Summary of a study on the TM01 radiation pattern of corrugated conical horn antennas with small flare angle

Scharten, T.

Published: 01/01/1970

Citation for published version (APA):
Summary of a study on the $T_{M_{01}}$ radiation pattern of corrugated conical horn antennas with small flare angle.

by

ir.Th. Scharten
Summary of a study on the $TM_{01}$ radiation pattern of corrugated conical horn antennas with small flare angle

Th. Scharten

This summary is based on the technical reports Nr ETA-3-1969 and Nr ETA-30-1969 (Eindhoven University of Technology)

1. The configuration and its electromagnetic description

1.1 Corrugated conical horn antenna

We assume a simple harmonic time dependence $e^{j\omega t}$.

We suppose $t_1, t_2 \ll \lambda_{guide}$ and $R_o \gg a \gg \lambda_{guide}$. Furthermore we suppose the flare angle to be so small that if a $TM_{0n}$ mode is excited in the horn, the aperture field can be approximated by the $TM_{0n}$ mode field of a circular cylindrical, corrugated waveguide with inner radius $a$, but now with a quadratic phase distribution in the radial aperture direction. For that case the longitudinal phase factor of the aperture field, $e^{j\beta_n z}$, has to be replaced by the factor $e^{j\beta_n (R_t - R_o)}$,

where $R_t = R_o \sqrt{1 + \frac{c^2}{R_o^2}} \approx R_o + \frac{c^2}{2R_o}$. So this factor reads

$e^{j\beta_n z} e^{j\beta_n \frac{c^2}{2R_o}}$.
2. Fields on the horn surfaces

2.1 Outer surface. On the outer surface we assume the wall currents to be zero; so we have

\[
\vec{n} \times \vec{E} = 0 \\
\vec{n} \times \vec{H} = 0 \\
\text{on } S_c.
\]

2.2 Inner surface. The surface impedance on the inner horn surface is determined by the relation

\[
Z_r = jX_r = \frac{t_1 Z_{\text{groove}} + t_2 Z_{\text{dam}}}{\epsilon_1 + \epsilon_2} = \frac{t_1}{\epsilon_0} Z_{\text{groove}} , \quad \text{on } S_r,
\]

where

\[
Z_{\text{groove}} = jX_{\text{groove}} = \frac{\mu_0}{\epsilon_0} \left( \frac{J_0(k\alpha)}{J_1(k\alpha)} \right) \left( \frac{Y_0(k\alpha + k\delta) - J_0(k\alpha + k\delta)}{J_0(k\alpha) - J_0(k\alpha + k\delta) Y_0(k\alpha)} \right),
\]

being the input impedance of a shortened radial waveguide, \( \alpha \leq \phi \leq \alpha + \delta \), propagating a TEM wave.

2.3 Aperture. The aperture fields are found by considering a circular cylindrical waveguide (radius \( a \)) having the reactance wall condition, given above, and propagating a TM_{0n} mode (\( \exp[+i\beta_0 z] \)). Taking into account the quadratic phase distribution we find

\[
E_{\varphi n} = -i\beta_n \lambda_n \lambda J_n(\lambda_n \rho) \exp \left( i\beta_n \rho^2/2R_0 \right) \\
H_{\varphi n} = -i\omega \epsilon_0 \lambda_n \lambda J_n(\lambda_n \rho) \exp \left( i\beta_n \rho^2/2R_0 \right) , \quad \text{on } S_a,
\]

where \( \lambda_n \) is the \( n \)th root of the TM_{0n} characteristic equation

\[
\lambda J_n(\lambda a) - \omega \epsilon_0 \lambda X_n J_n(\lambda a) = 0.
\]

In our case \( k\alpha \gg 1 \); so the surface reactance can be approximated by

\[
X_r = \frac{\mu_0}{\epsilon_0} \frac{t_1}{\epsilon_1 + \epsilon_2} \tan kd.
\]

It is shown that the characteristic equation has two imaginary solutions (surface wave solutions) if

\[
k \tan kd > \frac{2(t_1 + t_2)}{t_1 a},
\]

and real, single valued solutions if

\[
k \tan kd < \frac{2(t_1 + t_2)}{t_1 a}.
\]
3. The radiation pattern

The energy-flow density can now be evaluated in the usual way, giving

\[ S_n(R, \theta) \propto (k \cos \theta + \beta_n)^2 \left| \int_0^\infty J_1(\lambda_n \rho) J_1(k \rho \sin \theta) e^{i \beta_n \rho \rho^2} \rho d\rho \right|^2 \]

4. Results

Horn dimensions

- \( a = 0.132 \) (m)
- \( d = 0.009 \) (m)
- \( t_1 = 2.5 \) (mm)
- \( t_2 = 4.0 \) (mm)
- \( R = 0.493 \) (m)
- \( \theta_0 = 15^\circ \)

Dimensions of the 'equivalent' waveguide

Figure 3: Dispersion curves for the circular cylindrical, corrugated waveguide

Figure 4-10: Theoretical and experimental TM\(_{01}\) radiation patterns in the frequency range 8.5 - 11.5 GHz

Figure 11: Half beamwidth characteristics
\[ a = 0.137 \ (m) \]
\[ d = 0.009 \ (m) \]
\[ t = 0.385 \]
$TM_{01}$ radiation patterns
theoretical ——
experimental ----
frequency: 8,5 GHz

Figure 4
Figure 5

\( \text{TM}_{01} \) radiation patterns
theoretical ———
experimental ----
frequency: 9 GHz

---

0 10 20 30 40

relative power db

0 12 24 36 48 60

degrees
Figure 6

TM\(_{01}\) radiation patterns
theoretical ———
experimental ----
frequency: 9.5 GHz
$\text{TM}_{01}$ radiation patterns
theoretical ---
experimental ----
frequency: 10 GHz

Figure 7
$T_{M_{01}}$ radiation patterns
- theoretical
- experimental

frequency: 10.5 GHz

Figure 8

Relative power

db

0 12 24 36 48 60

Degrees
Figure 9

TM$_{01}$ radiation patterns
theoretical ——
experimental ———
frequency: 11 GHz
Figure 10

$TM_{01}$ radiation patterns

theoretical

experimental

frequency: $11.5 \, \text{GHz}$
Half beamwidth at 0, 10, 20, 30 db level respectively, as a function of frequency.