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Thermo-Mechanical Behaviour of a CMM

Design and evaluation of a model based on a linear temperature distribution field

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13 APR. 1992
SUMMARY

A study on the thermo-mechanical behaviour of a CMM is presented in this report. Large source of measurement errors with CMMs is due to the non-controlled temperature conditions. Software correction which is the aim of the described research seems to be an economical solution to this problem.

The temperature profile of the CMM is assumed to have a linear temperature distribution field and is taken as a basic assumption for the development of a model by which the angular mechanical deformations (bending) of the CMM can be described.

The model is derived on the base of thermal-stress analyses. The X-beam behaviour is described by the deformation field for beams, Y-beam structure is treated by the deformation field for a three dimensional structure.

The presented model is divided into three parts. In the first part, the thermal gradients are calculated from the temperature measurements at certain points of the CMM structure. In the second part of the model the angular deformations of the X-beam are calculated by the method of "separate elements" and converted into output comparable with the measurable rotational deviation values. The third part express the calculation of the angular deformations of the Y-beams.

The model is verified by measurements performed on the X-beam and Y-beams separately. An experimental temperature field was created with heating panels radiating onto the structure of the X-beam and the Y-beams. The developed measurement set-up allowed to measure the thermal deformations and their cause, the temperature field at the same time. Laserinterferometry was used to measure the rotational deviations (bending). The different placement configuration of Pt-100 sensors were used to detect the temperature of the observed structure.

Comparison of the measured and calculated results for the X-beam show that the presented model is able to give a close prediction of the deformations. In the range of the verified thermal gradients, 3-15°C/m, the model predicts the measured results by 80-100%. From the statistical determination of the confidence intervals for the calculations and the measurements is shown that the remaining differences between them can be explained by the inaccuracies of the measurements.
For the Y-beams, however, rather big relative differences between the measured and calculated results are found. These differences are due to incorrect thermal gradient input data into the model, caused by acquiring corrupted temperatures. Due to the construction of the Y-beams of the CMM, the temperatures could not be measured at most suitable positions. Direct radiation onto the sensors and/or local heat accumulation due to bad ventilation has lead to erroneous temperature measurements.

The recommendations and the discussion about sensor reduction for further research is presented.
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1 INTRODUCTION

Coordinate Measuring Machines (CMMs) have replaced many conventional measurement methods in production processes in recent years because of their versatility and capability to measure accurately and efficiently. At present time, because of an increasing demand for quality control a large number of CMMs are being placed into an industrial environment, like open production halls. CMMs operating in a manufacturing area are frequently integrated into material flow systems and are thus exposed to varying temperature conditions. Hence, the requirement of high precision measurements with CMMs in production halls, while maintaining or even improving its accuracy, requires the reduction of their thermal sensitivity. It has been found that thermal effects are a large source of geometrical deviations. Geometrical deviations as a result from thermal deformations of the structure may be of the same order or an even higher order of magnitude as errors due to the manufacturing process and the dynamic behaviour. These experiences and the demand for high precision measurements have led to research of the thermal behaviour of CMMs.

The main goal of the investigation published in this report was to find a description of the influence of the linear temperature distribution field on the structure of a CMM. This research is part of the Thermal Research Project of two partners:
- Precision Engineering Laboratory of Eindhoven University of Technology (TUE)
- Mitutoyo Nederland B.V. (MLE).

The work was performed at the Technical University on a Mitutoyo FN905 CMM.

![Diagram](image-url)

**Figure 1.1** Report concept.
The basic structure of this report is depicted in figure 1.1. It illustrates the two different paths leading to a model for the thermal behaviour of the CMM. One path is based purely on theory to derive a descriptive model. The second path expresses the experimental evaluation of the model.

In chapter 2 a qualitative analysis of the deviations affecting the CMM accuracy is presented. The established nomenclature of the geometrical deviations is quoted. Some methods for thermal sensitivity reduction are mentioned and the assumptions for the mathematical model are given.

A summary of the thermal-stress analysis and the derivation of the mathematical model for the thermo-mechanical behaviour of the CMM are presented in chapter 3.

The experiments for measuring the thermo-mechanical behaviour on the CMM are described in chapter 4. In this chapter all preparatory experiments, developed measurement set-ups and the results of the measurements are presented.

An evaluation of the mathematical model and the experiments is given in chapter 5. A comparison between the measurements and the calculated results from the model is given. The results for the X-beam and Y-beam are analysed separately. For the X-beam the possible sensor reduction is discussed.

Conclusions and recommendations for further research are presented in chapter 6.
2 DEVIATIONS AFFECTING THE CMM ACCURACY

2.1 Geometrical deviations

A CMM is a multi axis machine used for the verification of the dimensions of products. The measurement results can be used for the correction of systematic deviations in the production process. In principle the demand on the CMM’s accuracy should therefore be one order of magnitude more accurate than a machine tool. The basic structure of the Mitutoyo FN905 consists of three perpendicular guides; the X-beam, the left- and right-hand Y-beams and the Z-beam, as depicted in figure 2.1. On each of these guides carriages provide the translation movements. In the ideal situation an error free movement of the carriages and an exact squarness between the guides is expected. This is not the case in reality. The imperfect geometrical shape of the guides and carriages and their imperfect movement are caused by the non-ideal manufacturing process or by external and internal influences on the structure of the CMM. These influences result in a deviation of the probe position and thus in an erroneous detection of the dimensions of the product under measurement. These accuracy deviations of the CMM are called geometrical deviations, besides them various other deviation sources affect the accuracy of the CMM in addition.

![Diagram of Mitutoyo FN905 CMM](image)

Figure 2.1 Basic structure of the Mitutoyo FN905.
In general, the deviations can be divided into two main groups: machine related and external deviations, which are caused by the following sources (Teeuwsen, 89):

- **machine related:**
  - measuring system
  - probing system
  - machine coordinate system

- **external:**
  - environment
  - software
  - measuring object

In this publication, the attention is directed to geometrical deviations. The various geometrical deviations for x-guide and their notations are presented in figure 2.2. X-guide guide with carriage provides six degrees of freedom. These degrees of freedom are represented in the principle definition by six geometrical deviation parameters:

- three translation deviations: xtx, xty and xtz (linearity, straightness in two directions)
- three rotational deviations: xrx, xry and xrz (roll, pitch, yaw).

Figure 2.2  The geometrical deviations of the x-guide with carriage and their notation.
The notation and definition of these parameters is according to the German VDI 2617 Blatt 3, standard. Geometrical deviations are generally denoted by lower case characters, for translation and rotation by $ij$ resp. $irj$. The first character $i$ represents the axis of movement. The second character refers to the type of geometric deviation, in this case translation resp. rotation. The last character $j$ represents the axis along or about which the geometrical deviation is acting.

A number of twenty one sources of geometrical deviations may occur on the structure of a CMM with three guides and three carriages. These deviations are nine translations, nine rotations and three squareness deviations. The latter ones result from an imperfect perpendicular link between the guides.

2.2 Thermally induced deformations of the CMM

A different surrounding temperature than the reference temperature of 20 °C, as defined in DIN 102, causes changes in the CMM's geometry. Geometrical deviations of the CMM may be caused by manufacturing errors, adjustment errors, and by thermal effects. The latter are discussed in this report. Basically, the magnitude and the type of geometrical deviation depend on the temperature field in the structure of the CMM. The temperature field depends on two variables:

- **time**: When the temperature field varies with time, the geometrical deviations do not only depend on the temperatures in the structure at a certain moment, but also on the temperature history. This makes the thermal deviation model very complex. However, when the temperature is only changing slowly in time, the temperature field can be considered stable (quasi-static). This case is assumed in this report.

- **position**: The type of deformation is depending upon whether or not the temperature is changing with position.

* If the temperature is not changing with the position it is called *uniform*. When the uniform temperature differs from the reference temperature of 20 °C it results in a three dimensional uniform expansion of the constructional parts and the measuring scales of the CMM. During this thermo-mechanical effect no stresses arise in the structure. The expansion along the three axis can be described by $iti$. 

- 5 -
If the temperature is changing with the position it is non-uniform and thermal gradients exist. The effect of this temperature on a simple beam can be split into two parts:

- expansion along the longitudinal axis denoted generally by $\epsilon_i$
- bending around both lateral axes, denoted generally by $\epsilon_j$ and $\epsilon_r$.

The expansion of the beam results in the linearity deviation and both bending effects cause the rotational and also straightness deviations described in paragraph 2.1 and figure 2.2.

In the case of CMMs, the measuring accuracy is generally limited by five main thermal sources. In figure 2.3 a diagram of these sources and their influence is depicted:

1. temperature changes provided by the room environment
2. the heat generated by external sources
3. the effect of the people
4. heat generated by internal sources of the machine
5. the initial thermal situation of the machine

1. The room environment without any special local external heat sources usually creates a linear temperature distribution field with constant thermal gradients, affecting the structure of the CMM with expansion and bending effects. During this quasi-static temperature field no stresses occur in the mechanically free structure.

2. The situation changes to a non-uniform temperature distribution field when the CMM is subjected to local external heat sources. These are for example: radiation from sunshine, radiation from a neighbouring welding machine, sources of illumination like lamps when switched on or off, radiation from walls and ceilings, inappropriately placed air-conditioning ventilators. External heat sources may include also system controller electronics usually placed close to the machine.

3. Another minor influence has the presence of people and operators touching the machine.

4. Typical internal heat sources of the machine include motor drives, internal electronic systems, friction in the slideways and bearings. In the case of the Mitutoyo FN905, the air bearings contribute to cooling down or heating up of the place of their momentary location. The magnitude of this effect depends on the conditions of the supplied air. Also the scale readers may upset local thermal equilibrium due to their inappropriately placement. Included in this group of sources also design factors are counted, such as different materials for the constructional parts of the CMM causing different rates of the heat transfer and different expansion rates.
The initial thermal situation reflects the thermal history of the machine structure. Included are memory effects caused from a previous environment, such as moving large measurement objects to or from the CMM.

In particular situations the different corresponding thermal effects together form the temperature distribution field of the machine structure, the workpiece, and of the measuring system of the machine. Those together form the total thermal deformation.

![Diagram of possible thermal sources and their effects.](image)

**Figure 2.3** Diagram of possible thermal sources and their effects.
2.3 Thermal sensitivity reduction of CMMs

In order to reduce the thermal sensitivity of the CMM different methods can be applied:

- Improvement of the structural design of the CMM such as:
  * The selection of materials with respect to the demands of good heat conduction and equal expansion coefficients with low uncertainties.
  * The development of principles for the fixation of the scales, the linkage of all structural parts and their shape in consideration with a minimal reaction to thermal effects.

- Insulation of the CMM from external heat sources:
  * by means of a temperature-controlled protective cabin build around the CMM

- Software correction:
  * based on theoretically modeled thermal effects
  * based on the "black-box" methodology, combining the use of experimentally founded effects to develop a theoretical model

The application of a protective cabin solves a great deal of the problem, but it is not always accepted due to the costs involved and the complications in loading the CMM. The software correction solution is becoming of more importance for industrial use. The purely theoretical model for the thermal effects can be divided into two parts: the derivation of a model for the temperature distribution field and the derivation of a model for thermally caused deformations of the CMM's structure. These two parts are independent of each other, except that the temperature calculation must precede the deformation calculation. In order to solve these two tasks a Finite Element Method (FEA) may be used. This method enables to analyse the thermal behaviour of the structure by idealizing the structure with an assembly of finite elements. Since the input information into FEA expresses a large number of variables (such as the basic topological structure and main dimensions, geometrical configurations of the structure and the height, width, wall thicknesses and other dimensions of each component in the structure, material of the structure, the properties of the foundation, interfaces between components, external static and dynamic forces, heat sources with corresponding variables description) which cannot be exactly (effectively) described due to the complexity of the structure, uncertainties in the coefficients of expansion, radiation, convection and conduction properties etc, the FEA is becoming for this task complex and time consuming and has only a limited practical use (Theuws, 91).
2.4 Assumptions for the geometrical deviation model

As mentioned in the previous chapters the temperature distribution field of the CMM's structure is assumed to be a given function of the coordinate system and the model will be derived on the basis of the thermo-elasticity theory. The model presented in this report assumes a stable temperature distribution fields with constant gradients. This is based on the assumption that the CMM's structure has the same temperature field as its environment, due to the good heat conduction properties of steel. As already mentioned the temperature distribution field of the room is assumed to be linear. Such a temperature field results in expansion and bending of the constructional parts without any thermally introduced stresses.

- Expected geometrical deviations

In the table in figure 2.4 an overview of the expected geometrical deviations of the investigated CMM which may arise as a deformation effect due to a linear temperature distribution field is given.

<table>
<thead>
<tr>
<th>X-beam</th>
<th>X-scale</th>
<th>Y-beam</th>
<th>Y-scale</th>
<th>Z-beam</th>
<th>Z-scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-gradient</td>
<td>xtx</td>
<td>yrz,ylx</td>
<td></td>
<td>zry,ztz</td>
<td></td>
</tr>
<tr>
<td>Y-gradient</td>
<td>xrx,xty</td>
<td>yrx</td>
<td>yty</td>
<td>zrx,xty</td>
<td></td>
</tr>
<tr>
<td>Z-gradient</td>
<td>xry,xtz</td>
<td>yrx,ytz</td>
<td></td>
<td>ztz</td>
<td></td>
</tr>
</tbody>
</table>

- 3 squareness deviations between x,y and z-beams
- 3 zero drifts of x,y and z-scales

Figure 2.4 Table of the expected geometrical deviations caused by linear temperature field

The deformation effect of the X-beam and Y-beams are depicted in figure 2.5 and 2.6. The detailed geometrical deviations of the X-beam and Z-beam are depicted in the figure 2.7a and 2.7b. The notation of the thermal gradients corresponds with the coordinate direction along which axis the temperature is dependent. The expected deformations of the X-beam comprises of two rotational deviations xrx and xry, two translation deviations xty and xtz and the translation deviation xtx of the X-scale. The Y-beam deformations comprises of two rotational deviations yrz and yrx, two translation deviations ytx and ytz are expected and in addition the translation deviation yty of the Y-scale. A third expected rotational deviation of the Y-beams is denoted as yrx, which expresses the rotation of the narrow z-directed foot (middle support) of the Y-beam. For the Z-beam two rotational
deviations $z_{ry}$ and $z_{rx}$, two translation deviations $z_{tx}$ and $z_{ty}$ are expected. The Z-scale will expand in a $z_{tz}$ translation deviation. Due to the expansion and bending effects of the structure, three squareness deviations between the X,Y, and Z-beam will occur in addition. At last, also the zero points of all three scales will drift.

Figure 2.6  Expected geometrical deviations of Y-beams caused by X-gradient and Z-gradient.
Figure 2.7a  Expected geometrical deviations of the X-beam caused by Y-gradient and Z-gradient.

Figure 2.7b  Expected geometrical deviations of the Z-beam caused by X-gradient and Y-gradient.
3 THERMO - MECHANICAL MODEL

3.1 General theory

The derivation of a model for the thermo-mechanical behaviour of the CMM is based on the fundamental "thermal stress analysis for elastic systems", given in well known literature (Boley, 60). The basis of the model is formulated as the determination of thermal deformations in solid bodies under a prescribed temperature distribution field. The formulations of this model are based on three principal assumptions:

- that the temperature can be determined independently of the deformations,
- that the deformations are small,
- that the material is homogeneous, isotopic and behaves elastically at all times.

The principle of this model was derived in the report "Thermal behaviour of the CMM" (Dunovska, 91). Therefore in this chapter only a short summary is derived.

The stresses are identified as acting on the edge of an elemental square. The complete three dimensional description of the stress in a point is illustrated in figure 3.1.

![Figure 3.1](image_url)

Figure 3.1 The stress in the point.

The stress is generally denoted by $\sigma_{ij}$. The subscript notation identifies the direction of the stress at the face, or area, of the cube on which its acts. The first subscript $i$ identifies the face, or area of the cube. The second subscript $j$ identifies the direction of the stress. For example, the stress $\sigma_{xx}$ is the normal stress acting on the face perpendicular to the $x$ axis.
A shear stress on a face of the cube normal to the y axis and acting in the x direction would be \( \sigma_{yx} \), as shown in figure 3.1.

The shear stresses on mutually perpendicular planes are equivalent:

\[
\sigma_{xy} = \sigma_{yx} ; \sigma_{yz} = \sigma_{zy} ; \sigma_{xz} = \sigma_{zx}
\]  

(1)

The nine components shown in figure 3.1 can be presented as a stress component matrix:

\[
\sigma = \begin{pmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{pmatrix}
\]  

(2)

\( \sigma \) stress in a point  
\( \sigma_{ij} \) shear stress  
\( \sigma_{ii} \) normal stress

Similar as the stress component matrix, a strain component matrix can be derived as:

\[
\varepsilon = \begin{pmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{pmatrix}
\]  

(3)

\( \varepsilon \) strain in a point  
\( \varepsilon_{ij} \) shear strain  
\( \varepsilon_{ii} \) normal strain

For strain the use of the subscripts i and j are identical as mentioned above for the stresses.
The linear relationship between stress and strain is given by Hooke's law and for a bar in simple tension or compression has the form:

\[ \sigma_{ii} = E \varepsilon_{ii} \]  \hspace{1cm} (4)

\( E \) Young's modulus [N/m²]

For more complicated rates of stresses, a generalized Hooke's law is required. This so-called Hooke's law for plane stress in a body is given by equations for normal strains. The normal strains \( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} \) represent the changes in the dimensions of an infinitesimally small cube. These strains can be expressed in terms of the stresses by superimposing the effects of the individual stresses. For example, the stress \( \sigma_{xx} \) produces strain \( \varepsilon_{xx} \) equal to \( \sigma_{xx}/E \), and the stress \( \sigma_{yy} \) produces a strain \( \varepsilon_{xx} \) equal to \(-\nu \sigma_{yy}/E\). Of course, the shear stress \( \sigma_{xy} \) produces no normal strain in the x direction. The resultant strain \( \varepsilon_{xx} \) is:

\[ \varepsilon_{xx} = \frac{1}{E} \left( \sigma_{xx} - \nu \sigma_{yy} \right) \]  \hspace{1cm} (5)

Similarly, the strains in the y and z direction are obtained:

\[ \varepsilon_{yy} = \frac{1}{E} \left( \sigma_{yy} - \nu \sigma_{xx} \right) \]  \hspace{1cm} (6)

\[ \varepsilon_{zz} = \frac{1}{E} \left\{ -\nu \left( \sigma_{xx} + \sigma_{yy} \right) \right\} \]  \hspace{1cm} (7)

\( \nu \) Poisson's ratio, which gives the relation between lateral and axial strain for unaxially loaded members

The shear stress is related to the shear strain in a similar manner by Hooke's law in shear:

\[ \sigma_{ij} = G \varepsilon_{ij} \]  \hspace{1cm} (8)

\( G \) shear modulus [N/m²]
The generalized Hooke's law in shear has the form:

\[
\varepsilon_{xy} = \frac{1}{G} \sigma_{xy} ; \quad \varepsilon_{yz} = \frac{1}{G} \sigma_{yz} ; \quad \varepsilon_{zx} = \frac{1}{G} \sigma_{zx}
\]  

Thermal stresses may arise in a heated body because of either a non-uniform temperature distribution, or external constraints, or a combination of these two causes. The overview of the temperature effects on the mechanically free body is depicted in the figure 3.2.

**Figure 3.2** Overview of the temperature-stress-strain effects on the mechanically free body.

When the mechanically free body is subjected to any general non-uniform temperature field the structure will react by dimensional changes. The constant part of the temperature field causes an uniform expansion of the structure with no stresses. The non-constant part of the temperature field causes a non-uniform expansion which may result in thermal...
stresses in the structure, an exception is a linear temperature field in which there are also no stresses (Boley,60). The strains in each point of a heated body are thus made up of two parts:

The first part is an uniform expansion proportional to the constant temperature $T$. Since this expansion is the same in all directions for an isotropic body, only normal strains and no shear strains arise. If the coefficient of linear thermal expansion is denoted by $\alpha$, this normal strain in any direction is equal to $\alpha T$.

The second part is a non-uniform expansion and are the strains required to maintain the continuity of the body as well as those arising because of external loads. These strains are related to stress by Hooke's law of linear elasticity using equations (5) and (9). In this case also shear strains and thus torsion may arise.

Then the resulting strains at each point of the orthogonal coordinate system $x, y, z$, (Boley,60), are the sum of these two components and are related to the stresses and to the temperature by:

$$
\varepsilon_{xx} = \frac{1}{E} \{ \sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz}) \} + \alpha T
$$

$$
\varepsilon_{yy} = \frac{1}{E} \{ \sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx}) \} + \alpha T
$$

$$
\varepsilon_{zz} = \frac{1}{E} \{ \sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy}) \} + \alpha T
$$

$$
\varepsilon_{xy} = \frac{1}{2G} \sigma_{xy} ; \quad \varepsilon_{yz} = \frac{1}{2G} \sigma_{yz} ; \quad \varepsilon_{zx} = \frac{1}{2G} \sigma_{zx}
$$

$E$ Young's modulus

$G$ the shear modulus

$\nu$ Poisson's ratio, which gives the relation between lateral and axial strain for unaxially loaded members

The stress can be expressed as a function of the strain, as derived in (Boley,60):

$$
\sigma_{xx} = \lambda \varepsilon_{xx} + 2\mu \varepsilon_{xx} - (3\lambda + 2\mu) \alpha T
$$

$$
\sigma_{yy} = \lambda \varepsilon_{yy} + 2\mu \varepsilon_{yy} - (3\lambda + 2\mu) \alpha T
$$

$$
\sigma_{zz} = \lambda \varepsilon_{zz} + 2\mu \varepsilon_{zz} - (3\lambda + 2\mu) \alpha T
$$

$$
\sigma_{xy} = 2\mu \varepsilon_{xy} ; \quad \sigma_{yz} = 2\mu \varepsilon_{yz} ; \quad \sigma_{zx} = 2\mu \varepsilon_{zx}
$$
\( \lambda \) and \( \mu \) are called the Lame constants.

Applying laws of mechanics about the equations of equilibrium of the body under acting forces and moments, the equations of equilibrium for stress analysis are derived. In an orthogonal coordinate system with axes \( x, y, z \), these equations have the following form for the case of no body forces:

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\
\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= 0
\end{align*}
\]  

(12)

The relationship between the strains and the deformations in the orthogonal coordinate system \( u,v,w \) resp. \( x,y,z \) is:

\[
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} ; \varepsilon_{yy} = \frac{\partial v}{\partial y} ; \varepsilon_{zz} = \frac{\partial w}{\partial z} \\
\varepsilon_{xy} &= \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
\varepsilon_{yz} &= \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\
\varepsilon_{zx} &= \frac{1}{2} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)
\end{align*}
\]  

(13)

where \( u, v, w \), are the components of the deformation vector in respectively the \( x, y, z \), direction.

The boundary conditions of the deformations at every point of the bounding surface are described by the following equations:

\[
\begin{align*}
u &= f(P) \\
v &= g(P) \\
w &= h(P)
\end{align*}
\]  

(14)

\( f,g,h \) are prescribed functions.
3.2 Modelling methodology

The CMM is divided into the following observed parts for the purpose of mathematical description as well as for the measurements:

X-beam
Y-beam:
\hspace{1cm} Y-beam-right
\hspace{1cm} Y-beam-left

These observed parts are depicted in figure 3.3, according to the general theory, some assumptions are made for modelling these parts:

X-beam: considered as a 3D massive beam structure free of external loads and stresses (linear temp. field)
Y-beam: considered as a 3D massive volume structure free of external loads and stresses

Figure 3.3: Coordinate measuring machine with the observed parts for modeling.
Each of these observed parts has its own local reference coordinate system. Since the relative angular changes are verified by measurements for each part separately, the modeled structure can be compared directly with these measurements.

The diagram in figure 3.4 gives an overview of the modelling methodology. In model 1 the thermal gradients are calculated from a selected temperature distribution field. As already stated in chapter 2.4, the temperature field will be chosen linear and valid for all parts of the CMM. With the known thermal gradients of model 1 and based on the above mentioned assumptions about the observed parts, the angular thermal deformations of elements of respectively the X and Y-beam are calculated. These calculations are later transformed to the total rotational deviations, both in model 2 and 3.

Figure 3.4 Diagram of the overview of the modelling methodology.
3.2.1 Model 1: The thermal model

In the model presented in this chapter only a static temperature distribution field is considered. The temperature distribution field of the room is assumed to be linear. Also the temperature distribution field of each observed part has a form of a continuous linear function of a local orthogonal coordinate system $x,y,z$. This is based on the assumption that the structure of the CMM has the same temperature field as its environment, due to the good heat conduction properties of steel.

$$T(x,y,z) = a + bx + cy + dz$$

(15)

$T$ temperature

$x,y,z$ axis of the local coordinate system of observed parts

$a$ constant representing a uniform temperature

$b,c,d$ coefficients representing the constant gradients of a linear temperature function

Coefficients $a,b,c,d$ have to be identified by the temperature measurements at discrete points of the observed parts. The physical representation of the coefficients $b,c,d$ is a constant thermal gradient and they can be derived as:

$$b = \frac{\partial T(x,y,z)}{\partial x}; \quad c = \frac{\partial T(x,y,z)}{\partial y}; \quad d = \frac{\partial T(x,y,z)}{\partial z}$$

(16)

In the thermal model, the temperatures are measured in $n$ $yz$-cross-sectional planes of the beams, see figure 3.5.

Figure 3.5 X-beam and the configuration of the cross-sections and elements.
The constant thermal gradients are calculated at the locations of those cross-sections. The following assumption about these thermal gradients is used in model 1. This assumption allows to split the main axis of a beam into \( n \) so-called "independent discrete elements of the beam", further referred to as "elements". The beam is split such that the elements are laying around the cross-sections where the temperatures are measured, see figure 3.5. The thermal gradients at these cross-sections is considered to be constant for the whole length of the interval \( e_x \) of the element. Of each element the local dimensions \( e_x, e_y, e_z \) are known, the coefficient of expansion \( \alpha \) is selected value, and at six discrete positions the temperatures \( T_1...T_6 \) are scanned, as depicted in figure 3.6a and 3.6b. In figure 3.6a the positions of the temperature sensors are given for the determination of the thermal rotation \( xrz \), in figure 3.6b for the determination of \( xry \).

![Figure 3.6a](image)

The positions of the temperature sensors in one element for the determination of the thermal rotation \( xrz \).

![Figure 3.6b](image)

The positions of the temperature sensors in one element for the determination of the thermal rotation \( xry \).
To be able to investigate the influence of the location of the temperature sensors on the beams and for averaging purposes, the temperatures are measured in three planes \((s = 1,2,3)\) at each element, see figure 3.6a, 3.6b. For each plane the thermal gradient is calculated, so every temperature measurement delivers three coefficients for the thermal gradient. Also for averaging purposes every measurement is repeated a number \(m\) times. Furthermore the temperatures are recorded for three different heating levels. Thus the amount of temperature acquisitions for every rotational deviation add's up to: \(6 \cdot n \cdot m \cdot 3\).

The temperatures scanned by the sensors represent relative temperature data because of the relative sensor calibration discussed in chapter 4. These temperatures have to be corrected with the reference temperatures of the temperature distribution field of the unheated machine. Then the relative thermal gradient for each element can be calculated, for example coefficient of the thermal gradient in \(y\) direction:

\[
c_n = \frac{\Delta T_{ns} - \Delta T_{ref ns}}{e_y} \tag{17}
\]

- \(n\) the element number
- \(s\) the plane in the element
- \(e_y\) element length in \(y\) direction

When concerning the whole observed part for all \(m\) measurements and all three temperature levels, the resultant gradient matrix would take a 3 dimensional form. When omitting the different heating levels the gradient matrix takes the following form:

\[
C = \begin{pmatrix}
c_{11} & c_{1n} \\
c_{m1} & c_{mn}
\end{pmatrix} \tag{18}
\]

and resp. for the coefficient of the thermal gradient in \(z\) direction:

\[
D = \begin{pmatrix}
d_{11} & d_{1n} \\
d_{m1} & d_{mn}
\end{pmatrix} \tag{19}
\]
The elements of the matrix represent the gradient of each of the \( n \) "elements" with respect to the reference, \( m \) is the number of repeated temperature measurements.

### 3.2.2 Model 2: X-beam deformation model

The group of equations linking the temperature distribution field to the deformations are expressed in model 2. The equations for the thermal deformations are derived from thermo-elasticity theory under quasi-static conditions for the assumed geometrical representations of the observed parts as already discussed. In the methodology of the calculations the analogy between the thermal deformations in a heated body and the deformations in an unheated body subject to equivalent body forces is used. The X-beam is then defined with its boundaries as a sum of independent elements as depicted in figure 3.7. Each separate element is affected by local constant thermal gradient.

![Figure 3.7](image)

Figure 3.7 The X-beam defined as a sum of independent elements.

The thermal deformation field for the X-beam in a linear temperature distribution field, with notation is given in (Dunovska, 91, pp19, eq41-47) and is described as:

\[
\begin{align*}
\mathbf{u}(x) &= a (ax + \frac{1}{2}bx^2) + c_0 \\
\mathbf{v}'(x) &= -a c \\
\mathbf{v}''(x) &= -a c x + c_1 \\
\mathbf{v}(x) &= -\frac{1}{2}a cx^2 + c_1 x + c_2
\end{align*}
\]
\[ w''(x) = -\alpha d \]
\[ w'(x) = -\alpha d x + c_1 \quad \text{(xry)} \]
\[ w(x) = -\frac{1}{2}\alpha dx^2 + c_1 x + c_2 \]

- axes of the coordinate system
- axes of the deformation field
- primes are first and second differentiations with respect to \( x \)
- \( \alpha \) coefficient of the thermal expansion
- \( a \) uniform temperature
- \( b, c, d \) coefficients representing the constant gradients of a linear temperature field
- \( c_0, c_1, c_2 \) constants

The constants \( c_0, c_1, c_2 \) can be derived from the boundary conditions. For the measurements it is assumed that the angular deformation in position \( x = 0 \) is zero. To adapt to this measurement property the boundary conditions have to be taken a little different than in (Dunovska, 91, pp20).

\[ u(0) = 0 \quad c_0 = 0 \]
\[ v(0) = v'(0) = 0 \quad c_1 = 0 \]
\[ w(0) = w'(0) = 0 \quad c_2 = 0 \quad \text{(23)} \]

Then the angular thermal deformations affecting the X-beam become:

\[ v'(x) = -\alpha c x \quad \text{(xrz)} \quad \text{(24)} \]
\[ w'(x) = -\alpha d x \quad \text{(xry)} \quad \text{(25)} \]

These thermal deformations are then calculated for each of the \( n \) separate elements with the \( x \)-dimension \( x_n \) and local gradients \( c_n \) and \( d_n \).
where the vector \( \mathbf{x} = [x_1, \ldots, x_n] \) describes the discrete length dimensions of the separate elements of the X-beam, see figure 3.7. The matrix describing the angular thermal deformations of the separate elements has the form:

\[
V' = V'_{11} V'_{1n} \begin{pmatrix} \mathbf{c}_{11} & \mathbf{c}_{1n} \\ \mathbf{c}_{m1} & \mathbf{c}_{mn} \end{pmatrix} \begin{pmatrix} \mathbf{e}_{x_1} & 0 \\ \vdots & \vdots \\ 0 & \mathbf{e}_{x_n} \end{pmatrix}
\]

\[
W' = W'_{11} W'_{1n} \begin{pmatrix} \mathbf{d}_{11} & \mathbf{d}_{1n} \\ \mathbf{d}_{m1} & \mathbf{d}_{mn} \end{pmatrix} \begin{pmatrix} \mathbf{e}_{x_1} & 0 \\ \vdots & \vdots \\ 0 & \mathbf{e}_{x_n} \end{pmatrix}
\]  

(26)  

(27)

\[ \text{resp.} \]

\[
W' = W'_{11} W'_{1n} \begin{pmatrix} \mathbf{v}'_{11} & \mathbf{v}'_{1n} \\ \mathbf{v}'_{m1} & \mathbf{v}'_{mn} \end{pmatrix} \begin{pmatrix} \mathbf{e}_{x_1} & 0 \\ \vdots & \vdots \\ 0 & \mathbf{e}_{x_n} \end{pmatrix}
\]

\[
V' = V'_{11} V'_{1n} \begin{pmatrix} \mathbf{w}'_{11} & \mathbf{w}'_{1n} \\ \mathbf{w}'_{m1} & \mathbf{w}'_{mn} \end{pmatrix} \begin{pmatrix} \mathbf{e}_{x_1} & 0 \\ \vdots & \vdots \\ 0 & \mathbf{e}_{x_n} \end{pmatrix}
\]

\[ \text{resp.} \]

\[
\begin{pmatrix} \mathbf{v}'_{11} & \mathbf{v}'_{1n} \\ \mathbf{v}'_{m1} & \mathbf{v}'_{mn} \end{pmatrix}
\]

\[
\begin{pmatrix} \mathbf{w}'_{11} & \mathbf{w}'_{1n} \\ \mathbf{w}'_{m1} & \mathbf{w}'_{mn} \end{pmatrix}
\]

\[
\begin{pmatrix} \mathbf{e}_{x_1} & 0 \\ \vdots & \vdots \\ 0 & \mathbf{e}_{x_n} \end{pmatrix}
\]

\[ \text{the number of discrete elements} \]

\[ \text{the number of measurements} \]

Once the angular thermal deformations of the discrete elements of the X-beam are calculated, in the matrix form of (28), resp. (29), the relationship between the total rotational deviation and its location on the X-beam can be derived. This total rotational
deviation will be calculated from the discrete angular thermal deformation as a matrix of cumulative sums of the angular thermal deformations of each element.

\[ x_{rz} \] is the cumulative sum of the elements of the row of the matrix \( V' \)

\[
\begin{align*}
x_{rz_{11}} &= v'_{11} \\
x_{rz_{mn}} &= x_{rz_{mn-1}} + v'_{mn}
\end{align*}
\]

\[ x_{rz} = \begin{pmatrix}
x_{rz_{11}} & x_{rz_{1n}} \\
x_{rz_{m1}} & x_{rz_{mn}}
\end{pmatrix}
\]

\[ x_{ry} = \begin{pmatrix}
x_{ry_{11}} & x_{ry_{1n}} \\
x_{ry_{m1}} & x_{ry_{mn}}
\end{pmatrix}
\]

The position of the rotational deviations in the matrix, with respect to the zero point of the X-scale is dependent upon the distance \( x_p \) of the cross-sections to the zero point. Then the graphical interpretation of the relationship between the rotational deviation \( x_{rz_{mn}} \) and \( x_{ry_{mn}} \) with the positions \( x_p \) gives the predicted results in the same way as the scanned results of the measurements of these deviations. Therefore the predicted results can be easily compared with the measured results.

**Relation between the theoretical angular thermal deformations and the actual deformations measured on the end of the Z-beam**

In the theoretical description derived above, the angular deformations of the X-beam is found in the equations for \( w', \) resp. \( v' \) (25), (24). Actually these angular deformations can be related to the rotational deviation \( x_{rz} \) and \( x_{ry} \) only in the case if the angle of the Z-carriage \( \beta(x) \) is equal to the deformation angle \( w'(x) \) as depicted in the figure 3.8.
The angle $\beta(x)$ is the actually measured angle in the $xrz$ measurement. If the deformation $w(x)$ is described by the quadratic function (22) for the point $x = x_0$ as:

$$w(x_0) = -\frac{1}{2} \alpha d \ x_0^2$$

and for the point $x = x_0 + 1$, where 1 is the length of the Z-carriage as:

$$w(x_0 + 1) = -\frac{1}{2} \alpha d \ (x_0 + 1)^2$$

then the difference between the deformation of both points is:

$$w(x_0 + 1) - w(x_0) = -\alpha d \ (x_0 + \frac{1}{2})$$

The angle $\beta(x)$ of the Z-carriage can be calculated from the geometrical relation from the figure 3.8 as:

$$\beta(x) = \frac{w(x_0 + 1) - w(x_0)}{1} = -\alpha d \ (x_0 + \frac{1}{2})$$

The difference between the angle $\beta(x)$ and $w'(x)(\ )$ at the point of contact $x = x_0 + \frac{1}{2} 1$:

$$\beta(x) - w'(x_0 + \frac{1}{2}) = -\alpha d \ (x_0 + \frac{1}{2}) - \{ -\alpha d \ (x_0 + \frac{1}{2}) \} = 0$$

is equal to zero. This result concludes that the angle $\beta(x)$ of the carriage can be related to the theoretical deformations of the X-beam as long as the deformation $w(x)$ has the form of the quadratic function.
Figure 3.8 The angle of the Z-carriage $\beta(x)$ and the deformation angle $w'(x)$ of the X-beam.
3.2.3 Model 3: Y-beam deformation model

Similarly as for the X-beam, once the temperature gradients at certain locations of the Y-beam are calculated, the equations describing the thermal deformations can be derived by model 3. These equations make use of the thermo-elasticity theory under quasi-static conditions same as for the X-beam. The distinction between the X-beam and the Y-beam is the geometrical representation, the Y-beam is assumed to be a massive volumetric body, as depicted in figure 3.9, free of external loads and stresses. The deformation field of a volumetric body is described by a function with three variables \( f(x,y,z) \), unlike the X-beam which is described by a function of one dependent variable \( f(x) \).

![Geometrical representation of the Y-beam.](image)

Figure 3.9 Geometrical representation of the Y-beam.

For the calculation of the deformation field the analogy between the thermal deformations in a heated body and the deformations in an unheated body due to equivalent body forces is used. The Y-beam is defined, with its boundaries, as a sum of independent volumetric elements as depicted in figure 3.10. Each separate element is affected by local constant thermal gradients.

The thermal deformation field for the Y-beam (frame) in a linear temperature distribution field, with notation is given in (Dunovska, 91, pp27, eq86-88) and is described as:
\[ u(x,y,z) = \alpha (ax + \frac{1}{2}bx^2 + cxy + dxz) + U(y,z) \]

\[ v(x,y,z) = \alpha (ay + bxy + \frac{1}{2}cy^2 + dyz) + V(y,z) \]  \hspace{1cm} (38)

\[ w(x,y,z) = \alpha (az + bxz + cyz + \frac{1}{2}bz^2) + W(y,z) \]

- \( x,y,z \): axes of the coordinate system
- \( u,v,w \): axes of the deformation field
- \( \alpha \): coefficient of the thermal expansion
- \( a \): uniform temperature
- \( b,c,d \): coefficients representing the constant gradients of a linear temperature field

The functions \( U(y,z) \), \( V(y,z) \) and \( W(y,z) \) can be derived from the boundary conditions. The resulting thermal deformation field for this three dimensional problem is given in \((\text{Dunovska, 91, pp28, eq102-103})\) and is described as:

\[ u'(x,y,z) = \alpha (cx - by) \quad (yrz) \]  \hspace{1cm} (39)

\[ w'(x,y,z) = \alpha (cz - dy) \quad (yrx) \]  \hspace{1cm} (40)

**Figure 3.10** The Y-beam defined as a sum of independent volumetric elements.
When considering a CMM only the actual angular deformation which affects the movement of the carriage along the guide is important. So only the deformation as a function of the axis of movement can be considered. In the case of the Y-beam the deformation is a function of $y$. The dependency on $x$ and $z$ can be considered as a constant for the rotational deviation. This influence on the deformation is constant for all $y$ positions. See an example on the figure 3.11. Therefore the parts with dependency on the variables $x$ and $z$ in the theoretical equations can be considered constant and can be neglected. Then the angular thermal deformations affecting the Y-beam will be:

$$u'(y) = -\alpha b y$$  \hspace{1cm} (yrz) \hspace{1cm} (41)$$

$$w'(y) = -\alpha d y$$  \hspace{1cm} (yrx)  \hspace{1cm} (42)$$

Figure 3.11  \hspace{1cm} The position dependency of the rotational deviation measurement $y_{rx}$.

These thermal deformations are then calculated in the same way as for the X-beam, for each of the $n$ separate elements with $y$-dimension $e_{yn}$ and local gradients $b_n$ and $d_n$:

$$U' = -\alpha BY I = -\alpha$$

$$\begin{pmatrix}
    b_{11} & b_{1n} \\
    \vdots & \vdots \\
    b_{m1} & b_{mn}
\end{pmatrix}
\begin{pmatrix}
    e_{y_1} & 0 \\
    \vdots & \vdots \\
    0 & e_{y_n}
\end{pmatrix}$$  \hspace{1cm} (43)$$

$$W' = -\alpha DY I = -\alpha$$

$$\begin{pmatrix}
    d_{11} & d_{1n} \\
    \vdots & \vdots \\
    d_{m1} & d_{mn}
\end{pmatrix}
\begin{pmatrix}
    e_{y_1} & 0 \\
    \vdots & \vdots \\
    0 & e_{y_n}
\end{pmatrix}$$  \hspace{1cm} (44)$$
where the vector \( y = [e_1, ..., e_n] \) describes the discrete length dimensions of the separate elements of the Y-beam. The matrix describing the angular thermal deformations of the separate elements has the form:

\[
U' = 
\begin{pmatrix}
  u'_{11} & u'_{1n} \\
  u'_{m1} & u'_{mn}
\end{pmatrix}
\]  

resp.

\[
W' = 
\begin{pmatrix}
  w'_{11} & w'_{1n} \\
  w'_{m1} & w'_{mn}
\end{pmatrix}
\]  

\( u_{ll} \) the number of discrete elements

\( w_{mm} \) the number of measurements

The thermal angular deformations of the separate elements of the Y-beam are presented in matrix form in (45), resp. (46). This calculation is performed for both Y-beams; the right as well as the left part. From these discrete elemental angular deformations the total rotational deviation will be calculated, separately for the right and left part, as a matrix of cumulative sums of the angular deformation of each element.

\( yrz \) is the cumulative sum of the elements of the row of the matrix \( U' \)

\[
\begin{align*}
    yrz_{11} &= u'_{11} \\
    yrz_{mn} &= yrz_{mn-1} + u'_{mn}
\end{align*}
\]  

\[
yrz = 
\begin{pmatrix}
  yrz_{11} & yrz_{1n} \\
  yrz_{m1} & yrz_{mn}
\end{pmatrix}
\]
The position of the rotational deviations in the matrix, with respect to the zero point of the Y-scale is dependent upon the distance $y_p$ of the cross-sections to the zero point. Then the graphical interpretation of the relationship between the rotational deviation $yrz_{mn}$ and $yrx_{mn}$ with the positions $y_p$ gives the predicted results in the same way as the scanned results of the measurements of these deviations. Therefore the predicted results can be easily compared with the measured results.

$$yrx = \begin{pmatrix}
    yrx_{11} & yrx_{1n} \\
    yrx_{m1} & yrx_{mn}
\end{pmatrix}$$
4 EXPERIMENTAL DETERMINATION OF THE THERMO-MECHANICAL BEHAVIOUR OF THE CMM

4.1 Introduction

The main task of the experimental determination is to develop such a measurement set-up that allows to measure the thermal deformations and their cause, the corresponding temperature field of the structure of the CMM, at the same time.

The temperature data are used as the input into the theoretical model for the calculation of the thermal deformations. Since the difference between two temperature states causes a deformation shift, only relative temperatures are important and not the actual absolute temperatures. Therefore all measured temperature states are referring to a measured reference temperature state. Furthermore, because of the condition of a quasi-static field for the observation, both of these states are created with long time constants. For the reference state 10 hours and for the heated state 4 hours were allowed for the temperature field to stabilize, before measurements were made.

In the following paragraphs the equipment, the creation of the desired temperature field, the measurement of the temperatures and the measurement of the thermal deformations are discussed.

The complete measurement set-up consisted of the hardware of the following commercially available instruments:

- **Coordinate meas. machine:** Mitutoyo, FN905
- **Thermometer:** Process-Periphery-instrument, GSSE PP2 interfaced with 28 Pt-100 conductive sensors (resistance thermometers)
- **Laser interferometer:** Renishaw Laser Interferometer System:
  - ML10 laser with tripod
  - PC10 display/control option
  - appropriate optical reflectors
- **Personal Computer:** Hewlett Packart, Vectra ES/12
- **Radiation heater:** Watlow Radiant Panels, Raymax 1120
- **Heating regulator:** Triac, connected to an automatic switch-clock
The control unit of the CMM is connected by an IEEE488 interface to the PC. Through this link the CMM can be operated and the measured values of the scales can be read by the PC. The temperature sensors are connected to the Process Periphery-instrument, which is also connected to the PC by the IEEE488 link, and the measured temperature values can be read by the PC. The temperature sensors are fixed to the inside surface of the machine's structure by means of heat conducting paste, for assured thermal contact. The optical reflector is mounted on the tip of the Z-beam. The interferometer is on the granite table and together with the reflector they have to be aligned with the laser. The laser is connected to a special interface card, the PC10 interface card, in the PC. The triac regulator is a converter with three outputs and two ranges of regulation: 0 - 5A and 5 - 10A. The outputs are connected with the radiant panels which are projected to a surface of the CMM. The regulator is connected to an automatic switch-clock. The regulator output current is selected manually and for each measuring cycle adjusted to three different levels.
4.2 The temperature field creation

An important task for the performance of the measurements was to create a temperature distribution field in the structure of the machine, or in the separate observed parts of the machine. The required temperature field, as indicated in chapter 3, should have a linear distribution. Thus the theoretically selected temperature field requires three constant thermal gradients \( b,c,d \) in the coordinate axis \( x,y,z \) of the structure. This temperature field was created by power-controlled radiant heating panels, which had a uniform radiation characteristic over the surface of the panel. The radiant panel(s) were always placed on one side of the part under observation, so from this side to the opposite side a thermal gradient was created. A detailed explanation of the measuring set-ups is given in chapter 4.5.

4.3 Temperature field measurement

4.3.1 Temperature measurement equipment

- Thermometer: Process-Periphery-instrument, GSSE PP2 interfaced with 28 Pt-100 conductive sensors (resistance thermometers)

Because of the above mentioned necessities and from experience, Pt-100 sensors were selected, since they show a stable and good linear behaviour in the measurement range. To correct for differences between the sensors a calibration routine was performed.

4.3.2 Relative calibration of the temperature measurement equipment

To prepare for the experiment, the digital thermometer with the 28 Pt-100 resistance sensors connected had to be calibrated. The following method for relative calibration was chosen. This calibration corrects for relative deviations between the sensors with respect to one reference sensor. As a reference sensor one of the 28 connected sensors was taken. The set of 28 sensors was placed parallel on the basement plate of a special calibrating unit of the TUE metrology laboratory of which the temperature can be changed. The plate was cooled down to 8° Celsius and then continuously, during 24 hours, heated up to 40° Celsius. The graphical representation of part of the measured heating cycle is depicted in appendix 1. During the heating period, the resistance of the sensors was scanned. The measured data of each sensor was corrected with the data of the reference sensor,
which gives correction data for each sensor. A second order polynome was fitted through the correction data to describe the deviation of each sensor as a function of its resistance, relative to the reference sensor:

\[ R_x - R_{14} = a R_x^2 + b R_x + c = f(R_x) \]  \hspace{1cm} (50)

\[ R_x^{corr} = R_x - f(R_x) \]  \hspace{1cm} (51)

\[ R_x^{corr} = -a R_x^2 - (1 + b) R_x - c = -a_0 R_x^2 - a_1 R_x - a_2 \]  \hspace{1cm} (52)

The coefficients of the polynomials were fed into an automatical data acquisition program. This program calculates the resulting temperature from the corrected measured resistance as given by the following relationship, which was specified for the used GSSE PP2:

\[ T_x^{corr} = \{ 3.36785046 - (11.3424167 - R_x^{corr}/58.0195) \} 1000 \] [K]  \hspace{1cm} (53)

With these calibrated (equalized) sensors, the thermal state of the machine was determined. The sensors were placed at certain positions of the machine in order to measure the temperature field which causes the deformations. Due to the limited number of sensors, 28, the observed parts of the CMM were measured separate. The configuration of the sensors for the different measurements will be discussed in the chapter 4.5. In the reference state the structure has a temperature of 20°C Celsius. To control the temperature environment, a plastic insulation cabin with ventilation unit was placed around the machine (all placed in the temperature controlled measurement room). In practice the requirement of a uniform temperature of 20°C Celsius was defined to be the temperature state of the machine in the temperature controlled room after 10 hours of rest with the CMM switched on but with no beam movements, followed by one hour with continuous beam movements without any use of the radiant heating panels. The described reference state of the machine was always measured on the beginning of each measurement cycle.
4.4 Deformation field measurement

4.4.1 Deformation measurement equipment

- Laser interferometer: Renishaw Laser Interferometer System:
  - ML10 laser with tripod
  - PC10 display/control option
  - appropriate optical components

As a method to determine the thermal deformations after heating, interferometry was used because it is also used for the calibration of CMMs. For the current research only angular thermal deformations were observed. The laser interferometer is measuring this deviation with two optical components, namely an angular interferometer and an angular reflector as depicted in figure 4.2.

![Figure 4.2](image)

Figure 4.2 Principle of the rotational measurement.

The principle of angular measurement is the comparison of path length differences of two beams. The laser beam is split in two by the beam splitter within the angular interferometer. One part of the beam, reference (A1), passes straight through the interferometer and is reflected from one half of the angular reflector, which is on the measuring tip of the CMM, back to the laser head. The other beam (A2) passes through the periscope of the angular interferometer to the other half of the angular reflector, which returns it through the interferometer to the laser head. In the actual experiment the end point of the Z-pinole is moved along the observed axis while the other two axes are fixed. The angular interferometer as a stationary part is aligned with the laser and the reflector is moving in the direction of the axis of movement. If the angular reflector following the
observed axis of movement, will deflect from the aligned direction by some rotation, it will cause a change in path length difference between the measurement and reference beam, see figure 4.2. This change in path length difference is determined by the fringe counting circuitry in the laser unit and is converted to an angular measurement by the accompanied software. The range of such a measurement is ±0.175 rad, the resolution is 0.1μrad.

The main sources affecting the accuracy of the interferometry measurement method for angular thermal deviations are:
- a systematic error due to the uncalibrated laser interferometer system
- thermal unstability of the optical components.

Taking these effects in account, two different checks were performed. First a comparison of the angular measurement of the laser interferometer with a calibrated laser interferometer system and second a check of the optical thermal drift.

4.4.2 Calibration of the deformation measurement equipment

The calibration was realized by means of a sine bar and a displacement sensor. Both laser interferometer systems, The Renishaw Transducer System and a calibrated HP Laser system, were aligned with their own stationary angular interferometer and angular reflector which were placed on the sine bar. By moving one side of the bar an angle is created. The angular movement of the bar results simply as the arcsine of the displacement divided by the length of the sine bar. Both these dimensions are accurately known. Figure 4.3 shows the measurement set up of this experiment.

\[ \sin \theta = \frac{h}{l} \]

- \( h \) horizontal displacement
- \( l \) length of the sine bar

For the experiment \( l = 1850 \text{mm} \) and \( h_{\text{max}} = 10 \text{mm} \). The maximum deviation in \( l \) due to temperature effects and length measurement uncertainty was: \( \delta l_{\text{tot}} = 0.102 \text{mm} \), the maximum deviation in \( h \) due to temperature effects, uncertainty of the displacement meter and due to not exact perpendicular placement of the meter was: \( \delta h_{\text{tot}} = 0.0017 \text{mm} \). The resulting accuracy for this calibration process came down to: \( \delta \theta_{\text{max}} = 0.2 \text{arcsec} \).
The calculated angular value from the displacement of the sine bar was taken as a reference value for the calculation of the angular deviations. The difference between the measured angular value of both laser interferometer systems and the calculated value gave the angular deviation characteristic. On the measured range, $\theta = 0$-1200arcsec, the angular measurement of the HP Laser System showed a linear deviation with a maximum $\delta \theta = -0.4$arcsec and the Renishaw Laser Transducer System also linear with a maximum $\delta \theta = 1.5$arcsec, see figure 4.4. The presented data can be used for the correction of the angular measurements with these laser systems.

![Diagram](image)

Figure 4.3 The measurement set-up for the calibration of the Renishaw laser.

![Graph](image)

Figure 4.4 Deviation of the angular measurement for two different laser systems
4.4.3 Thermal drift of the optical components

Changes in the temperature of the optical components of the laser interferometer system during the measurement may cause deviations. Expansion of the optical parts due to the temperature changes may result in a change in optical path length which could be detected as an angular change (for the angular measurement). When a change in temperature occurs the physical size of the optical elements and their refractive index will change, and thermal stresses will arise.

Figure 4.5 The configuration of the laser interferometer system for the measurement of thermal drift.

A measurement check was performed with the interferometer and reflector clamped together on one stand placed on the granite table of the CMM, as depicted on the figure 4.5. The room was continuously heated with a radiant panel placed in the upper part of the room. The laser and optical components were isolated from direct radiation. The temperature of the optical elements was scanned during the measurement, figure 4.6b. The thermal drift in the first two hours of the heating process is about $9\mu$m/m°C = 1.8 arcsec/°C. After this time constant, the value of the drift stabilizes at approximately $\pm1\mu$m/m°C = $\pm0.2$ arcsec/°C. It can be derived from the optical drift characteristics in figures 4.6a, 4.6b and 4.6c.

Since the actual angular measurement is reset every time in the zero point of the axis of measurement, the maximal time interval of 8 minutes between two resets gives an inaccuracy of 0.1arcsec. During the execution of the actual measurement, the required time constant of 2 hours and a time interval of 8 minutes was always fulfilled.
Figure 4.6a  Optical thermal drift.

Figure 4.6b  Time dependency of the temperature change during the measurement of the thermal drift.

Figure 4.6c  Temperature dependency of the thermal drift.
4.5 Measurement Set-up for the different rotational deviations

4.5.1 Set-up for xrz

For the measurement of xrz, the following set-up is needed:
- creation of a constant thermal y-gradient in the X-beam
- placement of the 28 sensors in yz-planes in the X-beam structure
- alignment of the rotational interferometer system along the X-beam.

A photograph of the actual set up is presented in figure 4.9.

Ad 1. A constant thermal y-gradient was created by two radiant panels positioned parallel beside the X-beam at a horizontal distance of approximately 1m. The actual configuration is depicted in figure 4.7.

![Figure 4.7 Configuration of the heating panels for xrz measurement.](image)

Ad 2. The configuration of the sensors in the X-beam structure is displayed in figure 4.8. The sensors were placed inside the X-beam in five cross-sections on the inner surface of plates oriented in yz direction. The y-gradient is calculated from the temperature difference between two sensors laying in y direction of each other.

![Figure 4.8 Configuration of the sensors for xrz measurement.](image)
Ad3, the laser was placed on an aluminium plate extending over the granite table of the CMM. The optical reflector was attached to the z-pinole and aligned along the x-axis together with the laser and optical interferometer. The laser, the interferometer and the optical path were insulated by insulation plates in order to protect them from excessive heating. The X-beam was positioned in the zero Y-beam scale position and then the y and z position were fixed. The measurement was performed over the whole range of the x measuring scale.

![Image](image-url)  
*Figure 4.9  xrz measurement with laserinterferometry.*

**4.5.2 Set-up for xry**

For the measurement of xry, the following set-up is needed:
- creation of a constant thermal z-gradient in the X-beam
- placement of the 28 sensors in zy-planes in the X-beam structure
- alignment of the rotational interferometer system along the X-beam.

A photograph of the actual set up is presented in figure 4.12.

Ad 1. Creating a constant thermal z-gradient was realized by placement of two heating panels in a parallel configuration with experimentally adjusted angle $\alpha$. This configuration was chosen due to lack of upper space in the measurement room and due to the limitations caused by the high Z-pinole. In this configuration the heat was allowed to radiate freely
onto the upper yx-surface while the side zx-surfaces of the X-beam were isolated. The vertical distance between the heating panels and the surface of the X-beam was approximately 1m. The actual configuration is depicted in figure 4.10.

Figure 4.10 Configuration of the heating panels for xry measurement.

Ad 2. The configuration of the sensors was chosen in the same way as for xrz, and is displayed in figure 4.11. The z-gradient was calculated from the temperature difference between two sensors lying in z-direction of each other.

Figure 4.11 Configuration of the sensors for xry measurement.

Ad 3. The laser was placed in the same position as as used for xrz. Only the reflector and the interferometer have to be turned 90°.
4.5.3 Set-up for yrz

For the measurement of yrz, the following set-up is needed:
- creation of a constant thermal x-gradient in the Y-beam
- placement of 14 sensors in xz-planes in the right hand Y-beam structure
- alignment of the rotational interferometer system along the Y-beam.

Ad 1. Creating a thermal x-gradient was realized only for the right hand Y-beam, since this part can be considered as the single source of yrz deviations when considering the bearing construction of the CMM. Two radiant panels were positioned parallel beside the Y-beam at a horizontal distance of approximately 0.5m. The radiant panels heated the upper Y-guides, where carriage moves and also the supporting part under it. The actual configuration is depicted in figure 4.13.

Figure 4.13 Configuration of the heating panels for yrz measurement.
The right hand side surface of the Y-beam which should be subjected to the radiation was shielded by the y-drive spindle and motor, both placed in front of it. The steel box which covers the spindle and motor was not removed, because it was used as a protection against direct radiation onto the set of sensors placed at the outside surface of the supporting Y-beam.

Ad 2. The placement of the 14 sensors in (on) the right hand Y-beam is displayed in figure 4.14. The positioning of the sensors was rather limited due to the impossible placement of sensors on or into the Y-guides where the carriage moves and where the deformation is actually measured. Therefore the hollow supporting part under the guides was used for the placement of the sensors. Even here the positioning of the sensors was rather limited, due to the bad access to the inner surface of the Y-beam. Only one set of sensors could be placed into the inner structure, on a plate in the middle of the hollow structure of the Y-beam. The second set of sensors was positioned on the outside surface of the supporting Y-beam part. The outside sensors were protected from direct radiation by the covering steel box as already pointed before. Sensors were placed in five cross-sections oriented in xz direction. The x-gradient was calculated from the temperature difference between two sensors laying in x-direction of each other.

Figure 4.14  Configuration of the sensors for yrz measurement.

Ad 3. The laser was placed on an aluminium plate extending over the granite table of the CMM in the middle x-position. The Z-beam was positioned in the middle of the X-beam position and then the x and z position were fixed. The measurement was performed over the whole range of the y measurement scale.
4.5.4 Set-up for yrx

For the measurement of yrx, the following set-up is needed:
- creation of a constant thermal z-gradient in the Y-beam (left and right)
- placement of 28 sensors in zx-planes in the Y-beam structure
- alignment of the rotational interferometer system along the Y-beam.

Ad 1. The z-gradient was created for both Y-beams, right and left, since both parts are considered as a source for yrx deviation. One radiant panel was positioned above each Y-beam, with a vertical distance of approximately 1m. The actual configuration is depicted on figure 4.15. The rest of the CMM structure (X-beam, Z-pinole, laser and the optical path) were insulated by insulation plates.

![Figure 4.15](image)

Figure 4.15 Configuration of the heating panels for yrx measurement.

Ad 2. The placement of the 28 sensors was equally divided over both beams. The same limitation of positioning of the sensors raised due to the impossible placement sensors on or into the Y-guides where the carriage moves and where the deformation is actually measured. Also here the hollow supporting Y-beam parts had to be used. Because of the bad access to the inner surface, both sets of sensors were placed at the inner plate, positioned in the middle of the hollow structure of both Y-beams. The configuration of the sensors in the Y-beams is displayed in figure 4.16. The z-gradient was calculated from the temperature difference between two sensors laying in z direction of each other.
Figure 4.16  Configuration of the sensors for \( y_{rx} \) measurement.

Ad 3. The alignment of the laser interferometer was the same as for \( y_{rz} \). Only the reflector and interferometer have to be turned 90°.
4.6 Measurement execution

The movement of the CMM's pinole and the collection of measurement data from the CMM, the thermometer and laser interferometer are carried out automatically by a data acquisition software package called DAQ. This software package, developed by Mitutoyo Nederland B.V. (MLE), is installed in the HP Vectra. Some of the main items which are requested by the DAQ program before performing a specific measurement are in figure 4.17:

![Diagram showing the main items of the DAQ software program.]

Figure 4.17 Main items of the DAQ software program.

The software automatically creates files for all data storage for the defined measurements. A graphical interpretation of all measured data is displayed during the measurements.
For every rotational deviation a measurement cycle according to the following time schedule was performed, see also figure 4.18:

0. rest  "rest-period": 10 hours, the CMM switched on, but no movements
1. up  "up-period": 1 hour, the CMM moving, performing the rotational measurement without any heat
2. ref  "reference": 3 repetitions of the rotational measurement without any heat
3. up1  "up-period": 4 hours, the CMM moving, heat level nr. 1
4. h1  3 repetitions of the rotational measurement on heating level nr.1
5. up2  "up period": 4 hours, the CMM moving, heat level nr. 2
6. h2  3 repetitions of the rotational measurement on heating level nr.2
7. up3  up-period": 4 hours, the CMM moving, heat on level nr.3
8. h3  3 repetitions of the rotational measurement on heating level nr.3

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4.18.png}
\caption{Measurement conditions during one measurement cycle.}
\end{figure}

For all measurements the displacement step was 50mm, the speed of movement 50mm/s. The number of samples per position were 1 for the CMM, 5 for the thermometer and 5 for the laser interferometer.
4.7 Measurement results

4.7.1 Tests on the temperature distribution field

To get an impression about the created temperature distribution field some tests were performed. On the X-beam the following dependencies were investigated: Y-grad(t), Z-grad(t), Y-grad(x), Y-grad(z), Z-grad(x) and Z-grad(y). On the Y-beam: X-grad(y) and Z-grad(y). Some representative graphs are given in figures 4.19 to 4.23.

The variation of the y and z gradient during time was found as depicted in figure 4.19. This variation increases with increasing heating level and reaches a maximum magnitude of 2°C/m, which is 0.4°C over the width of the X-beam.

The major influences of this effect can be the temporary shielding of the X-beam surface by the moving carriage and in addition a relatively small cooling effect from the air bearings in the carriage which disturb the stability of the created temperature field. The variation of the y-gradient reflects the thermal history from the previous thermal environment and therefore follows the variation of the measured position with a time delay of about 150s, see figure 4.19. To exclude this effect in the measurements averaging of all thermal gradients was performed.

![Graphs of temperature gradients](image)

Figure 4.19 Time dependency of the y-gradient, y-grad(t), and the corresponding positions of the measuring Z-beam.

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The other mentioned measured dependencies of the thermal gradients show that the thermal gradient does not have a constant value over the measured length and width resp. height of the observed parts. This result is in contradiction with the expected situation, where a constant gradient was assumed, as was already mentioned.

For the X-beam, several different sensor configurations were tried to get an impression of the actual temperature field. The main limitation of this experiment was the impossibility of placing of all sensors under the same conditions, due to the complicated inner structure containing welded plates. Placing sensors close to those welded cross-sections gave a different picture then placing them at the inner surface of the simple hollow structure. On the X-beam, the y-gradient was measured in three different positions of the z-coordinate, as shown in figure 3.6. These three different positions show a different coefficient of the y-gradient also dependent on the heating level used. In figure 4.20 two different heating levels each with four gradient corresponding to z-position from figure 3.6 are depicted. The highest gradient arose in the middle z-position of the X-beam in this experiment. The reason for this effect is mainly the direct radiation from the heating panels. The close projection of the panels to the radiated surface causes local temperature rise resulting in higher local gradients. In order to minimize these effects the heating panels were placed as far as possible from the surface which was directly radiated. For the input into the model also the average gradient was calculated. The same was found for z-grad(y) on the X-beam, where the adjustment of the angle between the panels showed rather good results in the reduction of the z-gradient variation with y-position, see figure 4.21. Therefore the slanted configuration of the heating panels is preferable for the creation of a more stable temperature field with constant gradients.

For the Y-beam, the measured dependency of x-grad(y) showed very different values of the gradient from one cross-section to the other, figure 4.22. The maximum value read over 40°C/m. Such large thermal gradients cannot physically exist in the structure subjected to similar heat conditions as in the case of the X-beam. The corrupted results from this measurement are evidently caused by the limitation of the placement of the sensors, which was described in paragraph 4.5.3, figure 4.13. The steel box shielded one set of sensors from direct radiation, but on the other hand caused bad ventilation of accumulated heat. Therefore these sensors scanned very high temperatures of the heated inner air. The calculation of the gradient between these sensors and the sensors located on inner plate results in very high values. The low gradients at the end points of the Y-beam were caused by the created radiation field from the panels, which was not covering the whole length of the Y-beam.
The conclusion from this measurement is that the shielding of the sensors by the steel box doesn't solve the problem of measuring distorted temperature values. Since time for the preliminary experiments was limited, these measured values were nevertheless used as the input into the model.

The measured dependency for $z$-grad($y$), figure 4.23, shows rather constant characteristics, closer comparable with the characteristics of $z$-grad($x$) on the X-beam. However, the measured gradient is also here a substitution of the actual gradient which could not be measured at the Y-guides as already discussed in paragraph 4.5.4. This gradient was measured with all the sensors placed on the inner plate, the accumulated heat could have distorted the actual values also in this case. Nevertheless these gradient values were used as the input into the model, because of the limited time for additional tests.

Figure 4.20 Dependency of $y$-grad($x$) for different z-positions at two different heat levels.
y-positions of Z-Gradients: $s_1$, $s_2$, $s_3$, average gradient

Figure 4.21 Dependency of $z$-grad$(x)$ for different $y$-positions at three different heat levels.

Figure 4.22 Dependency of $x$-grad$(y)$.  

- 55 -
number of the cross-section positioned in y-positions on the Y-beam, fig.4.16

Figure 4.23  Dependency of z-grad(y).
4.7.2 Result of \textit{xrz} measurement

In figure 4.24 a graphical interpretation of the thermal rotation measurement \textit{xrz} is given. The results are presented as a relative measurement (the reference measurement is subtracted), and for the three different heating levels corresponding with an output current (power) of 5A (1.1kW), 7.5A (1.65kW) and 10A (2.2kW) of the regulator. At each heating level the final measurement is realized by three repetitions of a forward-backward measurement cycle. The deviation can be characterized by a smooth linear dependency on the x-position of the movement. The measurement is evidently not influenced by the periodic variation of the gradient value in the time which was found in the figure 4.19. This is valid for all performed measurements.

![Figure 4.24 \textit{xrz} measurement.](image)
The resulting thermal deviations corresponding to three different heating levels are depicted in figure 4.25. The three different heating levels correspond with an output current (power) of 4A (0.88kW), 5A (1.1kW) and 7.5A (1.65kW) of the regulator. Because the radiating panels were hanging close to the plastic cabin, the heat levels were taken a little lower as for the \textit{xrz} measurement. On each heating level the final measurement is realized by three repetitions of a forward-backward measurement cycle. The deviation can be characterized by smooth linear dependency on the x-position of the movement.

![Graph](image_url)

\textbf{Figure 4.25} \hspace{1cm} xry measurement.
4.7.4 Result of yrz measurement

The characteristic of the thermal rotational deviation measurement looks rather smooth and linear with the position at all three created heating levels. Therefore the temperature measurement method indicating the measured deformations seems to be incorrect, probably because of the bad placement of the sensors. In figure 4.26 the graph of the thermal rotation measurement yrz is shown. The results are presented as a relative measurement (reference measurement is subtracted), and for the three different heating levels corresponding with an output current (power) of 5A (1.1kW), 7.5A (1.65kW) and 10A (2.2kW) of the regulator. At each heating level the final measurement is realized by three repetitions of a forward-backward measurement cycle.

Figure 4.26 yrz measurement.
4.7.5 Result of yrx measurement

In figure 4.27 a graph of the thermal rotation measurement yrx is shown. The results are presented as a relative measurement (reference measurement is subtracted), and for the three different heating levels corresponding with an output current (power) of 4A (0.88kW), 5A (1.1kW) and 7.5A (1.65kW) of the regulator. At each heating level the final measurement is realized by three repetitions of a forward-backward measurement cycle.

![Graph of yrx measurement](image)

Figure 4.27 yrx measurement.
5 COMPARISON OF THE PREDICTED AND MEASURED RESULTS

In the next paragraphs comparisons between the measurements and calculations are made. After completion of all measurements and calculations, a final check on the correctness of thermal model is performed.

5.1 Linear regression of the data

Due to the quadratic curvature of the bend structure of the CMM, because of the assumption of constant thermal gradients, the angular dependency on the position is expected to be linear. Therefore a linear function was fitted through the set of data (rotational deviation versus position) from relevant experiments and calculations.

The approximated linear function $\hat{y}$ to data, describing the relation between the position $x$ and deformation $y$ is given by:

$$\hat{y} = \hat{a}_0 + \hat{a}_1 x$$  \hspace{1cm} (54)

- $y$ the response variable: measured or calculated rotational deviations $\text{irj}$
- $x$ the regressor: certain positions for the measured or calculated $\text{irj}$
- $\hat{a}_0$ the absolute parameter of the regression line
- $\hat{a}_1$ the slope parameter of the regression line

A general method of estimating the parameters of a regression line is by the method of least squares (Chatfield, 83). The parameter $\hat{a}_1$ expressing the slope of the regression line was calculated in this way and used for the purpose of comparing the measured and calculated data. Therefore the parameter $\hat{a}_1$ found from the calculated data was divided by parameter $\hat{a}_1$ found from the measurement data. The ratio obtained in this way is indicating the percentage of the (measured) deformation which can be explained by the model. The table in figure 5.1 are given these percentage results. The presented values are corresponding to three different heating levels during which the gradients of the structure reached the values in the range of $3 - 15 \, ^\circ\text{C/m}$ for the X-beam and $3 - 40 \, ^\circ\text{C/m}$ for the Y-beam. The range of the resulting rotational deviations for these gradients is $0 - 150 \, \mu\text{m/m}$. From this table it can be seen that the X-beam rotational deviations can be successfully explained by the model.
The results for the Y-beam, however, fit not so well with the modelled results. The deformations predicted by the model are too high for all heating levels. This is due to the high gradients which were calculated for the Y-beam (max $40^\circ$C/m for the Y-beam vs $15^\circ$C/m for the X-beam). In paragraph 5.4 it will be explained that the gradients for the Y-beam are too big, because of erroneous temperature measurements. These errors are induced because of the impossibility to measure the temperatures causing the deformation of the Y-guide. Furthermore the CMM under test was only available for a limited period of the time. So the measurements for the Y-beam could not be repeated with the different sensor configuration.
5.2 Analyses of the results for the X-beam

As already shown in figure 5.1, the presented model is adequate to describe the deformations of the X-beam and therefore is useful. In order to find out whether the remaining differences between the measured and calculated values are caused by the model itself or by inaccuracies in the numerous measurements, the uncertainty of the measurements were determined and compared with the differences.

5.2.1 Determination of the uncertainty of the measured results

Since the measured and the calculated data are compared by the slope parameter \( a_1 \), the uncertainty of the measured data will be expressed as the variance of this parameter. The variance is derived as (Chatfield, 83):

\[
S^2 = \frac{\sigma_{y,x}^2}{\sum (x_i - \bar{x})}
\]  

(55)

\[
S^2 = \text{variance of the slope parameter of the regression line}
\]

\[
\sigma_{y,x}^2 = \text{residual variance of given observations from a linear regression function: deviations versus positions}
\]

\[
x_i = \text{certain positions of the observation during the measurement}
\]

\[
x = \text{mean value of } x_i
\]

The sum of squared deviations of the observed points from the regression line is given by the residual sum of squares:

\[
\Sigma (y_i - \hat{a}_0 - \hat{a}_1 x_i)^2
\]

(56)

It can be shown that an unbiased estimate \( \sigma_{y,x}^2 \) can be obtained by dividing this residual sum of the squares by \( n-2 \), where \( n \) is the number of the measurements.

\[
S_{y,x}^2 = \frac{\Sigma (y_i - \hat{a}_0 - \hat{a}_1 x_i)^2}{\frac{n - 2}{n - 2}}
\]

(57)
the denominator, n-2, shows that two degrees of freedom have been lost. This is because \( \hat{a}_0 \) and \( \hat{a}_1 \) were estimated from the data.

In figures 5.2-5.7 the variance of the slope parameter \( \hat{a}_1 \) of the measured rotational deviations is depicted as an area between dashed lines.

### 5.2.2 Determination of the uncertainty of the calculated results

The calculated results are based on the values of the temperatures measured at certain points of the structure. The measurements of the temperature therefore introduce additional uncertainty to the model, which are not caused by the model itself. The determination of the standard deviation of the thermal gradient must precede the final standard deviation of the calculated rotational deviation. For the determination the same method is used for both cases.

**Standard deviation of the thermal gradient**

The thermal gradient is derived mathematically from equation (17).

As an example the standard deviation for the thermal gradient \( c \) is performed:

\[
\begin{align*}
  c &= \frac{T_1 - T_2}{e_y} = f(T_1, T_2, e_y) \\
  \text{(58)}
\end{align*}
\]

The variance of the thermal gradient \( c \) is:

\[
S_c^2 = (\frac{\delta f}{\delta T_1})^2 S_{T_1}^2 + (\frac{\delta f}{\delta T_2})^2 S_{T_2}^2 + (\frac{\delta f}{\delta e_y})^2 S_{e_y}^2 \\
\text{(59)}
\]

\[
\frac{\delta f}{\delta T_1}, \frac{\delta f}{\delta T_2}, \frac{\delta f}{\delta e_y} \quad \text{partial derivatives of the function } f(T_1, T_2, e_y)
\]

The value of the gradient is found as a functional dependency on three variables as noted in the equation above. These three variables are obtained by measurements and their standard deviations are known (estimated).

The standard deviations \( S_{T_1}, S_{T_2} \) of the calibrated thermal sensors were for the measurement conditions \( \pm 0.05^\circ\text{C} \).

The standard deviation \( S_{e_y} \) for the determination of the length dimension \( e_y \) estimated from the adjustments of the sensors was dependent on the beam structure. In this case estimated to \( \pm 0.0025 \text{ m} \).
The confidence interval for the gradient c can be described as:

\[ c \pm 2S_c \quad (95\%) \tag{60} \]

In the figures 5.8-5.9 these confidential intervals are depicted as a bar in the points on the X-beam for which the gradients are calculated.

**Standard deviation of the calculated rotational deviation**

Determination of the standard deviation of the rotational deviation follows from the above described method for the gradient. An example of the calculation for the geometrical deviation \( x_{rz} \) caused by thermal gradient \( c \) is derived here. The rotational deviation for one element is described by a function of three variables as was explained in chapter 3, eq.(24):

\[ x_{rz} = -acx = f(a, c, x) \tag{61} \]

The variable \( x \) representing the length of the elements of the X-beam has a constant chosen value which is not influenced by the measurements. But it should be noted that the choice of the interval length \( x \), i.e. the number of intervals in which the X-beam is divided, will influence the calculated deformations. However this is part of the model itself and as already mentioned before the aim of current calculation is to decide about the influence of the inaccuracies which are not part of the model. Therefore \( S_x = 0 \).

The variance of the rotational deviation \( x_{rz} \) is:

\[ S_{x_{rz}}^2 = (\frac{\delta f}{\delta a})^2 S_a^2 + (\frac{\delta f}{\delta c})^2 S_c^2 \tag{62} \]

\[ \frac{\delta f}{\delta a}, \frac{\delta f}{\delta c} \] partial derivatives of the function \( f(a, c) \)

The standard deviation of the coefficient of expansion \( S_\alpha = 1.1 \times 10^{-6} \, ^\circ C^{-1} \).

The standard deviation of the thermal gradient \( S_c \) was calculated above.

The confidence interval of the rotation deviation \( x_{rz} \) can be described as:

\[ x_{rz} \pm 2S_{x_{rz}} \quad (95\%) \tag{63} \]

In the figures 5.2-5.7 these confidence intervals are depicted as a bar in the points on the X-beam for which the rotational deviations are calculated.
5.3 Comparison of the measured and calculated values for the X-beam

Figures 5.2 - 5.7 show the rotational deviations of the X-beam for three heating levels, figures 5.2-5.4 show the rotational deviation \( x rz \) and the figures 5.5-5.7 the deviation \( x ry \). The calculated thermal gradients which were used as input data for the model are depicted in figures 5.8 and 5.9.

All figures are showing both the measured values with the fitted least square lines and the calculated values also with the fitted lines. As already mentioned before also the confidential intervals, which indicate the part of the deformation due to the inaccuracy of the calculation of the gradients (bars) are shown. Finally the confidence interval in which the measured values are located are shown (dashed lines).

All heating levels show a good prediction of the deformations by the model. Furthermore the position of the confidence intervals show, that the remaining differences between the measurements and the model can be explained by the inaccuracies in the measurements. So for this situation the model itself is accurate enough. The development of a more detailed model for the X-beam is only useful when the gradients can be determined more accurately.

![Figure 5.2: Rotational deviation \( x rz \) and the confidence intervals for the calculation and the measurement for heating level 1.](attachment:image.png)
Figure 5.3: Rotational deviation $\theta_{xz}$ and the confidence intervals for the calculation and the measurement for heating level 2.

Figure 5.4: Rotational deviation $\theta_{xz}$ and the confidence intervals for the calculation and the measurement for heating level 3.
Figure 5.5: Rotational deviation $x_{ry}$ and the confidence intervals for the calculation and the measurement for heating level 1.

Figure 5.6: Rotational deviation $x_{ry}$ and the confidence intervals for the calculation and the measurement for heating level 2.
Figure 5.7: Rotational deviation $\alpha_{xy}$ and the confidence intervals for the calculation and the measurement for heating level 3.
Figure 5.8  Y-thermal gradient at five cross-sectional positions of the X-beam from figure 4.10 for xrz calculation for three heat levels.

Figure 5.9  Z-thermal gradient at five cross-sectional positions of the X-beam from figure 4.13 for xry calculation for three heat levels.
5.4 Comparison of the measured and calculated values for the Y-beam

The measured and calculated rotational deviations of the Y-beam are plotted in figures 5.10-5.11. Figure 5.10 is showing the results of the yrx deviation (deformation due to the z-gradient) and figure 5.11 of the yrz deviation (deformation due to the x-gradient). In both cases, again three heating levels were used. The dashed lines represent the calculation results (not averaged), the solid lines are the actual measurements. The calculated thermal gradients which were used as input data for the rotational deviation calculations are depicted in figures 4.22-4.23 in chapter 4. The figures 5.10-5.11 and comparison of the slope parameter $a_1$ in the table in figure 5.1. show that the calculated results and the measured results differ for the Y-beam. As already mentioned in paragraph 5.1 this is due to the fact that the gradients are estimated too high. In chapter 4 it is explained that the sensor configuration and heating conditions were very limited and therefore the scanned temperatures probably don't represent correct input data into the model. Due to the construction of the CMM the temperatures could not be measured directly at the guides (the sensors on the guide would prevent the CMM from moving) as also pointed in chapter 4. Therefore the temperatures were measured at inside plate of the hollow construction which is supporting the guides. The temperatures, and also gradients calculated by these temperatures, were obviously not the same as the actual temperatures and gradients causing the bending of the guides. Beside this fact, also the method of the heating has been of influence. In some parts of the machine the air temperature has reached higher values than the local machine temperature. The temperature sensors, which were connected at the CMM in those places, were also heated by the air and by this indicating probably wrong temperatures.

Figures 4.22-4.23 representing the gradients, x-grad(y), z-grad(y), also indicate erroneous temperature measurements, because the gradients shown are not constant. Since a similar beam construction as Y-beam, namely X-beam, under similar heating conditions, does show a relatively constant gradient, it is unlikely that there will be such a different temperature field in the Y-beam.

To overcome these problems for the Y-beam there are some possible solutions:

1. The temperatures should be measured at or in the Y-guides themselves to perform the measurement in the same way as for X-beam. Measuring at a Y-guide will constrain the CMM from moving, which means that the measurements of the deformations cannot be performed by the CMM itself. However this leads to the major disadvantage of this measurement. Such a measurement setup cannot measure the influence of the movement'shielding on the local beam temperature field.
2. Measuring at the Y-guide or Y-support would be preferable, but this means that changes in the concept of the design of the machine itself have to be made. The possible access into the inner structure and placement sensors at the inner surface of the guides or Y-supports should be included in the new concept of the machine design.

3. When there is no access into the inner structure made, the relation between the actual temperatures of the Y-guides, Y-supports and temperatures of the inner trunk plate of Y-support should be examined (theoretically or experimentally). By this the gradients in the Y-guides might be determined more accurately from the measured temperatures in the inner trunk plate of Y-supporting construction. However, this would make a relative simple model rather complicated.

Figure 5.10 Rotational deviation \( y_{rx} \), calculation and the measurement for three heat levels.
Figure 5.11  Rotational deviation $y_{rz}$, calculation and the measurement for the three heat levels.
5.5 Sensor reduction for the X-beam

Reduction of the number of sensors is limited by the deviation of the temperature distribution from a linear distribution field. So at first, it has to be proven that the temperature field in the CMM is linear in practice.

In the ideal case, such a temperature field holds three constant thermal gradients and each of them can be measured by two sensors positioned in the direction of dependency of the gradient. Therefore in such a case only three sensors are enough to indicate the constant value of two gradients influencing the angular deformation of one beam. This ideal configuration is depicted in figure 5.12.

![Diagram](image)

Figure 5.12 Reduced sensor configuration in one cross-section for a linear temperature distribution field on the X-beam.

In the experiments of the temperature distribution field, paragraph 4.7.1, the dependencies \( y\text{-grad}(z) \) resp. \( z\text{-grad}(y) \) were measured for the X-beam. The measurements were performed for three different \( y \) resp. \( z \) positions, figure 3.6. The best results for the prediction of rotational deviations were found when for the input data of the model were taken the gradient measured in the middle position \( s=3 \), see figure 4.21, figure 3.6. It can be concluded from these experimental experiences that the middle (\( s=3 \)) \( y \), resp. \( z \) position in the cross-section of the X-beam gives the most actual temperature values for the model input. Therefore it is advisable to use four sensors placed in these middle positions in the cross-section, as depicted in figure 5.13, instead of three sensors.
A decision about the number of necessary measurement cross-sections in the X-beam is dependent on the accuracy demand of the deviation prediction. From this demand could be drawn the requirement on the accuracy of the gradient and experimentally estimated the number of the cross-sections and the sampling time for the gradient determination. It can be seen from the measurements which were performed, that two sensors would be enough to predict closely the measured value of the rotational deviation but with higher inaccuracy, an example from one of the measurements $x_{rz}$ at the end position $x = 920$ mm:

- the measured deviation: $x_{rz} = 36 \pm 0.8 \mu m/m$
- the calculated deviation (by five cross-sections, see figure 4.10):
  
  $x_{rz} = 33 \pm 6.6 \mu m/m$
- the calculated deviation (by two sensors in the middle cross-section, see figure 4.10):
  
  $x_{rz} = 33 \pm 14.5 \mu m/m$

Figure 5.13 Advised, reduced sensor configuration in one cross-section for a linear temperature distribution field.
CONCLUSIONS

The model presented in this report showed good results for the prediction of rotational deviations on the X-beam exposed to a linear distribution field. The model was tested for thermal gradients in the range of 3 - 15 °C/m. Even if the gradients measured at different positions of the structure didn't match with each other, the beam bend smoothly and the rotational deviation had a linear dependency on the position of the measuring probe. The range of the rotational deviations during the measurement was 0 - 150 μm/m.

The results from the corresponding mechanical behaviour of the X-beam showed that the X-beam bends according to a mean constant gradient and additional internal heat sources of the CMM (described in chapter 2), have probably only a minor influence on the deviation characteristic in the range of the presented thermal gradients.

For the Y-beams, however, there are relatively big differences between the measured and calculated deviations. The calculated results are about a factor 2 higher than the measured results. These differences are due to the incorrect calculation of the gradient, caused by the erroneous temperature data measurement. This leads to incorrect input data into the model. Since the model is useful for the X-beam, it should be investigated why the results for the Y-beam are a bit disappointing. As already pointed out before, the temperature data was unreliable, therefore a lot of attention should be paid to measurements of the temperature distribution field of the Y-beam. In order to make the model also useful for the predictions of both Y-beam deviations there are some principle solutions:

- The model has to be extended in such a way that the gradients in Y-guides and Y-supports can be converted from the temperatures in the points of the Y-beam which are "easy" to measure.

- The temperatures of the Y-guides or Y-supports themselves have to be measured. This leads to a change in the concept of the design of the Y-beams. The possible access into the inner structure and the placement of temperature sensors at the inner surface in the same way as for the X-beam should be made.

- The measuring set-up has to be changed:
  The use of different temperature sensors, which are smaller size than currently used Pt-100 and can be screwed into the Y-beam structure itself, see figure 6.1. For such a placement of temperature sensor small holes should be drilled in the side of the Y-beam.
The model and the measurements could be also converted from the temperature inputs into the strain inputs. An investigation whether the strain measurement performed by the strain gages get better results of deviation prediction can be examined.

Figure 6.1 Advised construction of the temperature sensors and possible mounting onto the Y-beam

From the positive results on the X-beam and also the results on Y-beam general conclusions about the conditions under which the model is valid can be drawn. The most important for a successful application of the model are correct input data. Therefore two conditions must be fulfilled for the use of the model in practice:

1) the temperature field in the machine must be linear;
2) the placement of the sensors must be chosen at positions which can guarantee the correct reading of the temperatures which cause the bending effect.

At the moment there is no knowledge about the use of the model for non linear temperature fields. Therefore, as a recommendation for the further research, the real temperature field of the CMM placed on the work-shop floor should be tested to give direction for further research. The question about the range of the real thermal gradients and whether they are constant non constant should be answered.

The presented model should be tested for different circumstances of the heating and also for a different magnitude of gradients, probably smaller than what was measured in this research.
When there is sufficient know how about the validity of the model and the practical temperature situations in which CMMs are operating, software correction for the thermal deformations can be applied based on this model. In order to make software correction economically interesting, only a small number of temperature sensors should be applied. Therefore it is important to reduce the number of sensors. There should be more extended research about possibilities for the reduction of the number of sensors.

When the assumptions of a linear temperature distribution field turns out to be valid, also under practical working conditions, there are good possibilities for sensor reduction as discussed in paragraph 5.5 for the X-beam. For measurement of three constant gradients only four sensors per beam may be enough, three of them positioned as depicted on the figure 5.12 and one extra in x-direction. Dependent on the required accuracy more sets of sensors may be applied.

For the further research the special temperature sensors should be developed which can be screwed on the outside surface of the beams, but which do not suffer from direct radiation. Making the sensors mountable from the outside also makes the software correction possible for already existing systems, easier to assemble and service.
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LIST OF SYMBOLS

a constant due to the uniform temperature distribution field (constant expansion)

\(a_0, a_1\) parameters of regression line

b, c, d constant temperature gradients due to the nonuniform temperature distribution field (linear expansion)

B,C,D gradient matrix

c_0, c_1, c_2 integration constants

e_y, e_z dimension of the elements

E Young's modulus

f_0, f_1, f_2 functions

f'_0, f'_1, f'_2 differentiations of the functions \(f_0, f_1, f_2\)

G shear modulus

irj rotational deviation of the structure

itj translation deviation of the structure

n number of the element

R resistance

S standard deviation

T temperature

t time

u, v, w axes of the deformation field in x, y and z direction

U', V', W' matrices of angular thermal deformation

x, y, z axes of coordinate system of the structure, axes of the temperature distribution field

\(\beta\) regression line

\(\alpha\) coefficient of thermal expansion of the CMM

\(\beta(x)\) angle of the carriage on the X-beam

\(\varepsilon\) strain matrix

\(\varepsilon_{ij}, \varepsilon_{ii}\) strain components (shear, normal)

\(\sigma\) stress matrix

\(\sigma_{ij}, \sigma_{ii}\) stress components (shear, normal)

\(\lambda, \mu\) Lame constants

\(\nu\) Poisson's ratio
Appendix I:

Graphical representation of the sensor calibration measurement
Heating cycle before the calibration

![Graph showing the heating cycle before the calibration. The x-axis represents the number of measurements, ranging from 0 to 800, and the y-axis represents resistance, ranging from 106 to 118.]
Appendix II:

Example of the programs for the modelling:

- **ZGRYX1.M** calculation of $z$-gradient
- **PRYX1.M** calculation of $y_{rx}$ deviation and comparison with the $y_{rx}$ measurement
- **FRYX1.M** fitting linear regression line through the calculated and measured data and the calculation of the parameters of the regression line.
This program calculates the gradients through the observed temperature in the 8 cross-sections of the 8 beam.

Temperatures are loaded by the 8-gradient variable; graph through the lower length and grad3-through the upper length and average-grad3 average gradient calculated from grad1, grad2, grad3.

If grad3 is calculated from the 1st heating level:

```c
load ryst1, ryst2 = ryst1(:,1:5); 
```

If grad3 is calculated from the 2nd heating level:

```c
t1(:,15»)/l80*1000, 
```

```c
start to load data from the reference measurement
```

clear a; 
load ryst1, ryst2 = ryst1(:,15»); 
```

```c
start to load data from the first heating level on
```

clear a; 
load ryst1, ryst2 = ryst1(:,1:5); 

```c
save grad1 grad3; 
```

```c
save grad2 grad3; 
```

```c
calculate 8-gradient grad1(gradient) from 1 heating level
```

```c
for i = 1:18, 
```

```c
grad1(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
plot(i,i,grad1(:,i)); 
```

```c
xlabel(('8-beam cross-sections-lam-grade1')); 
```

```c
ylabel('meta name'); 
```

```c
eval('gammaנפתח, name, /apa op'); 
```

```c
end; 
```

```c
save grad2 grad3; 
```

```c
calculate 8-gradient middle grad3(gradient) from 1 heating level
```

```c
grad3(:,i) = grad1(:,i); 
```

```c
clear grad3; 
```

```c
grad3(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
grad3(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
clear grad3; 
```

```c
save grad2 grad3; 
```

```c
for i = 1:18, 
```

```c
plot(i,i,grad3(:,i)); 
```

```c
title(('8-beam cross-sections-middle-grade3')); 
```

```c
xlabel('time [s]'); 
```

```c
ylabel('meta name'); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
average gradient avggrad from 1 heating level
```

```c
for i = 1:18, 
```

```c
avggrad(:,i) = grad1(:,i) + grad3(:,i); 
```

```c
avggrad(:,i) = grad1(:,i) + grad3(:,i); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
load ryst2, c = ryst2(:,1:5); 
```

```c
clear t2; 
```

```c
t2 = c(:,1:5); 
```

```c
calculate 8-gradient grad2(gradient) from 2 heating level
```

```c
for i = 1:18, 
```

```c
grad2(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
plot(i,i,grad2(:,i)); 
```

```c
xlabel('time [s]'); 
```

```c
ylabel('meta name'); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
save grad2 grad3; 
```

```c
calculate 8-gradient grad3
```

```c
for i = 1:18, 
```

```c
grad3(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
plot(i,i,grad3(:,i)); 
```

```c
title(('8-beam cross-sections-middle-grade3')); 
```

```c
xlabel('time [s]'); 
```

```c
ylabel('meta name'); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
average gradient avggrad from 2 heating level
```

```c
for i = 1:18, 
```

```c
avggrad(:,i) = grad2(:,i) + grad3(:,i); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
load ryst2, c = ryst2(:,1:5); 
```

```c
clear t2; 
```

```c
t2 = c(:,1:5); 
```

```c
calculate 8-gradient grad2(gradient) from 2 heating level
```

```c
for i = 1:18, 
```

```c
grad2(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
plot(i,i,grad2(:,i)); 
```

```c
title('8-beam cross-sections-middle-grade2'); 
```

```c
xlabel('time [s]'); 
```

```c
ylabel('meta name'); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
calculate 8-gradient grad3
```

```c
for i = 1:18, 
```

```c
grad3(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
plot(i,i,grad3(:,i)); 
```

```c
title('8-beam cross-sections-middle-grade3'); 
```

```c
xlabel('time [s]'); 
```

```c
ylabel('meta name'); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
average gradient avggrad
```

```c
for i = 1:18, 
```

```c
avggrad(:,i) = grad2(:,i) + grad3(:,i); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
load ryst2, c = ryst2(:,1:5); 
```

```c
clear t2; 
```

```c
t2 = c(:,1:5); 
```

```c
calculate 8-gradient grad3
```

```c
for i = 1:18, 
```

```c
grad3(:,i) = ((t1(:,i) - t1(:,15) - t1(:,15) + t1(:,1)))/180*1000; 
```

```c
plot(i,i,grad3(:,i)); 
```

```c
title('8-beam cross-sections-middle-grade3'); 
```

```c
eval('gamma_tcb, name, /apa op'); 
```

```c
end; 
```

```c
change of the gradient dependency on the change of rotational axis error
```

```c
laser reading of rotational error - reorganization of data from reference and heating levels measurements
```

```c
```

```c
save ryst1 ryst2; 
```

```c
clear ryst1; 
```

```c
```

```c
save ryst2; 
```

```c
clear ryst2; 
```

```c
clear ryst2; 
```

```c
```
```matlab
save rel3 rel3;
end;

plot(plt(t,3).rel(t,3).b2(t,3).rel(t,2).b1(t,2).rel(t,3));

plot(b1(t,1),rel(t,1),b2(t,1),rel(t,2),b3(t,2),rel(t,3));

grid;

title('Rys error 16-07-91');
xlabel('position on the 3-beam [mm]');
ylabel('Rys error [micrometers/beam]');

eval(['meta',name]);

eval(['egps36',name,'/egps)];
```

-86-
Grad11 from KBsys1.M
load grad11.mat
C11 = grad11;

% calculate rotational error
for i = 1:115,
    rot11(i,1) = alpha*c11(i,1)*(-0.1);% (1)
end;

% summation of the rotational error
rot11 = sum(rot11,1);

% translation(rotation) of the system origin into a real measurement situation
rot21(:,1) = vmp11(:,1);
rot21(:,2) = vmp11(:,2)+vmp11(:,1);
rot21(:,3) = vmp11(:,3)+vmp11(:,1);
rot21(:,4) = vmp11(:,4)+vmp11(:,1);
rot21(:,5) = vmp11(:,5)+vmp11(:,1);
rot21(:,6) = vmp11(:,6)+vmp11(:,1);

% Grad21 from KBsys1.M
load grad21.mat
C21 = grad21;

% calculate rotational error
for i = 1:116,
    rot21(i,1) = alpha*c21(i,1)*(-0.1);% (1)
end;

% summation of the rotational error
rot21 = sum(rot21,1);

% translation(rotation) of the system origin into a real measurement situation
rot31(:,1) = vmp21(:,1).*6;% (1)
for i = 1:118,
    rot31(:,1) = rot31(:,1) + vmp21(:,i);
end;

% Grad31 from KBsys1.M
load grad31.mat
C31 = grad31;

% calculate rotational error
for i = 1:119,
    rot31(i,1) = alpha*c31(i,1)*(-0.1);% (1)
end;

% summation of the rotational error
rot31 = sum(rot31,1);

% translation(rotation) of the system origin into a real measurement situation
rot31(:,1) = vmp31(:,1).*6;% (1)
for i = 1:119,
    rot31(:,1) = rot31(:,1) + vmp31(:,i);
end;

load Gradewl from KBsys1.M
load apgrad.mat
calculation of the correlation parameters of and cl
fitting 1st order polynomial through calculated and measured data

FIRST HEATING LEVEL

load vmp11.mat
load vmp21.mat
load vmp31.mat
load vmp12.mat
load vmp22.mat
poly111(1,:)=polyfit([11,1],vmp11(1:1,1));
poly121(1,:)=polyfit([11,1],vmp12(1:1,1));
poly131(1,:)=polyfit([11,1],vmp13(1:1,1));
poly112(1,:)=polyfit([11,1],vmp11(1:1,1));
poly122(1,:)=polyfit([11,1],vmp12(1:1,1));
poly132(1,:)=polyfit([11,1],vmp13(1:1,1));
plot([11,1],vmp11(1:1,1),'-','-',vmp12(1:1,1),'-','-',vmp13(1:1,1),'-');
hold on;
plot([11,1],vmp11(1:1,1),'-','-',vmp12(1:1,1),'-','-',vmp13(1:1,1),'-');
grid;

test(["d01",numstr(poly11(1,1),10')"-3"],["uh"]);
test(["d01",numstr(poly12(1,1),10')"-3"],["uh"]);
test(["d01",numstr(poly13(1,1),10')"-3"],["uh"]);

hold off;

SECOND HEATING LEVEL

load vmp12.mat
load vmp22.mat
load vmp32.mat
load vmp13.mat
load vmp23.mat
poly211(1,:)=polyfit([11,1],vmp21(1:1,1));
poly221(1,:)=polyfit([11,1],vmp22(1:1,1));
poly231(1,:)=polyfit([11,1],vmp23(1:1,1));
poly212(1,:)=polyfit([11,1],vmp21(1:1,1));
poly222(1,:)=polyfit([11,1],vmp22(1:1,1));
poly232(1,:)=polyfit([11,1],vmp23(1:1,1));
plot([11,1],vmp21(1:1,1),'-','-',vmp22(1:1,1),'-','-',vmp23(1:1,1),'-');
hold on;
plot([11,1],vmp21(1:1,1),'-','-',vmp22(1:1,1),'-','-',vmp23(1:1,1),'-');
grid;

test(["d01",numstr(poly21(1,1),10')"-3"],["uh"]);
hold off;

THIRD HEATING LEVEL

load vmp13.mat
load vmp23.mat
load vmp33.mat
load vmp14.mat
load vmp24.mat
poly311(1,:)=polyfit([11,1],vmp31(1:1,1));
poly321(1,:)=polyfit([11,1],vmp32(1:1,1));
poly331(1,:)=polyfit([11,1],vmp33(1:1,1));
poly312(1,:)=polyfit([11,1],vmp31(1:1,1));
poly322(1,:)=polyfit([11,1],vmp32(1:1,1));
poly332(1,:)=polyfit([11,1],vmp33(1:1,1));
plot([11,1],vmp31(1:1,1),'-','-',vmp32(1:1,1),'-','-',vmp33(1:1,1),'-');
hold on;
plot([11,1],vmp31(1:1,1),'-','-',vmp32(1:1,1),'-','-',vmp33(1:1,1),'-');
grid;
hold off;

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Appendix III:

Example of the measurement input data of the DAQ program:

RYX1R.FMS xry measurement
RZX0R.FMS xrz measurement
RXY1R.FMS yrx measurement
RZY1R.FMS yrz measurement
DACframe
Module = 3;
Type = 10;
Interval = 0;
Operator = "William"
Commens = "Nystria"
DisplayMode = graphemic
ONIVai low = -6.000000;
ONIVai high = 20.000000;
OFFVal low = 20.000000;
OFFVal high = 25.000000;
Date = 17-07-91;
Time = 16:17:01
Version = DAQV1.0;
Menu-Mode = enabled;
Save-Mode = enabled;
End;

ANKframe
Comm reading = enabled;
GNI reading = disabled;
STEPpos(m) = 0.000000;
STEP(m) = 100.000000;
End;

ONIframe
Module = enabled;
Device = PD75;
Controller = ONIC-1;
Step mode = equidistant;
Type = line;
Plane = 0;
Line = 1;
Input Xpos = manual;
Speed(m/s) = 50.000000;
Hess = 0;
Delay (ms) = 5000;
Mode = 16;
NxtVal = 900.0000, 89.1340, 16.1310;
NxtVal = 200.0000, 88.1310, 16.1310;
LastVal = 900.0000, 89.1340, 16.1310;
LastVal = 200.0000, 88.1310, 16.1310;
Lable = 0.000000;
NxtSet = 16;
Fixed-1 = (1);
NxtSet = 16;
NxtSet = 16;
Fixed-0 = 0;
Fixed-1 = 0;
NxtSet = 16;
NxtSet = 16;
End;

ONIframe
Module = enabled;
TX device = non;
TY device = non;
RX device = non;
RT device = Renishaw;
RS device = non;
Hess = 51;
Delay (ms) = 10;
Speed(m/s) = 50.0000, 0.0000, -75.0000;
Speed(m/s) = 0.000000;
SGP (m/s) = 0.000000;
Sag sign = 1;
TX sign = 1.000000;
TY sign = 1.000000;
RX sign = 1.000000;
RT sign = 1.000000;
RS sign = 1.000000;
End;

NIIframe
Mode = sequential;
Device = 772;
Hess = 35;
P-Counter = 1;
Channels = 1, 28;
Ty-Channels = ;
Ty-Channels = ;
Ty-Channels = ;
End;

HI fram e
ID Address = 753;
Time(m) = 2000;
Cls. Linear = 1000.000000;
Cls. Linear = 1000.000000;
Cls. Nodat = 1000.000000;
End;

PR1frame
Calib. Code = CFI;
I settle = 01;
I delay = 100;
I interval = 500;
I sample = 200;
Bref chan# = 0;
Bref Smp# = 5;
Bref Value = 99.986000;
Bref chan# = 17;
Bref Smp# = 5;
Bref Value = 115.513000;
Bref chan# = 0;
Bref Smp# = 9;
Bref Value = 99.994000;
Bref chan# = 11;
Bref Smp# = 5;
Bref Value = 115.516000;
Max Value = 100.000000;
Max Value = 120.000000;
End;

End.
<table>
<thead>
<tr>
<th>Step</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.000000</td>
<td>16.000000</td>
<td>23.000000</td>
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