 = \text{Euro-dollar interest rate.} \]
\[ R_i = \text{local money market interest rate for currency } i; \ i = 1, 2, \ldots, N; \text{ with } R_1 = \text{US interest rate.} \]

### 2. The one-currency case

To illustrate the derivation of the model, we consider the three hedging alternatives (forward market, Euro-currency market, local money market) consecutively and this for only one foreign currency (e.g. DM). Hence, \( X \) denotes the exposure in DM, assuming a long position (\( X > 0 \)).

The end-of-period value of the firm (ignoring the fraction expressed in domestic currency, i.e. US $) is then: \( V = Xs_1 \)

**Hedging in the forward market.** The value of the foreign currency (DM) exposure at the end of the planning period as a function of the amount hedged can be represented as:

\[
V = Xs_1 + (h_1s_1 - h_1TC_1s_0)
\]
or,

\[
V = Xs_1 + h_1(s_1 - TC_1s_0)
\]

**Hedging in the Euro-currency market:** Hedging a long position in a foreign currency consists of borrowing the foreign currency, converting these funds into the base currency ($) and investing them in the money market at the going rate (i.e. T-bill rate).

\[
V = Xs_1 + [h_2s_0(1 + r_2) - h_2(1 + r_{DM})s_1 - h_2TC_2s_0]
\]
\[
= Xs_1 + h_2[s_0(1 + r_2) - (1 + r_{DM})s_1 - TC_2s_0]
\]

Similarly, hedging a short position in a foreign currency consists of converting the base currency ($), converting them into local currency and investing the proceeds in the money market. Hence,

\[
V = Xs_1 + [h_2s_0(1 + r_2) - h_2(1 + r_3)s_1 - h_2TC_3s_0]
\]
\[
= Xs_1 + h_2[s_0(1 + r_2) - s_1(1 + r_3) - TC_3s_0]
\]

Assuming the long-DM exposure \( X \) can be covered by both hedging alternatives simultaneously, then:

\[
V = Xs_1 + [h_2s_0(1 + r_2) - h_2(1 + r_3)s_1 - h_2TC_3s_0]
\]
\[
= Xs_1 + h_2[s_0(1 + r_2) - s_1(1 + r_3) - TC_3s_0]
\]

**Hedging in the local money market.** The procedure is exactly similar to hedging in the Euro-currency market, only the notation changes, i.e. \( h_1, R_1, R_{DM}, TC_3 \) instead of \( h_2, r_2, r_{DM}, TC_2 \).

The \( V \) function is:

\[
V = Xs_1 + [h_3s_0(1 + R_{DM}) - h_3(1 + R_3)s_1 - h_3TC_3s_0]
\]
\[
= Xs_1 + h_3[s_0(1 + R_{DM}) - s_1(1 + R_3) - TC_3s_0]
\]

Considering the three hedging alternatives simultaneously, it can be shown that

\[
V = Xs_1 - h_1[s_1 - s_0 + TC_1s_0]
\]
\[
- h_2[(1 + r_{DM})s_1 - (1 + R_3)s_0 + TC_2s_0]
\]
\[
- h_3[(1 + R_{DM})s_1 - (1 + R_3)s_0 + TC_3s_0]
\]

The expected value of \( V \) is the same, except that \( s \) needs to be substituted for \( s \) and the variance is

\[
W = [X - h_1 - (1 + r_{DM})h_2 - (1 + R_{DM})h_3]^2s_1(W)
\]

Under the assumption of efficient markets and no transaction costs, however,

(a) \( TC_1 = TC_2 = TC_3 = 0 \)

(b) \( s_1 = s_0 \) (the forward rate is the unbiased forecast of the future spot rate)

(c) Interest Parity holds: \( s_1 = s_0 \left( \frac{1 + r_g}{1 + r_{DM}} \right) \)

\[
\rightarrow s_1(1 + r_{DM}) = s_0(1 + r_g)
\]

and similarly

\[
s_1(1 + R_{DM}) = s_0(1 + R_3)
\]
The 'HEDGE' strategy leads to:

\[ V = X_s \]
\[ V = X_s = X_0s_1 \]
\[ W = [X - h_1 - (1 + r_{DM})h_2 - (1 + R_{DM})h_3]^2s_1(W) \]

The 'NO HEDGE' strategy implies:

\[ V = X_0s_1 \]
\[ V = X_0s_1 \]
\[ W = X^2s_1(W) \]

Comparing the 'HEDGE' with the 'NO HEDGE' policy under the assumption of efficient markets and no transaction costs, it can be deduced that the right policy is to hedge all exposure all the time. The expected value of the firm is the same, but the variance of the firm's end-of-period value is reduced under the 'HEDGE' strategy. The N-currency case can be similarly developed and, again, under efficient market conditions and zero transaction costs,

(a) \( TC_1 = TC_2 = TC_3 = 0 \)

(b) \( \hat{s}_{i,j} = \hat{s}_{i,j} \)

(c) \( \begin{cases} 
\hat{s}_{ij}(1 + r_i) = \hat{s}_{ij}(1 + r_i) \\
\hat{s}_{ij}(1 + R_i) = \hat{s}_{ij}(1 + R_i) 
\end{cases} \)

(d) \( \begin{cases} 
\hat{s}_{ij}(1 + r_i) = \hat{s}_{ij}(1 + r_i) \\
\hat{s}_{ij}(1 + R_i) = \hat{s}_{ij}(1 + R_i) 
\end{cases} \)

The 'HEDGE' strategy implies:

\[ V = \sum_{i=2}^{N} X_i s_{1,i} \]
\[ \bar{V} = \sum_{i=2}^{N} X_{i0} s_{1,i} \]
\[ W = \text{same expression as before} \]

Under the assumption of efficient markets and no transaction costs, ex-ante costs of hedging are zero, i.e. one can reduce the variance \( W \) to zero at no costs. However, ex-post hedging costs might differ from zero, but will still be relatively small since deviations occur on both sides of the Interest Parity line. In practice, transaction costs are also non-zero. Figure 2 represents the relation between the expected value of the firm and the variance of this value for both ex-ante and ex-post hedging costs under the efficient market hypothesis. The 'NO HEDGE' strategy implies:

\[ V = X_1 + \sum_{i=2}^{N} X_i s_{1,i} \]
\[ \bar{V} = X_1 + \sum_{i=2}^{N} X_{i0} s_{1,i} \]
\[ W = \sum_{i=2}^{N} \sum_{j=2}^{N} X_{i0} s_{1,i} X_{j0} s_{1,j} \text{Cov}(s_{1,i}; s_{1,j}) \]

Hence, under the efficient market hypothesis and no transaction costs, the recommended strategy is to hedge all exposed currency positions, because compared with the 'no hedge' strategy, the variance of the value of the firm at the end of the period is lower while the expected value remains unchanged.

**CONCLUSIONS**

The portfolio approach to foreign exchange exposure management makes an explicit consideration of the inherent relationships among the currencies in the company's foreign exchange portfolio. It also considers the costs of hedging per individual currency and this for three alternative methods of hedging (i.e.

![Fig. 2. (a) Ex-ante costs of hedging and zero transaction costs. (b) Ex-post costs of hedging and positive transaction costs.](image-url)
forward market, Euro-currency market, local money market).

The quadratic programming model traces out an 'efficient frontier' between the variance of the company's currency portfolio and the expected value of this portfolio at the end of the planning period. For every point on the frontier the program selects the optimal mix of hedging activities, i.e. the amount of exposure to be hedged and the method of hedging to be used. In doing so, the model chooses the optimal amount and method of hedging in each currency, minimizing the variance of the company's foreign exchange portfolio, corresponding to a selected level of total hedging costs (end-of-period expected value of the company's foreign currency portfolio) to be incurred.

ACKNOWLEDGEMENTS

The author wishes to thank Professor W White, R Glauber, and P Jones of the Harvard University Graduate School of Business Administration for their advice and assistance in the course of this study. Of course, I alone bear full responsibility for its content and shortcomings.

ADDRESS FOR CORRESPONDENCE: LA Soenen, Assistant Professor, Department of Industrial Engineering and Management Science, Eindhoven University of Technology, 5600 MB, Eindhoven, The Netherlands.