The quantitative effect of tool geometry and strain-hardening on the critical punch force in cup drawing

Citation for published version (APA):

Document status and date:
Published: 01/01/1970

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.

Download date: 02. Nov. 2019
THE QUANTITATIVE EFFECT OF TOOL GEOMETRY AND STRAIN-HARDENING
ON THE CRITICAL PUNCH FORCE IN CUP DRAWING

by

J.A.G. Kals

Eindhoven University of Technology

Presented to the
XX General Assembly of C.I.R.P.
Torino
1970
SUMMARY

A relation between tensile curves and critical punch force in deep drawing of cylindrical products is developed. Both the work hardening effect and the geometry of the drawing punch are taken into account. A reasonable correspondence between the analytical results and the experimental data can be established. Finally, the practical significance of the mathematical model is shown by giving a criterion for the minimum corner radius of the punch. Moreover, the usefulness of the model is confirmed on the basis of some observations on deep drawability and geometric similarity in formability tests.

1. INTRODUCTION

Deep drawability can be radically influenced by many factors, which may constitute the difference between successful production of a stamping and breakage during pressworking operations. Many individual drawing steps may be required to produce a stamping. In order to reduce the number of drawing operations, the drawing ratio, defined between the blank diameter and the average cup diameter, has to be chosen as high as possible. The limit of deformation is reached when the load, required to deform the flange, becomes greater than the load-carrying capacity of the cup wall.

The required punch load depends upon a large number of drawing conditions, such as forming properties of the sheet material, sheet thickness, drawing ratio, blank diameter, die profile radius, hold-down pressure and friction conditions. On the other hand the critical punch load is influenced by the punch profile radius, the punch diameter and by lubrication, sheet thickness and material properties as well. Changes of lubricant and material characteristics caused by speed fluctuations are other factors that may influence formability.

The actual value of the limiting drawing ratio is fixed by all these coinciding forming conditions.
In this paper, a theory is described which enables a calculation of the critical punch load and of a favourable dimension of the corner radius of the punch. In order to limit the complexity of the mathematical problem to a practical minimum, a number of validity restrictions has to be made with respect to the following theory:

- it is assumed that deformation speed effects can be neglected,
- the working sheet materials are homogeneous, plastic-rigid and isotropic,
- friction effects can be neglected,
- comparatively thin sheet material is only considered, so that bending effects have not to be taken into account,
- a relatively small punch edge radius in relation to the punch diameter.

The direct practical significance of this theory may be based on the fact that special literature of objective information concerning the selection of a useful punch profile radius in relation to formability limits is lacking.

2. THE CURRENT STRESS AND STRAIN STATE IN THE CRITICAL CROSS-SECTION

In radial drawing of the flange region the material is being upset in tangential direction. This results in an increasing sheet thickness and a hardening of the material. These effects are stronger according as a volume element is moved further into the direction of the die cavity. So, the increase in sheet thickness is restricted to the outer flange areas. On the contrary and especially under critical drawing conditions, the inner flange area is stretched very strongly during the initial increase of the punch force. Particularly, this holds for the material originally over the die wall. Therefore, the failure will be located exclusively in the stretched area near the bottom of the cup wall.

The exact location of the failure, caused by exceeding the stability limit in stretching, depends on the material and the forming conditions, particularly on friction.
Preliminarily to the analysis of the stretching limit, the failure location is assumed to be exactly on the border-line between the cup wall and the rounded edge of the punch. Lacking friction conditions and a relatively large edge radius excepted, the foregoing will be a fair approximation of reality (Fig. 1.). A laborious procedure can be avoided by representing the rounded cup area as a part of a torus. According to the simplifying assumptions, failure takes place in a symmetry plane of the torus (Fig. 2.)

Fig. 1. Failure occurs usually in the rounded edge, close to the cylindrical wall area.

Let $\sigma_\phi$ and $\sigma_t$ be the average axial and circumferential stress components in the critical cross-section and $p$ the local normal pressure between the punch and the cup wall. The equation of equilibrium is
Fig. 2. Schematic stress state in the critical cross-section of the cup wall.

\[ \frac{P}{s} = \frac{\sigma_{ph}}{\rho_{st}} (1 + \frac{s}{2r_{st}}) + \frac{\sigma_{ts}}{r_{st}} (1 + \frac{s}{2\rho_{st}}) \]  

(1)

where

- \( s \) = the momentary cup wall thickness
- \( r_{st} \) = the punch radius
- \( \rho_{st} \) = the punch profile radius.

An immediate simplification of equation (1) can be achieved by using
the restriction $s \ll \rho_{st}$. In this case, equation (1) reduces to

$$\frac{P}{s} = \frac{\sigma_\phi}{\rho_{st}} + \frac{\sigma_t}{r_{st}}$$  \hspace{1cm} (2)

In the first instance the normal stress component $\sigma_n$ depends on the inner wall pressure $p$. Thus far ($0 \leq i \leq 1$):

$$\sigma_n = -ip = -is \left(\frac{\sigma_\phi}{\rho_{st}} + \frac{\sigma_t}{r_{st}}\right)$$  \hspace{1cm} (3)

The axial stretching of the cup wall during the initial increase of the punch load is compensated by a reduction in wall thickness exclusively, as the punch effectively precludes straining in the circumferential direction. The decrease of the average cup radius $r_{ss}$ (Fig. 2.) by the reduction in thickness may be neglected in connection with $s \ll r_{st}$. As a consequence

$$d\delta_t = 0$$  \hspace{1cm} (4)

Let $d\delta_t$, $d\delta_\phi$ and $d\delta_n$ be the principal components of an increment of strain. Since there is no change of volume the following relation exists

$$d\delta_t + d\delta_\phi + d\delta_n = 0$$  \hspace{1cm} (5)

Hence

$$d\delta_\phi = -d\delta_n$$  \hspace{1cm} (6)

The Lévy-von Mises equations, as they are known, may be expressed for the normal and the axial direction respectively

$$d\delta_n = d\lambda \left(\sigma_n - \frac{\sigma_t + \sigma_\phi}{2}\right)$$

$$d\delta_\phi = d\lambda \left(\sigma_\phi - \frac{\sigma_n + \sigma_t}{2}\right)$$  \hspace{1cm} (7)

where $d\lambda$ is a scalar factor of proportionality. If this is combined with the straight strain-path as expressed in equation (6), we obtain the following necessary condition for the stress state
\[
\sigma_n = 2\sigma_t - \sigma_\phi \tag{8}
\]

Now, the average normal stress \(\sigma_n\) can be eliminated from equation (3). Thus

\[
\sigma_t = j\sigma_\phi \tag{9}
\]

where

\[
j = \frac{r_{st} \rho_{st} - is}{\rho_{st}^2 r_{st} + is} \tag{10}
\]

Finally, the equations (8) and (9) may be combined to

\[
\sigma_n = (2j-1) \sigma_\phi \tag{11}
\]

It seems fair to regard the equations (9) and (10) as a reasonably good first approximation of the complete current stress state in the critical cross-section.

For applications, requiring a high accuracy, it will eventually be necessary to exclude the simplifications from the theoretical framework. At present, however, a practical approximation is wanted.

So, for the time being an additional mathematical complexity does not seem to be worthwhile.

3. THE CURRENT LOAD OF THE CUP WALL

Von Mises suggested that yielding occurs when the second stress tensor invariant reaches a critical value \(\sigma\). In connection with our problem this criterion may be written in terms of the principal components of the stress state. Thus

\[
2\bar{\sigma}^2 = (\sigma_t - \sigma_\phi)^2 + (\sigma_\phi - \sigma_n)^2 + (\sigma_n - \sigma_t)^2 \tag{12}
\]

where \(\bar{\sigma}\), the so-called effective stress, is a parameter depending on the amount of strain. For the concept of a yield criterion is not restricted merely to loading directly from the annealed state, as is sometimes thought. In combination with equation (12) we have from (9)
In order to include the strain hardening effect in the theoretical model, $\bar{\sigma}$ has to be related to a certain measure of the total plastic deformation. A quantity $d\bar{\delta}$, known as the generalized or effective plastic strain increment, is defined in terms of the principal strain increments by the equation

$$d\bar{\delta} = \sqrt{\frac{2}{3} (d\delta_1^2 + d\delta_2^2 + d\delta_3^2)}$$

Apart, from the numerical factor, $d\bar{\delta}$ is the same invariant function of the plastic strain increment tensor as $\bar{\sigma}$ is of the components of the deviatoric stress tensor. The use of the previous equations (4) and (6) and integration of (14) result in

$$\bar{\delta} = \int d\bar{\delta} = \frac{2d\phi}{\sqrt{3}}$$

This integration is the simplest and most natural way to satisfy the obvious requirement that the measure of total distortion must involve the summation of some continually positive quantity over the whole strain path. In this case integration is very simple, because the components of any strain increment bear constant ratios to one another. Besides it is worth noting that this strain model has the additional advantage that the general requirement of minimum dissipation of specific strain energy is satisfied automatically.

Turning now to the strain hardening relation between $\bar{\sigma}$ and $\bar{\delta}$, it is assumed that the following generalized form of an early empirical power law, due to Nadai, fits well to many sheet materials:

$$\bar{\sigma} = C (\bar{\delta} + \bar{\delta}_0)^n$$

where $C$ (characteristic stress) and $n$ (strain hardening exponent) are
material constants. The quantity $\overline{\delta}_o$ may be considered to include the strain history. Extending Nadai's equation with $\overline{\delta}_o$, results in $C$ and $n$ being independent of strain history essentially. According to the results taken from many tensile tests on different sheet materials, the introduction of $\overline{\delta}_o$ has the additional advantage of a considerably higher accuracy in approximating real stress-strain curves of materials with a (unknown) strain history. Typical examples are given in Figs. 3. and 4.

![Graph showing stress-strain curves](image)

**Fig. 3.** The usual form of Nadai's equation in comparison with the generalized one and the results of tensile tests.

With the use of the equations (13) and (15) the actual form of (16) becomes

$$\sigma_\phi = \frac{C}{(1-j)\sqrt{3}} \left(\frac{2}{\sqrt{3}} \overline{\delta}_\phi + \overline{\delta}_o\right)^n$$

(17)
Fig. 4. The usual form of Nadai's equation in comparison with the generalized one and the results of tensile tests.

Substitution of $\sigma_\phi$ in the general expression for the cup wall load (Fig. 2.),

$$ F = 2\pi s (r_{st} + \frac{s}{2}) \sigma_\phi = 2\pi s r_s s \sigma_\phi $$

results in

$$ F = \frac{2\pi}{\sqrt{3}} \cdot \frac{C s r_{ss}}{1-j} \left( \frac{2}{\sqrt{3}} \delta_\phi + \delta_o \right)^n $$

According to the general definition of a logarithmic strain we can write

$$ s = s_o e^n $$
where \( s_0 \) is the initial sheet thickness. Combining equations (6) and (20), we find

\[
s = s_0 e^{-\phi}
\]

(21)

The wanted relation between the load \( F \) and the axial strain \( \delta_\phi \) is obtained by substituting this formula in equation (19).

\[
F = \frac{2\pi}{\sqrt{3}} \cdot \frac{C r_{ss} s_0}{1 - \frac{1}{J}} e^{-\phi} (\frac{2}{\sqrt{3}} \delta_\phi + \delta_0)^n
\]

(22)

Finally, it is to be remarked that the present expression for the axial load on the critical cross-section of the cup wall is applicable for calculating the punch force too, with the limitation that friction forces can be neglected. This simplification has previously been assumed.

4. THE CRITICAL PUNCH LOAD

The elongation of the partially formed cup wall is accompanied by a reduction in thickness, i.e. a decrease in the cross-sectional area \( A \), and thereby strengthened by strain hardening. Initially the strain hardening effect is dominating in view of the stretching force.

\[
\frac{dF}{d\delta_\phi} = \frac{dA}{d\delta_\phi} (\sigma_\phi A) = \sigma_\phi \frac{dA}{d\delta_\phi} + A \frac{d\sigma_\phi}{d\delta_\phi} > 0
\]

(23)

Therefore the cup wall can now support the larger deep drawing load, so flange forming can continue. With only a few exceptions the strain hardening effect \( d\sigma_\phi / d\delta_\phi \) decreases with increasing strain level (Figs. 3. and 4.). The deep drawing process going on, an ultimate strength of the cup wall will be reached when both the strain hardening and the stretching term in equation (23) cancel each other:

\[
\frac{dF}{d\delta_\phi} = 0
\]

(24)

When the chosen drawing ratio implies a further increase of the drawing
force being necessary for continuous deformation of the flange region, this load cannot be transmitted through the lower cup wall any more. Finally, the load carrying capacity of this structurally weak link of the system appears to be decreasing with the punch going on continuously. Now, the stamping starts releasing elastically with the exception of the lower region of the cup wall. This plastic region is shrinking into a circumferential constriction.

If the stability limit is once exceeded, plastic straining continues only in the necked part of the cup wall and consequently no further straining will take place in the remaining part. Thus, equation (24) is the limiting condition of forming and in general it seriously reduces the achievable amount of overall deformation in those processes where stretching occurs. It is therefore the deep drawability limit.

For our purposes it may be sufficient to consider \( r_{ss} \) and \( j \) being constant during differentiating equation (22). Otherwise no explicite solution for the critical amount \( \delta_{pk} \) of the axial component of strain can be obtained. Then introducing the criterion of necking by differentiating (22) and setting to zero, we may write

\[
\delta_{pk} = n - \frac{\sqrt{\delta}}{2} \delta_0 \tag{25}
\]

as a good approximation. The material with the higher \( n \)-value is characterized by a steeper stress-strain curve (Figs. 3 and 4). The critical strain value at maximum punch load is larger for higher \( n \)-values. Generally the \( n \)-value primarily influences stretchability. The most important effect of a high \( n \)-value is to improve the uniformity of the strain distribution in the presence of a stress gradient, and necking happens to be a strong non-uniformity of the strain distribution.

According to equation (25) and to practical experience pre-straining diminishes formability.

Inserting this strain ceiling in combination with equations (10) and (21) in the expression of the cup wall load, (22), we obtain
The last term in the numerator may be neglected according to the previous assumption of relatively thin sheet materials. Furthermore, this equation may be simplified, by the introduction of dimensionless quantities to

$$F_k = \frac{2\pi}{\sqrt{3}} C r_{st} s_o \rho_{st} (2n)_{ev3} n \frac{2 r_{st} e^{n + i s_o \exp (\frac{\sqrt{3}}{2} \frac{\delta}{o})}}{\rho_{st} r_{st} \exp (n - \frac{\sqrt{3}}{2} \frac{\delta}{o}) + i s_o (r_{st} + \rho_{st})}$$

(26)

and

$$F_k^{*} = \frac{4\pi}{\sqrt{3}} (2n)_{ev3} n \frac{1}{\rho_{st} r_{st}} \exp (n - \frac{\sqrt{3}}{2} \frac{\delta}{o})$$

(27)

where

$$F_k^{*} = \frac{F_k}{C r_{ss} s_o} ; \rho_{st}^{*} = \frac{\rho_{st}}{s_o} ; r_{st}^{*} = \frac{r_{st}}{s_o}$$

(28)

and where $r_{ss}$ is the average local cup radius at maximum wall load (equations (20) and (25)):

$$r_{ss} = r_{st} + \frac{s}{2} = r_{st} + \frac{s_o}{2} \exp (\frac{\sqrt{3}}{2} \frac{\delta}{o} - n)$$

(29)

A problem still to be solved concerns the numerical value of the stress parameter $i$ (equation (3)). The normal stress distribution may be approximately linear. So the value of $i$ that we are looking for seems to be 0.5. Nevertheless it is better to choose the maximum value $i = 1$, for it is evident that instability must be initiated at the punch side of the cup wall according to the assumed uniformly distributed axial and tangential stresses. If a constant value $i = 1$ is combined with equation (27) the following expression is finally obtained

$$F_k^{*} = \frac{4\pi}{\sqrt{3}} (2n)_{ev3} n \left[ \frac{1}{\rho_{st}^{*}} + \frac{1}{r_{st}^{*}} + \exp (n - \frac{\sqrt{3}}{2} \frac{\delta}{o}) \right]^{-1}$$

(30)

A representation of this relation is given in Fig. 5.
Fig. 5. Theoretical curves according to equation (30) for $\delta_0 = 0$; 
$r_{st} = 38.6\, mm; \ s_0 = 2\, mm.$

5. THEORETICAL RESULTS

Of course, the present solution is only a simplification of a more complex process, but this first step may shed some light on the mechanism of failure in deep drawing. Equation (30), as shown in Fig. 5., permits some interesting conclusions:

- Obviously, the load carrying capacity of the cup wall is vanishing very rapidly with decreasing edge radius below a definable limit of $\rho_{st}^*$. Practically, this effect implies the punch cutting into the cup wall.

According to OEHLER and KAISER [1] the minimum value of the edge radius
has preferably to be chosen equal to five times the initial sheet thickness. A value $\rho_{st}^* = 15 - 25$ is judged being still more recommendable. These empirical data are supporting our foregoing theory clearly. Nevertheless, experimental investigations are necessary in order to compare theoretical results with reality more systematically.

- Strain hardening is only slightly effecting a change of the critical $\rho_{st}^*$-value.

- The effects of $\rho_{st}^*$ and $r_{st}^*$ on the critical punch force are quite identical. Considering this fact may be useful in detecting failures of small stampings.

- A noteworthy phenomenon being observed is that of the critical punch force being smaller for larger $n$-values, due to larger stretchability until instability occurs. The corresponding curves appear to pass through a minimum value at about $n = 0.8$. It can be shown (vide 7.2) that the maximum punch load being necessary to deform the flange region also decreases with increasing $n$-values. The corresponding curves $F_{\text{max}}(n)$ appear to decline steeper than $F_k(n)$. So, ultimately, the limiting drawing ratio shows a slightly progressive increase with increasing $n$-values.

- The opposite influence of the "strain-history" parameter $\delta_o$ (equation (25)) is shown in Fig. 6.

Finally a restriction has to be made with regard to the practical validity of equation (30). At very low values of the punch edge radius in relation to sheet thickness, i.e., where the edge is cutting into the wall, the validity of the presupposed deformation model may become doubtful. So Fig. 5. has to be understood merely as a representation of the mathematical relation in this region.

According to the previous assumption of relatively small values of the edge radius, the validity of the theoretical equation has to be restricted in this respect too. It has been observed, that the instability region is moving towards the punch center at increasing edge radius.
6. EXPERIMENTAL RESULTS

In order to obtain the discussed material data, tensile tests were carried out intermittently at a mechanical tensile test machine. So the local plastic strains could be measured separately by measuring the cross-sectional area of the test specimen after discharging the material every now and then.

The material constants have been computed according to the least squares criterion. A number of ten sheet materials \((s_0 = 2 \text{ mm})\) has been selected on the base of small earing in deep drawing. Nevertheless this planair anisotropy effect is increasing slightly in the direction of increasing test numbers (Table 1.). Tensile tests were carried out
at 0° as well as 45° to the rolling direction. The results are given in Table 1.

<table>
<thead>
<tr>
<th>Nr</th>
<th>sheet material</th>
<th>in rolling direction</th>
<th>45 degrees to rolling direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma_0$ [N/mm²]</td>
<td>C [-]</td>
</tr>
<tr>
<td>1</td>
<td>72/28 brass</td>
<td>1.97</td>
<td>791</td>
</tr>
<tr>
<td>2</td>
<td>stainless steel</td>
<td>2.09</td>
<td>1418</td>
</tr>
<tr>
<td>3</td>
<td>stainless steel</td>
<td>2.01</td>
<td>1512</td>
</tr>
<tr>
<td>4</td>
<td>63/37 brass</td>
<td>1.96</td>
<td>719</td>
</tr>
<tr>
<td>5</td>
<td>63/37 brass</td>
<td>1.93</td>
<td>697</td>
</tr>
<tr>
<td>6</td>
<td>alum (Si)</td>
<td>1.90</td>
<td>437</td>
</tr>
<tr>
<td>7</td>
<td>alum (99.5%)</td>
<td>1.96</td>
<td>140</td>
</tr>
<tr>
<td>8</td>
<td>nickel</td>
<td>2.06</td>
<td>1166</td>
</tr>
<tr>
<td>9</td>
<td>copper</td>
<td>1.95</td>
<td>408</td>
</tr>
<tr>
<td>10</td>
<td>steel (Cu)</td>
<td>1.98</td>
<td>895</td>
</tr>
</tbody>
</table>

Table 1. Results of tensile tests and deep drawing tests (sheet materials as received).

The best fitting stress-strain curves on the base of the original Nadai equation (without strain history parameter) can be reconstructed with the values in Table 2.

The deep drawing tests were carried out at a hydraulic press with low punch velocities and a rather arbitrary chosen tool geometry: $r_{st} = 38.6$ mm and $p_{st} = 12.0$ mm. It is a well-known fact that the load carrying capacity
of the cup wall decreases slightly according as the drawing ratio is exceeding further the limiting value.

This is due to introducing local instability before forming of the bottom rounding has been completed. In this case necking occurs nearer to the flat bottom and additionally the critical cross-section is not perpendicular to the moving direction of the punch. Therefore, the critical drawing load has to be measured exactly at the limiting drawing ratio. In order to obtain these values of $F_k$, both the maximum drawing force $F_{\text{max}}$ and the critical punch load $F_k$ have been measured as a function of the drawing ratio. The wanted value of $F_k$ can be taken at the intersection of both of these curves. The results are given in

<table>
<thead>
<tr>
<th>Nr</th>
<th>material</th>
<th>in rolling direction</th>
<th>45 degrees to rolling direction</th>
<th>plastic anisotropy parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$C , [N, \text{mm}^2]$</td>
<td>$n$</td>
<td>$C , [N, \text{mm}^2]$</td>
</tr>
<tr>
<td>1</td>
<td>72/28 brass</td>
<td>754</td>
<td>0.45</td>
<td>724</td>
</tr>
<tr>
<td>2</td>
<td>stainless steel</td>
<td>1230</td>
<td>0.31</td>
<td>1219</td>
</tr>
<tr>
<td>3</td>
<td>stainless steel</td>
<td>1346</td>
<td>0.37</td>
<td>1387</td>
</tr>
<tr>
<td>4</td>
<td>63/37 brass</td>
<td>553</td>
<td>0.13</td>
<td>583</td>
</tr>
<tr>
<td>5</td>
<td>63/37 brass</td>
<td>566</td>
<td>0.18</td>
<td>618</td>
</tr>
<tr>
<td>6</td>
<td>alum (Si)</td>
<td>410</td>
<td>0.22</td>
<td>408</td>
</tr>
<tr>
<td>7</td>
<td>alum (99.5%)</td>
<td>137</td>
<td>0.30</td>
<td>133</td>
</tr>
<tr>
<td>8</td>
<td>nickel</td>
<td>1132</td>
<td>0.43</td>
<td>1069</td>
</tr>
<tr>
<td>9</td>
<td>copper</td>
<td>335</td>
<td>0.06</td>
<td>339</td>
</tr>
<tr>
<td>10</td>
<td>steel (Cu)</td>
<td>778</td>
<td>0.17</td>
<td>714</td>
</tr>
</tbody>
</table>

Table 2. Experimental results according to the engineering form of the Nadai equation and measured values of the plastic anisotropy parameter.
the last column of Table 1. Fig. 7. shows a satisfying correspondence between the calculated values $F_k$ and the experimental values $F_{kw}$ of the critical punch force.

Fig. 7. Theoretical versus experimental values of the critical punch load.

According to equation (28) the characteristic stress $C$ is taking up a rather dominant position with relation to the absolute value of the critical punch load. By eliminating this quantity the effect of strain hardening can be made clear. Therefore in Fig. 8. the theoretical and experimental values of the dimensionless critical load number are compared.

A stronger scattering can be observed in this representation. Nevertheless the theoretical effect of strain hardening may be considered being verified as well. It is probable that the divergence may be partly attributable to plastic anisotropy, especially in the case of the plotted points for the materials 8, 9, and 10 (see Table 2).
In order to compare experimental and theoretical results (Fig. 6.) with regard to the hardening effect on the critical load number as well, equation (30) has been evaluated according to the standard Nadai equation ($\tilde{\delta}_0 = 0$), with the aid of the values from Table 2. Fig. 9. shows the results.

Every deep drawing experiment so far mentioned has been carried out with a constant punch geometry. In order to verify the theoretical effect of the punch edge radius (Fig. 5.) separately, a series of experiments, therefore, had to be carried out additionally.

The experimental results and the corresponding theoretical curves according to equations (27) or (30), are shown in Fig. 10.
Equations (27) and (28) have been evaluated with the following data from tensile tests.

rolling direction: \( C = 798 \, \text{N/mm}^2 \)
\( n = 0.54 \)
\( \bar{\delta}_0 = 0.06 \)

45° to rolling direction: \( C = 760 \, \text{N/mm}^2 \)
\( n = 0.57 \)
\( \bar{\delta}_0 = 0.08 \)
Fig. 10. Experimental and theoretical relationship between the critical load number and the punch edge radius for a relatively thin sheet material.

From Fig. 10. it is found again that equation (30) is a satisfying approximation of reality.

These experiments have been repeated for the larger relative sheet thickness \( s_o/r_{st} \) as practized in the former series of experiments. The results are given in Fig. 11.

From this graph in comparison with Fig. 10. it appears that the validity restriction to comparatively thin sheet materials (See 1.) may not be overlooked.

Additionally it is worth noting that the divergence of the plotted points in both of the figures equals approximately the initial sheet thickness.

Even though some other variables exercise control over the deep drawing process to some extent, equation (30) seems to be giving a true picture of the main conditions effecting the load carrying capacity of the cup wall. Of course this study was only a first attempt to analyse the deep
drawing process and greater accuracy could probably be achieved with the aid of numerical calculation procedures. Many useful purposes, however, appear not to be served by the application of rigor in an analysis for the sake of exactness.

![Diagram](image)

**Fig. 11.** Experimental and theoretical relation of the critical load number and the punch edge radius for a larger relative sheet thickness.

**7. APPLICATIONS**

Finally, some significant engineering aspects of the foregoing theoretical failure model will be elucidated briefly. In trying out stamping tools, it is often necessary to change to a more formable material, to modify the die design and even to change the stamping design in order to form a new product successfully. This takes time and money, and illustrates the need for a better understanding of sheet metal formability and for objective formability testing methods. Of course, formability alone is not the sole criterion which has to be taken into consideration when sheet metal, tool geometry and production conditions have to be selected, but it is an inevitable one.
7.1. Punch geometry and formability

It is convenient to introduce a parameter

\[ \eta = \frac{F^*_k}{(F^*_k)^{\max}} \]  

(31)

defining a practically useful value of \( F^*_k \) in proportion to an imaginary maximum value

\[ (F^*_k)^{\max} = \frac{4\pi}{\sqrt{3}} \left( \frac{2n}{\sqrt{3}} \right)^n \left[ \frac{1}{r^*_{st}} + \exp \left( n - \frac{\sqrt{3}}{2} \delta_o \right) \right]^{-1} \]  

(32)

which results from equation (30) for \( \rho^*_{st} \rightarrow \infty \). Substitution of (30) and (32) in (31) results in

\[ \rho^*_{st} = \frac{\eta}{1-\eta} \left[ \frac{1}{r^*_{st}} + \exp \left( n - \frac{\sqrt{3}}{2} \delta_o \right) \right]^{-1} \]  

(33)

![Graph showing experimental values of the necessary drawing force as a function of the relative die edge radius.](image)

Fig. 12. Experimental values of the necessary drawing force as a function of the relative die edge radius.
This expression enables evaluating a favourable punch edge rounding as a function of the initial sheet thickness, the strain hardening exponent, the punch diameter and the chosen $\eta$ - value. In the case represented in Fig. 10., for example, the following values are obtained from equation (33):

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\rho_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>5</td>
</tr>
<tr>
<td>0.86</td>
<td>10</td>
</tr>
<tr>
<td>0.90</td>
<td>15</td>
</tr>
</tbody>
</table>

Another more complex criterion might be defined in terms of a steepness limit:

$$\frac{\partial F^*_k}{\partial \rho^*_{st}} \leq q$$

(34)

In general, the admissible slope tangent $q$ has to be selected in dependence of the maximum drawing force in proportion to the critical punch load. Though this criterion would be a better one it is not going to be developed here. At present the experimental data appear to be to slight to make the additional mathematical complexity worthwhile.

As indicated in the introduction, the present study is a part of a study that is directed to a theoretical analysis of some factors influencing deep drawability. In deep drawing the overall deformation limit - Limiting Drawing Ratio $S_o$ - can be defined as the ratio of the maximum blank diameter that can be drawn into a cup without failure to the average diameter of the cup wall. This limit of deformation is reached when the load $F_{\text{max}}$, required to deform the flange, becomes equal to the load carrying capacity $F_k$ of the cup wall. A noteworthy aspect of taking $F_{\text{max}}$ into account is that the die edge radius $\rho_{\text{ZR}}$ has an effect on it that is opposite to the effect of the punch edge radius on the critical punch load. Experimental values illustrating this are shown in Fig. 12. Several experimental curves are shown in Fig. 13. for different drawing ratios $S_o$. The corresponding measured $F_k$ values are plotted additionally. In the particular case that equal values $\rho_{st}$ and $\rho_{\text{ZR}}$ are selected - as often happens to be done in praxis - the limiting drawing ratios are fixed in
dependence of the tool geometry by the intersections of the $F_{\text{max}}$-curves and the $F_k$-curves. Experimental and theoretical research in this field is going on in order to find a useful expression for $F_{\text{max}}$ and finally for the limiting drawing ratio as a function of tool geometry and strain hardening behaviour of sheet metals.

Finally, looking at Fig. 13., the observation can be made that the limiting drawing ratio has a practical maximum with respect to optimization of tool geometry.

![Graph showing the required drawing force $F_{\text{max}}$ as a function of the relative die edge radius $p_{zT}/s_0$ and the critical punch load $F_k$ as a function of the relative punch edge radius $p_{st}/s_0$.]

Fig. 13. Experimental curves representing the required drawing force $F_{\text{max}}$ as a function of the relative die edge radius $p_{zT}$ for different values of the drawing ratio $\beta_0$ and the critical punch load $F_k$ at a function of the relative punch edge radius $p_{st}$. 
7.2. Strain hardening and formability

It was pointed out already (Chapter 5) that the required drawing force $F_{\text{max}}$ decreases slightly stronger than its critical value with increasing n-value. This results in larger values of the limiting drawing ratio according as the strain hardening exponent is larger.

This proposition still has to be made acceptable in order to give an outlook on the importance of the n-value as a basic material quantity affecting deep drawability. Let $\sigma_\phi$ and $\sigma_t$ be the radial and circumferential stress components in the flange at radius $r$. Under the restriction that friction effects and the blank holder pression may be disregarded the equation of equilibrium is

$$\frac{d}{dr} (\sigma_\phi s r) = \sigma_t s$$  \hspace{1cm} (35)

where $s$ is the local thickness of the blank. From many experiments the strain state in the annulus appeared not to be a plane one, as is sometimes thought. The sheet thickness was found to be independent of $r$ as a reasonable good first approximation. This leads to

$$\frac{d\sigma_\phi}{dr} = \frac{\sigma_t - \sigma_\phi}{r}$$  \hspace{1cm} (36)

The relation between the radial stress component $\sigma_\phi$ and the circumferential one $\sigma_t$ is given by ($r_a$ = external blank radius)

$$\sigma_t = \sigma_\phi \frac{r^2 + r_a^2 - r_a^2}{r^2 - r_a^2}$$  \hspace{1cm} (37)

as can be shown with the aid of the Lévy - von Mises equations.

Substitution in the equation of equilibrium, followed by integration, leads to

$$\sigma_\phi = k \left( \frac{r_a^2}{r^2} - 1 \right)$$  \hspace{1cm} (38)

where $k$ is the integration constant.

The analytic expression for $k$ can be obtained by using the boundary condition of a uniaxial peripherical stress state. Hence, with the tensile
stress-strain relation (16), we may write

\[(\sigma_t)_{r=r_a} = - (\sigma)_{r=r_a} = - C \left( \ln \frac{r_{o}}{r_{a}} + \delta_0 \right)^n \]  

(39)

where \( r_{o} \) is the initial radius of the blank and \( r_{a} \) the external radius at a certain moment.

Substitution of equation (38) in (37), followed by combination with equation (39) gives

\[ k = \frac{C}{2} \left( \ln \frac{r_{o}}{r_{a}} + \delta_0 \right)^n \]  

(40)

and

\[ \sigma_f = \frac{C}{2} \left( \frac{r}{r^2} - 1 \right) \left( \ln \frac{r_{o}}{r_{a}} + \delta_0 \right)^n \]  

(41)

To investigate the influence of work-hardening on the drawing force we must find the sheet thickness.

With the restriction of \( s \) being independent of \( r \) and further of a uniaxial peripheral stress state in combination with the condition of constant volume and the Lévy–von Mises equations, the current flange thickness appears to be

\[ s = s_0 \sqrt{\frac{r_{o}}{r_{a}}} \]  

(42)

Since we are interested in the work-hardening effect only, within this scope, the effect of the punch edge and the – for the rest important – local friction may be represented in a strongly simplified way.

Let \( r_s \) be the average radius of the drawing clearance. Then, the equation for the current drawing force is

\[ F = 2\pi r_s e^{\frac{\mu \pi}{2}} s(\sigma_f)_{r=r_s} \]  

(43)

where \( \mu \) is the friction coefficient. Substitution of equation (41) and (42) delivers

\[ F = \pi s_0 r_s C \frac{r_{o}}{r_{a}} \left( \ln \frac{r_{o}}{r_{a}} + \delta_0 \right)^n \left( \frac{r}{r_{a}} - 1 \right) \]  

(44)
or
\[ F^* = C \left( \frac{\rho_{ao}}{\rho_a} \right)^{\frac{1}{n}} \left( \ln \frac{\rho_{ao}}{\rho_a} + \frac{\rho_o}{\rho_a} \right) \left( \frac{\rho a}{\rho_s} \right)^2 - 1 \]  
(45)

where
\[ F^* = F/(s_0 \rho_a \rho_s C) \]  
(46)

The punch force reaches its maximum value for \( \rho_a = \rho_{ak} \).

Then, with
\[ \beta_o = \rho_{ao}/\rho_s \]  
("drawing ratio")

\[ \beta_k = \rho_{ak}/\rho_s \]

we obtain
\[ F^*_{max} = \pi \rho \left( \frac{\rho_o}{\rho_k} \right)^{\frac{1}{n}} \left( \ln \frac{\rho_o}{\rho_k} + \frac{\rho_o}{\rho_s} \right) \left( \beta_k^2 - 1 \right) \]  
(48)

where \( \beta_k \) can be calculated with
\[ \beta_o = \beta_k \exp \left( 2n \frac{\beta_k^2 - 1}{3 \beta_k^2 + 1} \right) \]  
(49)

This expression has been obtained by differentiating equation (49) with respect to \( \rho_a \), followed by equalizing to zero.

Now, the nature of the work-hardening effect on deep drawability can be studied by evaluating the general condition \( F^*_k = F^*_{max} \) with the aid of the equations (30) and (48). The theoretical values - represented by the curves in Fig. 14. - are obtained by means of a digital computer omitting the geometrical terms in equation (30) and for \( \delta = 0 \). Thus, both calculated force numbers \( F^*_k \) and \( F^* \) may be considered maximum values with respect to tool geometry.

The substantial correctness of the theoretical tendency of the work-hardening effect may be demonstrated by the experimental work of ARBEL [3] (1950). His results (Tab. 3.) are shown in Fig. 14. as well.

In order to eliminate friction effects, these tests were carried out without a blank-holder. Therefore, it was essential to use a sheet thick enough to prevent folding. Contrary to the original values, the limiting drawing ratios have
Fig. 14. Theoretical work-hardening effect on the limiting drawing ratio in comparison with experimental data of Arbel.

Table 3. Experimental data of Arbel, showing the limiting drawing ratio $\rho_0\text{max}$ as a function of the work-hardening exponent $n$. ($r_{st} = 1.1$ in., $s_o = 0.125$ in.)
been recalculated according to the following relation (see equation (29)):

$$\beta_{\text{max}} = \frac{r_{ao}}{r_{ss}} = r_{ao} / (r_{st} + \frac{s_{o}}{2 \epsilon_n})$$  \hspace{1cm} (50)

The last metal of Tab. 3. had a very marked directionality and was tested to assess the results obtained with a metal of low formability. From the form of the dotted line (Fig. 14.) ARBEL concluded that little progress, from the deep-drawing point of view, can be expected from new alloys of a high work-hardening exponent. Though being an approximation, our foregoing theory brings to light that too much importance has presumably been attached to the last metal. In that case ARBEL's conclusion should have to be reversed to the opposite sense. Recent studies in superplasticity \cite{4} support our conclusion. Research activities are going on in order to analyze the additional effects of friction, anisotropy and the drawing edge on formability.

### 7.3. Simulative testing methods

There exist three main methods for determining the forming characteristics of sheet metal:

- testing the fundamental plastic properties of the sheet metal. The use of the determined quantities has been demonstrated in this study.

- comparative testing on the base of arbitrary chosen formability parameters. The use of the resulting values should be restricted to make sure that properties do not vary from coil to coil etc.

- testing by simulating forming operations. Even in the case of carefully controlled geometric similarity there is the problem of the scale factors. Whether or not a small diameter punch - the Swift flat-bottom cup test for example - can truly represent a punch used to draw a geometric similar cup 10 or 20 times larger in diameter is questionable.

Complete similarity exists in the case that the limiting drawing ratio obtained from a scale test equals the value observed under production conditions.

A free choice of the material characteristics and the initial sheet thickness can be skipped for practical reasons, a controlled change in friction conditions as well. Thus, the rules of similarity can be obeyed.
only by adjusting the testing tool geometry. Hence, if equation (30)
holds - and under the simplifying restriction that the load numbers
$F_k$ and $F_{\text{max}}$ under testing conditions must be equal to the values
under production conditions - one of the rules of geometrical similarity
can be formulated from (30):

$$\frac{1}{r_{st}} + \frac{1}{r_{st}} = \frac{1}{c} = \text{constant} \quad (51)$$

Solutions are shown in Fig. 15. for different $c$-values. Owing to the
diminishing steepness of the practically interesting part of the curves,
it is clear that it will be impossible to realize the right geometrical
scale conditions in most of the cases. It has to be noticed that common
testing conditions are situated far below left in the graph.

![Diagram](image-url)

Fig. 15. Curves representing the theoretical condition (51) for
geometrical similarity in scale testing.

It appears that no matter how much any simulative test is perfected, no
single deep-drawing test is presumably sufficient to evaluate formability
in an accurate way. Similar findings have been expressed by SHAWKI 5
(1965) on the basis of many attempts to correlate results from different
tests. Nevertheless, it is evident that there is a real need for a way to predict or evaluate the formability of sheet metal in combination with tool geometry and working conditions. For the time being a careful theoretical analysis of deep-drawing on the basis of fundamental plastic properties seems to be the only way.

REFERENCES


