Hierarchical quantification of arterial vasculature based on Strahler ordering

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Chapter 1

Introduction

Blood perfusion is a complex mechanism that is responsible for nutrition and drainage in biological tissue. Blood flows through an extensive network of blood vessels. This vascular system has a hierarchical architecture: blood flows from one or a few large supplying arteries through a diverging, arteriolar vascular bed, reaches the capillaries and is drained by a converging venous vasculature. Blood perfused tissue can be described as a mixture of a fluid, which represents the blood, and a solid, which represents the surrounding tissue. A mixture description of blood flow through a hierarchical network was first developed by [Huyghe et al (1989a, 1989b)]. Blood flow was described by a spatial component and a hierarchical component. The latter quantified the flow between the vascular compartments, i.e. from the arterial towards the venous vessels. Huyghe and van Campen (1995) included this mixture description of blood perfusion in a finite deformation theory of hierarchical porous media.

An integrated Finite Element (FE) description of finite deformation and blood perfusion of biological tissue was developed by Vankan et al (1996b) and it was implemented in the software package DIANA. The displacements of the solid, the mixture’s hydrostatic pressure and a number of hierarchical fluid pressures are taken as the nodal degrees of freedom. In the mixture theory, fluid flow through porous solids is described by a Darcy equation. In this equation, a permeability tensor relates volume averaged fluid flow to the gradient of volume averaged fluid pressure. In [Huyghe et al (1989a, 1989b)], an extended Darcy equation is presented, which accounts for both spatial and hierarchical components of fluid flow. Also, a quantitative relationship between the geometry of the microvasculature in a representative elementary volume and the fluid viscosity on one hand and the permeability tensor belonging to this representative elementary volume on the other hand is given [Huyghe et al (1989a, 1989b), Huyghe & Van Campen (1995)]. This geometry-permeability relationship tends to be exact when the number of vessels within the averaging volume goes to infinity.

The description of the hierarchical flow in the extended Darcy equation requires that the vascular hierarchy is quantified. This is achieved by a hierarchical parameter $z_0$, which assigns a value between 0 and 1 to each hierarchical level. In [Huyghe et al (1989a)], a hierarchical quantification procedure is presented. In this procedure, the $z_0$-value was based on the diameters of the vessels within the vascular system.

The reliability of the geometry-permeability relationship was investigated [Vankan et al (1996c)]. The number of vessels within the averaging volume was limited. FE blood pressure and flow results were compared to volume averages of network (NW) analysis results. Two different hierarchical quantification procedures ($z_0$-definitions) were used; one was based on the blood pressure calculated in the network analysis and the other was based on vessel diameters according to [Huyghe et al (1989a)]. For both $z_0$-definitions, differences between the calculated network flows and pressures and the results of the FE calculations were found. The first $z_0$-
definition (pressure-based) gave better results than the second $x_0$-definition (geometry-based). Two important conclusions were that the FE results were very sensitive to the employed $x_0$-definition, and that a good correspondence between FE results and NW results required a good correlation between the $x_0$-value and the blood pressure [Vankan et al (1996c)].

The pressure-based $x_0$-definition requires knowledge of the blood pressure values in the whole vascular tree. As this information is usually not available, the $x_0$-definition must be based on other information, for example on structural data of the vasculature. However, the diameter-based definition of Huyghe [Huyghe et al (1989a)] resulted in poor correspondence between FE results and NW results. Therefore, in this study, an alternative $x_0$-definition is developed, which is based on a hierarchical classification method for tree-like structures that was developed for flow through rivers [Strahler (1952)].
Chapter 2

Methods

This chapter elucidates methods for analysis of blood flow through vascular trees. In section 2.1, the use of network models for the simulation of vascular flow and their limitations are discussed. Section 2.2 deals with the hierarchical mixture model, which is advantageous with respect to the network models in the case of large vascular trees. The demands for the applied \( x_0 \)-definitions in case of the FE calculation are described in section 2.3, together with two \( x_0 \)-definitions that were used in previous studies. Other quantification methods, based on the so-called Strahler ordering scheme, are described in section 2.4. The arterial tree model that is used in this study is dealt with in section 2.5 and the principles of the FE simulation are described in section 2.6.

2.1 Network models for the vascular blood flow

Network models are often based on simple approximations, such as Poiseuille flow in the vessels, straight vessels parts (segments) with a constant diameters and a constant fluid viscosity inside the vessels.

Blood flow through a vascular tree can be calculated by network analysis. Pressure boundary conditions are prescribed at the proximal end of the feeding arteries and the distal end of the terminal capillaries: high pressure at the arterial inflows and low pressure at the capillary outflows. A part of a vessel which is contained between two bifurcations is called a segment.

According to Poiseuille’s law, the pressure-flow relation in each segment can be written as:

\[
Q = \frac{\pi d^4}{128 \eta l} \Delta P
\]

where \( Q \) represents the volumetric flow and \( \Delta P \) is the pressure drop in a segment. From the connectivity of the segments, a system of linear equations can be derived for the pressure-flow relation in the whole network.

For the numerical analysis of blood flow through a certain hierarchical vasculature, detailed computer-generated models of branching arterial and venous trees have been developed [Schreiner (1993), Schreiner & Buxbaum (1993)]. Demands for the structure of network models of vascular trees can be determined by measurements of general network features in a specific piece of tissue, for example in a rat spinotrapezius muscle [Skalak & Schmid-Schönbein (1986)].

2.2 The hierarchical mixture model

For small vascular trees, network models can be used successfully in order to determine the pressure and flow characteristics in the vasculature. The complexity of the network models
increases with increasing number of vessels.
The hierarchical mixture model contains a continuum description of blood perfusion, in which volume averaged values of blood pressure and flow are used. Due to the volume averaging procedure, the complexity of the hierarchical mixture model is independent of the complexity of the vasculature, which makes the use of the mixture model advantageous for analysis of blood flow through large vascular trees.
In the hierarchical mixture model, blood perfusion is described as fluid flow through a hierarchical system of distensible pores in a deforming porous solid. This model is based on the local conservation equations of fluid mass, solid mass and momentum. The constitutive behaviour of the fluid in the hierarchical mixture is described by an extended Darcy equation. In [Huyghe et al (1989a, 1989b)], the extended Darcy equation is derived, assuming Poiseuillian fluid flow on the level of individual vessels.
In this study, only two-dimensional, rigid, non-distensible vascular trees in non-deforming tissue are considered in which a steady Poiseuillian flow is assumed. Due to these simplifications, the conservation equations, together with the extended Darcy equation, are reduced to one governing differential equation, which represents local conservation of fluid mass [Vankan et al (1996c)]:

$$4 \hat{\nabla} \cdot 4 \hat{K} \cdot 4 \hat{\nabla} \tilde{\mu}^f = 0$$

(2.2)

where $\tilde{\mu}^f$ represents the volume averaged fluid pressure and $4 \hat{K}$ is the hierarchical Darcy permeability tensor. A tilde ' ~' above a quantity indicates that this quantity depends on the hierarchical position in the vasculature. The 4-dimensional operator $4 \hat{\nabla}$ and the 4-dimensional permeability tensor $4 \hat{K}$ are denoted as:

$$4 \hat{\nabla} = \left[ \begin{array}{c} \frac{\partial}{\partial x_0} \\ \nabla \end{array} \right]$$

(2.3)

$$4 \hat{K} = \left[ \begin{array}{cc} \tilde{k}_{00} & \tilde{k}_{0}^T \\ \tilde{k}_{0} & \tilde{K} \end{array} \right]$$

(2.4)

where $\tilde{k}_{00}$ represents the hierarchical permeability, $\tilde{k}_{0}$ is the cross-term permeability vector and $\tilde{K}$ is the spatial permeability tensor.

The Slattery-Whitaker averaging theorem was used [Huyghe et al (1989a, 1989b)] to derive a volume averaged representation of the fluid flow through the vasculature. For $4 \hat{K}$, the following expression was derived:

$$4 \hat{K} = \frac{\tilde{n}^f}{32} \left( \frac{d^2}{\eta(d)} \frac{d\hat{x}}{d\hat{s}} \frac{d\hat{x}}{d\hat{s}} \right)^* + \frac{\tilde{n}^f}{32} \left( \frac{d^2}{\eta(d)} \frac{d\hat{x}}{d\hat{s}} \frac{d\hat{x}}{d\hat{s}} \right)^*$$

(2.5)

where $\tilde{n}^f$ denotes the fluid volume fraction per unit $x_0$, $\hat{s}$ is the curvilinear coordinate representing the distance along the vessel axis, $d$ is the vessel diameter, $\eta(d)$ the diameter dependent fluid viscosity, $\hat{x} = (x_0, \hat{x})$ the hierarchical and spatial positions of the vessels and $(f)^*$ the real volume average of quantity $f$ [Huyghe et al (1989a, 1989b)].

The permeability tensor in a point $(x_0, \hat{x})$ is uniquely defined by the fluid viscosity $\eta(d)$ and the geometry of the vasculature within the averaging volume $V_{avg}$ surrounding $\hat{x}$ and belonging to the hierarchical compartment $\partial x_0$ surrounding $x_0$. In the case of straight segments and a diameter independent viscosity, a permeability tensor $4 \hat{K}_{vol}$ for volumetric fluid flow can be derived from Eq. 2.5 by taking the circular vessel cross sections into account. The components of this permeability tensor can be written as:
\[
K_{ij} = \frac{\pi}{128 V_{avg} \partial x_0 \eta} \sum_{n_s} \frac{d^4 \Delta x_i \Delta x_j}{l} ; \ i, j = 0, \ldots, 2
\]  

(2.6)

where \(n_s\) is the number of segments belonging to the hierarchical compartment \(\partial x_0\) and within the \(V_{avg}\) surrounding \(\vec{x}\) (Figure 2.1).

Figure 2.1: Contribution of a segment within \(\partial x_0\) and \(V_{avg}\) surrounding \(\vec{x}\) (shaded area within \(V_{avg}\)) to \(4K_{vol}\)

### 2.3 Quantification of the vascular hierarchy

In the previous section, an expression for the permeability tensor for volumetric flow was derived (Eq. 2.6). In this equation, \(\partial x_0\) denotes one hierarchical compartment (for example the arterial compartment), which is a representative elementary part of the hierarchical range. The subdivision of the hierarchy into compartments requires that the whole hierarchical range is quantified. This is achieved by assigning a hierarchical parameter \(x_0\) to each hierarchical level. This parameter runs from 0.0 at the proximal end (e.g., arterial level) to 1.0 at the distal end (e.g., capillary level). The quantified hierarchical range, together with the spatial quantification of the vascular tree, i.e., geometry, are used for the calculation of \(4K_{vol}\) (Eq. 2.6). This permeability tensor is used in the FE simulation.

In [Vankan et al (1996c)], two \(x_0\)-definitions were used to calculate the \(x_0\)-value for each point in the vascular hierarchy. One \(x_0\)-definition used was directly related to the fluid pressure:

\[
x_0 = -P_{nw}
\]  

(2.7)

where \(P_{nw}\) is the computed network pressure. The negative value of the fluid pressures was used to obtain increasing \(x_0\)-values in the direction from the proximal end of the feeding artery (\(x_0 = 0.0\)) towards the distal end of the capillaries (\(x_0 = 1.0\)). With this definition, further referred to as the pressure definition, a total FE flow of \(1.089 \cdot 10^{-5} \ m^3/s\) was calculated, which was 96% of the total NW flow. The vascular tree that was used, will be used in this study also.
The quality of the FE model was investigated in the case that \( x_0 \) was derived from structural parameters, such as vessel diameters in branching points:

\[
x_0 = -\sqrt[3]{\frac{d_1 + d_2}{2d_n}}
\]

where \( d_1 \) is the largest diameter and \( d_2 \) is the second largest diameter in a branching point, and \( d_n \) is a characteristic diameter, for example the capillary diameter. If the branching point was a capillary terminal node, it was assumed that \( d_2 = d_1 = d_n \). In between these branching points, the value of \( x_0 \) varied linearly. This definition is called the diameter definition. The FE simulation with this diameter definition gave a total flow of \( 0.705 \cdot 10^{-5} m^3/s \), which was 62% of the total network flow. Mainly at the higher \( x_0 \) levels, a wide range of fluid pressures was found for both the FE and the NW analysis. This was due to the \( x_0 \)-definition, which corresponded poorly to the fluid pressure.

The primary unknown quantity in the hierarchical mixture model, \( \tilde{\mu}(x_0, \bar{z}) \) (Eq. 2.6), represents the fluid (blood) pressure, averaged over the segments within \( \partial x_0 \) and \( V_{avg} \) surrounding \( (x_0, \bar{z}) \). Therefore \( \partial x_0 \) should, just like \( V_{avg} \), only contain segments in which the fluid pressure is within a relatively narrow range about the average pressure \( \tilde{\mu} \). As a consequence, the \( x_0 \)-value should be closely related to the fluid pressure in the vasculature in order to obtain a representative average value of the fluid pressure within the spatial averaging volume.

### 2.4 Strahler ordering of vascular trees

With the diameter definition, the main problem was that the connectivity in the arterial tree was not taken into account. This connectivity is important to make a distinction between two vessels of the same order, one of which is connected to a large artery (low hierarchical position) and the other is connected to a small arteriole (high hierarchical position). To avoid this problem, a hierarchical ordering scheme can be used to incorporate the connectivity of the vascular tree. In such a hierarchical ordering scheme, the most peripheral vessels are assigned order one. The method of ordering presented by Strahler [Strahler (1952)] was found to be particularly suitable. The major advantage of the Strahler ordering is that it is purely topological, referring only to node-node interconnections and not to lengths, diameters, or orientations of the various vessels comprising a network.

With the use of the Strahler method, vascular orders are assigned. The number of vessels, mean vessel diameter, and mean vessel length typically form a geometric progression in successive orders. The Strahler method has been applied to the entire pulmonary arterial and venous trees [Koller et al (1987)].

**Order 1** is assigned to the arteriole terminating in the capillary network as described above. When two vessels of **order 1** join, a second-order segment is formed. The order of a segment is increased by one only if its two daughter segments are of equal orders; if the two daughter segments are of different orders, the parent segment retains the higher of the two orders. The bifurcations (nodes) at which the adjacent segments change their Strahler order are called main-nodes (Figure 2.2, where the highest Strahler order equals **order 3**).

When the Strahler orders of all the segments in the tree are determined, the corresponding \( x_0 \)-values are calculated in three steps:

**Step 1:** The distal ends of the most distal capillaries, so vessels with **order 1**, get an \( x_0 \)-value of 1.0, while the proximal end of the feeding artery, so a vessel with the highest Strahler order in the whole tree, gets an \( x_0 \)-value of 0.0. The result of this step is that all the in- and outflow points of the vasculature have an \( x_0 \)-value and that the \( x_0 \)-range is fixed.
Step 2: Next, the $x_0$-values of the main-nodes are determined. First, a linear relation between the highest Strahler order in a main-node ($h_{stranod}$) and the $x_0$-value is applied by:

$$x_0 = \left( \frac{h_{stratree} + 1}{h_{stratree}} \right) - \left( \frac{h_{stranod}}{h_{stratree}} \right)$$

where $h_{stratree}$ denotes the highest Strahler order in the tree. This relation is depicted in Figure 2.2 with $h_{stratree} = 7$. This $x_0$-definition is further referred to as the linear Strahler definition.

Also a quadratic relation between $h_{stranod}$ and $x_0$ was used to optimize the pressure-$x_0$-relation, as will be explained later, and to obtain this, the following formula was used:

$$x_0 = 1.0 - \left( \frac{(h_{stranod})^2}{(h_{stratree} + 1)^2} \right)$$

See also Figure 2.2. With this quadratic formula, further called the quadratic Strahler definition, the $x_0$-values in the main-nodes are different, resulting in other FE flow and pressure distributions.

Step 3: After fixing all the $x_0$-values of the terminal nodes (Step 1) and the main-nodes (Step 2), the $x_0$-values of all the other nodes are determined by linear interpolation: first, all the adjacent segments with the same Strahler order are called an element, that means for example that an element of order 3 can contain one or more adjacent segments of order 3, with the restriction that these segments are coupled in series. Next, all the $x_0$-values of the nodes of one specific element are calculated step by step using linear interpolation, starting with the highest order elements.

At the beginning, one node of the highest order elements is picked, let’s assume that this is node $j$. Then a search for the neighbouring nodes begins from node $j$ in two opposite directions along

Figure 2.2: Illustration of the Strahler quantification method when the highest Strahler order equals order 3
the element towards its neighbouring nodes $j_1$ and $j_2$. The neighbouring nodes of node $j$ are the nodes which already have an $x_0$-value and are attached to the same element as node $j$ with the most nearby positions. These neighbouring nodes can be main-nodes or terminal nodes, but also other nodes which $x_0$-values are already calculated. The two $x_0$-values of these neighbouring nodes are determined and in the same time the distance from node $j$ to node $j_1$ and node $j_2$ are calculated ($l_1$ and $l_2$ respectively). With the following expression (linear interpolation) the $x_0$-value of node $j$ can be calculated:

$$x_0(j) = \frac{x_0(j_2)l_1 + x_0(j_1)l_2}{l_1 + l_2}$$

(2.11)

This expression is applied to node $j$ and after this, the whole procedure will be repeated, continuing with all the other nodes on the highest order element. When all the $x_0$-values of the highest order element are determined, this procedure is repeated for all the nodes on the second highest elements, except for these nodes which already have an $x_0$-value. When this is finished too, the procedure is continued for the third highest order elements and so on, till all the nodes of the lowest order elements have an $x_0$-value.

When the whole process of calculating the $x_0$-values of all the nodes of the tree is finished, the hierarchical positions of all the vessels are quantified. By comparing the FE results with the NW calculations, the quality of the $x_0$-definition can be judged.

### 2.5 Arterial tree model

The computer-generated arterial tree model which is used in this study is depicted in Figure 2.3.

![Figure 2.3: Model for the arterial tree](image)

This model of the vascular tree contains 7999 segments, 4000 of which are capillaries, and is embedded in a square piece of tissue of $7.854 \times 10^{-5} \text{ m}^2$ weighing 100 $\text{g}$. The blood is modeled by a Newtonian fluid with a dynamic viscosity of $3.6 \times 10^{-3} \text{ Pa s}$. According to the network analysis, it conveys a total blood flow of $1.133 \times 10^{-5} \text{ m}^3/\text{s}$, which results from the network pressure boundary conditions of $13.3 \text{ KPa}$ arterial pressure and $8.0 \text{ KPa}$ capillary pressure.
2.6 Finite Element analysis

The blood perfused, square piece of tissue is modeled by a FE mesh of $12 \times 12$ equal, square linear elements, each containing 5 fluid compartments of $\delta x_0 = 0.2$. Permeability tensors are calculated according to Eq. 2.6 for each compartment of each FE. As a result of this calculation, a three-dimensional field of permeability tensors is created, each of them constant per FE and per compartment. A circular averaging volume with a diameter of three times the element edge size is used to achieve smooth variation of the element permeability tensors. Volume averages of the network pressure (Eq. 2.12) at the lowest and the highest hierarchical level of the tree (artery and capillary level respectively) are prescribed as boundary conditions in the nodal points of the FE model (Figure 2.4).

Hierarchical fluid pressures on the boundaries ($x^b_{0i}$) of the compartments are calculated in the nodal points of the mesh, where $i$ represents an index belonging to a specific boundary of a compartment. The FE results for fluid pressure represent the real volume averages of the fluid pressure.

The corresponding volume averaged NW pressures ($P^I_{nw_i}$) can be determined as follows (Eq. 2.12):

$$P^I_{nw_i} = \frac{1}{V^f} \sum_{n_{seg}} P_{seg} \bar{V}_{seg}$$  \hspace{1cm} (2.12)

where $\sum_{n_{seg}}$ can be explained as the sum over all segments ($a_s$) within the averaging volume $V_{avg}$ that surrounds nodal point $I$ and within the $x_0$-range $[x^b_{0i} - (\delta x_0)/2, x^b_{0i} + (\delta x_0)/2]$. $P_{seg}$ represents the average pressure in a segment, $\bar{V}_{seg}$ is the volume of a segment and $V^f$ is the total fluid volume within $V_{avg}$:

$$V^f = \sum_{n_{seg}} \bar{V}_{seg}$$  \hspace{1cm} (2.13)

The nodal values of the FE fluid flow are calculated by taking the integral over the domain of the nodal shape function $\phi^I$, where $I$ represents a specific nodal point, multiplied by the fluid flow passing a compartment boundary $x^b_{0i}$ per unit of volume.

The corresponding value of the NW flow ($Q^{I}_{nw_i}$) can be expressed by Eq. 2.14:

$$Q^{I}_{nw_i} = \sum_{n_{seg}} \phi^I(x) Q_{seg}(x^b_{0i})$$  \hspace{1cm} (2.14)
This previous equation can be explained as a summation over \( n_s \), segments of the product of the nodal shape function value at the position of the segment (\( \phi^I(x) \)) and the segment flow passing the compartment boundary (\( Q_{seg}(x_0) \)).

To evaluate the effect of the hierarchical quantification on the FE results, different \( x_0 \)-definitions are used. First, the pressure definition and an alternative diameter definition, in which each of the terminal outflow nodes is assigned \( x_0 = 1.0 \), are applied. Next, the linear and quadratic Strahler definitions are used. In the vascular tree of Figure 2.3, Strahler order runs from 1 to 7.
Chapter 3

Results

In this chapter, the results of the FE simulations will be compared with the results of the NW analysis. In the first part, the results for the pressure definition and the alternative diameter definition will be presented. After that, the correlation between the network pressure and the \( z_0 \)-value at one specific hierarchical level will be dealt with. Finally, the results of the linear and quadratic Strahler definitions are discussed.

For the pressure definition, a total FE flow of \( 0.1089 \times 10^{-4} \text{ m}^3/\text{s} \) was obtained, which is 96% of the total NW flow. For the alternative diameter definition, the FE simulations result in a total flow of \( 0.0738 \times 10^{-4} \text{ m}^3/\text{s} \), which corresponds to 65% of the total NW flow.

The distributions of the fluid pressure at the pre-capillary level (\( z_0 = 0.8 \)) and the fluid flow at the capillary level (\( z_0 = 1.0 \)) are considered inside the FE domain. At this level, the distributions of these quantities are the most dependent on the \( z_0 \)-definition. The pre-capillary pressure is used, because the capillary pressure is prescribed as a boundary condition. The distribution results for the pressure definition and the alternative diameter definition are presented in Figure 3.1 and Figure 3.2 respectively.

In case that \( z_0 \) is based on the pressure definition, the results for the FE and the NW simulations correspond well. The locations of the area where the NW capillary flows are large are well predicted by the FE simulations. The maximum NW flow is less than the maximum FE flow. The FE pre-capillary pressure distribution doesn’t correspond well with the NW pressure distribution.

For the alternative diameter definition, the prediction of the NW flow distributions by the FE simulation is worse, compared to the results that are obtained with the pressure definition. The correspondence between the FE and NW pressure distributions is better for the alternative diameter definition, although the FE-pressures are lower.

To optimize the correspondence between the NW and the FE pressure and flow distributions, as well as the total flow values, the relation between the dimensionless network pressures and the applied \( z_0 \)-definitions has to be as good as possible. For the pressure definition and the diameter definition, the \( z_0 \)-values versus the NW pressure values are presented in Figure 3.3 and Figure 3.4 respectively.

The pressure definition results in a straight line, because the \( z_0 \)-values are directly related to the network pressure. For the alternative diameter definitions, a relatively wide range of fluid pressures exists at the higher \( z_0 \)-levels.

For relatively small \( z_0 \)-values (\( z_0 \leq 0.1 \)), the relationship between the \( z_0 \)-values and the dimensionless network pressure is uniquely defined in all the cases. This means that in the proximal part of the tree (the larger vessels), the pressure differences between the vessels with the same hierarchical position is very small. The more the hierarchical parameter \( (z_0) \) increases, the more
Figure 3.1: Comparison between NW and FE pre-capillary pressure (left) and NW and FE capillary flow (right) in the case that $x_0$ is calculated with the pressure definition

Figure 3.2: Comparison between NW and FE pre-capillary pressure (left) and NW and FE capillary flow (right) in the case that $x_0$ is calculated with the alternative diameter definition
the pressures range of all the vessels at the same hierarchical level increases, and this increase in the pressure range is probably caused by the small capillaries which are attached to the larger arteries, in other words, small capillaries with relatively high pressure values. A large pressure range at one specific hierarchical level can affect the FE calculations.

This is one of the reasons to try another quantification method, the Strahler method, which is not based on the diameter of the vessels, but purely on the hierarchical positions of these vessels. As a first trial, the linear Strahler definition was applied. For this definition, the $x_0$-values versus the NW pressure values are presented in Figure 3.5.

The wide range of fluid pressures at one hierarchical level is reduced compared to the results of the alternative diameter definition. The difference between the mean pressure value in a specific hierarchical range decreases with an increasing hierarchical position. This phenomenon is clearly visible in Figure 3.5 ($x_0$ versus NW pressure) and in Figure 3.7 (Strahler order versus NW pressure). A low Strahler order corresponds to a high $x_0$-value. The relationship between the NW pressure and the Strahler order can not be changed. The only way to improve the correlation between the NW pressure and the hierarchical parameter $x_0$ is the introduction of a non-linear dependence of the $x_0$-values on the Strahler order. A quadratic dependence is chosen.
in quadratic Strahler definition. In this $x_0$-definition, the boundary conditions $x_0 = 0$ at the arterial inflow and $x_0 = 1$ at the capillary outflows were taken into account.

Figure 3.7: Strahler order versus mean value (o) and standard deviation (—) of the NW pressure values for each Strahler order

If the quadratic Strahler definition is applied instead of the linear Strahler definition, the correlation between the NW pressure and, indeed, the $x_0$-values is improved (Figure 3.6). This increased correlation can cause an improvement of the results of the FE calculations. This was the main reason to try this quadratic Strahler definition instead of the linear Strahler definition. The NW and FE total flows are presented in Table 3.1 for all $x_0$-definitions.

<table>
<thead>
<tr>
<th>$x_0$-definition</th>
<th>NW flow ($*10^{-4} m^3/s$)</th>
<th>FE flow ($*10^{-4} m^3/s$)</th>
<th>FE/NW *100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>pressure</td>
<td>0.1133</td>
<td>0.1089</td>
<td>96%</td>
</tr>
<tr>
<td>alternative diameter</td>
<td>0.1133</td>
<td>0.0738</td>
<td>65%</td>
</tr>
<tr>
<td>linear Strahler</td>
<td>0.1133</td>
<td>0.0708</td>
<td>63%</td>
</tr>
<tr>
<td>quadratic Strahler</td>
<td>0.1133</td>
<td>0.0961</td>
<td>85%</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison between NW and FE total flow results

The best total flow agreements between NW and FE simulations can be obtained by taking the pressure definition. The alternative diameter definition gives larger differences in the total flow values. The linear Strahler definition seems to result in approximately the same total flow as the alternative diameter definition does. The quadratic Strahler definition shows a better correspondence between the FE total flow and the NW total flow, which is probably caused by the improved relationship between the hierarchical parameter $x_0$ and the network pressure. The pre-capillary pressure and capillary flow distributions for the linear and quadratic Strahler definition are presented in Figure 3.8 and Figure 3.9.

The FE pre-capillary pressure and capillary flow distributions for the linear Strahler definition does not differ much from the FE results of the alternative diameter definition. The positions and the values of the maximum FE flows are approximately the same for both $x_0$-definitions. The quadratic Strahler definition obviously gives better FE pre-capillary pressure and capillary flow distributions than the other $x_0$-definitions. The FE pressure distributions and their maximum values correspond well with the NW distributions, as well as the maximum values of the capillary flow and their positions.
### Figure 3.8: Comparison between NW and FE pre-capillary pressure (left) and NW and FE capillary flow (right) in the case that $\sigma_0$ is calculated with the linear Strahler definition

<table>
<thead>
<tr>
<th></th>
<th>Pre-capillary pressure</th>
<th>capillary flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FE</strong></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td><strong>NW</strong></td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
</tbody>
</table>

### Figure 3.9: Comparison between NW and FE pre-capillary pressure (left) and NW and FE capillary flow (right) in the case that $\sigma_0$ is calculated with the quadratic Strahler definition

<table>
<thead>
<tr>
<th></th>
<th>Pre-capillary pressure</th>
<th>capillary flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FE</strong></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td><strong>NW</strong></td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
</tr>
</tbody>
</table>
Summarizing, the best comparison between network and FE results can be obtained by using the quadratic Strahler definition for the hierarchical quantification, in the case of a vascular geometry which corresponds to the geometry of the vascular model.
Chapter 4
Discussion and conclusions

The hierarchical mixture model has been used for quantitative analysis of fluid flow through vascular tree models. The Darcy permeabilities are derived from the micro-structure of the vasculature. Therefore, the hierarchy and the spatial geometry of the vasculature have to be quantified. For the hierarchical quantification, a hierarchical parameter \( z_0 \) is used.

The hierarchical parameter has to correlate to the fluid pressure as good as possible in order to be able to calculate the fluid flows in the vasculature accurately. To evaluate the influence of the hierarchical quantification on the FE calculations, several \( z_0 \)-definitions are applied on a model of a vascular tree.

The results of the FE calculation with the pressure definition correspond well with the results of the NW analysis. Although, to apply this definition on the vascular tree, the pressure values in the whole tree have to be available, which is generally not the case. Therefore it is necessary to base the \( z_0 \)-definition on structural data of the vasculature. For the alternative diameter definition, the correlation between the fluid pressure and the segment diameter is very poor, mainly in the arteriolar and capillary range, which causes a worse correspondence between the FE and NW total flows and pressure and flow distributions.

In order to obtain a better correlation between the \( z_0 \)-value and the fluid pressure, a different approach of the hierarchical parameter definition is applied in this study: the Strahler ordering method. The advantage of this method is the capability to take the connectivity of the successive vessels into account.

The correspondence between the FE simulation and the NW analysis is not improved with the linear Strahler definition, compared to the results of the alternative diameter definition, although the correlation between \( z_0 \) and the fluid pressure is increased. In order to increase this correlation further, the quadratic Strahler is applied. The results for both the FE total in- and outflows, as well the FE flow and pressure distributions at capillary and pre-capillary level respectively, show an increased correspondence with the NW results. This is caused by the improved correspondence between the hierarchical parameter and the fluid pressure. The range of fluid pressures at the capillary level is reduced.

In case of the Strahler quantification method, the dependence of the hierarchical parameter \( z_0 \) on the Strahler order at the main-nodes has a great influence on the FE pressure and flow results. Therefore, other (non-linear) relations between \( z_0 \) and the Strahler order may be derived which are even better than the quadratic Strahler definition.

In literature, a modified Strahler method is described, and this method is called the diameter-defined Strahler method. The only differences are several restrictions, that take the diameters of the vessels into account. Previous studies [Jiang et al (1994)] showed that the pressure range at one specific level of the capillaries can be further reduced by the diameter-defined Strahler ordering method in comparison to the conventional Strahler method. For this reason it would...
be interesting to use this method as another trial to improve the results of the FE calculations. In order to be able to evaluate the influence of the hierarchical parameter on the FE pressure and flow results better, it is recommendable to carry out FE simulations on other vascular trees. The FE model of the vascular tree predicts the pressure and flow distributions in this vascular tree. The results of the simulations with this FE model depend on the method that is used to quantify the hierarchy of these trees. The Strahler ordering method offers a good point of departure for a further improvement of the quantification of vascular trees.
Chapter 5

References


Huyghe J.M. (1986), Non-linear finite element models of the beating left ventricle and the intramyocardial coronary circulation, PhD-desertation, Eindhoven University of Technology, Department of Mechanical Engineering.


