Debonding along a fiber/matrix interface in a composite

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Debonding along a fiber/matrix interface in a composite

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Summary

Composites are widely used in many different applications. The use of fiber-composites for structural applications is restricted because of their inferior and unpredictable strength characteristics. To analyze their behavior during design a criterion for damage initiation and growth is necessary. There are two ways to take cracking into account: using fracture mechanics or applying material strength procedures. A fracture mechanics approach assumes pre-existing defects and is based on micro-mechanics. The material strength methods consider chemical bonds on an atomic scale. In this study research on cracking in a composite is carried out using fracture mechanics. According to fracture mechanics damage propagation takes place if stress or strain concentrations at microcracks or other defects reach a critical level. If the fiber/matrix behavior is brittle, Linear Elastic Fracture Mechanics (LEFM) for composites can be used, otherwise Non-Linear Fracture Mechanics (NLFM) has to be employed. The decision whether to use LEFM or NLFM depends on the relation between the properties of matrix and fiber.

To study the use of LEFM a finite element model (FEM) of a cracked body with an existing crack is used to determine certain quantities of fracture mechanics. The first cracked body is a commonly used test specimen in fracture mechanics (see Fig.1) and the second a model of a composite (see Fig.2).
1. Introduction

It is possible to study damage propagation in a composite employing concepts from fracture mechanics for an anisotropic material. In so far as the fracture mechanics of anisotropic bodies is concerned, the stress intensity factors are in most practical cases the same as for isotropic bodies. In particular, if no unbalanced loads act on the crack faces, the stress intensity factors will be independent of the material constants. It seems to be promising, however, to study the composite on a micro-level, where the microstructure, the component properties and their interaction is taken into account. It is observed that damage in a composite material is mostly initiated by fiber-matrix debonding, resulting in matrix-cracking and finally in fiber-rupture. The fiber-matrix interface is thus considered to be the weak spot, especially when the material is subjected to off-axial loads. This is the reason why research concentrates on the mechanical behaviour of the interface, with the ultimate goal to optimize its properties for a certain application. For a detailed study of the fiber-matrix interface it is necessary to fabricate a model-material. Advanced strain measurement techniques can be used to study the interface behaviour under longitudinal and transversal loading. Such experiments are very time consuming, as the influence of a rather high number of parameters must be investigated. Numerical simulation of the interface behaviour is much more effective and may indicate which experiments have to be carried out. For this numerical analysis a model of the composite material must be composed. Some choices concerning the geometry and the loading have to be made. At the TUE attention is focussed on a unidirectional composite which is loaded transversally. In the transversal plane the material is considered to be periodic with a rectangular fiber-stacking, which results in approximately isotropic transversal properties. The representative element to be considered consists of one fiber embedded in a matrix. The numerical model to be analyzed, can be confined to a quarter of this element if the appropriate kinematic boundary conditions are applied to satisfy the periodicity of the material (see Fig.3). The following assumptions are made:  
- the fiber and the matrix are homogeneous and have isotropic transversal properties  
- the fibers within the composite are perfectly straight  
- the analysis dimension in the plane of the cross section is plane strain

![Fig.3 idealized composite model](image-url)
Both perfect bonding and no bonding are considered between fiber and matrix. Also the interaction is described with an interface layer, consisting of springs or a continuous medium. In this study we consider the above mentioned fiber-matrix interface to be a boundary between the fiber and the matrix material. The behaviour of an initial crack at this boundary is studied using concepts of fracture mechanics.

A crack can grow from its original length $a$ to $a+\Delta a$ if the necessary energy to create new crack surface is available. This energy may be stored as elastic energy in a loaded body or be provided by increasing the external load. Critical crack growth parameters, e.g. energy release rate $G_c$ or stress intensity factor $K_c$, depend on type of loading, geometry and material. Crack propagation can occur in three different modes: opening mode (mode I), sliding mode (mode II) and transverse mode (mode III).

This work focusses on:
- the influence of mode I and mode II over crack growth along an interface,
- the direction of crack growth,
- the changing behaviour in an elastic-plastic case.

Quantities of fracture mechanics which are studied, are:
- $K$ and $G$ of a simple test specimen (single edge notch) (see Fig.1) consisting of a homogeneous material (model A)
- $G$ and $J$ for the same test specimen consisting of two materials (model B)
- $G$ and $J$ for a crack growing along a curved interface (model C) (see Fig.2)

The material properties of the epoxy matrix and the fiber, which were used in the calculations, are listed below. First the single edge notch test specimen was modelled with the material properties of the epoxy matrix (model A). Then the single edge notch test specimen was modelled as a bimaterial body (model B), one half as fiber and the other as matrix. The same type of finite element model was used for the homogeneous as well for the bimaterial configuration. In one half of the model the material properties were just changed into the transversal properties of the fiber to calculate the bimaterial model. Finally one quarter of a single fiber cross-section was modelled (model C). All models were two-dimensional, and thus the material properties are considered to be isotropic in the transversal direction for model C.

**Table 1: material properties**

<table>
<thead>
<tr>
<th>composite</th>
<th>epoxy matrix</th>
<th>carbon fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus long.</td>
<td>3.2 GPa</td>
<td>305 GPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.37</td>
<td>0.2</td>
</tr>
<tr>
<td>Young's modulus trans.</td>
<td>3.2 GPa</td>
<td>14 GPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.37</td>
<td>0.25</td>
</tr>
</tbody>
</table>
2. An overview in Literature

In literature several fracture mechanics analysis of fiber/matrix configurations are reported.

In 1992 Hermann and Ferber published their research on failure mechanisms of interface and matrix cracks under thermal loading. They used a elementary hexagonal model with six fibers surrounded by matrix. The matrix consists of epoxy and the fibers of araldite. The matrix behaviour was assumed fully elastic. Strain energy release rates were determined via numerical calculation with finite element method for mixed mode. They used global and local energy methods depending on the modified crack closure integral using continuum mechanics. The K-values were related to the strain energy release rates according to the normal linear elastic equations.[1]

In 1992 Naik and Crews published a paper about a three-dimensional numerical continuum model of a fiber/matrix configuration under normal and longitudinal loadings. They determined strain energy release rates with virtual crack closure techniques (VCC) for mode I and mode II. The corresponding K-values were calculated from numerical K-values by using a curve fitting procedure. The fiber (AS4) and matrix (3051-6) were both linear-elastic. Additionally they made a non-linear analysis of friction and slip on the interface.[2]

In 1974 Karlak, Grossman and Grant published an article about a two-dimensional numerical model of a fiber/matrix element. They studied AL 6061-T6 with boron fibers and Ti6A1-4V with beryllium fibers. They used an elastic-plastic formula (Yamada) and v.Mises and Prandtl-Reuss equations for loadings in the plastic range. To describe debonding a maximum normal stress criterion was used. The crack grew along the interface. It was found that the interface Stress-Intensity-Factor (SIF) is relative independent of filament volume fraction.[3]

In 1992 M.Toya published a work about mode I and mode II energy release rates of an interface crack along a straight line. The opening and sliding components of the energy that is released during an incremental extension of an interface crack between two different elastic materials are evaluated by Irwin’s crack closure method. Each component of the energies (G_l and G_n) is expressed in terms of the length of the incremental crack extension(Δa) and the real and imaginary part of the complex stress intensity factor, defined by Malyshev and Salganik. Toya found out that values of G_l/Δa and G_n/Δa are oscillating violently if Δa approaches zero.[4]

In 1991 J.W.Ju reported results, which he got using a micro-mechanical damage model for uniaxially reinforced composites weakened by interfacial microcracks. He used a bimaterial fracture criterion for mixed mode loading, analytically derived by Toya in 1974. He also used a fourth-rank damage tensor and the compliance method to show the influence of arc cracks on stress and strain state.[5]
In 1991 C.G. Sih published a book on mechanics of crack initiation and propagation. The application of the strain energy density method in fracture mechanics is discussed. The strain energy density is determined around the crack-tip at a pre-existing defect. A minimum value can be found in a certain direction and crack initiation will take place in that direction, if the load is high enough. The strain energy density criterion assumes that rapid crack growth will occur in this direction, if the minimum value of the strain energy reaches a critical value. The critical value of the strain energy density has to be determined by experimental tests. The function of strain energy density near the crack-tip has a relative minimum depending on a certain direction with a constant radius around the crack-tip.[6]

R. Sethuraman and S. K. Maiti showed in 1988 how to determine the strain energy release rate using Irwin's crack closure integral and the FE-method. The advantage of using this method is that it allows a separation of the energies in $G_I$ and $G_{II}$ with the different modes in a mixed mode problem. The virtual crack closing $\Delta a$ is equal to the size of the crack-tip element. The crack-tip element has to describe the stress singularity and needs therefore a certain geometry depending on the type of elements and linear or non-linear analysis. To calculate $G_I$ and $G_{II}$, forces to close the crack to its original length and the corresponding displacements at the crack-tip elements are determined. The results show a good agreement with analytical values.[7]

Finally of all listed methods, the FE-method of R. Sethuraman and S. K. Maiti to determine energy release rates and the theory of C.G. Sih about strain energy density seems to be the most suitable tools to be used for numerical micro-mechanics of composites in this case.
3. FE-modelling of a cracked body

3.1 The single edge notch test specimen

To study the agreement between theory and numerical calculation, a simple fracture mechanics test specimen was modelled. The test specimen consists of a thin plate infinite in one direction and with one crack in one side, the so called single edge notch test specimen (SEN), (see Fig.1). Geometrical conditions for reference tests and the belonging analytical solutions of stress fields are well known.

The SEN configuration was used to optimize the mesh and boundary conditions. Several crack lengths were considered: \( a = 2, 5, 10 \text{ [mm]} \).

The geometrical parameters were:

\( l = 206 \text{ [mm]}, b = 20 \text{ [mm]} \) (see Fig.1). The FE-mesh for the SEN is shown in figure 4.

The analyses were carried out with the MARC program [8]. The meshes were generated with MENTAT II [9].

In the first part of the calculations all elements had the material properties of matrix (model A) and in the second part in one half of the mesh the material properties were changed into the fiber properties, making use of symmetry (model B).

The edge A-A was kept straight by tying nodes to one corner node in y-direction and edge B-B was kept straight by setting the degrees of freedom in y-direction to zero.

The applied strain was in longitudinal direction to create pure mode I conditions.

The applied strain was 10e-3.

The analysis dimension was plane stress.

The crack-tip (see Fig.5) was modelled with 16 distorted 6-node elements arranged in a cylindrical manner, each element with an opening angle of 22.5 deg.

The mesh was expanded radially with 8-node quadrilateral elements and the element size increased from row to row in radial direction with 1.4. A cylindrical arrangement of elements was chosen because the stress field shows the same behaviour and it is easier to determine certain quantities along node paths in radial and axial directions.

Fig.4 FE-model of single edge notch
If 8-node quadrilateral elements are used, one edge is distorted to one point, reducing a quadrilateral element to a triangular element. If the mid-side nodes of two unchanged edges are moved to the ¼-points and the third edge is kept straight, then in every direction of \( \theta \) a singularity in the stress field exists (see Fig. 6a, b). If three nodes of one edge are reduced to one node in one point, a \( 1/\sqrt{r} \) singularity is approached. If the same three nodes coinciding in one point, a \( 1/r \) singularity is approached. A \( 1/\sqrt{r} \) singularity can also be obtained, with a 6-node quadrilateral triangle element (see Fig. 6c). Two mid-side nodes are moved to the ¼-point of the elements size and the third side remains unchanged.[10,11]
3.2 Crack-tip analysis

3.2.1 The stress intensity factor K

A simple way to determine stress intensity factors, is to use the direct method, where K-values are evaluated from the stress field near a crack-tip, which can be determined with the FE-method. A mesh has to be fine enough to get accurate results. According to literature [12] the K-value for the SEN of pure mode I loading is:

\[ K_I = \sigma_n \frac{\sqrt{\pi a}}{\left[ 20 - 13\left( \frac{a}{b} \right) - 7\left( \frac{a}{b} \right)^2 \right]^{0.5}} \quad \text{with} \quad \frac{a}{b} \leq 0.7, \quad \frac{l}{b} \to \infty \]

The end edges of the test specimen are moved parallel during loading.

In case of pure mode I loading a numerical \( K_I \) can be calculated from the stress \( \sigma_{yy} \) obtained by FEM with:

\[ K_I = \frac{\sigma_{yy} \sqrt{2\pi r}}{\cos\left(\frac{\theta}{2}\right) \left(1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right)} \]

The angle \( \theta \) and the radius \( r \) are defined in figure 7.

The normalized stress intensity factor is:

\[ Y = \frac{K}{\sigma_n \sqrt{\pi a}} \]

where \( \sigma_n \) is the applied load.

The numerical \( K_I \) was estimated in radial directions at several angles from the crack tip. The results show in the case of homogeneous SEN (model A), that the \( K_I \) values are well approximated (see Fig.8b-10b). The results show also, that the formula is getting inaccurate, if the angle \( \theta \) is bigger than 0-deg. The same results were found in former researches at the TUE [13]. In the case of a bimaterial SEN (model B) no representative \( K_I \)-value can be found for \( r \to 0 \) (see Fig.11b-13b). Other parameters have to be used. In the next chapter the energy release rate \( G \) is presented as a parameter to analyze the stress state.
Figure 8a

2 mm crack SEN of matrix

sigma_y [N/mm²]

0-deg

distance from crack tip [mm]

1000
800
600
400
200
0

10^3
10^2
10^1
10^-1
10^-2
10^-3

distance from crack tip [mm]

Figure 8b

Stress intensity factor at 2 mm crack SEN

Y

0-deg

analytic

distance from crack tip [mm]

0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

1.35
1.3
1.25
1.2
1.15
1.1

22.5-deg

analytic

distance from crack tip [mm]

0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
5 mm crack SEN of matrix

Stress intensity factor at 5 mm crack SEN

Figure 9a

Figure 9b
Figure 11a

Figure 11b
Figure 12a

Figure 12b
Figure 13a

Figure 13b
3.2.2 The energy release rate $G$

The energy release rate $G$ which was defined by Irwin is a measure of the energy available for an increment of crack extension to create new crack surface:

$$G_{tot} = -\frac{d\Pi}{dA}$$

with $d\Pi$ as the change in potential energy of an elastic body and $dA$ the change in crack surface. The elastic potential $\Pi$ is defined as: $\Pi = W - F$, where $W$ is the strain energy stored in a body and $F$ is the work done by external forces. In the case of fixed displacement $u$ for the loaded edges $F$ is equal to zero and $\Pi = W$. Thus

$$G_{tot} = -\frac{1}{B}(\frac{dW}{da})_u$$

with $B$ as the thickness. The total energy release rate $G_{tot}$ can be separated in $G_I$ of mode I and $G_{II}$ of mode II crack opening. Refering to Sethuraman and Maiti [7] which use the virtual crack closure technique (VCC), $G_I$ and $G_{II}$ can be written as:

$$G_I = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} \int_0^{\Delta a} \sigma_{yx} U_x \, dx; \quad G_{II} = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} \int_0^{\Delta a} \sigma_{yy} U_y \, dx$$

The VCC-technique allows to determine the work required to close a crack with a virtual extension $\Delta a$ to its initial length. For the FE-method using $\frac{1}{4}$-point distorted elements as crack-tip elements, Sethuraman and Maiti give the following formulas for $G_I$ and $G_{II}$:

$$G_I = \frac{U_{y4}}{\Delta a} \left( F_{y2}+((1.5\pi-4)F_{y3}) \right); \quad G_{II} = \frac{U_{x4}}{\Delta a} \left( F_{x2}+((1.5\pi-4)F_{y3}) \right)$$

For an inplane crack $G_I$ and $G_{II}$ can be obtained from the forces and displacements of the node at the crack-tip and the node next to it. To obtain the values of the forces, very stiff springs are assigned at these nodes in both degrees of freedom. The displacements and forces are defined in Fig.14. The virtual crack extension $\Delta a$ is chosen equal to the size of the crack-tip element. To compare $K_I$-values with $G_I$ the following formula can be used in LEFM for a plane stress situation:

$$G_I = \frac{K_I^2}{E}$$

![Fig.14 the crack-tip](image)
3.2.3 The J-integral

A third possibility of treating stress states is to use a path independent line integral around the crack-tip like the J-integral. J is defined as:

\[ J = \oint_{\Gamma} (W \, dy - T \, \partial u / \partial x \, ds) \]

where \( w \) is the strain energy density, \( T \) is the traction vector, \( u \) is the displacement vector and \( ds \) is an increment along the path \( \Gamma \) around the crack-tip.

For linear elastic materials \( J \) is equal to \( G_{\text{pot}} \). The MARC program offers an optional evaluation of the J-integral according to Lorenzi [8] using virtual crack extension technique. The program estimates the change in strain energy, which has to be divided by the virtual crack extension \( \Delta a \) to obtain the value of \( J \). The purpose of using the J-integral estimation in this study is to verify \( G_{\text{pot}} \) for the bi-material configuration of model B.

In case of non-linear elastic material behaviour an analysis of cracks is more difficult. The stress-strain relation has to be known and it should be proved that the material behaviour in the micro-mechanics view follows this behaviour. To apply methods of fracture mechanics, the order of the singularity has to be determined for example by using the finite element method. If the type of the singularity is known, elements which describe the singularity of order \( r^n \) can be created by displacing element nodes to change the approximating functions [11].

To simulate a non-linear material the load can be applied stepwise and held constant during a single step. The strain is separated into an elastic part and a plastic part. To describe the elastic behaviour Hooke’s law is used and for the plastic behaviour may be used the law according to Prandtl and Reuss. For example, if the stress-strain curve can be expressed by a power law [14]:

\[ \frac{\sigma}{\sigma_0} = \left( \frac{\varepsilon}{\varepsilon_0} \right)^n \]

\( n \) is the hardening exponent, \( \varepsilon_0 \) and \( \sigma_0 \) are the limits of proportionality, the stress field can be described with:

\[ \sigma_y = \sigma_0 \left[ J(r\sigma_0\varepsilon_0 l_n) \right]^{n-1} f(\beta) \]

\( f(\beta) \) is a dimensionless function, \( l_n \) is a numerical parameter for plane strain or plane stress. A yield criteria has to be stipulated to find the point of plastic deformation. The estimated work to close a crack to its initial length consists of work used for plastic deformation and fracture work and cannot be separated.
3.3 Comparison of different methods

For model A of the SEN, the Y-values using the direct method were extrapolated to $r \to 0$. These Y-values were converted into $G_{\text{num}}(Y)$. Further $G_I$ was evaluated by the VCC-technique. For model B of the SEN, $G_I$ and $G_{II}$ were determined using the VCC-technique. Additionally the J-integral was calculated to verify $G_{\text{tot}}$ in the case of bimaterial SEN (model B). The results are summarized in table 2.

Table 2: results

<table>
<thead>
<tr>
<th>model A</th>
<th>$a = 2$</th>
<th>$a = 5$</th>
<th>$a = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{\text{analyt.}}$</td>
<td>1.1584</td>
<td>1.238</td>
<td>1.4586</td>
</tr>
<tr>
<td>$Y_{\text{num. 0°}}$</td>
<td>1.1542</td>
<td>1.2625</td>
<td>1.778</td>
</tr>
<tr>
<td>$Y_{\text{num.22.5°}}$</td>
<td>1.1627</td>
<td>1.2716</td>
<td>1.792</td>
</tr>
<tr>
<td>$Y_{\text{num.45°}}$</td>
<td>1.1765</td>
<td>1.2863</td>
<td>1.815</td>
</tr>
<tr>
<td>$G_I$ analyt.</td>
<td>2.70e-2</td>
<td>7.70e-2</td>
<td>2.139e-1</td>
</tr>
<tr>
<td>$G_I$ num(Y)</td>
<td>2.77e-2</td>
<td>8.23e-2</td>
<td>0.322</td>
</tr>
<tr>
<td>$G_I$ (VCC)</td>
<td>2.77e-2</td>
<td>8.25e-2</td>
<td>0.330</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>model B</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_I$ (VCC)</td>
<td>3.502e-2</td>
<td>9.503e-2</td>
<td>0.3204</td>
</tr>
<tr>
<td>$G_{II}$ (VCC)</td>
<td>1.33e-2</td>
<td>7.613e-3</td>
<td>0.2157</td>
</tr>
<tr>
<td>$G_{\text{tot}}$</td>
<td>4.832e-2</td>
<td>0.1026</td>
<td>0.5361</td>
</tr>
<tr>
<td>J-integral</td>
<td>4.838e-2</td>
<td>0.1353</td>
<td>0.526</td>
</tr>
</tbody>
</table>

$a$: crack length [mm]

$G$ and $J$ in [Nmm/mm$^2$]

Conclusion: The table shows a good agreement for the numerical values of $G_I$ with the direct method and the VCC-technique in the homogenous and for $G_{\text{tot}}$ and $J$ in the bimaterial case. That means that the estimation of $G$ using VCC - technique seems to be an applicable tool for a curved crack at a bimaterial interface.
3.4 The strain energy density theory

Referring to C.G. Sih [6] the strain energy density function can be expressed as follows:

\[
\frac{dW}{dV} = \frac{1}{2E} \left[ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right] \frac{\nu}{\nu + 1} \left( \frac{\sigma_1 + \sigma_2 + \sigma_3}{\nu + 1} \right)
\]

\( \sigma_1, \sigma_2, \sigma_3 \) are the principal stresses, \( \nu \) the Poisson's ratio and \( E \) the Young's modulus.

The strain energy density factor \( S \) depending on the distance \( r \) from the crack-tip is defined as:

\[
S = r \left( \frac{dW}{dV} \right)
\]

Crack initiation can take place in a direction \( \theta_0 \) determined by the relative minimum of the strain energy density factor \( S \):

\[
\frac{\partial S}{\partial \theta} = 0 \quad \text{and} \quad \left( \frac{\partial^2 S}{\partial \theta^2} > 0 \right) \quad \text{at} \quad \theta = \theta_0
\]

Rapid crack growth occurs if this minimum \( S_{\text{min}} \) reaches a critical value:

\[
S_{\text{min}} = S_c \quad \text{at} \quad \theta = \theta_0
\]

The critical value is material dependent and can be estimated by experimental tests. The strain energy density factor \( S \) around the crack-tip was determined. A local minimum \( S \) can be found and is under 0-deg in the direction of cracking as expected in the homogeneous case (see Fig.14a-16a). This means that the crack will grow in its initial direction. If the load is such high that \( S_{\text{min}} \) can reach the critical value \( S_c \), rapid crack growth occurs. In the case of two materials two local values for \( S_{\text{min}} \) can be found (see Fig.14b-16b); one in the matrix and one in the fiber, now \( \theta_0 \) is not 0-deg anymore.

There are two possibilities for a crack extension: the crack can grow along the interface or into one material. The critical values for \( S_{\text{min}} \) have to be known to predict the crack path. The crack will grow into that material, in which the critical value is reached first, while the load is increased.
Figure 14a

S around matrix crack a=2 (SEN)

Figure 14b

S around fiber/matrix crack a=2 (SEN)

Figure 15a

S around matrix crack a=5 (SEN)

Figure 15b

S around fiber/matrix crack a=5 (SEN)

Figure 16a

S around matrix crack a=10 (SEN)

Figure 16b

S around fiber/matrix crack a=10 (SEN)
4. FE-modelling of a idealized composite

4.1 A fiber/matrix configuration of a composite

The model to investigate debonding along a curved bimaterial interface consists of the same type of crack-tip mesh as in the former calculations. Symmetry allows to look only at one quarter of the cross section. The total amount of elements was reduced after it could be shown, that the numerical results remain constant while increasing the size of the smallest element. This is in contrast to the determination of $G$ which demands very small element sizes at the crack-tip. Small elements lead to a better approximation, according to the formula in chapter 3.2.2.

Table 3 lists the different element sizes $\Delta a$ and the corresponding $G_{\text{m}}$-values.

Table 3:

<table>
<thead>
<tr>
<th>$\Delta a$ [(\mu m)]</th>
<th>1e-4</th>
<th>6e-4</th>
<th>1e-3</th>
<th>5.5e-3</th>
<th>J-integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G*e4$</td>
<td>4.55</td>
<td>4.55</td>
<td>4.56</td>
<td>4.55</td>
<td>4.7</td>
</tr>
</tbody>
</table>

For the calculation 5.5e-3 was chosen as smallest element size. That seems to be a good compromise. An enlargement of the crack-tip element leads to a better numerical balance in convergence and singularity rates. The total mesh (see Fig.18a) consisted of two single meshes: a basic mesh for the cross section of 147 elements (see Fig.18b) and a fine mesh for the crack in the interface of 136 elements (see Fig.18c). The mesh of the cross section described a fiber to matrix volume content of 50:50%. The mid-line of the finer crack-tip mesh was attached to the fiber radius, which was 3\(\mu m\). To simulate debonding the fine mesh was inserted at 15 positions into the basic mesh and at each position calculations were carried out. In this way the growth of the circular arc crack along the interface was simulated.

Fig.18a model C
Fig. 18b basic mesh

Fig. 18c crack-tip mesh
All boundary conditions infer from the idealized composite model with repeating units (see Fig.19). Two edges are kept straight because of symmetry in the cross section. The two other sides were tied to node H because of the imposed periodicity condition. The load-case of transverse normal strain is regarded. At node H a fixed displacement \( u \) of 0.001 in \( y \)-direction was applied, which led to a composite strain of \( \varepsilon = 0.13\% \). The analysis dimension was assumed plane strain.

![boundary conditions](image1)

Fig.19 boundary conditions, links and load

Although the fiber is orthotropic, it can be considered as isotropic in the transversal plane and the transversal material properties are therefore used. The fiber is 20 times stiffer in the longitudinal direction and effects on the crack behaviour would be marginal for this transversal load case. The material properties for the matrix and the fiber were chosen according to table 1. All angles used for the calculations, are defined in the coordinate system printed below. \( \beta \) is the angle which describes debonding and \( \theta \) is the angle around the crack-tip measured from the tangent of the fiber contour (see Fig.20).

![coordinate systems](image2)

Fig.20 coordinate systems
4.2 Calculations and results

At each simulated step of debonding, the energy release rate of the interface, the force necessary for constant displacement and the strain energy density around the crack-tip were determined. The deformed mesh is printed for $\beta=10.4$ deg with a magnification factor of 500, to make the displacements visible (see Fig. 21).

![Fig. 21 deformed mesh](image)

To get an imagination of the crack contour, the zone near the crack-tip was zoomed and is printed in figure 22.

The elements in the upper half represent the matrix and the elements in the lower half the fiber. It can clearly be seen, that the deformation in the matrix is much higher than in the fiber.
To study the influence of different crack modes during debonding, we can look at the total and the separated energy release rates (see Fig.23a,b). Debonding initiates in pure mode I and continues up to $\beta=45$ deg in total mixed mode. For $\beta>45$ deg the influence of mode I decreases rapidly and for $\beta>70$ deg pure mode II conditions exist for crack growing. For $G_{\text{tot}}$ we can find a maximum at $\beta=16$ deg.
The energy release rate $G$ for an inplane crack extension and constant edge displacement can be written as:

$$ G = -\frac{B}{B} \left( \frac{dW}{da} \right)_{\text{u}} = -\frac{u^2}{2C^2} \left( \frac{dC}{da} \right)_{\text{u}} $$

where $B$ is the thickness, $W$ is the strain energy, $C$ is the compliance, $a$ the crack length and $u$ the displacement. We can see that $G$ is proportional to $u^2$ and consequently proportional to the strain $\varepsilon^2$. A crack can just grow if a critical value of $G$ is reached.

Let us consider now the critical stress and strain for debonding along the interface. The critical values for $\sigma$ and $\varepsilon$ at the stress state when $G_c$ is reached, can be substituted as follows:

$$ \frac{\sigma_c}{\sqrt{G_c}} = \frac{\sigma}{\sqrt{G}} \quad ; \quad \frac{\varepsilon_c}{\sqrt{G_c}} = \frac{\varepsilon}{\sqrt{G}} $$

where $\varepsilon$ is the global strain of the cross section, which is assumed to be constant during debonding and $\sigma$ is the global stress of the cross section. $\sigma$ is obtained with $\sigma=E_*\varepsilon$, where $E$ is the modulus of the composite unit (see Fig.24a).

The modulus of the cross section can be calculated with the formula:

$$ E = \frac{\sigma}{\varepsilon} = \frac{F}{b \frac{1}{\Delta u}} = \frac{F}{\Delta u} $$

Here $F$ is the reaction force at node H, $b$ and $l_o$ are the edge lengths of the quarter unit and they are equal.

The relation between $\sigma_c$ and $\varepsilon_c$ can now be plotted qualitatively by plotting the substituted magnitudes (see Fig.24b). The graphs show that the stress increases linearly until it reaches a critical value and debonding is initiated. After that, the stress and strain for debonding decreases and it can propagate at a lower stress level, it is unstable. Similar results were found using a material strength approach (see Fig.24c)[15].
Modulus of composite

debonded arc [deg]

modulus [N/mm^2]

eps = 0.13%

Fig. 24a
critical stress-strain of interface

**Fig. 24b**

**Fig. 24c**
The strain energy density at the second row of elements around the crack-tip is shown in Figure 25a. The curve is similar to that for a straight crack. In both materials are local minima of \( S_{\text{min}} \). In both cases, curved and straight crack, \( S_{\text{min}} \)(matrix) is higher than \( S_{\text{min}} \)(fiber). A large deviation exists at the interface. The strain energy density in the fiber is small compared to the strain energy density in the matrix. Further details are focussed on the matrix material. To determine \( \theta_0 \) of \( S_{\text{min}} \) in the matrix, a polynom of 4th order was fitted to the data with MATLAB [16]. The graphs are shown in figure 25b, all local points of \( S_{\text{min}} \) are marked. Table 4 lists \( \theta_0 \) of the strain energy density and the debonded angle \( \beta \).

**Table 4**: debonding angle \( \beta \) and \( \theta_0 \) of \( S_{\text{min}} \)

<table>
<thead>
<tr>
<th>( \beta ) deg</th>
<th>5.6</th>
<th>10.4</th>
<th>16</th>
<th>21.6</th>
<th>27.3</th>
<th>33.2</th>
<th>39.1</th>
<th>45</th>
<th>51</th>
<th>56.8</th>
<th>62.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_0(S_{\text{min}}) ) deg</td>
<td>68.4</td>
<td>74</td>
<td>79.5</td>
<td>84.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_0(S_{\text{min}}) ) deg</td>
<td>19.1</td>
<td>23.3</td>
<td>26.1</td>
<td>29.9</td>
<td>34.1</td>
<td>38.9</td>
<td>44.6</td>
<td>52.3</td>
<td>59</td>
<td>66.1</td>
<td>72</td>
</tr>
</tbody>
</table>

To clarify the direction of a probable crack bifurcation into the matrix material, at all 15 crack positions an arrow is plotted into the direction of \( \theta_0 \) (see Fig.26).

![direction of crack initiation in matrix](image)

**Fig.26 probable crack bifurcations**
Finally the energy release rate of an assumed bifurcating crack into the matrix was determined. It was assumed that the crack initiates in pure mode I and that the direct method can be applied. The result is plotted together with $G_{\text{m}}$ of the interface (see Fig. 27). To predict the crack path the load has to be increased until a critical value of $G$ is reached. In the case of linear elastic materials we just have to shift the graphs of the energy release rates. If we assume the same critical $G_c$ for the interface and for the matrix, the crack will grow along the interface until 13 deg and while the applied strain is held constant, will bifurcate into the matrix. If we assume that $G_c(\text{interface}) < G_c(\text{matrix})$ the crack will bifurcate at a higher degree of debonding into the matrix.

![Diagram](image-url)

Fig. 27 $G$ of the interface and the matrix
5. Conclusions

Using methods for determination of energy release rates in fracture mechanics as proposed by Sethuraman and Maiti, is not limited to homogenous materials or straight cracks. It can also be applied to bimaterial crack surfaces and curved crack geometries. Because the equations for $G$ are based on energy formulation and at the crack-tip the curved line is approached with small elements as a straight line. By using the finite element method and special crack-tip elements, the energy release rate can easily be determined. The energy release rates can be separated into fractures of mode I and mode II crack opening mechanisms. The results show, that debonding along a curved bimaterial interface initiates in mode I and propagates in mixed mode crack opening.

All $G_{ct}$-values were verified with the J-integral, which is a general tool to describe stress singularities near crack tips, particularly for elastic-plastic material behaviour. The disadvantage of the J-integral is, that the different crack modes can not be separated, which may be important for a better understanding of the interface behaviour.

The results presented in the foregoing chapters, show a similar composite behaviour for transversal loadings, as obtained by a material strength approach. These two different ways of modelling of composites, can be used to verify the other and to make qualitatively comparisons.

By combining the strain energy density theory with micro-mechanics, it is possible to study crack growing and crack paths in homogeneous materials as well as in composites. Particularly, if we consider debonding between two materials, we can say, after debonding has initiated, it will continue in an unstable way and lead to an decrease of the transversal strength. Unstable, because the stress necessary for crack propagation decreases for higher degrees of debonding. Debonding can also initiate matrix cracking, which can affect new micro-defects in the area close to the crack. A crack will propagate in that direction, in which most energy can be set free. This means that a crack can bifurcate from the interface into the matrix, if the critical value $G_c$ in the matrix is reached and the value $G$ for the interface is lower.

All results support a better understanding of the micro-mechanics in composites and can help for an improved design of interface materials.
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