A Portfolio Model for Foreign Exchange Exposure Management

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This paper summarizes the results of our research into applications of decision analysis and portfolio theory to the management of foreign exchange exposure. In contrast with much current practice in foreign exchange management, the portfolio approach takes into explicit consideration the inherent relationships among the currencies in the company's foreign currency portfolio. The hedging model developed in this article traces out an 'efficient frontier' or trade-off curve between expected value and variance of the foreign currency portfolio at the end of the planning period. In doing so, the model chooses the optimal amount and method of hedging for each currency in the portfolio.

INTRODUCTION

Current exchange exposure management is usually executed on a currency by currency basis failing to recognize the statistical relationships among the changes in the exchange rates of the currencies maintained by the company, not to mention exploiting the reduction of exchange risk holding a diversified portfolio of currencies. Although traditional exposure analysis has been conducted for individual currencies separately, portfolio theory has taught us that individual risks should not be measured independently. Because of diversification, the risk of a portfolio composed of various currencies is not equal to the sum of the risks of each currency. Just as unsystematic risk can be reduced via diversification by the number of stocks in an investment portfolio, so exchange risk can be reduced in a diversified portfolio of currencies. Consideration of the statistical relationship among currencies allows exposure netting as a technique to reduce exchange risk. If one believes that exchange rate movements of two currencies are highly positively correlated, then a short position in one currency will largely offset a long position in the other currency. If the currencies are negatively correlated, long positions in both currencies (or short positions) will tend to balance each other out.

Traditional techniques of foreign exchange risk management (monetary balance, swaps, forward markets etc.) do not explicitly recognize the risk element. However, this kind of exposure netting (i.e. balancing of foreign currency positions on the basis of their co-movement) can be fully exploited by applying portfolio theory to the management of foreign exchange. The portfolio approach will allow the international company to determine correctly its overall exchange exposure, to build a well-diversified (minimum variance) currency portfolio, and to assess the impact of major investments on its company-wide foreign exchange exposure.

OBJECTIVE OF FOREIGN EXCHANGE MANAGEMENT

After an international company has consolidated the foreign exchange exposure of all subsidiaries and netted out assets and liabilities per currency, its net exposure will consist of long/short positions in various currencies. This set of foreign currency positions constitutes the
currency portfolio of the company. The objective of the international company's foreign exchange management is then to protect the company and its subsidiaries against exchange losses, but not to seek windfall gains through currency speculation or similar hedging activities unrelated to the nature of its business. The company's profitability should be based on the return on the goods and services it produces and sells, not on the return of its currency portfolio. Protection against currency risk is the objective, not speculative profits. The multinational company can reduce its foreign exchange risk by engaging in hedging transactions.

Hedging reduces the exposure and consequently the variance of the foreign currency portfolio, but also reduces its expected end-of-period value by the hedging costs incurred. In other words, the company should minimize the variance of its foreign currency portfolio given a maximum amount of hedging costs it is willing to incur, and not maximize the expected end-of-period return on the foreign exchange portfolio. Under efficient market conditions, only the contribution of the currency to the total risk of a portfolio should influence the value of the currency. All other risks (unsystematic risk) can be eliminated by holding a diversified portfolio of currencies. Unless the price movements of two currencies are perfectly positively correlated, any variation in the exchange rate of one that is not accompanied by an equivalent and simultaneous change in the rate of the other will lead to a reduced combined variation between both currencies. In other words, the combined variation of the two currencies will be less than the sum of the fluctuations in both exchange rates individually. The total risk of the currency portfolio is thus a function of the level of correlation among all currencies, the volatility of all currencies individually, and the proportion of each currency to the base currency equivalent sum of all currencies in the portfolio (i.e. the relative importance of each currency in the portfolio). In this portfolio-context, the objective of foreign exchange management is defined as the minimization of exchange risk to the company, i.e. to minimize the variance of the company's foreign currency portfolio subject to the costs incurred by hedging.

Hedging reduces variance caused by exchange rate changes but entails costs to the company. Therefore, one should concentrate on the relationship between the expected value and the variance of the company's foreign exchange portfolio at the end of the planning horizon. A portfolio model will now be developed to trace out the expected value-variance frontier for the company's foreign currency portfolio.

STATEMENT OF THE PROBLEM

The hedging problem can be formulated and solved as a decision problem. The data inputs required for the solution of the problem are: (a) a set of known variables (spot and forward exchange rates, domestic and foreign interest rates) and a set of variables assumed to be known (the foreign exchange exposure of the company); and (b) a set of unknown random variables (the future spot exchange rates for every currency in the portfolio at the end of the planning period). The decision variables are the amounts to be hedged in every currency in order to reduce the portfolio's variance to a selected level. These three sets of variables produce an uncertain value, \( V \), i.e. the value of the company's foreign exchange portfolio at the end of the planning horizon, with expected value \( \bar{V} \) and variance \( W \). The hedging problem is then set up as the minimization of the variance of the portfolio subject to a set of operational constraints reflecting a specified maximum level of hedging costs and bounds on the amounts of hedging transactions.

Since the risk factor is expressed in quadratic terms, the solution technique has a quadratic objective function and hence requires quadratic programming. The model then chooses the amounts to be hedged in every currency to produce a desirable combination of expected value and variance. The flowchart in Fig. 1 illustrates the inputs and outputs of the model.

MODEL FORMULATION

1. General notation

The following notation will be used in the formulation of the model:

\[ V = \text{value of the firm at the end of the period.} \]
\[ X_i = \text{projected local currency denominated exposure in currency } i \text{ at the end of the period.} \]

\( i = \text{subscript denoting the currencies in which the firm is conducting its business; i.e. } i = 1, 2, \ldots, N; \text{ with } i = 1 \text{ denoting the base currency of the company which is assumed to be the US dollar.} \)

In order to be able to make a distinction between long and short positions in different currency exposures, we add the following conventional notation:

\( i = 1 \text{ for the base currency } \]

\( = 2, 3, \ldots, n \text{ for currencies in which the company has a long exposure } \]

\( = n + 1, n + 2, \ldots, N \text{ for currencies in which the company has a short exposure} \]

\( s_{0,i} = \text{spot rate for currency } i; \]

\( i = 2, \ldots, N. \]

\( s_{1,i} = \text{future spot rate for currency } i; \]

\( i = 2, \ldots, N. \]

\( s_{0,i} = \text{forward exchange rate for currency } i; \]

\( i = 2, \ldots, N. \]

\( h_{1,i} = \text{amount of exposure in currency } i \text{ hedged in the forward market; } i = 2, \ldots, N. \]

\( h_{2,i} = \text{amount of exposure in currency } i \text{ hedged in the Eurocurrency market; } i = 2, \ldots, N. \]

\( h_{3,i} = \text{amount of exposure in currency } i \text{ hedged in the local money market; } i = 2, \ldots, N. \]

\( [1_{1,i}; u_{1,i}] = \text{lower and upper bounds of hedging transactions in the forward market for currency } i. \]

\( [1_{2,i}; u_{2,i}] = \text{lower and upper bounds of hedging transactions in the Eurocurrency market for currency } i. \]

\( [0; u_{3,i}] = \text{lower and upper bounds of hedging transactions in the local money market for currency } i. \]

\( TC = \text{transaction costs per unit amount of hedging (i.e. brokerage fees, information costs and administrative costs).} \)
\[ TC_1 = \text{unit } TC \text{ in the forward market, expressed as a percentage of the hedging volume.} \]

\[ TC_2 = \text{unit } TC \text{ in the Euro-currency market, expressed as a percentage of the hedging volume.} \]

\[ TC_3 = \text{unit } TC \text{ in the local money market, expressed as a percentage of the hedging volume.} \]

**Note:** We assume that all hedging transactions are taken at the beginning of the period. It is also assumed that there are no odd-day forward contracts and that all contracts are held to maturity.

\[ r_i = \text{Euro-currency interest rate for currency } i; \ i=1, 2, \ldots, N \text{ with } r_1 = \text{Euro-dollar interest rate.} \]

\[ R_i = \text{local money market interest rate for currency } i; \ i=1, 2, \ldots, N; \text{ with } R_1 = \text{US interest rate.} \]

### 2. The one-currency case

To illustrate the derivation of the model, we consider the three hedging alternatives (forward market, Euro-currency market, local money market) consecutively and this for only one foreign currency (e.g. DM). Hence, \( X \) denotes the exposure in DM, assuming a long position (\( X > 0 \)).

The end-of-period value of the firm (ignoring the fraction expressed in domestic currency, i.e. US $) is then: \( V = XS_1 \)

**Hedging in the forward market.** The value of the foreign currency (DM) exposure at the end of the planning period as a function of the amount hedged can be represented as:

\[ V = XS_1 + (h_1 o s_1 - h_1 s_1 - h_1 TC_1 s_0) \]

or,

\[ V = XS_1 + h_1 (o s_1 - s_1 - TC_1 s_0) \]

**Hedging in the Euro-currency market:** Hedging a long position in a foreign currency consists of borrowing the foreign currency, converting these funds into the base currency ($) and investing them in the money market at the going rate (i.e. T-bill rate).

\[ V = XS_1 + [h_2 s_0(1 + rs) - h_2(1 + r_{DM})s_1 - h_2 TC_2 s_0] \]

\[ = XS_1 + h_2 [s_0(1 + rs) - (1 + r_{DM})s_1 - TC_2 s_0] \]

Similarly, hedging a short position in a foreign currency consists of converting the proceeds in the money market and investing them into the local currency.

\[ V = XS_1 + [h_2 s_0(1 + r_{DM}) - h_2(1 + r_s)s_1 - h_2 TC_2 s_0] \]

\[ = XS_1 + h_2 [s_0(1 + r_{DM}) - s_1(1 + r_s) - TC_2 s_0] \]

Assuming the long-DM exposure \( X \) can be covered by both hedging alternatives simultaneously, then:

\[ V = [X - h_1 - (1 + r_{DM})h_2]s_1 \]

Assuming the long-DM exposure \( X \) can be covered, it is shown that

\[ V = [X - h_1 - (1 + r_{DM})h_2]s_1 \]

\[ = XS_1 + h_2 [s_0(1 + r_{DM}) - s_1(1 + r_s) - TC_2 s_0] \]

### Under the assumption of efficient markets and no transaction costs, however, the equation becomes:

\[ V = XS_1 + (h_1 o s_1 - h_1 s_1 - h_1 TC_1 s_0) \]

\[ = XS_1 + h_1 (o s_1 - s_1 - TC_1 s_0) \]

\[ V = X s_1 + (h_2 s_0(1 + rs) - h_2(1 + r_{DM})s_1 - h_2 TC_2 s_0) \]

\[ = XS_1 + h_2 [s_0(1 + rs) - (1 + r_{DM})s_1 - TC_2 s_0] \]

\[ = XS_1 + h_2 [s_0(1 + r_{DM}) - s_1(1 + r_s) - TC_2 s_0] \]

\[ Assumptions: \]

- \( TC_1 = 0 \)
- \( h_1 = 0 \) (the forward rate is the unbiased forecast of the future spot rate)
- Interest Parity holds: \( s_1 = s_0 \left( \frac{1 + rs}{1 + r_{DM}} \right) \)
- \( s_1(1 + r_{DM}) = s_0(1 + rs) \)

\[ \text{and similarly} \]

\[ s_1(1 + r_{DM}) = s_0(1 + R_s) \]
The 'HEDGE' strategy leads to:
\[ V = Xs_1 \]
\[ V = Xs_1 = Xos_1 \]
\[ W = [X - h_1 - (1 + r_{DA})h_2 - (1 + R_{MA})h_3]^2s_1(W) \]

The 'NO HEDGE' strategy implies:
\[ V = Xs_1 \]
\[ V = Xos_1 \]
\[ W = X^2s_1(W) \]

Comparing the 'HEDGE' with the 'NO HEDGE' policy under the assumption of efficient markets and no transaction costs; it can be deduced that the right policy is to hedge all exposure all the time. The expected value of the firm is the same, but the variance of the firm's end-of-period value is reduced under the 'HEDGE' strategy. The N-currency case can be similarly developed and, again, under efficient market conditions and zero transaction costs,

(a) \( TC_1 = TC_2 = TC_3 = 0 \)

(b) \( \hat{s}_{i,j} = os_{i,j} \)

(c) \( \begin{cases} s_{0,i}(1 + r_j) = \hat{s}_{i,j}(1 + r_j) \\ s_{0,j}(1 + R_i) = \hat{s}_{i,j}(1 + R_i) \end{cases} \)

(d) \( \begin{cases} s_{0,i}(1 + r_j) = \hat{s}_{i,j}(1 + r_j) \\ s_{0,j}(1 + R_i) = \hat{s}_{i,j}(1 + R_i) \end{cases} \)

The 'HEDGE' strategy implies:
\[ V = X_1 + \sum_{i=2}^{N} X_is_{1,i} \]
\[ \bar{V} = X_1 + \sum_{i=2}^{N} X_os_{1,i} \]
\[ W = \text{same expression as before} \]

Under the assumption of efficient markets and no transaction costs, ex-ante costs of hedging are zero, i.e. one can reduce the variance \( W \) to zero at no costs. However, ex-post hedging costs might differ from zero, but will still be relatively small since deviations occur on both sides of the Interest Parity line. In practice, transaction costs are also non-zero. Figure 2 represents the relation between the expected value of the firm and the variance of this value for both ex-ante and ex-post hedging costs under the efficient market hypothesis. The 'NO HEDGE' strategy implies:
\[ V = X_1 + \sum_{i=2}^{N} X_is_{1,i} \]
\[ \bar{V} = X_1 + \sum_{i=2}^{N} X_os_{1,i} \]
\[ W = \sum_{i=2}^{N} \sum_{j=2}^{N} X_is_{1,i}X_js_{1,j} \text{Cov}(s_{1,i}; s_{1,j}). \]

Hence, under the efficient market hypothesis and no transaction costs, the recommended strategy is to hedge all exposed currency positions, because compared with the 'no hedge' strategy, the variance of the value of the firm at the end of the period is lower while the expected value remains unchanged.

CONCLUSIONS

The portfolio approach to foreign exchange exposure management makes an explicit consideration of the inherent relationships among the currencies in the company's foreign exchange portfolio. It also considers the costs of hedging per individual currency and this for three alternative methods of hedging (i.e.

![Fig. 2](image-url)
forward market, Euro-currency market, local money market).

The quadratic programming model traces out an 'efficient frontier' between the variance of the company’s currency portfolio and the expected value of this portfolio at the end of the planning period. For every point on the frontier the program selects the optimal mix of hedging activities, i.e. the amount of exposure to be hedged and the method of hedging to be used. In doing so, the model chooses the optimal amount and method of hedging in each currency, minimizing the variance of the company’s foreign exchange portfolio, corresponding to a selected level of total hedging costs (end-of-period expected value of the company’s foreign currency portfolio) to be incurred.

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