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On the order of simultaneously stabilizing compensators

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Abstract

In this paper, simultaneous strong stabilization problem is considered and it is shown that there is no upper bound for the minimal order of a simultaneously strongly stabilizing compensator, in terms of the plant orders. A similar problem was also considered in [10], where it was shown that such a bound does not exist for the strong stabilization problem of a single plant. But the examples given in [10] were *forcing* an approximate unstable pole-zero cancellation, or *forcing* the distance between two distinct unstable zeros to go zero. In this paper it is shown that: (i) if approximate unstable pole-zero cancellation does not occur, and the distances between distinct unstable zeros are bounded below by a positive constant, then it is possible to find an upper bound for the minimal order of a strongly stabilizing compensator; (ii) and for the simultaneous strong stabilization problem (even for the two plant case), such a bound cannot be found.

1. Introduction

It is known that, a plant P of order n can be internally stabilized[1] by a compensator C of order at most $n-1$. As shown in [10], if the compensator is required to be stable, no such bound exists in terms of the plant order. The examples given in [10] are

$$G_\epsilon(s) = \frac{(s-1)(s-2)}{s(s-2-\epsilon)(s-3)}, \quad \text{and}$$

$$H_\epsilon(s) = \frac{(s-(1+j\epsilon))(s-(1-j\epsilon))}{s^2(s-2)},$$

and as $\epsilon \to 0^+$, the minimal order of a strongly stabilizing compensator, goes to infinity, [10]. It is clear that the first example *forces* an approximate unstable pole-zero cancellation and the second one *forces* the distance between two distinct unstable zeros to go to zero.

In Section 2, it will be shown that as long as approximate unstable pole-zero cancelation is not *forced*, and the distances between distinct unstable zeros, are not *forced* go to zero i.e. as long as we know a positive lower bound, δ_o for $\delta(P) = \max\{\delta_1(P),\ \delta_2(P)\}$ where $\delta_1(P) = 1$, if $P(s) = \frac{k}{s^n}$, otherwise

$$\delta_1(P) = \frac{\min\{|z_o - z_p| \ :\ z_o \text{ is a } \mathbb{C}_+ \text{ zero }, z_p \text{ is a } \mathbb{C}_+ \text{ pole of } P\}}{\max\{|z| \ :\ z \text{ is a } \mathbb{C} \text{ pole or zero of } P\}}$$

Similarly, $\delta_2(P) = 1$, if $P(s) = \frac{k}{s^n}$, otherwise

$$\delta_2(P) = \frac{\min\{|z_1 - z_2| \ :\ z_1, z_2 \text{ are distinct}_+ \text{ zeros of } P\}}{\max\{|z| \ :\ z \text{ is a pole or zero of } P\}}$$

and $\mathbb{C}_+$ is the closed right half plane, then it is possible to find an upper bound on the minimal order of a strongly stabilizing compensator. Namely, there exists an upper bound $M(n, \delta_o)$ such that, if P is strongly stabilizable then P is strongly stabilizable by a compensator of order at most $M(n, \delta_o)$.

In Section 3, it will be shown that, as $\beta \to (4\pi^2/\Gamma^4(1/4))^+$ the minimal order of a compensator, which simultaneously strongly stabilizes

$$P_{1,\beta}(s) = \frac{(s-1)^2}{(1+\beta)(s+1)(s-\frac{1-\beta}{1+\beta})}, \text{and}$$

$$P_{2,\beta}(s) = \frac{(s-1)^2}{(1-\beta)(s+1)(s-\frac{1+\beta}{1-\beta})}, \tag{2}$$

goes to infinity. Note that $\delta(P_{1,\beta})$ and $\delta(P_{2,\beta})$ are bounded below by a positive number. The results of Section 3 are based on [3].

Simultaneous stabilization problem of n plants is equivalent to the simultaneous strong stabilization problem of $n-1$ plants, [12]. Stabilization by a stable compensator problem is also called the strong stabilization problem. For the one plant case, strong stabilizability is equivalent to the so called parity interlacing property, [16]. But the problem of stabilizing two plants with a stable compensator (equivalently the problem of simultaneously stabilizing three plants), seems to be more difficult. Several necessary conditions [5, 14] and sufficient conditions [15, 2, 8] are known, as well as necessary and sufficient ones which involve untractable transcendental equations [6, 4]. Recently, Blondel and Gevers showed

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[1] Throughout this paper, only proper plants is considered and stability means internal stability [12].

that simultaneous stabilization problem of three plants (equivalently the simultaneous strong stabilization problem of two plants) is rationally undecidable, [3]. This means that, it is impossible to find a necessary and sufficient condition which involves only rational operations (i.e. addition, subtraction, multiplication and division) on the coefficients of the plants, inequalities (i.e. $>, \geq, <, \leq$) and logical connectives (i.e. AND, OR, NOT). For example, the well known Routh-Hurwitz test for the stability of a polynomial $p(s)$, involves only rational operations on the coeeficients of $p(s)$, inequalties and logical connectives, hence the stability of a polynomial $p(s)$, is rationally decidable. Similiarly, the strong stabilization problem is rationally decidable, because parity interlacing property can be checked by using only rational operations, [1]. The result presented in [3] shows that no such simple iff type of condition can be found for the simultaneous strong stabilization problem of two plants. Using these observations, in Section 3, it will be shown that the minimal compensator order may be unbounded for the simultaneous stabilization problem of three plants, equivalently for the simultaneous strong stabilization problem of two plants even if $\delta(P)$'s are bounded below by a positive number.

2. Strong stabilization: one plant case

In this section, it is shown that given the order n of a plant P and a positive lower bound δ_o for $\delta(P)$, it is possible to find an upper bound $M(n, \delta_o)$ for the minimal order of a strongly stabilizing compensator. Let $v(P)$ denote the minimal order of a compensator which strongly stabilizes $P(s)$. Without loss of generality, we may assume that $P(s) \neq k/s^n$. Because, for $k \neq 0$, $C(s)$ strongly stabilizes $1/s^n$ iff $C(s)/k$ strongly stabilizes k/s^n, so $v(k/s^n) = v(1/s^n)$ for all $k \neq 0$. Therefore, $\sup_{k\neq 0} v(k/s^n) = v(1/s^n) < \infty$. Define

$$\kappa = 2\max\{|z| \ : \ z \text{ is a pole or zero of } P(s)\} \neq 0.$$

Since $C(s)$ strongly stabilizes $P(s)$ iff $C(\kappa s)$ strongly stabilizes $P_1(s) := P(\kappa s)$, we have $v(P(s)) = v(P_1(s))$. Note that, all $\mathbb{C}$ poles of $P_1(s)$ have absolute value $\leq 1/2$ and the distance between a $\mathbb{C}_+$ pole and a $\mathbb{C}_+$ zero of $P_1(s)$ as well as the distance between two distinct $\mathbb{C}_+$ zeros of P_1 is greater that $\delta_o/2$. Let $p_1(z) := P_1(\frac{1+z}{1-z})$, then $c_1(z)$ strongly stabilizes $p_1(z)$ iff $C(s) := c_1(z)|_{z=\frac{s-1}{s+1}}$ strongly stabilizes $P(s)$. Note that the orders of P_1 and p_1 are the same as well the orders of C_1 and c_1. The distance between a $\overline{\mathbb{D}}$ pole and a $\overline{\mathbb{D}}$ zero of $p_1(z)$ as well as the distance between two distinct $\overline{\mathbb{D}}$ zeros of p_1 is greater than $\widehat{\delta}_o := \min\{1, \delta_o/4\}$. Because for $s_1, s_2 \in \mathbb{C}_+$ with $|s_1|, |s_2| \leq 1/2$ and $|s_1 - s_2| > \delta_o/2$,

$$\left|\frac{s_1-1}{s_1+1} - \frac{s_2-1}{s_2+1}\right| = \frac{2|s_1-s_2|}{|(s_1+1)(s_2+1)|} > \frac{\delta_o}{4}.$$

Furthermore, the point $s = \infty$ in the s-plane is mapped to $z = 1$ in the z-plane. If $s_1 \in \mathbb{C}_+$ with $|s_1| \leq 1/2$, then

$$\left|\frac{s_1-1}{s_1+1} - 1\right| = \frac{2}{|s_1+1|} > 1$$

hence the positive lower bound $\widehat{\delta}_o := \min\{1, \delta_o/4\}$ is justified. Furthermore, poles and zeros of $p_1(z)$ have absolute value ≤ 4, because if $s \in \mathbb{C}$ and $|s| \leq 1/2$, then $|(s-1)/(s+1)| \leq 4$ and $s = \infty$ is mapped to $z = 1$ which is also ≤ 4 in absolute value.

Let $p_1(z) = n_1(z)/d_1(z)$, where $n_1(z)$ and $d_1(z)$ are polynomials with no common root (hence coprime), with leading coefficient of $d_1(z)$ equal to 1. The distance between a $\overline{\mathbb{D}}$ root of $n_1(z)$ and a $\overline{\mathbb{D}}$ root of $d_1(z)$ as well as the distance between two distinct $\overline{\mathbb{D}}$ roots of $n_1(z)$ are greater than $\widehat{\delta}_o$. To prove that the controller order remains bounded, we will use the unit construction procedure given in [12] and show that it is possible to find a bound on the order of the associated unit. This clearly implies that it is possible to find a bound on the controller order too. And a bound on the controller order in z-domain implies the existence of a similiar bound on the controller order in the s-domain.

Let $n_1(z) = (z - z_1)^{r_1} ... (z - z_t)^{r_t} (z - z_{t+1})^{r_{t+1}} ... (z - z_n)^{r_n}$ with $z_1, ..., z_t \in \overline{\mathbb{D}}$ and $z_{t+1}, ..., z_n \in \mathbb{C} \setminus \overline{\mathbb{D}}$. First, assume that we know t and $r_1, ..., r_t$ but don't know the exact location of $z_1, ..., z_t$ in $\overline{\mathbb{D}}$. If it is possible to find a bound on the order of the associated unit with this much of information, then it is possible to find a bound which depends only on n and δ_o, because there are finitely many possibilities for t and $r_1, ..., r_t$. In the remaining part of the section, it is assumed that n and δ_o is known as well as t and $r_1, \cdots, r_t$. Let us look at more closely the unit construction procedure given in Section 2.4 (Interpolation in the disc algebra) of [12]. Namely, we would like to construct a unit $f(z)$ such that $(f(z) - d_1(z))/n_1(z)$ is stable. Once, such a unit $f(z)$ is constructed, $c(z) = (f(z) - d_1(z))/n_1(z)$ is a strongly stabilizing controller for $p_1(z) = n_1(z)/d_1(z)$. First, construct a polynomial $g(z)$

of order $r_1 + \cdots + r_t$ such that

$$\frac{d^j}{dz^j} e^{g(z)} \Big|_{z=z_i} = \frac{d^j}{dz^j} d_1(z) \Big|_{z=z_i},$$

for $i = 1, ..., t, j = 0, ..., r_i - 1$. Namely,

$$\begin{array}{rcl} g(z_i) & = & \log(d_1(z_i)) \\ g'(z_i) & = & d_1'(z_i)/d_1(z_i) \\ g''(z_i) & = & (d_1''(z_i) - [d_1'(z_i)]^2)/d_1(z_i) \\ \vdots & \vdots & \vdots \end{array} \tag{3}$$

This gives a linear system of equations for the coefficients of $g(z)$. Since $d_1(z)$ is an n^{th} order polynomial with leading coefficient equal to 1 and all of the roots of $d_1(z)$ have absolute value ≤ 4, distance of a root of $d_1(z)$ to any of the z_i's is at least $\widehat{\delta}_o$, and z_i's are in $\overline{\mathsf{D}}$, the right hand sides of the linear system of equations given in (3) are bounded from above (in absolute value) by numbers which depend only on $n, \widehat{\delta}_o, t, r_1, \cdots, r_t$, but independent of the exact location of z_i's. Furthermore, the determinant of the left hand side of (3) is bounded from below (in absolute value) and the entries of the adjoint matrix of the left hand side of (3) is bounded from above (in absolute value) by numbers which depend only on $n, \widehat{\delta}_o, t, r_1, \cdots, r_t$, but independent of the exact location of z_i's. This is because all of the z_i's are in $\overline{\mathsf{D}}$ and the distance between distinct z_i's is at least $\widehat{\delta}_o$. Therefore, it is possible to find upper bounds for the absolute values of the coefficients of $g(z)$ which depend only $n, \widehat{\delta}_o, t, r_1, \cdots, r_t$, but independent of the exact location of z_i's.

At this point, we know the existence of an upper bound for the absolute values of the coefficients of $g(z)$, independent of the exact location z_i's. Therefore $\exists M_1(n, \delta_o)$ such that for all $z \in \overline{\mathsf{D}}$

$$\left|\frac{d^j}{dz^j} g(z)\right| \leq M_1(n, \delta_o), \quad j = 0, \cdots, n-1 \tag{4}$$

therefore

$$\left|\frac{d^j}{dz^j} g(z)^k\right| \leq k^j M_1(n, \delta_o)^k.$$

Since the infinite series

$$\sum_{k=1}^{\infty} \frac{k^j M_1(n, \delta_o)^k}{k!}$$

converges for all $j = 0, \cdots, n-1$, there exist $N_1(n, \delta_o, \epsilon)$ which is independent of the exact locations of z_i's, such that for all $z \in \overline{\mathsf{D}}$,

$$\left|\frac{d^j}{dz^j} \sum_{k=N_1(n,\delta_o,\epsilon)}^{\infty} \frac{g(z)^k}{k!}\right| \leq \epsilon, \quad j = 0, \cdots, n-1. \tag{5}$$

Now, by again Lagrange interpolation, there exists a polynomial $q(z)$ (which of course depends on the exact location of z_i's) such that

$$\frac{d^j}{dz^j} q(z) \Big|_{z=z_i} = \frac{d^j}{dz^j} \sum_{k=N_1(n,\delta_o,\epsilon)}^{\infty} \frac{g(z)^k}{k!} \Big|_{z=z_i},$$

for $i = 1, ..., t, j = 0, ..., r_i - 1$. Since the distance between distinct z_i's are at least $\widehat{\delta}_o$ and (5) holds, by a similiar *linear system of equations* argument, it follows that the coefficients of $q(z)$ are bounded from above (in absolute value) by bounds which depend only on $n, \delta_o, t, r_1, \cdots, r_t, \epsilon$ and all of the upper bounds go to zero as ϵ goes to zero. Therefore, $\exists \Delta(n, \delta, \epsilon)$ such that for all $z \in \overline{\mathsf{D}}$

$$|q(z)| \leq \Delta(n, \delta, \epsilon), \tag{6}$$

and $\lim_{\epsilon \to 0} \Delta(n, \delta, \epsilon) = 0$. Define,

$$f(z) = q(z) + \sum_{k=0}^{N_1(n,\delta,\epsilon)-1} \frac{g(z)^k}{k!}.$$

By (5) and (6), $|e^{g(z)} - f(z)| \leq \epsilon + \Delta(n, \delta, \epsilon)$. Since $e^{g(z)}$ is a unit, if

$$\sup_{z\in} |e^{g(z)} - f(z)| < (\sup_{z\in} |e^{-g(z)}|)^{-1} \tag{7}$$

then f is a rational unit of order at most $nN_1(n, \delta, \epsilon)$ which interpolates $d_1(z)$ at z_i's up to $(r_j - 1)^{\text{st}}$ derivates, for $j = 1, \cdots, t$. Note that, by (4)

$$e^{-M_1(n,\delta_o)} \leq (\sup_{z\in} |e^{-g(z)}|)^{-1}. \tag{8}$$

Now choose $\epsilon = \epsilon(n, \delta)$ such that $\epsilon + \Delta(n, \delta, \epsilon) < e^{-M_1(n,\delta)}$ (This is possible because $\Delta(n, \delta, \epsilon)$ goes to zero as $\epsilon \to 0$). Then, because of (8), (7) holds, so f is a unit of order $\leq M_f(n, \delta) := nN_1(n, \delta, \epsilon(n, \delta))$ satisfying the appropriate interpolation conditions. With this construction, $c_1(z) = (f(z) - d_1(z))/n_1(z)$ is a strongly stabilizing compensator for $p_1(z)$ and an upper bound on the order of the unit gives a similiar upper bound $M(n, \delta)$ on order of the compensator.

The above results can be summarized in:

Theorem 1: For a given n and δ_o, there exists a bound $M(n, \delta_o)$ such that, a plant $P(s)$ of order n and $\delta(P) > \delta_o$, is strongly stablizable iff it is strongly stabilizable by a compensator of order at most $M(n, \delta_o)$.

3. Strong stabilization: two plant case

In this section, we show that, as $\beta \to (4\pi^2/\Gamma^4(1/4))^+$, the minimal compensator order, which simulatenously stabilizes

$$P_{0,\beta}(s) = 0 \ , \ P_{1,\beta}(s) = \frac{(s-1)^2}{(1+\beta)(s+1)(s - \frac{1-\beta}{1+\beta})} \text{ , and}$$

$$P_{2,\beta}(s) = \frac{(s-1)^2}{(1-\beta)(s+1)(s-\frac{1+\beta}{1-\beta})}$$

equivalently the mininal compensator order which strongly stabilizes $P_{1,\beta}$ and $P_{2,\beta}$, goes to infinity. Note that orders of $P_{1,\beta}$ and $P_{2,\beta}$ remain the same as $\beta \rightarrow (4\pi^2/\Gamma^4(1/4))^+$ and neither approximate unstable pole-zero cancelation is *forced*, nor the the distances between distinct unstable zeros are not *forced* to go to zero, i.e.

$$\lim_{\beta \rightarrow (4\pi^2/\Gamma^4(1/4))^+} \delta(P_{1,\beta}) > 0,$$

$$\lim_{\beta \rightarrow (4\pi^2/\Gamma^4(1/4))^+} \delta(P_{2,\beta}) > 0.$$

In [3], it is shown that if $|\beta| > 4\pi^2/\Gamma^4(1/4))$, then there exists a stable compensator which stabilizes $P_{1,\beta}$ and $P_{2,\beta}$, and if $|\beta| < 4\pi^2/\Gamma^4(1/4))$, then no such compensator exists. By the transcendence of $4\pi^2/\Gamma^4(1/4)$, it follows that simulataneous stabilization problem by a stable compensator, is rationally undecidable even for the pairs of plants $(P_{1,\beta}, P_{2,\beta})$, [3].

Theorem 2: For $\beta > 4\pi^2/\Gamma^4(1/4)$, define

$$R(\beta) = \text{minimal order of a compensator which}$$
$$\text{strongly stabilizes } P_{1,\beta} \text{ and } P_{2,\beta}.$$

Then

$$\lim_{\beta \rightarrow (4\pi^2/\Gamma^4(1/4))^+} R(\beta) = \infty.$$

Proof: Define

$$N = \sup_{\beta > 4\pi^2/\Gamma^4(1/4)} R(\beta),$$

if N is finite, then in order to check simulatenous stabilizability by a stable compensator, it is enough to consider compensators C with a state-space realization (A, b, c^T, d) (possibly nonminimal), where A is an N by N matrix, b and c are N dimensional column vectors and d is a scalar. Let (A_i, b_i, c_i^T, d_i) be controllable canonical form realizations of $P_{i,\beta}$ for $i = 1, 2$. Then the condition "C strongly stabilizes $P_{1,\beta}$ and $P_{2,\beta}$" is equivalent to the stability of A, $1 + dd_i \neq 0$, and the stability of the following two matrices:

$$K_i := \begin{bmatrix} A_i - \frac{1}{1+dd_i} b_i d c_i^T & -\frac{1}{1+dd_i} b_i c^T \\ \frac{1}{1+dd_i} b c_i^T & A - \frac{1}{1+dd_i} b d_i c^T \end{bmatrix}, \quad i = 1, 2$$

(These matrices correspond to the "A-matrix" of the closed loop systems). Note that, stability of a matrix is rationally decidable in terms of the entries, i.e. first find the coefficients of the characteristic polynomial, which will be polynomial expressions in terms of the entries and then apply the Routh-Hurwitz criterion which will give a rational decision test in terms of coefficients of the characteristic polynomial hence in terms of the entries of the matrix. So, if we apply this rational decision test to the stability of A, K_1 and K_2 we will get a rational decision test for the simultaenous strong stabilizability of $(P_{1,\beta}, P_{2,\beta})$ in terms of the entries of A, b, c, d and A_i, b_i, c_i, d_i, $i = 1, 2$. At this point, we can use the Tarski's theorem [11, 7] or the Seidenberg elimination algorithm [7] to eliminate the entries of A, b, c, d and obtain a new rational decision test in terms of only the entries of A_i, b_i, c_i, d_i, $i = 1, 2$. Therefore, this procedure will give a rational decision test for the simultaneous strong stabilizability of $(P_{1,\beta}, P_{2,\beta})$ in terms of the coefficients of their numerators and denominators. But, this contradicts with the rational undecidability result of [3]. Therefore N must be infinite, i.e.

$$\sup_{\beta > 4\pi^2/\Gamma^4(1/4)} R(\beta) = \infty.$$

Remark: Tarski's theorem and the Seidenberg elimination algorithm are rather deep theoretical results. These results show that algebraic statements over real numbers [7] are decidable in finitely many steps. Both the Tarski's theorem and the Seidenberg elimination algorithm suffer from exponential growth and hence aren't quite practical methods.

Now we know that, $\sup_{\beta > 4\pi^2/\Gamma^4(1/4)} R(\beta) = \infty$. The compensator $C(s)$ simultaneously strongly stabilizes $P_{1,\beta}(s)$ and $P_{2,\beta}(s)$ iff the discrete time compensator $c(z) = C(\frac{1+z}{1-z})$ simultaneously strongly stabilizes the discrete time plants $p_{i,\beta}(z) = P_{i,\beta}(\frac{1+z}{1-z})$ for $i = 1, 2$. By [3], we know that $c(z)$ simultaneously strongly stabilizes $p_{i,\beta}(z)$ for $i = 1, 2$ iff $c(z)$ is stable and

$$\pm\beta + z + z^2 c(z) \text{ has no zeros on the closed unit disc.}$$

This condition shows that, if $c(z)$ is a simultaneously strongly stabilizing compensator for $p_{1,\beta}(z)$ and $p_{2,\beta}(z)$, then for $\Delta > 0$, $\frac{\beta}{\beta+\Delta} c(\frac{\beta}{\beta+\Delta} z)$ is a simultaneously strongly stabilizing compensator for $p_{1,\beta+\Delta}(z)$ and $p_{2,\beta+\Delta}(z)$. Hence, $\tilde{C}(s) = \frac{\beta}{\beta+\Delta} c(\frac{\beta}{\beta+\Delta} z)|_{z=\frac{s-1}{s+1}}$ is a simultaneously strongly stabilizing compensator for $P_{1,\beta+\Delta}(s)$ and $P_{2,\beta+\Delta}(s)$. Since the orders of $C(s)$ and $\tilde{C}(s)$ are the same, we obtain $R(\beta+\Delta) \leq R(\beta)$. Hence $R(\beta)$ is a non-increasing function of β. This result together with $\sup_{\beta > 4\pi^2/\Gamma^4(1/4)} R(\beta) = \infty$ implies that

$$\lim_{\beta \rightarrow (4\pi^2/\Gamma^4(1/4))^+} R(\beta) = \infty. \qquad \square$$

The above result means that, for two arbitrary plants $P_1(s)$ and $P_2(s)$, with given orders n_1 and n_2 and a given positive lower bound δ_o for $\delta(P_1)$ and $\delta(P_2)$, it is impossible to find an upper bound $M(n_1, n_2, \delta_o)$ for the minimal order of a simultaneously strongly stabilizing compensator. Similary no bound exists for the simultaneous stabilization problem of three plants, even if we know the plants orders and a positive lower bound for $\delta(P_i)$'s.

4. Concluding remarks

In [10], it was shown by examples that, *forcing* an approximate unstable pole-zero cancelation or *forcing* the distance between two distinct unstable zeros to go to zero, may *force* the minimal order of a strongly stabilizing compensator, to go to infinity. In this paper it is shown that, as long as we know a positive lower bound δ_o for $\delta(P)$ and know the order n of P, it is possible to find an upper bound $M(n, \delta_o)$ for the minimal order of a strongly stabilizing compensator. But such a bound cannot be found for the simultaneous strong stabilization problem of two plants, and for the simultaneous stabilization problem of three plants.

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