Mechanical Aspects
of
High-Field Magnet Coil-Systems

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Summary

This report primarily deals with the mechanical design of the coil-system in a 60 tesla magnet. This design is supported by numerical simulations within the framework of the Finite Element Method (FEM). An axisymmetric model is developed to calculate the stresses and deformations in the entire coil volume. Validation of the model is achieved by comparing the stresses in the mid-plane of the coil with results from an analytical approach. In this analytical approach the insulation material, separating the windings and stabilising the position of the windings, is considered as relatively weak compared to the winding material. More realistic material modelling, however, can be performed rather easily within the FEM.

After validation of the model the following influences on the stresses and the strains in the winding material were studied:
- additional shear and tensile stiffness of the insulation material
- strain-hardening of the wire material
- thermal loading of the wire material
- imperfections, local reduction of strength
- the application of steel shields

Besides, the FEM is also used to analyse coil designs generated by other institutes. The simulation of those coils, results in stress solutions that are comparable to the data supplied by the various designers. The differences which are observed can be attributed to the different assumptions made in various models.

Complementary, the results of experimental work on the fatigue properties of the hard-drawn wire material CuAg are presented. To facilitate the interpretation of these results and to validate the translation of the behaviour in a stress-controlled uniaxial push-pull test to the strain-controlled triaxial load condition in a real coil, this study was extended to the fatigue properties of hard-drawn Cu as used for over 25 years in the existing 40 tesla installation.

keywords: magnet, fatigue, copper
1 Introduction

1.1 General introduction

This report concludes the research project of the design engineer's programme for computational mechanics at the Eindhoven University of Technology, Faculty of Mechanical Engineering. The project involved participation in the 60 tesla Project at the Van der Waals-Zeeman Laboratory at the University of Amsterdam.

The 60 tesla project group is working on the design of a magnet installation capable of momentarily (100 ms) generating a magnetic field of 60 tesla. This high field will be used for fundamental materials research, on mostly metallic materials, in the Faculty of Physics and Astronomy. The entire project includes the design and construction of a magnet coil-system, a power supply and regulation unit, a cryogenics installation, instrumentation and a housing for the complete installation.

The research project of the design engineer’s programme is a subproject of the mechanical design of the 60 tesla coil-system. In the design of this system, there are restrictions on power, produced heat, and mechanical load [1, 2, 3]. The volume of the coil, for a given bore, is governed mainly by energy and heating considerations. The maximum admissible field depends on the tensile strength of the winding material. Small volume coils need a high current-density to generate the field, leading to a rapid heating of the coil. Larger coils can have a lower current-density, but the self-inductance increases the rise-time of the field; this also causes heating. With an optimum coil volume, heating can be kept within acceptable limits. This optimum volume requires materials with high strength and good electrical conductivity.

The aims of the present project are:

1. To develop a Finite Element Model (FEM) to verify the analytical model [4] for the calculation of the mid-plane stresses. This FEM will also produce the stresses and deformations outside the mid-plane, and it gives the opportunity to use more realistic material parameters and material descriptions.

2. To perform low-cycle fatigue experiments, in the for the magnet design relevant load-range, on the winding material to be used.

3. To compare the results for the Amsterdam design method with results on similar coils obtained at various institutes by slightly different approximations.

Before continuing with the outline of the report it is necessary to give an introduction into the physical behaviour of a magnet coil. Figure 1.1.1 shows a simplified model of a high-field magnet coil-system. The Kapton taped wire, cross-section about 4x6 mm², is wound with a small pitch around a spine, in about 25 turns. At the ends the pitch and radius changes cause some imperfections in the winding compactness. During the winding process the space between adjacent windings is filled with a resin insulation material, to avoid electrically contacts and to reduce the displacement of the windings. After several layers a reinforcing (stainless) steel shield can be placed. This shield acts like a stiff ring to be a resistance to radial displacements of the coil. A number of these concentric coils, of different heights, can be assembled to form a magnet coil-system.

The coil is connected to a regulated power supply. The current through the windings generates a magnetic field in the coil, that is almost the highest in the centre of the bore. The field decreases quadratically to the radial outside of the coil, and is symmetric with respect
to the \( r\theta \)-mid-plane. The combination of current-density \( J \) and magnetic field \( B \) causes a Lorentz-force (per unit of volume) \( F \), with components: \( F_r = JB_z \) and \( F_z = -JB_r \).

![Diagram of a magnet consisting of concentric wire-wound coils, loaded by a Lorentz-force into the radial \((F_r)\) and axial \((F_z)\) direction \((J = \text{current-density})\). Typical data are: bore 0.025 m, outer diameter 0.56 m, height 0.34 m. Temperature rise 200K during pulse of 100 ms. Maximum power 85 MW. Weight 500 kg.](image)

The axial field component gives an outwards directed radial force that causes a tangential stress in the winding material. The radial field gives a compressive directed axial force that is causing an "independent" (compressive) axial stress, symmetric with respect to the \( r\theta \)-mid-plane. The stresses in the winding are independent of the wire cross-section for identical current-densities and fields. Figure 1.1.2 shows the tangential and radial stresses in the winding material of a single coil for some simple modelling cases.

For a single coil with a large \( \alpha \) (the ratio between outer and inner radius) three different deformation characteristics can be found. On the inside the radial Lorentz-force separates the elastically deforming windings from each other. On the outside this force makes that the elastically deforming windings touch each other. In the middle part the equivalent stress in the windings is so high that the touching windings are plastically deforming, that is, situation 4 in figure 1.1.2.

For an assemblage of mechanically independent concentric coils the same deformation characteristics can be found, distributed over all coils. The stresses, however, are adapted to fulfill the correct boundary conditions. Each concentric coil now has a small \( \alpha \) and only the innermost coil deforms with separated elastic windings on the inside and plastic deformations on the outside of the shielded coil. The outermost coil can show elastic deformations with touching windings. Intermediate coils are reinforced by steel shields, because the windings are plastically deforming up to the outside of the coil.
1. Unsupported elastic windings (no radial force transmission).
2. Supported elastic windings (tensile and compressive radial force transmission).
3. Supported elastic windings, separating under tensile radial forces (only compressive radial force transmission).
4. Supported and separating windings, including plastic deformation.

1.2 Outline

The outline of the report is as follows. In section 2 it is explained that the materials the coil consists of, can be regarded as a continuous orthotropic material. The Halpin-Tsai composite theory can be used to express the elastic behaviour of the composite in the material properties of winding and insulation material. The formulae for the two-dimensional ply used by Halpin-Tsai are extended, with ordinary series/parallel connections of the stiffnesses, to a three-dimensional orthotropic material for the coil. An analytical model is presented to calculate the mid-plane stresses.

Section 3 deals with the FEM that can be derived for the composite coil. The elastic behaviour of the coil is modelled using the composite theory, whereas the plastic behaviour is calculated from the stresses in the winding material only. Variations in material modelling for the winding material as well as for the insulation material give essential information about the influence of the different parameters. Knowledge of these influences can give confidence in a design.

Section 4 compares the FEM of section 3 with calculations performed by other institutes, based on different assumptions and slightly different designs.

Section 5 deals with the experimental part of the project, and includes the fatigue measurements of the winding material to be used: copper silver (CuAg) and hard-drawn pure copper (Cu).

Section 6 summarises the results found in the earlier sections and concludes the investigations.
References


2 Composite Model and Analytical Model

2.1 Coil as a composite

Coils consist of two materials, strong and stiff windings and relatively weak insulation. For calculations, analytical as well as numerical (Finite Element Method), it is preferable to describe the mechanical properties of the magnet coil as an orthotropic composite material. Windings are regarded as fibres and the insulation material as the matrix. Neglecting the pitch and the radius change of the windings the coil's geometry is axisymmetric, just as the loading condition. There are therefore no shear components in the \( r\theta \)-plane and the \( \theta z \)-plane. The winding material is considered as isotropic in the analyses; the anisotropy of the hard-drawn wire is neglected. Isotropy is chosen because the main load is in "wire direction" and because the composite's behaviour "perpendicular to the wire" is strongly bounded by the insulation material. The orthotropic axisymmetric strain-stress relationship is written as:

\[
\begin{pmatrix}
\varepsilon_{zz} \\
\varepsilon_{rr} \\
\varepsilon_{\theta\theta} \\
\gamma_{rz}
\end{pmatrix}
= \begin{pmatrix}
1 & -\nu_{rz} & -\nu_{\theta r} & 0 \\
\frac{1}{E_{zz}} & E_{rr} & E_{\theta\theta} & 0 \\
\frac{1}{E_{rr}} & \frac{1}{E_{\theta\theta}} & \frac{1}{E_{\theta\theta}} & 0 \\
\frac{1}{G_{rz}} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\sigma_{zz} \\
\sigma_{rr} \\
\sigma_{\theta\theta} \\
\tau_{rz}
\end{pmatrix}
= D^{-1}
\]

For the calculation of the orthotropic moduli and Poisson's ratios, the Halpin-Tsai \(^1\) composite theory is chosen. This theory describes the composite material not only as a set of parallel and serial connected stiffnesses, but it also allows adaptation to experimental results by fitting the parameter \( \xi \). The Halpin-Tsai equations are originally for a unidirectional ply (figure 2.1.1), but they are extended for the axisymmetric situation of windings embedded in insulation material (figure 2.1.2). For the ply in figure 2.1.1 the original Halpin-Tsai equations are written as:

\[
\begin{pmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\gamma_{12}
\end{pmatrix}
= \begin{pmatrix}
1 & -\nu_{21} & 0 \\
\frac{1}{E_{11}} & \frac{1}{E_{22}} & 0 \\
\frac{1}{E_{22}} & 0 & \frac{1}{G_{12}}
\end{pmatrix}
\begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\tau_{12}
\end{pmatrix}
\]

With the indices \( m \) for the matrix material and \( f \) for the fibre, the components of the compliance matrix are:
\[ E_{11} = \frac{1 + \xi_E \eta_E \lambda}{1 - \eta_E \lambda} E_m, \quad \text{with} \quad \eta_E = \frac{E_f - E_m}{E_f + \xi_E E_m} \quad \text{and} \quad 0 \leq \xi_E \leq 2 \quad 2.3a \]

\[ E_{22} = E_f \lambda + E_m (1 - \lambda) \quad 2.3b \]

\[ v_{21} = v_f \lambda + v_m (1 - \lambda) \quad 2.3c \]

\[ G_{12} = \frac{1 + \xi_G \eta_G \lambda}{1 - \eta_G \lambda} G_m, \quad \text{with} \quad \eta_G = \frac{G_f - G_m}{G_f + \xi_G G_m} \quad \text{and} \quad 0 \leq \xi_G \leq 2 \quad 2.3d \]

Figure 2.1.1: Unidirectional ply, schematic view and transformed to a unit cell with fibre fraction \( \lambda \) and matrix fraction \((1-\lambda)\).

In (2.3) \( E \) denotes Young's modulus, \( G \) shear modulus and \( v \) Poisson's ratio. The multipliers \( \xi_E, \xi_G \) are measures of the fibre reinforcement of the composite that depend on the fibre geometry, packing geometry, and load conditions. Formula (2.3a) changes for \( \xi_E = 0 \) into the expression for standard serial connected stiffnesses, and for \( \xi_E \to \infty \) into the expression for standard parallel connected stiffnesses. For usual applications \( \xi_E \) is limited to approximately 2.

Figure 2.1.2: Unit cell of winding material embedded in insulation material.
Comparing the ply in figure 2.1.1 with the simplified unit cell for winding material embedded in insulation material (figure 2.1.2), the following equations for the moduli can be derived (with the indices \( w \) for the windings and \( i \) for the insulation material).

In the tangential direction holds, analogous to (2.3b):

\[
E_{00} = \lambda_r \lambda_z E_w + (1 - \lambda_r \lambda_z) E_i
\]

\[2.4a\]

In the axial direction over the width \((1 - \lambda_r)\) there is insulation material parallel (like 2.3b) to a series (like 2.3a) of winding and insulation material over the width \( \lambda_r \). Therefore:

\[
E_{zz} = \left( \lambda_r \frac{(1 + \xi_E \eta_E \lambda_z)}{(1 - \eta_E \lambda_z)} + (1 - \lambda_r) \right) E_i
\]

\[2.4b\]

with:

\[
\eta_E = \frac{E_w - E_i}{E_w + \xi_E E_i}
\]

\[0 \leq \xi_E \leq 2 \]

\[2.4c\]

Analogous to (2.4b) the modulus in the radial direction is:

\[
E_{rr} = \left( \lambda_z \frac{(1 + \xi_E \eta_E \lambda_r)}{(1 - \eta_E \lambda_r)} + (1 - \lambda_z) \right) E_i
\]

\[2.4d\]

The multiplier \( \xi_E \) can be used to match the Halpin-Tsai theory to experimental results. With an increasing \( \xi_E \) the influence of the winding stiffness on the total stiffness increases too. No experimental results are available to fit \( \xi_E \); it is chosen for \( \xi_E = 0 \) in all the analyses because then the winding stiffness has the lowest influence on the total stiffness.

The shear deformation is assumed to be mainly located in the insulation material because at low shear stresses the windings will slide over each other instead of that they are exposed to shear deformations. Also the shear stiffness of the insulation material is much lower than the stiffness of the winding material. Analogous to (2.3d) the shear modulus can be written as:

\[
G_{rz} = \frac{1 + \xi_E \eta_G \lambda_r \lambda_z}{1 - \eta_G \lambda_r \lambda_z} G_I ,
\]

with \( \eta_G = \frac{G_w - G_i}{G_w + \xi_E G_i} \) and \( 0 \leq \xi_G \leq 2 \)

\[2.4e\]

For the lack of experimental results \( \xi_G \) is chosen equal to 0. Due to the sliding windings and because \( G_w >> G_i \) the value of \( \eta_G \) is equal to 1. The shear stiffness of the composite is then derived from the shear stiffness of the insulation material by application of a correction for the fraction of insulation material in the unit cell. This leads to:

\[
G_{rz} = \frac{G_i}{(1 - \lambda_r \lambda_z)}
\]

\[2.4f\]
The Poisson’s ratios for the unit cell depend strongly on the Young’s moduli and the Poisson’s ratios of the winding and the insulation material. The derivations of the ratios cannot be found directly from the formulae (2.3), and are therefore described in appendix A. The results are:

\[
\nu_{0z} = (1 - \lambda_z) \nu_i + \lambda_z \left( \frac{\lambda_r \nu w E_w + (1 - \lambda_r) \nu_i E_i}{\lambda_r E_w + (1 - \lambda_r) E_i} \right) \quad 2.4g
\]

\[
\nu_{0r} = (1 - \lambda_r) \nu_i + \lambda_r \left( \frac{\lambda_z \nu w E_w + (1 - \lambda_z) \nu_i E_i}{\lambda_z E_w + (1 - \lambda_z) E_i} \right) \quad 2.4h
\]

\[
\nu_{rz} = \frac{\nu_i (1 - \lambda_r) \{\lambda_z E_w + (1 - \lambda_z) E_i\} + \mu \lambda_r E_i \{\lambda_z \nu w + (1 - \lambda_z) \nu_i\}}{\{\mu \lambda_r + (1 - \lambda_r)\} \{\lambda_r E_i + (1 - \lambda_r) (\lambda_z E_w + (1 - \lambda_z) E_i)\}} \quad 2.4i
\]

with:

\[
\mu = \frac{(E_w + \xi \lambda E_i)}{(E_w + \xi \lambda E_i) - \lambda z (E_w - E_i)} \quad 2.4j
\]

The orthotropic composite matrix in (2.1) filled with components as given by the formulae (2.4) will be used in section 3.

2.2 Analytical model for the mid-plane stresses

In the analytical model of the magnet coil only the stresses in the cross-sectional mid-plane of the coil can be calculated. These mid-plane stresses are expected to be the highest stresses of all cross-sections. As in the previous section, the coil is assumed to be axisymmetric. Gersdorf [2] has shown that the mid-plane stresses can be derived from the equilibrium conditions in the radial and axial direction (in cylindrical co-ordinates) and a constitutive relationship, by an iterative solution procedure, as described in appendix B. The relevant equations for this section are summarised in (2.6), (2.7) and (2.11). Gersdorf has not used a composite theory, but assumed an elastic strain-stress relationship (Hooke’s law) for the windings, while the insulation material between adjacent windings is expected to be hydrostatically loaded. The shear stiffness in the coil is governed by the relatively weak insulation material only, because the windings will slide over each other when they are exposed to shear deformations. Therefore the shear stresses in the coil can be neglected compared to the tangential, radial and axial stresses. Likewise the shear gradients can be neglected.

\[
\tau_{rz} = \frac{\partial \tau_{rz}}{\partial z} = \frac{\partial \tau_{rz}}{\partial r} = 0 \quad 2.5
\]

For the equilibrium equations in the mid-plane \((z = 0)\) in radial (2.6) and in axial direction (2.7) it can be derived (appendix B):
The stresses in (2.6) and (2.7) are the stresses in the winding material. The fractions of winding material in radial and axial direction are $\lambda_r$ and $\lambda_z$ respectively. The Lorentz-forces $F_r$ in radial and $F_z$ in axial direction do only act on the windings. The axial stress can be calculated by integrating (2.7) over the half-height of the coil. For the calculation of the tangential and the radial stresses, three (radially) different concentric zones of the magnet coil have to be considered.

A. The innermost zone where the windings are separated and the radial stress $\sigma_{rr,w}$ is zero. The tangential stress follows from equation (2.6):

$$\sigma_{\theta\theta,w} = r F_r$$  
(2.8)

B. This zone starts at the radius where the tangential and axial stress combination, calculated using the von Mises yield-criterion, reaches the yield-strength $\sigma_{c,w}$ of the winding material. Due to the (perfectly) plastic deformation, the stiffness in the tangential direction decreases (to zero), stiffness against radial displacements is then given by the elastically deforming windings on the radial outside only. In this situation the plastically deforming windings as well as the elastically deforming windings on the outside (zone C) will touch each other, resulting into a compressive radial stress. The tangential and radial stresses are continuous at the boundary of zones A and B. The differential equation (2.6) is solved using a simple integration scheme:

$$\Delta \sigma_{rr,w} = (\lambda_r \sigma_{\theta\theta,w} - \sigma_{rr,w} - r \lambda_r F_r) \frac{\Delta r}{r}$$  
(2.9)

The tangential stress follows from the von Mises yield-criterion, because $\sigma_{rr,w}$ and $\sigma_{zz,w}$ are known in a point $\Delta r$ outwards:

$$\sigma_{\theta\theta,w} = \frac{1}{2} (\sigma_{rr,w} + \sigma_{zz,w}) + \sqrt{\frac{\sigma_{c,w}^2}{4} - \frac{3}{4} (\sigma_{rr,w} - \sigma_{zz,w})^2}$$  
(2.10)

C. On the radial outside of the coil the windings are not loaded to the yield-strength of the material. For the calculation of the tangential stress another relationship should be used now, it can be derived (appendix B):

$$\frac{\partial \sigma_{\theta\theta,w}}{\partial r} = \frac{1}{r} \left( \left( \lambda_r + \frac{(1 - \lambda_r) E_w}{3 B_i} \right) \sigma_{rr,w} - \sigma_{\theta\theta,w} - \lambda_r v_r W r F_r + (1 - \lambda_r) v_w \sigma_{zz,w} \right) + \frac{1}{r} \sigma_{zz,w} \frac{\partial v_w}{\partial r}$$  
(2.11)
In (2.11), $B_i$ is the bulk modulus of the insulation material, indicating that the stresses in the winding material are influenced by the insulation material. The differential equation for the tangential stress (2.11) can be solved with a simple integration scheme:

$$
\Delta \sigma_{\theta,w} = \left( \lambda_r + \frac{(1 - \lambda_r)E_w}{3B_i} \right) \sigma_{rr,w} - \sigma_{\theta,w} - \lambda_r \nu_r r F_r + (1 - \lambda_r) \nu_w \sigma_{zz,w} \right) \frac{\Delta r}{r} + 2.12
$$

The radial stress is calculated as in zone B. At the boundary of the zones B and C the radial stress is continuous, the tangential stress is also continuous because the plastic strain is almost zero at the radial outside of zone B.

An iterative solution procedure is used to determine the radius where zone B ends and zone C starts. A value for this radius is chosen, and the stresses in the mid-plane are calculated. This calculation is repeated with another radius when the radial stress at the radial outside of the coil does not respect the boundary condition, for a single coil $\sigma_{rr,w} = 0$. To be sure that the material in zone C is only elastically deforming, the equivalent stress, calculated with the von Mises criterion, is compared with the yield-strength of the material.

For the residual stresses after removing the field, it is assumed that the axial stress returns to zero in the three zones A, B and C, because the axial stress is only governed by the (to zero returning) axial Lorentz-force if the shear stress is zero. In zone A with only elastically deforming (separated) windings the radial and tangential stresses also return to zero. In zone B the windings are plastically deformed, and the coil has become somewhat larger. This means that although in zone C only elastic deformation had taken place, some stress remains in the windings. For the calculation of the elastic back motion of the windings in zones B and C the relationships (2.9) and (2.12) are used. The residual stresses are found by subtracting the stresses of the elastic back motion from the stresses at the maximum applied load. The boundary conditions for the zones B and C are a zero radial stress at the radial outside and at the border of the zones A and B, because of the narrow ‘gap’ between the formerly separated elastically deformed windings and the plastically deformed windings.

Analogous to the analytical model of Gersdorf, equilibrium relationships for the unit cell of the composite model (figure 2.1.2) can be derived. For the constitutive relationship the Halpin-Tsai equations should then be used, resulting into the composite stresses. When the stresses in the winding material are extracted from these composite stresses, similar formulae as (2.6), (2.7) and (2.11) are found. These new formulae, expressed in the winding fractions and in the winding and insulation moduli, are nearly identical to (2.6), (2.7) and (2.11), if an insulation material is used relatively weak compared to the winding material. The solution routine prepared by Gersdorf (a computer program), therefore, can be used for the calculation of the mid-plane stresses in the magnet coil.

The results of the analytical model for a representative test-coil are presented in section 3, where the analytical solution is compared with a finite element solution.
References


3 Finite Element Model of a High-Field Magnet Coil-System

3.1 Introduction in the finite element model

A magnet coil-system consists of several concentric coils, separated and therefore mechanically independent of each other. In this section the finite element model of a single coil without a surrounding shield and a steel shielded coil are considered. For the single coil the influences of the insulation properties and of changes in the material properties of the coil's windings are studied. The influence of the shield on the coil's behaviour is studied with the steel shielded coil. The effects of a temperature increase in the coil due to the high current over the wire resistance are also investigated.

To model the elastic behaviour of the winding and the insulation material the Halpin-Tsai composite theory [1] is used. The plastic deformations of the windings are calculated using an isotropic yield-law for the stresses in the windings.

The analysis of the coil is executed with the MARC finite element program [2] version K5.2 with user subroutines prepared for a magnet coil-system. The MENTAT II post-processor [3] version 1.2 is used in the standard version with some procedures to handle a double element mesh, introduced for the calculation of the stresses in the windings.

In sections 3.2 and 3.3 the single coil model is defined and the assumptions in material behaviour and coil geometry are explained. In section 3.4 the results of the Finite Element (FE) analysis are compared with the results of the analytical model [4] derived in section 2. If the same simplifying assumptions for the material behaviour are taken, both solutions result in almost the same stresses for the mid-plane of the coil. The single coil model is extended with a steel reinforcing shield around the coil, in section 3.5. In 3.6 the results for the shielded coil are presented for the situation in which the same simplified assumptions are taken as for the single coil. These results are used as a reference for the analyses, in section 3.7, with more realistic material modelling. The results of the modelling variations can be summarised as follows.

- If the insulation material in the coil has tensile and shear stiffness, then deformations are spread over a wider area of material, peak stresses in the winding material decrease.
- A homogeneous temperature increase only in the winding material of the single coil has hardly influence on the stresses, although the insulation material remains cold. Only the total deformation of the coil is increased with the thermal strain.
- Strain-hardening decreases the plastic deformation of the winding material. After the load-pulse, the residual stresses are decreased in the winding material.
- Imperfections in the winding material nearby the highest loaded part of the coil show that the total load can be redistributed in the rest of the coil.
- The results for the steel shielded coils show that a temperature increase in the winding material reduces the amount of plastic deformation in the coil. The load on the shield, however, (remaining at low temperature) is increased.

3.2 Discretization of the single coil

The symmetry regarding the cross-sectional mid-plane of the coil makes that only (the lower) half of the coil needs to be modelled. Ignoring the pitch and the radius change of the windings, the coil is also cylindrically symmetric. The FE mesh in figure 3.2.1 represents one axisymmetric quarter of the coil’s section. One element includes several windings with
surrounding insulation, because the elements are chosen larger than the cross-section of the windings. The kinematical boundary condition for a single coil is a zero axial displacement in the cross-sectional mid-plane. The coil is mechanically loaded by the Lorentz-force (a body force), acting into the radial and the axial direction. The element force depends on the fraction of winding material in the element. The Lorentz-forces [5] are calculated in a user subroutine using the geometry of the coil and the current-density through the windings. In the calculation of the forces in the coil, the influences of the fields from all coils in the magnet coil-system are taken into account. A more detailed description will be presented at the end of section 3.3.

For the single coil calculations, a coil with a relatively large $\alpha$ is used ($\alpha$ is the ratio between outer and inner radius). Specially in this large $\alpha$ coil radially three regions with different deformation characteristics can be distinguished: separation of elastically deformed windings on the inside, touching of elastically deformed windings on the outside and in-between plastic deformation of touching windings. The coil is modelled with MARC element number 10 [6] (axisymmetric linear 4-node quadrilateral). The mesh is built with two layers of rectangular elements connected to one set of nodes (explained in section 3.3), with refinements at the places where relatively large gradients in the stresses occur. Near the outer radius of the coil smaller elements are used, because of the considerable gradient in the magnetic field.

![Figure 3.2.1: Mesh of a single coil (axisymmetric), inner radius 0.1m, outer radius 0.55m and (half) height 0.328m.](image)

### 3.3 Modelling of the single coil's behaviour

The coil is made of strong windings and weak resin insulation material. For both materials the elastic mechanical properties are well known. To reduce the number of elements in the FE model, the elastic properties are combined in a composite model defined by Halpin-Tsai. This composite model is capable of calculating elastic deformations, but not valid when plastic deformations occur. In analytical calculations [4], it was assumed that the stiffness of the coil, in first approximation, is only governed by the stiffness of the windings and if present by the reinforcing shield. Stress transfer through the insulation material is possible because of the small ratio of insulation to winding material. Because no applicable
yield-law for the composite material is known, the FE model also assumes that the plastic
deformation of the coil is mainly governed by the plastic deformation of the windings. In the
FE model not only the overall stresses in the composite material are calculated, but, for the
calculation of the plastic deformation of the windings, the stresses in the windings are
complementary determined.

To derive the winding stresses from the composite stresses the unit cell in the rz-
plane is reconsidered. The dimension of the unit cell is assumed to be small compared to the
(inner) radius of the coil. Figure 3.3.1 shows the unit cell of windings surrounded by
insulation, with different winding fractions in the radial and the axial direction.

\[ 1 - \lambda_z \]
\[ \lambda_r \]
\[ \lambda_r \]
\[ \lambda_z \]

\[ = \text{winding fraction } \lambda_r \lambda_z \]
\[ = \text{insulation fraction } (1 - \lambda_r \lambda_z) \]

Figure 3.3.1: Unit cell of winding and insulation with winding fractions \( \lambda_r \) in radial direction and \( \lambda_z \) in axial direction (0.75 < \( \lambda_r \) < 0.95).

The winding stresses are extracted from the composite stresses by considering the
local equilibrium of forces over the sections A-A, B-B and the cross-section in the rz-plane.
The material indices used are: \( c \) for the composite, \( i \) for the insulation, and \( w \) for the
winding.

Over the section A-A the composite stress in the axial direction can be written as:

\[ \sigma_{zz,c} = \sigma_{zz,i} (1 - \lambda_r) + \sigma_{zz,w} \lambda_r \quad \text{3.1a} \]

Over the section B-B the radial composite stress can be written as:

\[ \sigma_{rr,c} = \sigma_{rr,i} (1 - \lambda_z) + \sigma_{rr,w} \lambda_z \quad \text{3.1b} \]

Over the cross section in the rz-plane the composite stress can be written as:

\[ \sigma_{\theta\theta,c} = \sigma_{\theta\theta,i} (1 - \lambda_r \lambda_z) + \sigma_{\theta\theta,w} \lambda_r \lambda_z \quad \text{3.1c} \]
The shear component of the composite stress is chosen as:

$$\tau_{rz,c} = \tau_{rz,i} / (1 - \lambda_r \lambda_z)$$  \hspace{1cm} \text{(3.1d)}

In the FE model $\tau_{rz,w}$ is taken zero, because the shear stress is much lower than the other stresses in the winding material, due to the low shear transfer between the unconnected adjacent windings.

The stress-strain relationship for the insulation material is given by a diagonal matrix representing, as a network of slices of insulation material, the extra stiffness into each principal direction, that is afforded by the insulation material besides the stiffness of the winding material. The stress-strain relationship for the insulation material is:

$$\begin{pmatrix}
\sigma_{zz} \\
\sigma_{rr} \\
\sigma_{\theta\theta} \\
\tau_{rz}
\end{pmatrix}_i =
\begin{pmatrix}
E_i & 0 & 0 & 0 \\
0 & E_i & 0 & 0 \\
0 & 0 & E_i & 0 \\
0 & 0 & 0 & G_i
\end{pmatrix}_i
\begin{pmatrix}
\varepsilon_{zz} \\
\varepsilon_{rr} \\
\varepsilon_{\theta\theta} \\
\gamma_{rz}
\end{pmatrix}_c$$  \hspace{1cm} \text{(3.2)}

Substitution of (3.2) in (3.1) results in:

$$\sigma_{zz,c} = \varepsilon_{zz,c} E_i (1 - \lambda_r) + \sigma_{zz,w} \lambda_r$$  \hspace{1cm} \text{(3.3a)}

$$\sigma_{rr,c} = \varepsilon_{rr,c} E_i (1 - \lambda_z) + \sigma_{rr,w} \lambda_z$$  \hspace{1cm} \text{(3.3b)}

$$\sigma_{\theta\theta,c} = \varepsilon_{\theta\theta,c} E_i (1 - \lambda_r \lambda_z) + \sigma_{\theta\theta,w} \lambda_r \lambda_z$$  \hspace{1cm} \text{(3.3c)}

$$\tau_{rz,c} = \gamma_{rz,c} G_i / (1 - \lambda_r \lambda_z)$$  \hspace{1cm} \text{(3.3d)}

The winding stresses can now be derived from the relationships (3.3) resulting into:

$$\begin{pmatrix}
\sigma_{zz} \\
\sigma_{rr} \\
\sigma_{\theta\theta} \\
\tau_{rz}
\end{pmatrix}_w = \Lambda (D_c - D_i)
\begin{pmatrix}
\varepsilon_{zz} \\
\varepsilon_{rr} \\
\varepsilon_{\theta\theta} \\
\gamma_{rz}
\end{pmatrix}_c$$  \hspace{1cm} \text{(3.4)}

with $D_c$ according to (2.1) and with $D_i$ defined by:

$$D_i =
\begin{pmatrix}
E_i(1 - \lambda_r) & 0 & 0 & 0 \\
0 & E_i(1 - \lambda_z) & 0 & 0 \\
0 & 0 & E_i(1 - \lambda_r \lambda_z) & 0 \\
0 & 0 & 0 & G_i / (1 - \lambda_r \lambda_z)
\end{pmatrix}$$  \hspace{1cm} \text{(3.5)}
and \( \Lambda \) defined by:

\[
\Lambda = \begin{pmatrix}
\frac{1}{\lambda_r} & 0 & 0 & 0 \\
0 & \frac{1}{\lambda_z} & 0 & 0 \\
0 & 0 & \frac{1}{\lambda_r \lambda_z} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

3.6

For the implementation of (3.4) in a common FE code it is chosen to use two layers of elements. The first choice is to implement in mesh1 the stiffness matrix \( \Delta(D_\sigma-D_\tau) \) necessary for the calculation of the winding stresses. For mesh2 then remains the rest stiffness \( \{D_\sigma - \Delta(D_\sigma-D_\tau)\} \), because the “parallel” meshes should result in the composite stiffness matrix. This choice results for mesh1 in a matrix stiffer than the composite stiffness matrix, because the value of the diagonal elements of \( \Delta \geq 1 \). For mesh2 this choice then results in a non-positive definite matrix, which is undesired numerically and physically, in the case that windings are separating, as will be described later on.

A more suitable choice is to assign for mesh1 \( (D_\sigma-D_\tau) \) and for mesh2 \( D_\tau \). The stresses in the windings are then calculated afterwards from the stresses in mesh1 by multiplying with \( \Lambda \). Mesh1 now represents the reduced stiffness of the winding material into each principal direction, while mesh2 represents the extra insulation contribution to the total stiffness. The splitting of the stiffness over both meshes is made in a user defined subroutine.

Now the stiffness of the composite matrix is split-up over two matrices and the winding stresses can be derived from mesh1, after scaling the stresses with the matrix \( \Delta \), the plastic deformation of the coil can be calculated. As already mentioned, the plastic deformation of the coil is assumed to be mainly governed by the deformation of the winding material. For the calculation of the plastic deformation of the winding material the isotropic von Mises criterion should be applied on the scaled stresses calculated from mesh1. The FE program, however, is not capable to determine the von Mises stress from the scaled stresses, because the program is based on the usual stress definitions. The FE program offers in a sub-routine, based on the Hill criterion [7], the possibility to implement problem adapted yield-ratios. Therefore the anisotropic Hill criterion with well-chosen yield-ratios is applied on the element stresses to mimic the von Mises criterion for the winding material as close as possible. According to Hill the equivalent stress can be written as:

\[
2\sigma^2_{\text{Hill}} = \left(\frac{1}{Y_{rz}^2} + \frac{1}{Y_{\theta}^2} - \frac{1}{Y_{r}^2}\right)(\sigma_{rr} - \sigma_{\theta\theta})^2 + \left(\frac{1}{Y_{\theta}^2} + \frac{1}{Y_{z}^2} - \frac{1}{Y_{r}^2}\right)(\sigma_{\theta\theta} - \sigma_{zz})^2 + \left(\frac{1}{Y_{z}^2} + \frac{1}{Y_{r}^2} - \frac{1}{Y_{\theta}^2}\right)(\sigma_{zz} - \sigma_{rr})^2 + \left(\frac{6}{Y_{rz}^2}\right)^2 \tau_{rz}^2
\]

3.7

With \( Y_z, Y_r, Y_{\theta} \) and \( Y_{rz} \) the yield-ratios representing the anisotropy. The Hill criterion is equivalent to the von Mises criterion for \( Y_z = Y_r = Y_{\theta} = Y_{rz} = 1 \).
The yield-ratios in (3.8) are chosen such that the equivalent Hill stress ($\sigma_{Hill}$) is nearly identical to the equivalent stress calculated with the von Mises criterion applied on the scaled stresses. $\sigma_{Hill}$ does not depend on the value of $Y_{n}$, because $\tau_{rz}$ is zero for the winding material. For $h$ about 0.9, the yield-ratios in (3.8) are almost equal to the yield-ratios for the von Mises criterion, therefore the actual yield-surface approaches the von Mises yield-surface. After all, it could have been considered to use the von Mises criterion, but that is not possible when $h$ is about 0.75, because then the yield-ratios in the Hill criterion as well as in the von Mises criterion applied on the scaled stresses differ from the original yield-ratios $(1,1,1,1)$ of the von Mises criterion.

In the calculation of the plastic deformation the winding material is assumed to yield perfectly plastic. In the FE model this behaviour is implemented by making the strain-hardening term in the yield-function (3.9) independent of the plastic strain. In the case of modelling variations hardening behaviour will be examined.

$$\sigma_{y} = \sigma_{y, initial} (1 + x \varepsilon_{p}) \quad 3.9$$

With: $x = 0$ for perfectly plastic behaviour, $x > 0$ for strain-hardening and $\varepsilon_{p}$ the effective plastic strain.

After the description of the elastic and plastic behaviour of the coil, a particular case of the elastic deformation, requiring adaptations, is considered now. On the radial inside of the coil, the windings separate during the field pulse, due to a different tangential stress (= elongation, as in formula (2.8) and figure 1.1.2), and because there is no tensile stiffness between two layers of windings. The composite theory, however, assumes that the material is a continuum, thus a correction should be made. In the separated windings the radial stress becomes zero. The compliance matrix of $mesh1 (D_{c} - D_{l})^{-1}$ is adapted to the reduced set of equations, valid for a plane-stress situation in an orthotropic material, because only in $mesh1$ the contribution of the winding stiffness to the composite stiffness is represented. Formula (3.10) shows that some components of the compliance matrix are set to zero, and therefore independent of the radial stress. The stiffness in radial direction is reduced to zero, to make sure that the radial stress becomes zero. The radial strain can still be calculated, because it is implicitly given by the tangential strain. The radial strain represents not only the radial strain in the winding material, but also the strain in the "gap" between the separated windings.

$$(D_{c} - D_{l})^{-1} = \begin{pmatrix} \sigma_{zz} & 0 & 0 \\ 0 & \sigma_{rr} & \sigma_{r\theta} \\ \sigma_{\theta\theta} & \tau_{rr} & \tau_{rz} \end{pmatrix} \begin{pmatrix} \sigma_{zz} \\ \sigma_{rr} \\ \tau_{rz} \end{pmatrix} \quad * = \text{unchanged} \quad 3.10$$

In a user subroutine of the FE program the algorithm (3.10) is implemented, which, if tensile radial stress appears in an integration point of $mesh1$, inverts the stiffness matrix.
and adapts the inverse. After the adaptation, the compliance matrix of mesh1 is inverted, and a new iteration is made with the reduced stiffness matrix. The position where the windings separate varies during an increment, until the solution converges. When the number of elements in the model is increased the convergence rate considerably reduces, because the number of possible solutions for the area of the separated windings increases too.

The Lorentz-forces $f$ acting on the winding material are implemented in the FE model as body-forces $b$. These body-forces are calculated, for coil $i$, in a user-subroutine using the current-density $J$ and the magnetic field components $B_r$ and $B_z$:

$$
\begin{pmatrix}
  b^l_r \\
  b^l_z
\end{pmatrix} = \lambda^l_r \lambda^l_z \begin{pmatrix}
  f^l_r \\
  f^l_z
\end{pmatrix} = \lambda^l_r \lambda^l_z J^l \begin{pmatrix}
  -B_r \\
  B_z
\end{pmatrix}
$$

3.11

The Lorentz-forces in each element have to be scaled with the fraction of winding material to correct for the element volume the force is acting on. The field $B$ generated by all $n$ coils in the system is calculated from:

$$
\begin{pmatrix}
  B_r \\
  B_z
\end{pmatrix} = \mu_0 \sum_{k=1}^{n} \lambda^k_r \lambda^k_z J^k \begin{pmatrix}
  Q_r^k (r,z) \\
  Q_z^k (r,z)
\end{pmatrix}
$$

3.12

with $\mu_0 = 4\pi \times 10^{-7}$ [Vs/Am] and with $Q_r^k$ and $Q_z^k$ special functions for the magnetic field solutions for coil $k$, depending on the coil geometry and functions of the position $(r,z)$ in the coil-system.

3.4 Results for the single coil

After the discretization and the modelling of the behaviour of the coil, the FE model can now be used for calculations. First the FE solution is compared with the analytical solution. Therefore the same assumptions are taken as in the analytical model, this should result in the same stresses for the cross-sectional mid-plane of the coil.

In the analytical model the shear stresses are omitted, because the shear stress is negligible compared to the other stresses. Also it is assumed that the weak insulation material, enclosed between relatively stiff windings, is hydrostatically loaded. In the FE model the insulation material is assumed to be elastic. Based on considerations about the analytical model applied on the unit cell (in section 2), it was noticed that the analytical solution of Gersdorf [8] and the solution for the unit cell are identical, when the compressive stiffness of the insulation material is multiplied with $1/(1-2v)$. This factor (as explained in appendix B) is just the distinction factor between the elastic modulus and the modulus for a material under a hydrostatic load condition. With the shear stiffness reduced to almost zero, the influence of deformations of material outside the mid-plane on the mid-plane stresses can be neglected, which compares with the analytical model. The stiffness of the insulation material, in mesh2, is reduced to a low dummy stiffness for all the integration points where windings are separated.
For the variations in material modelling it is necessary to have a reference situation, the above mentioned FE model with the same assumptions as in the analytical model is chosen for that purpose. For the calculation of the stresses and deformations in the reference situation the following data are used. The coil is made of hard-drawn copper wire, Young's modulus 140 GPa, yield-strength 360 MPa, $\lambda_x = \lambda_z = 0.92$. The resin insulation material has a Young's modulus of 2 GPa. The maximum magnetic field applied is 30.5 tesla. The inner radius of the coil is 0.1 m, the outer radius 0.55 m and the (half) height 0.328 m.

In figure 3.4.1.a the results for the calculated stresses in the mid-plane of the single coil are presented for the maximum load. Figure 3.4.1.b presents the stresses after the field pulse in the steady-state situation. The compressive tangential stress is smaller than the yield-strength, therefore no plastic deformation of the winding material occurs during the unloading of the coil. Repetitive load-cycles, starting from the residual stresses in the steady-state, give stresses between these residual stresses and the stresses for the 100% load-situation, without further plastic deformation. The analytical solution and the FE solution are almost the same over the entire cross-section, for the maximum load as well as for the steady-state solution.

Figure 3.4.2 presents the deformed mesh of the single coil at the maximum load. The nodal displacements are multiplied by a factor 10. The deformed mesh shows, on the radial inside of the coil, an irregular pattern of the elements. It is known from real coils that this irregularity is a “gap” between two windings.

Figure 3.4.1: Stresses in the mid-plane of the reference coil. a. At 100% applied field b. Residual stresses after removing the field. From a radius 0.10-0.13 m elastic deformation of separated windings, from 0.13-0.39 m plastic deformation of touching windings and from 0.39-0.55 m elastic deformation of touching windings.
The irregularity in the deformation pattern can also be observed in figure 3.4.3, in which the tangential strain in the mid-plane of the coil is presented.

The elastic 85% load curve shows that the separated windings on the inside deform differently from the touching windings on the outside. During the load increase from 85% to 100%, separated windings, near the radius with the maximum strain in the curve, deform plastically. During this plastic deformation the windings experience a large step in the (plastic) tangential strain. The strain difference between the points A and B, near the
maximum in the 100% strain curve, implies that two neighbouring windings are exposed to a discontinuous tangential strain. This difference in the tangential strain results in a separation of the two windings by a radial “gap”. This “gap” has moved through the coil during the plastic deformation.

3.5 Modelling of the steel shielded coil

In this section a coil surrounded by a steel shield is considered. The steel shields separate a large single coil in several sub-coils, with for each coil a resistance against the radial displacement on the outside. The tangential (plastic) strain in the windings that depends on this radial displacement decreases too.

The mesh for the steel shielded coil is made of the same quadrilateral elements (MARC element 10) as for the single coil, and is also double meshed in the coil’s part. The model is enlarged with elements representing the insulation layer between coil and shield, and elements for the shield. These extra elements are single meshed. Figure 3.5.1 shows the mesh for a coil surrounded by a steel shield. The elements have all the same size, because for this coil no gradients in the stresses are expected. The elements for the insulation layer between coil and shield are very small, and can hardly be observed. The kinematical boundary condition is a suppressed axial displacement of the nodes in the cross-sectional mid-plane. The Lorentz-force is only applied to the elements that contain winding material.

Figure 3.5.1: Mesh of a steel shielded coil, only the elements near the mid-plane are shown. Inner radius 0.2225m, outer radius 0.2713m, (half) height 0.4985m, insulation layer thickness 0.0004m and shield thickness 0.0261m.

The insulation layer between the coil and the shield is made of the same material as the insulation in the coil. This layer should be able to act like a compressive spring to transmit the outwards directed radial forces of the coil to the shield, therefore only stiffness in the radial direction is applied. It is assumed that the material remains elastic during the
transfer of the radial stress, therefore no yield-criterion is used. The equivalent stress, however, is still calculated from the principal stresses to give an impression of the total stress in the insulation layer.

The steel shield is made of a strong isotropic material. The necessary material properties are the Young’s modulus and the Poisson’s ratio. The steel shield is expected to remain elastic during the load-cycle. For calculations in which the shield is yielding the isotropic von Mises yield-law is used, without strain-hardening.

3.6 Results for the steel shielded coil

The steel shielded coil that will be described in this section is part of a magnet coil-system. The examined coil is not only loaded by the field produced in the coil itself, but also by the fields from the other coils in the system.

For the comparison of the results of variations in material modelling a reference situation is chosen. In this reference situation the coil’s materials are assumed to behave like the material in the reference situation of the single coil. The insulation layer between coil and shield is assumed to behave like a relatively weak material to transmit only the radial (compressive) force from the coil to the shield. The shear stiffness in the insulation layer is reduced to almost zero, so that the coil and the shield can deform relatively independent of each other in the axial direction.

![Graphs showing stress distribution](image)

**Figure 3.6.1:** Stresses in the mid-plane of the reference steel shielded coil. a. At 100% applied field, b. Residual stresses after removing the field. From a radius 0.2225-0.2713 m plastic deformation of touching windings, from 0.2717-0.2778 m plastically deformed shield and from 0.2778-0.2978 m elastically deformed shield.

The mesh in figure 3.5.1 is used for the analysis with a copper-alloy as winding material and also a different insulation material. Material data are for the windings a Young’s modulus of 143 GPa, yield-strength 745 MPa; for the insulation a Young’s modulus of 16.7 GPa; and for the shield a Young’s modulus of 192 GPa, yield-strength
1030 MPa. The radial winding fraction $\lambda_r$ is 0.77 and the axial winding fraction $\lambda_\theta$ is 0.93. Figure 3.6.1 presents the calculated mid-plane stresses for the steel shielded coil in the reference situation. In figure 3.6.1.a the stresses at the maximum applied load are visualised. The axial stress in the mid-plane shows that the coil is loaded by the axial component of the Lorentz-force. The windings near the mid-plane of the coil are yielding all over the cross-section. Although the shield is plastically deforming on the inside, it is strong enough to withstand the radial forces. The residual stresses after removing the load are presented in figure 3.6.1.b. The tangential stress shows that the elastically deformed part of the shield compresses the plastically deformed coil.

3.7 Variations in material modelling

In the previous sections the single coil and the steel shielded coil are studied, the results are taken as a reference. In the next sections (3.7.1 and 3.7.3) the influence of the properties of the insulation material on the stresses in and the deformation of the (single) coil and the shield are studied. Likewise the influence of a temperature increase in the coil is studied. Section 3.7.2 presents the influence of changes in the mechanical properties of the winding material, for the single coil only.

3.7.1 Influence of the insulation and temperature on a single coil

In the reference situation the insulation material was assumed to have no shear and tensile stiffness. Now the influence, on stresses and deformation, of an insulation material with shear and tensile stiffness is considered. All other material properties are taken as in the reference situation. The slices of insulation material are tensile loaded into the radial direction in the part of the coil where the windings separate. The radial strains can be up to 10% due to the "gap" between two adjacent windings. The insulation material is expected to become brittle at liquid nitrogen temperature, therefore the cross-section of material that transmits the tensile force is reduced. This is implemented in the FE code by a linear decrease of the Young's modulus, for a positive radial strain. For strains larger than 0.5% the modulus is set to almost zero. The shear force in this separated windings' part is reduced with the fraction $(1-\lambda_\theta)$, which is the cross-section of the radial directed slice of insulation material that transfers the (axial) shear deformation between the separated windings. In the FE code this is also implemented by reducing the modulus, in the touching windings' part the full modulus is applied.

Figure 3.7.1.a shows the stresses in the mid-plane at the maximum load, whereas figure 3.7.1.b shows the stresses in the steady-state situation after the field pulse. Compared with the results of the reference situation, the sharp peak in the tangential stress in the steady-state situation is spread over several windings. The tensile stress in the insulation material keeps the separated windings together and forces the innermost windings to deform plastically. The (zero) radial stress on the inside of the coil shows that some separation still exists, at 100% load. The shear stiffness in the insulation material spreads differences in the axial deformation of the coil over a wider area, which is reflected in the axial stress. The deformed mesh in figure 3.7.2 shows that there is no "gap" between the windings. The flat bottom part of the coil shows that the shear stiffness in the insulation material has spread the axial deformation of the coil over the entire area.
Figure 3.7.1: Stresses in the mid-plane of the coil with shear and tensile stiffness in the insulation.

a. At 100% applied field, b. Residual stresses after removing the field.

Figure 3.7.2: Deformed mesh (scale factor 10) of the single coil with shear and tensile stiffness in the insulation material. Dashed line is shape of the undeformed mesh.

The calculations for the reference situation are performed without a thermal load. The magnet coil-system, however, is cooled down to liquid nitrogen temperature (77K) before applying the field, to reduce the electrical resistance of the windings. During the load-pulse the current through the windings causes a considerable temperature rise, in some situations even up to room temperature ($\Delta T > 200K$). Therefore the influence of a considerable temperature increase in the coil is studied. The thermal load is applied to the
winding material only, because it produces the heat. The insulation material is expected to remain at low temperature during the pulse, due to the short pulse-time and the relatively low thermal conductivity.

The thermal expansion (3.13) of a unit cell (figure 3.3.1) depends on the winding fractions in the radial and the axial direction. The thermal expansion coefficient $\alpha_{\text{Cu}}$ of the winding material (copper) depends strongly on the temperature, and is therefore made a function of the temperature. Other material properties are chosen temperature independent, at the room temperature value, because the yield-strength and the ultimate tensile strength are expected to improve at lower temperatures.

$$
\begin{pmatrix}
\varepsilon_{zz} \\
\varepsilon_{rr} \\
\varepsilon_{\theta\theta} \\
\gamma_{zz}
\end{pmatrix}_{\text{thermal}} =
\begin{pmatrix}
\alpha_{\text{Cu}}(T) \\
1 \\
0
\end{pmatrix} DT.
$$

A temperature increase of the single coil, due to the current through the windings, leads to an overall heating of the coil. Because the coil expands homogeneously, the stresses in the windings remain almost the same as in the reference situation, and are hardly influenced by the cold insulation material.

### 3.7.2 Influence of the mechanical properties of the winding material

Changes in the properties of the winding material are subjects of study now, the properties of the insulation material are kept the same as in the reference coil. First strain-hardening is considered. In the yield-function (3.9) the value of $x$ is now chosen 10.

$$
\sigma_y = \sigma_{y,\text{initial}}(1 + 10 \varepsilon_p)
$$

The strain-hardening modulus is still low 3.6 GPa (initial yield-strength 360 MPa), compared to the elastic modulus of 140 GPa, but the influence on the tangential stress is already noticeable.

Figure 3.7.3 shows the stresses in the mid-plane of the coil at the maximum load and in the steady-state after unloading. In figure 3.7.3.a it can be observed that, due to the strain-hardening, the maximum tangential stress is increased with 20 MPa for the zone with the highest plastic strain. It can also be observed that the amount of plastic deformation is decreased, because less windings are plastically deformed. Due to the reduction in plastic deformation the tangential peak stress after unloading is decreased with 50 MPa.

The next subject is an imperfection in the winding material, modelled by reducing the yield-strength of four integration points to 70% of the initial yield-strength. The four integration points are chosen in three adjacent elements near the mid-plane of the coil, in the zone with the highest deformation in the reference situation. These weaker windings will start earlier with the plastic deformation. Because these windings lose their (tangential) stiffness, a part of the Lorentz-force produced in these windings is transferred to the windings on the outside. These outside windings are therefore higher loaded, and the plastically deformed region of the coil is enlarged to the outside, which can be observed in figure 3.7.4. During the unloading the weaker windings are again plastically deforming,
which is an undesired situation, because repeated plastic deformation reduces the lifetime of the coil.

The modelling of an imperfection by reducing the yield-strength in four integration points in an axisymmetric model may be exaggerated. However, the results show that the remaining material can take over the load from the imperfection without too much extra plastic deformation.

Figure 3.7.3: Stresses in the mid-plane of the coil, winding material with a strain-hardening modulus of 3.6 GPa. a. At 100% applied field. b. Residual stresses after removing the field.

Figure 3.7.4: Stresses in the mid-plane of the coil, winding material with reduced yield-strength of 70% in four integration points. a. At 100% applied field. b. Residual stresses after removing the field.
3.7.3 Influence of insulation and temperature in a steel shielded coil

In the previous sections 3.5 and 3.6 the steel shielded coil was considered with an insulation material without shear and tensile stiffness. In section 3.7.1 the properties of the insulation material in the single coil were changed to a (reduced) tensile and shear stiffness when the windings separated. In the steel shielded coil considered now, the insulation material has shear and tensile stiffness, in the coil as well as in the insulation layer between coil and shield.

The coil is axially loaded by the Lorentz-force, and not the shield, therefore in the reference situation only the coil is compressed into the axial direction. As the insulation material in the layer is able to transfer shear deformations, a part of the axial force in the coil is transferred to the shield, which is now also axially deforming. In the yielding winding material the axial stress is reduced, thus according to the yield-condition the tangential stress can increase. The higher tangential stress in the coil reduces the radial displacement of the coil, and that decreases the load on the shield. The results of the mid-plane stresses are shown in figure 3.7.5. The total plastic strain in the coil is decreased, which can be observed in figure 3.7.5.b, where the residual tangential stress is lower than in the reference situation.

The temperature increase of the coil during the field pulse gives a thermal expansion of the winding material only, as was already explained for the single coil. The shield remains on the liquid nitrogen temperature and forms a stiff ring around the thermally expanded coil. This temperature difference causes therefore an increase in the radial stress between coil and shield. The thermal load can be added to the mechanical load in different ways. Figure 3.7.6 shows three options.

Figure 3.7.5: Stresses in the mid-plane of a steel shielded coil, with tensile and shear stiffness in the insulation material. a. At 100% applied load. b. Residual stresses after removing the load.

The temperature increase of the coil during the field pulse gives a thermal expansion of the winding material only, as was already explained for the single coil. The shield remains on the liquid nitrogen temperature and forms a stiff ring around the thermally expanded coil. This temperature difference causes therefore an increase in the radial stress between coil and shield. The thermal load can be added to the mechanical load in different ways. Figure 3.7.6 shows three options.

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The thermal load can be added after the field pulse, the residual stresses in the coil are so low that the temperature increase causes elastic deformations only, in the coil as well as in the shield. This load condition, however, is not comparable with the real thermal load. When the temperature rise is applied at the maximum field, mainly the highly loaded shield is extra stressed, and (further) plastic deformation of the shield is introduced. The residual tangential stress after the pulse is increased, because the shield is now compressing the plastically and thermally deformed coil more than in the reference situation. During the unloading the residual stresses in the winding material can reach the yield-strength. This option leads therefore to the “worst case” situation in which both the shield and the coil are extra loaded.

The most realistic option, however, is to add the thermal load to the mechanical load, starting already during the increase of the mechanical load. At the maximum mechanical load a considerable thermal load is already added, and the rest of the thermal load is added afterwards. From a separate calculation program the temperature increase related to the generated field is known. The thermal load increase is implemented in the FE model by alternating load steps for the mechanical and thermal loads. The total amount of plastic deformation in the coil is reduced, because a part of the thermal load is added to the coil during the plastic deformation of the windings. The total tangential strain in the coil is a combination of partly a mechanical strain and partly a thermal strain. During the deformation process the amount of mechanical strain in the tangential direction is lower than in the reference situation, because a part of the total strain is thermal strain. If the mechanical load is up to 99%, the temperature increase can even cause a mechanical unloading of the winding material, and no further plastic deformation is added in the mechanical load step to 100%. Compared to the thermal load step at 100% mechanical load, the shield is now loaded less. Figure 3.7.7 presents the stresses in the shielded coil with a temperature increase (116K) added during the field pulse. In figure 3.7.7.b it can be observed that the cold shield is compressing the coil more than in the reference situation.
Figure 3.7.7: Mid-plane stresses in a thermally loaded steel shielded coil. a. At 100% applied mechanical load and 72.5% thermal load. b. Residual stresses after removing the mechanical load and applying 100% thermal load.

References

4 Comparison of Several Magnet Design-Tools

4.1 Introduction

Several institutes, around the world, are designing and building high-field magnet coil-systems [1,2,3]. After comparing the Finite Element Model (FEM), described in section 3, with an analytical approach and using it for more realistic materials modelling, now a comparison of this developed FEM with other institutes' models is presented. This comparative study can be of great interest, because each institute uses different models and assumptions in the design of their coils, and because it can show the capabilities of the developed design-tool. In this section the design-tools developed and used at the Faculty of Physics, Katholieke Universiteit Leuven (KUL), Belgium, and at the National High Magnetic Field Laboratory, Los Alamos National Laboratory (LANL), USA, are compared to the FEM of the University of Amsterdam (AFEM).

The comparison is performed by comparing the stress results as delivered by the institutes of a certain coil design, with the results as calculated on the same coil with the AFEM. Several assumptions can be made in material properties, thermal loading condition, and coil behaviour. In first approximation the assumptions about material properties and the addition of thermal loads are copied from the different models to the AFEM. Assumptions regarding the coil behaviour, that is, element mesh, winding separation, shear stiffness, etc., are applied according to the assumptions as made in the Amsterdam design.

4.2 KUL tool

4.2.1 Introduction in the KUL coil design

KUL coils are internally reinforced coils, with around each layer of winding material a layer of composite material containing 85% glass-fibre. The relatively thick outer shielding of the reinforced coil is also made of this glass-fibre composite. The coils are loaded with a high current-density, that results in a relatively small coil compared to the Amsterdam design. The coil used in this comparison produces 72.5 tesla, and can be regarded as ten subcoils characterised by the following data:

- The overall dimensions of the coil are: inner radius 0.0041 m, outer radius 0.07 m and half-height 0.033 m.
- Ten layers of soft copper windings, yield-strength 110 MPa and a parabolic strain hardening-curve to an Ultimate Tensile Strength (UTS) of 250 MPa at an elongation of 25%. The Young’s modulus is 125 GPa, the Poisson’s ratio 0.34. Thermal loads are not applied.
- Each copper layer is surrounded by a composite layer of S2 glass-fibre. The anisotropic composite properties in the fibre direction are: Young’s modulus 60 GPa, UTS 2500 MPa, and Poisson’s ratio 0.25 for transverse deformations caused by a deformation in the fibre direction. In section 4.2.2 it will be explained why no properties are necessary perpendicular to the fibre direction.
- Around the four inner subcoils a layer of Teflon tape is placed, to allow the windings to separate.
The outer six subcoils are touching each other, and a radial force transmission should be allowed for. This can be realised by making a rigid connection between the meshes of the touching subcoils, or by connecting two subcoils with a radially directed tying.

4.2.2 Model of the KUL design

In this section, it will be described how the KUL design is modelled with the MEM. In the KUL design-tool, a simple FEM, the shear stiffness in the insulation layer between winding and reinforcement can be chosen either as zero or as infinite (the axial strain is then equal in both parts). In-between these limits a shear stiffness is not possible.

It seems physically unrealistic to set the shear stiffness to infinite in all the insulation layers, because this results in a very stiff coil. If the shear stiffness is set to zero in all the layers then the coil is too weak. This weakness of the coil is obvious when the very high axial load is applied, that results in a considerable axial strain as the soft copper winding material is yielding. By deformations due to the Poisson's ratio the winding material near the cross-sectional mid-plane shows an inwards directed radial deformation, while the radial Lorentz-force component is still outwards directed. These deformations have not been reported in experimentally used coils of this type, and therefore the practical approach is adopted that these large deformations are artefacts due to the extreme values chosen in the model.

In the KUL design it is chosen to connect a winding with surrounding glass-fibre with the maximum shear stiffness within the subcoil, and to connect two subcoils with a zero shear stiffness. In the Amsterdam design two situations are considered for the shear stiffness behaviour. Firstly, the comparison with the KUL design with alternating maximum and zero shear stiffnesses. Secondly, the application of an intermediate (resin) shear stiffness in all the insulation layers.

The element-mesh of the coil model is produced with the mesh generator as used for the Amsterdam coils, resulting into ten separate subcoils. Each subcoil is built of one winding in radial direction, a glass-fibre reinforcement, and an insulation layer to realise the shear stiffness between the winding and the reinforcement. The glass-fibre reinforcement is approximated as an isotropic material, because isotropy is also assumed in the KUL design. Considering that the main stiffness against radial displacements is delivered by the tangential stiffness, and thus parallel to the fibre, this choice seems allowable. However, the axial stiffness is very high, resulting into probably unrealistic axial stresses. For the comparative study it is, however, chosen to use isotropy, also because in the AFEM the shield is modelled and assumed to be isotropic. A difference in the glass-fibre composite modelling is that the AFEM incorporates shear stiffness and it is not in the KUL tool. In the AFEM also some shear stiffness is applied in the resin insulation material in the windings' part of the coil.

As mentioned before, the inner four subcoils are allowed to separate during the applied load, therefore the meshes of these coils remain separated. The outer six subcoils are constrained with ties in the radial direction to allow a force transmission due to contact loads. As the ties can only transmit the radial directed forces, the shear stiffness is set to zero, according to the KUL design. For the calculation of the stresses in the coil where an intermediate shear stiffness is used among the six outer windings, extra elements are used to connect the meshes. These newly added elements are treated as the elements in the insulation layer, and are given a shear stiffness approximately equal to that of a resin.
4.2.3 Results and conclusions for the KUL design

The first situation considered is for the maximum shear stiffness between the copper winding and the glass-fibre reinforcement outside. The radial winding fraction \( \lambda_r \) is 1.0 (just one winding) and the axial winding fraction \( \lambda_z \) is 0.80. The mid-plane stress results are presented in figure 4.2.1 at the maximum load of 72.5 tesla, and in figure 4.2.2 are presented the residual stresses in the steady state.

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**Figure 4.2.1** Mid-plane stresses at the maximum applied load of 72.5 tesla, for the situation that the shear stiffness is applied according to the KUL design. The stresses in the reinforcements can be recognised by the stress values above 500 MPa.

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**Figure 4.2.2** Residual mid-plane stresses in the steady state after fully unloading, for the situation that the shear stiffness is applied, during the loading, according to the KUL design.
The stresses in the AFEM at the maximum load almost approach the stresses in the KUL design. The residual stresses are slightly different, because the KUL design assumes a zero shear stiffness in all the insulation layers, during the unloading of the coil. In the AFEM, it is assumed that the shear stiffness is still available, because the windings and reinforcements are all the time compressed in the radial direction. This compression in the plastically deformed windings is due to the elastic back-motion of the reinforcement outside.

Figure 4.2.3 Mid-plane stresses at the maximum applied load of 72.5 tesla, for the situation that a more realistic shear stiffness is applied than in the KUL design. The stresses in the reinforcements can be recognised by the stress values above 500 MPa.

Figure 4.2.4 Residual mid-plane stresses in the steady state after fully unloading, for the situation that a more realistic shear stiffness is applied than in the KUL design.
In figure 4.2.3 the results of the stresses at the maximum load are presented for the second and most realistic situation. An intermediate shear stiffness is applied in all the insulation layers, instead of the alternating maximum and zero stiffnesses as used in the first situation. Figure 4.2.4 shows the residual stresses. It can be observed that the axial load in the reinforcement of the six outer subcoils is redistributed. The thin reinforcements are less loaded axially and the thick outer reinforcement is heavier loaded than when the maximum shear stiffness was applied within the insulation layer of a subcoil. The overall situation for the loading of the coil does not seem to change remarkably.

It can be concluded that the stresses calculated with the KUL tool, although in the KUL design some simplified assumptions are taken.

4.3 LANL tool

4.3.1 Introduction in the LANL coil design

The LANL 60 tesla coil-system design is characterised by nine mechanically independent coils. The coils have a small $\alpha$ (ratio of outer to inner diameter), to reduce the amount of plastic deformation and to limit the residual compressive stresses in the windings. These thin coils also have the feature that they can be cooled to low temperatures rapidly. In the LANL design the strength of the coil is mainly governed by the shield, instead of by the windings as in the Amsterdam design. The shields, therefore, are relatively thick compared to the shields in the Amsterdam design. The coils are made of only a low number (2 to 6) of winding layers. The coil design is based on an analytical solution. Only three out of nine coils, coil one, five and seven have been analysed in more detail with a FEM. Coil one was modelled the most realistic, with gap elements between coil and shield. These gap elements were required because in the analyses the coils are first cooled down to low temperatures. This causes a gap between the coil and shield, due to different thermal expansion coefficients of the materials in both parts. Coil five was modelled with coil and shield elements tied together. Coil seven was modelled with averaged material properties in the coil elements.

4.3.2 Model of the LANL design

The three coils are modelled with the AFEM using the material properties supplied by the LANL data, and with the assumptions as used in the Amsterdam design. The coils are therefore generated with the mesh generator resulting into coils with winding elements, elements for the steel shield and elements for the insulation layer between the coil and the shield. To describe the stresses in the coil accurately the element size in the AFEM mesh is less or equal than the winding size. The insulation material in the coils is assumed to have merely compressive strength, the shear and tensile stiffnesses are reduced just as it was in the reference situations of the coils described in section 3.

For the material properties the values as used in the LANL are not taken temperature dependent, but these are taken at the room temperature values. This is because the AFEM does not support temperature dependence. In the AFEM perfect-plasticity is assumed for the winding material and for the shields. The LANL model uses strain-hardening for the shields, but this is not used in the AFEM because of the lack of data about
the strain-hardening modulus. The geometry and material data are summarised in table 4.1, for the LANL coils as analysed with the AFEM.

<table>
<thead>
<tr>
<th></th>
<th>Coil 1</th>
<th>Coil 5</th>
<th>Coil 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner radius [m]</td>
<td>0.0175</td>
<td>0.1038</td>
<td>0.2225</td>
</tr>
<tr>
<td>outer radius [m]</td>
<td>0.0287</td>
<td>0.1300</td>
<td>0.2713</td>
</tr>
<tr>
<td>half-height [m]</td>
<td>0.0796</td>
<td>0.3593</td>
<td>0.4986</td>
</tr>
<tr>
<td>thickness shield [m]</td>
<td>0.003</td>
<td>0.0323</td>
<td>0.0265</td>
</tr>
<tr>
<td>$\lambda_w [-]$</td>
<td>0.7679</td>
<td>0.7939</td>
<td>0.8238</td>
</tr>
<tr>
<td>$\lambda_s [-]$</td>
<td>0.9258</td>
<td>0.9210</td>
<td>0.9378</td>
</tr>
<tr>
<td>$E_w$ [GPa]</td>
<td>122</td>
<td>143</td>
<td>143</td>
</tr>
<tr>
<td>$\sigma_{yield,w}$ [MPa]</td>
<td>626</td>
<td>745</td>
<td>745</td>
</tr>
<tr>
<td>$\Delta T$ [K]</td>
<td>150</td>
<td>150</td>
<td>116</td>
</tr>
<tr>
<td>$E_I$ [GPa]</td>
<td>16.7</td>
<td>16.7</td>
<td>16.7</td>
</tr>
</tbody>
</table>

Table 4.1: Coil data for three of the LANL coils, as used in the AFEM. $w =$ winding, $i =$ insulation.

The thermal load is applied according to the LANL design, during the increase, at the maximum and during the decrease of the mechanical load. The total temperature increase, calculated with the computer routines used in Amsterdam, compares very well to the temperature increase supplied in the LANL data. The coils in the AFEM are not cooled down from room temperature to low temperatures, as in the LANL design. This is because the expected radial strain between coil and shield, due to the different thermal expansion if both are cooled down, can be neglected.

4.3.3 Results and conclusions for the LANL design

For the LANL results only some data for the maximum applied mechanical load are available. Results for the maximum von Mises stress in the windings and shields, and the maximum radial stress are summarised for each coil in table 4.2. The results calculated with the AFEM, for the maximum values, are also presented in table 4.2. In figure 4.3.1 and 4.3.2 the stresses in the mid-plane of the coils are presented, calculated with the AFEM.

<table>
<thead>
<tr>
<th></th>
<th>Coil 1</th>
<th>Coil 5</th>
<th>Coil 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>von Mises winding</td>
<td>426</td>
<td>553</td>
<td>610</td>
</tr>
<tr>
<td>von Mises shield</td>
<td>902</td>
<td>713</td>
<td>1020</td>
</tr>
<tr>
<td>radial pressure</td>
<td>106</td>
<td>59</td>
<td>220</td>
</tr>
<tr>
<td>AFEM</td>
<td>553</td>
<td>621</td>
<td>722*</td>
</tr>
<tr>
<td>LANL</td>
<td>1020</td>
<td>1030</td>
<td>1105</td>
</tr>
<tr>
<td>AFEM</td>
<td>59</td>
<td>217</td>
<td>1026</td>
</tr>
<tr>
<td>LANL</td>
<td>106</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Maximum stress data for the LANL and AFEM models of coil 1, 5 and 7 of the LANL design. * von Mises stress lower than the yield-strength because coil is already mechanically unloading, as is explained in section 3.7.3.

For the conclusions about the results calculated with the AFEM, it should be considered that only limited data for the stresses in the mid-plane of the coils are available. In coil 1, the radial stress and the von Mises stress for the shield indicate that the shield is higher loaded in the LANL calculation than in the AFEM calculation. The only elastically loaded winding material is just less loaded in the LANL design than in the AFEM. The
overall stresses in coil 1, however, are so low that even with the AFEM approach no problems regarding (repeated) plastic deformations are expected in the winding material.

Figure 4.3.1: Stresses for coil 1, 3 and 7 calculated with the AFEM, at the maximum mechanical load and a partly (80%) applied thermal load. The results of the LANL design-tool are indicated with a 'A'. Mind the 'radius' axis.

Figure 4.3.2: Residual stresses for coils 1, 3 and 7 calculated with the AFEM, after removing the mechanical load and after fully applying the thermal load. Mind the 'radius' axis. Coils are stressed in compression by the shields, because of the thermal expansion of the coils.
For coils 5 and 7 the results are reproduced very well. The radial stresses are nearly the same in both coils, and for coil 5 also the maximum von Mises stresses are the same. The shield in coil 7 is lower loaded in the AFEM because the shield in the LANL design is strain-hardened. The shield in the AFEM shows, in the mid-plane, plastic deformations almost all over the shield, from the inner radius to the outer radius. The stiffness against radial displacements of the shield in this situation is only governed by the shear stiffness, that couples this plastically deforming zone to the elastically deforming top and bottom zones of the shield. In figure 4.3.2, it can be observed that the residual stresses are not causing plastic deformations in the coils 5 and 7 during the unloading.

The LANL coil design calculated with the AFEM shows that the stresses in these coils according to the AFEM calculations are acceptable, although care should be taken of the yielding shields.

4.4 Conclusion comparative study

In this section two coil design-tools were described. One with a simple FEM background (KUL) and one with a more detailed background (LANL). The first aim of this comparative study was to compare the results of a composite material modelling to the more realistic modelling of windings and insulation material in separate elements. At the end this comparative study was used for a comparison with a simple model, with different assumptions as in the AFEM described in section 3. The AFEM is also used for the comparison with a more detailed FEM, however the supplied data were not detailed enough to discover the actual differences in both modelling approaches.

It is shown, however, that without much difficulty several types of coils can be modelled with the in Amsterdam developed FEM. The model of section 3 is accurate enough to reproduce the results of the KUL design, and is superior in the way materials can be modelled. For anisotropic shields the AFEM should be adapted to produce more realistic results. The developed model also reproduces the results of the LANL model for coils 5 and 7. If, however, more material data had been supplied the modelling could have been better, specially for coil 1 of the LANL.

Comparing the results of the KUL and the LANL design with the Amsterdam design, the following concluding remarks can be drawn.

- The KUL design allows plastic deformation of the winding material during the unloading of the coil, in contrast to the Amsterdam design. In the Leuven design the most emphasis is put on: designing a small coil, producing a high-field with a short pulse duration, due to the limitations in available energy. A limited lifetime, due to the repeated plastic deformations, is accepted.
- The designers of the LANL have put emphasis on very realistic material modelling, by using the material properties temperature dependent and by using the effects of strain-hardening. This strain-hardening effect is obvious, because if the materials are modelled assuming perfectly plastic behaviour, then the shields start yielding, for coil 7 even all over the cross-section in the mid-plane. The use of a very high strength winding material is avoided by using extensive external shielding, giving rise to a large increase in the required power and energy. On the basis of the design, small α coils were used to reduce the amount of plastic deformation in the coil. The designers therefore expect a lifetime of the coils of at least 10,000 load-cycles.
References


5 Fatigue Experiments on CuAg and Cu Wires

5.1 Introduction

For an optimised design, magnet coils require a winding material with a high-strength as well as an excellent electrical conductivity. Recently (begin 1990s) a hard-drawn copper-silver (CuAg) wire with 16 at.% Ag has been developed, which accommodates the required properties. Data on fatigue, however, are hardly existing until now. For the reliable use of this wire in the repetitively highly loaded magnet coil, not only tensile test data but also low-cycle fatigue data are required [1].

Testing a real magnet coil under equivalent load conditions as in the expected actual operation is unworkable, because of cooling-cycles down to 77K and to the needed power supply. Using a scaled-down magnet, with load conditions as in the real magnet, is as well impractical, although the needed power and cooling capacity can be much smaller.

Searching for an experiment that equivalently loads the winding material as in the coil, an appropriate choice seemed to be the tension-compression test. Other fatigue tests [2] like rotating-bending or torsion tests do not match the load situation in the coil, where the highest load direction is parallel to the wire. The other tests, also, do not show respect for the anisotropy of the hard-drawn wire. To facilitate the interpretation of the consequences of the CuAg fatigue test results to the lifetime of a real coil, similar fatigue tests were performed on the hard-drawn copper wire used for the existing 40 tesla magnet.

5.2 Fatigue test

For the low-cycle fatigue test a tensile test machine is used, in which the material is loaded in tension-compression with repeated load-cycles. To protect the load-cell against transverse displacements (forces) that arise from misalignment of the specimen, during compressive loads, a special equipment (figure 5.2.1) is constructed to avoid these displacements. The friction force in the bearings can be neglected compared to the load-forces.

![Figure 5.2.1: Equipment used to protect the load-cell against transverse forces. Load-cell can be connected at the top.](image)
The load-curve in figure 5.2.2 is derived from finite element analysis on the coil-design to be used, for the part of the magnet coil with the largest tangential stress-range. Initially the loading of a wire (winding) starts in the origin of the strain-stress plot. First a tensile stress is applied to the test-specimen, followed by a plastic deformation, in which perfectly plastic behaviour is assumed. During the unloading of the coil, the plastically deformed winding material is stressed in compression, due to elastic back motion of material on the outside. Compression therefore is also applied to the test-specimen.

![Strain-Stress Plot](image)

*Figure 5.2.2: Applied load-curve for the CuAg wire material, derived from finite element analysis on the coil-design to be used. Repetitive load-cycles are applied between $\sigma_{\text{min}}$ and $\sigma_{\text{max}}$. Perfectly plastic behaviour is assumed to the Ultimate Tensile Strength (UTS).*

After this first load-pulse, the minimum (compressive) load can be regarded as the load in the steady-state situation of the coil. Other windings, outside the region of the largest stress-range can still be exposed to tensile tangential stresses. Subsequent pulses result in repeated elastic load-cycles, between the minimum (compressive) load and the maximum (tensile) load.

During the repeated load-cycles the maximum load is limited to approximately 90% of the Ultimate Tensile Strength (UTS), because of overshoot in the control of the forces by the test machine. The overshoot in the force control strongly depends on the force increase rate, which depends on the stiffness of the tested material and the applied displacement speed (between $8.3 \times 10^{-5}$ and $2.5 \times 10^{-4}$ m/s).

Strain gauges are used on some of the test-specimen to measure the applied strain. During the plastic deformation the hard-drawn wire material shows nearly perfect plasticity, and the tensile test is stopped as the measured force starts to decrease. The decreasing force indicates that the cross-sectional area of the sample reduces and that necking starts. Repetitive measurements, on specimen with strain gauges, showed that the maximum applied plastic strain is about 0.95% at the start of the force decrease. For the test-specimen without strain gauges, and loaded with an equivalent load-curve it is assumed that the plastic deformation is also 0.95%. It can be expected that small deviations in the applied plastic strain, will have little or no influence on the crystal orientation and behaviour of the hard-drawn wire material. During the production, the anisotropic wire was already exposed to an area reduction of approximately 98%.
Fatigue measurements for only a single stress amplitude hardly give insight into the fatigue behaviour of a material. Therefore experiments are also performed with different applied stress amplitudes. The stress amplitude is defined as:

\[
\text{stress amplitude} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}
\]

5.1

In usual fatigue measurements, materials are loaded with a fixed mean stress and at different stress amplitudes, after that the mean stress is changed and again measurements are performed at different stress amplitudes. In the design of a magnet coil in which, during the first tensile load, plastic deformations are permitted, the maximum stress, however, is always equal or higher than the yield-strength of the material.

The maximum stress in the test is kept constant (~ 90% of the UTS), because the main purpose of the performed fatigue tests is to estimate the fatigue behaviour of winding material in the coil. The minimum stress is varied, in order to change the stress amplitude. This is the parameter in the magnet coil-system that can be easily modified in the design, to achieve by decreasing the \( \alpha \) (ratio of outer to inner radius) of the coil. The result is a reduction of the plastic deformation and with that a decrease of the compressive stress. The applied test method of a constant maximum stress and variable minimum stresses results in different stress amplitudes and in different mean stresses. The measured fatigue lives should now be regarded as single data-points on the mean stress curves of the usual fatigue measurements.

5.3 Results for the CuAg wire

Initial wire dimensions of 4x6 mm\(^2\) and the compressive load (possibility of buckling), restricts the shape of the specimens, that are therefore not designed like regular tensile test specimen. Buckling is also promoted by the initial curvature of the wires as delivered. The shape of the test-specimen (figure 5.3.1) was adapted several times, to extend the measured fatigue life. Stress-concentrations are avoided and the surface condition is improved by polishing.

\[
\text{strain gauges (Kyowa KFG-1-120-D16-16, gauge length 1 mm and grid width 1.2 mm)}
\]

Strain gauges (Kyowa KFG-1-120-D16-16, gauge length 1 mm and grid width 1.2 mm) are glued (cyano-acrylate CC-33A) to the specimen on the uniform cross-section in the axial mid-plane. The results, measured at room temperature, for the different stress amplitudes

\[\text{Figure 5.3.1: Fatigue test-specimen made by a lathe and by a milling machine. Wire initial cross-section: 4x6 mm}^2, \text{final cross-section: by lathe } \Theta 3 \text{ mm, by milling 1.75x4 mm}^2\]
are summarised in Table 5.1 and presented in Figure 5.3.2. A full list of all measurements data is presented in appendix C.

The lines in Figure 5.3.2 do not represent the regular fatigue life lines for a fixed mean stress and different stress amplitude. The lines, however, connect the average values for the given stress amplitudes, with different mean stresses at a fixed maximum stress of 800 MPa. A remark should be made for the results with a minimum stress of zero (stress amplitude for CuAg 400 MPa). These specimens are not pre-loaded with a plastic deformation, because they represent the load-situation for a coil in which no plastic deformation has occurred. Likewise are the literature data [3] for only elastically tensile loaded specimen, with different maximum stresses and a minimum stress of zero.

<table>
<thead>
<tr>
<th>Stress amplitude [MPa]</th>
<th>Stress amplitude/UTS [-]</th>
<th>Lifetime [cycles] [-]</th>
<th>Number of specimen [-]</th>
<th>Standard deviation [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>0.72</td>
<td>1257</td>
<td>8</td>
<td>462</td>
</tr>
<tr>
<td>550</td>
<td>0.61</td>
<td>3419</td>
<td>7</td>
<td>360</td>
</tr>
<tr>
<td>400</td>
<td>0.44</td>
<td>9196</td>
<td>5</td>
<td>5328</td>
</tr>
</tbody>
</table>

Table 5.1: Results of fatigue test CuAg, $\sigma_{\text{max}} = 800$ MPa, $\sigma_{\text{min}} = \text{variable}$. Measurements are performed at room temperature.

Figure 5.3.2: Relative stress amplitude versus number of cycles for CuAg (UTS 900 MPa) and for hard-drawn copper (UTS 400 MPa). The lines connect the average values for given stress amplitudes at a fixed maximum stress. Two sets* of copper-data are with a different maximum stress, and therefore not connected in the average line.
The large scatter in the measured fatigue lives, represented in the standard deviation, is caused by the small number of specimen that was used as well as by the tested experiment. Only a small number of specimen is used for the presented data, because many tests were performed on specimen not able to represent the fatigue life of the material, due to a bad manufacturing process. On the other hand fatigue life tests always result in a large scatter, because a small variation in the applied stress-range results in a large variation in the number of cycles.

5.4 Results for the copper wire

Identical fatigue tests (compared to CuAg) are performed on the copper wire material. The sizes of the specimen are adapted to the wire dimensions 2.7x6 mm². Only the lathe samples are used, due to problems in the fabrication process of the milled samples (uncontrollable heat production/surface temperature). The copper wire is loaded with a load-curve equivalent with that for the CuAg wire. First the specimen is loaded in tension, followed by a plastic deformation (1%) and after that a compressive load, resulting into the steady-state situation. The applied stresses, however, are adapted to the UTS of the hard drawn copper wire (400 MPa). The results, measured at room temperature, for different stress amplitudes are presented in figure 5.3.2 [4] and summarised in table 5.2. A full description of all measured data can be found in appendix C.

<table>
<thead>
<tr>
<th>Stress amplitude [MPa]</th>
<th>Stress amplitude/UTS [-]</th>
<th>Lifetime (cycles) [-]</th>
<th>Number of specimen [-]</th>
<th>Standard deviation [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>330</td>
<td>0.83</td>
<td>46</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>289</td>
<td>0.72</td>
<td>190</td>
<td>7</td>
<td>62</td>
</tr>
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<td>246</td>
<td>0.61</td>
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<td>7</td>
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</tr>
<tr>
<td>178</td>
<td>0.45</td>
<td>29164</td>
<td>3</td>
<td>14961</td>
</tr>
<tr>
<td>320*</td>
<td>0.80</td>
<td>410</td>
<td>4</td>
<td>248</td>
</tr>
<tr>
<td>260*</td>
<td>0.65</td>
<td>844</td>
<td>2</td>
<td>441</td>
</tr>
</tbody>
</table>

Table 5.2: Results of fatigue test copper, \( \sigma_{\text{max}} = 360 \text{ MPa}, \sigma_{\text{min}} = \text{variable}, \sigma_{\text{max}} = 320 \text{ MPa}. \) Measurements are performed at room temperature.

The results for the specimen tested at a lower maximum stress give confidence in the results of the measurements, because they indicate that fatigue life increases if the same stress-range is applied with a lower maximum stress.

5.5 Discussion

The tension-compression test, of course, is not fully representative for the load-situation in the coil. With the applied load-condition overestimated, the lifetime found will be a lower boundary, as indicated by the following three reasons.

1. For a practical reason, that is, the flexibility of the tensile test machine, the displacement (strain) controlled load condition is replaced by a force (stress) controlled experiment. In the coil, a wire is enclosed between neighbouring windings, localisation effects in the deformation are suppressed, because the deformation of a winding is bounded by the deformation of the neighbouring windings. Materials in uniaxial tests, however, are
sensitive to localisation of failure by imperfections. Due to these imperfections all the applied elongation during the uniaxial test is transmitted to the imperfect part. When arising damage (cracks) in the material reaches a certain threshold value, the force controlled test immediately ends, because the maximum prescribed force cannot be reached anymore. The displacement (strain) controlled test is expected to result in more load-cycles than the applied force (stress) controlled test, as indicated by figure 5.5.1.

Figure 5.5.1: Displacement controlled load versus a force controlled load. A force controlled load condition does not show respect for damage effects in the material, therefore the material is higher loaded than in the strain controlled load condition.

2. The experiments are performed at room temperature instead of low temperatures, $77K < T_{\text{actual}} < 300K$. Room temperature experiments were performed, because of obvious practical reasons and because measurements at a fixed low temperature are not representative enough, due to the temperature rise during the mechanical loading of the coil. From experiments on copper material it is known [4] that with decreasing temperatures the number of load-cycles until failure increases for a fixed stress amplitude. For the copper material described in [4] the number of load-cycles increased with a factor 100, when the temperature was decreased from 293K to 90K.

3. In the experiment a uniaxial load is applied instead of a triaxial load where compressive transverse loads support the deformation of the material. The effects of a multi-axial load on the fatigue life are not well known yet. This can be observed from the fact that fatigue tests are often applied on large and complex structures, for example parts of an aircraft, instead of on (representative) test specimen.

5.6 Conclusions

The real copper coil is loaded already more than 20,000 times to the maximum admissible load, so it seems that the winding material in the real coil is loaded milder than the material in the tension-compression test. As already mentioned, the following three effects could partly account for this difference.

- The wire is stress controlled in the test instead of strain controlled in the coil.
- Measurements are performed at room temperature instead of at low temperatures.
- The wire is uniaxially loaded in the test versus a triaxial stress state in the coil.
A safety margin in the design stresses is used. The safety margin in the stresses reduces the amount of plastic deformation, resulting into a decrease in the compressive stress. For example, a safety margin of 20% in the UTS (for a 30 tesla test coil) results in a decrease of the maximum plastic strain with 78%, and in a decrease of the stress amplitude with 12% from 345 MPa to 305 MPa.

From the fatigue results obtained so far there are no reasons to assume that the fatigue behaviour of CuAg is inferior to the fatigue behaviour of Cu under relatively similar load conditions. For the actual coil construction all the mechanical properties should, however, be taken into account. Special attention should be paid to the low ductility of CuAg compared with that of Cu at low temperatures, because the large ductility of the copper can give an extra safety margin against unexpected deformations.

References
6 Conclusions

6.1 General conclusions

This report describes the mechanical aspects in the design of a high-field magnet coil-system. The main issues are summarised and concluded here.

In section 1 the aims of the project are presented as:
1. The development of an axisymmetric Finite Element Model (FEM) for the verification of an analytical model for the calculation of the mid-plane stresses. This FEM will also produce the stresses and deformations outside the mid-plane, and it gives the opportunity to use more realistic material parameters and material descriptions.
2. The performance of low-cycle fatigue experiments. These experiments are performed in the for the magnet design relevant load-range, on the winding material to be used.
3. The comparison of results calculated with the Amsterdam design method with results on similar coils obtained at various institutes by slightly different approximations.

After the presentation of the physical behaviour of a magnet coil, the materials in the coil are considered as a composite material, in the first part of section 2. In the second part an analytical approach for the mid-plane stresses is introduced.

Section 3 deals with the introduction into the FEM, in the single coil and in a coil surrounded by a steel shield. This model allows the use of more realistic material modelling than used in the analytical approach, and it calculates the stresses and deformations in the entire coil volume. The comparison of the mid-plane stress-results of the FEM, for the single coil, with the analytical stress-results fulfills the first aim of the project. For an insulation material that is relatively weak compared to the winding material, the mid-plane stresses derived with both approaches are almost the same over the entire mid-plane.

The validated FEM is then used to study the influence of several properties on the stresses in the winding material, by the following modelling variations:
A. additional shear and tensile stiffness of the insulation material
B. strain-hardening of the wire material
C. thermal loading of the wire material
D. imperfections, local reduction of strength
E. the application of steel shields

These variations have shown:
A. With tensile and shear stiffness in the insulation material, the deformations in the coil are spread over a wider area of material, and the peak stresses in the winding material are decreased.
B. Strain-hardening decreases the amount of plastic deformation of the winding material. The residual stresses in the winding material, after the load-pulse, are decreased too.
C. Imperfections in the winding material, to 30% strength reduction in one turn nearby the highest loaded part of the coil, show that the total load is redistributed in the rest of the coil. This redistribution occurs without severe consequences for the stresses or deformations.
D. A homogeneous temperature increase only in the winding material of the single coil hardly influences the stresses in this winding material, although the insulation material
remains cold. The total deformation of the coil, however, is increased with the thermal strain.

E. Firstly, the results for the steel shielded coils show that the temperature increase in the winding material reduces the amount of plastic deformation in the coil. The load on the shield, however, (remaining at low temperature) is increased.

Secondly, the study to the influence of the insulation material behaviour has shown that shear stiffness in the insulation layer between coil and shield transfers the axial load, generated in the coil by the axial Lorentz-force, from the coil to the shield. This axial load transfer reduces the axial stress in the yielding winding material, and following from the yield-condition the tangential stress in the windings now increases. With the higher stress in the tangential direction the radial displacement of the coil decreases. Due to this displacement reduction the load on the shield in tangential direction is decreased now.

It can be concluded that, due to the considerable difference in the computing times, the analytical approach should be used for the initial mechanical design of the coil. In combination with routines calculating the heat production and the amount of necessary power, a coil can be designed that is optimised within the limits of heat, power and mechanical strength. Afterwards the FEM can then be used for more detailed analyses of the stresses and deformations in the whole coil volume. The analytical approach calculates the ‘worst case’ situation, because it was assumed that the stiffness in the coil is mostly governed by the stiffness in the winding material. The use of a more realistic material modelling can show the safety margin that is left in the actual stresses. This so called ‘safety margin’ can give confidence in the design.

The second aim of the project is fulfilled in section 5. Here the fatigue behaviour of the newly developed copper-silver (CuAg) wire material is compared with the copper wire material as used for more than 25 years in the existing 40 tesla magnet-installation. If both materials are loaded with comparable load conditions, a stress amplitude between 0.72 and 0.44 times the Ultimate Tensile Strength (UTS), the measured lifetimes are about the same and amount to be 1,000 to 10,000 cycles. For the copper material with a stress amplitude of 0.83 times the UTS, that is based on the design calculations for the existing 40 tesla coil, the lifetime is measured to be 46 cycles. The real copper coil, however, is loaded already more than 20,000 times to the maximum admissible load. It seems, therefore, that the winding material in the real coil is loaded milder than the material in the applied tension-compression test. The gap in lifetime between the measured and the actual lifetime of the material can be explained from the facts that:

- The loading of the wire in the test is stress controlled instead of strain controlled in the coil.
- The measurements are performed at room temperature instead of at low temperatures.
- A uniaxial load is applied in the test versus the triaxial stress state that is present in the coil.

Additionally, a safety margin in the design stresses of the copper coil is used, reducing the amount of plastic deformation, because the real yield-strength is higher and some strain hardening is possible. Less plastic deformation decreases the residual compressive stress in the winding material, because the plastically deformed material is less compressed by the elastic material on the radial outside. The stress amplitude is reduced and the lifetime of the winding material increases. For example if the stress amplitude decreases with 10% from 0.83 to 0.72, the lifetime increases with 400% to 190 cycles. The rest of the
gap in the lifetime should be explained by the three points mentioned above. These points shift the measured curves to a longer lifetime, thus an increase of 400% by the safety margin on the absolute number of cycles is remarkable. From the fatigue results obtained so far there are no reasons to assume that the fatigue behaviour of CuAg is inferior to the fatigue behaviour of Cu under relatively similar load conditions.

Finally the third aim of the project is fulfilled in section 4, where the FEM as described in section 3 is compared with various models developed at other institutes. The comparison of the Amsterdam design tool with the tool of the Katholieke Universiteit Leuven, shows that both models result in the almost the same solution for the mid-plane stresses. For the Los Alamos National Laboratory design tool, the supplied data for the maximum stresses in the several sub-coils of that design are approximately the same as the data calculated with the model developed in Amsterdam.

6.2 Impact of this study on the design of the 60 tesla magnet

In the report, thus far, less attention had to be paid to the actual design of the 60 tesla coil-system. This is because this report primarily describes the development of the design-tool that can be useful in the detailed design of a coil-system. After validating the analytical model to the FEM, this analytical method is the preferred way to do the basic design. Mechanical considerations are not the only limitation in the design of a coil. The actual design is performed, until now, by changing various parameters by hand (according to general scaling laws that are exactly valid in simple cases). Therefore a quick and reliable tool to calculate these stresses is needed.

The results of the variations in the material modelling (section 3) support the existing design and demonstrate that all simplifications, necessary in the quick analytical approach, give indeed rise to a conservative design. The results, regarding the stress amplitude in the highest loaded part of the coils, are already used in the study to the fatigue properties as described in section 5.

However, besides these reassuring results two aspects emerged from this study that need extra attention. Firstly: The analyses have shown that the effective plastic strain, about 3%, in some parts of the windings in the inner coils is larger than the plastic strain that can be measured in a tensile test, about 1%. Bending tests of the wire around a spine, where localisation effects in the deformation can be neglected, have shown that the actual plastic strain in the material can be about 20%. The basic design of the coils, therefore, can still be used for the construction, because it is to be expected that the winding material in the coil is not exposed to localisation of the deformation. Secondly: Attention should be paid to the fatigue test results, because a change in the geometry of the coil can reduce the stress amplitude and it can increase the lifetime of the winding material. However, the amount of cycles between the measured lifetime and the lifetime of the existing coil indicates that the winding material in the coil is milder loaded than during the performed test.

For these two aspects, because the coils are not under construction yet, it can be considered to reduce the effective plastic strain in the coil by changing the coil’s geometry. This means that a coil should be divided into two subcoils, mechanically independent of each other by a steel shield, or mechanically dependent by a (steel) fibre-reinforcement. This fibre-reinforcement is a newly development for the Amsterdam coil design, but is actually under construction in the Leuven coil designs.
The development of fibre-reinforced coils should be regarded with interest, because the construction of these coils will probably be easier than the construction of a steel shielded coil. In these steel shielded coils the dimension of the shield should be in very good agreement with the dimensions of the coil. To be sure that the coil will be really reinforced without too much plastic deformation during the first pulse, the shield should narrowly fit around the coil. The fibre-reinforcement, however, can be added during the winding process in the same manner as the coil is wound. The disadvantage of the fibre reinforcement is the lack of axial and radial stiffness. Therefore a good combination of external shields and internal reinforcements should be used.

After thus defining the basic design of the coil, the choice for actual realisation, for instance wire section, insulation and filling material, construction of the leads, coil mounting and centring, has still to be made. This phase of the design strongly needs input from coil winding experiments. These choices have to be made before and during the process of coil winding, on basis of existing expertise, experience to be gained with the CuAg wire and the insight based on the calculations with the FEM. It is foreseen that several of these decisions will demand for finite element analyses mainly mechanical, but also thermal, on structures with low symmetry, for instance the connection of the leads and the coil’s centring pieces.
Appendix A

Derivation of the Poisson’s ratios for the unit cell of the composite

For the derivation of $v_{\theta z,c}$ in the composite material, a (tensile) tangential stress is applied to the unit cell. This stress enlarges the material in the tangential direction, and due to the Poisson’s ratios the material also deforms in the axial and radial directions. It is assumed that the axial and radial stresses remain zero, therefore the tangential and axial strains caused by the tangential stress are given (see formula (2.1)) by:

$$
\varepsilon_{\theta\theta,c} = \frac{1}{E_{\theta\theta,c}} \sigma_{\theta\theta}, \quad \varepsilon_{z z,c} = -\frac{v_{\theta z,c}}{E_{\theta\theta,c}} \sigma_{\theta\theta}
$$

The Poisson’s ratio $v_{\theta z,c}$ is then found from:

$$
v_{\theta z,c} = \frac{-\varepsilon_{z z,c}}{\varepsilon_{\theta\theta,c}}
$$

The following indices are used: $i$ for the insulation material, $w$ for the winding material, $c$ for the composite and $h$ for the homogeneous material which will be defined later on. In formula (A.2) the tangential strain can be calculated from the tangential Young’s modulus (2.4a), and the applied tangential stress using (A.1). To determine the axial strain some assumptions have to be made.

![Figure A.1: Unit cell divided into two rectangles to derive $v_{\theta z,c}$. $\lambda_r$ = radial winding fraction, $\lambda_z$ = axial winding fraction.](image)

Figure A.1 shows that in step1 the unit cell is divided into two rectangles. In this step it is assumed that straight edges in the unit cell remain straight if $\sigma_{\theta\theta}$ is applied, and that the edges of the two rectangles will also remain straight. The division into these rectangles is chosen because it is assumed for rectangle2 that the small part of insulation material will sustain the same axial strain that is governed by the relatively stiff winding material.
The axial strain in rectangle 1 can be calculated using the insulation’s properties.

\[ \varepsilon_{zz,i} = \frac{-\nu_{0z,i}}{E_{\theta\theta,i}} \sigma_{\theta\theta,i} \]  \hspace{1cm} (A.3)

For rectangle 2 the axial strain cannot be calculated directly analogous to (A.3) because the overall Poisson’s ratio \( \nu_{0z,r2} \) of rectangle 2 still depends on the (different) axial deformations of the winding and the insulation material. Step 2 in figure A.1 is necessary now, it shows that the material in rectangle 2 can be considered as a homogeneous material, with a Poisson’s ratio \( \nu_{0z,h} \).

\[ \begin{array}{c}
\text{rectangle 2} \\
\text{homogeneous}
\end{array} \]

\[ \text{Figure A.2: Reaction-forces of the material in rectangle 2 and in the homogeneous material, due to the (same) applied tangential load.} \]

The Poisson’s ratio \( \nu_{0z,h} \) can be determined by considering that the edges with a normal in the axial direction are bounded by a zero displacement, as in figure A.2. The applied tangential stress will cause, through the axial deformation, the reaction-forces \( F_w \) and \( F_i \) in the winding and the insulation material respectively. For the homogeneous material it is assumed that a force \( F_h \) will be caused by an axial deformation due to the Poisson’s ratio \( \nu_{0z,h} \), when the same tangential load is applied as on rectangle 2. A balance of the reaction-forces of the two situations, described in figure A.2, should result in the unknown \( \nu_{0z,h} \) because the reaction-forces in both situations should be equal.

The reaction-forces are calculated by a multiplication of the axial stresses in each material and the area the stress is acting on. The axial stress is calculated by considering that the winding and insulation material are isotropic, and that the homogeneous material is anisotropic. The strain-stress relationship can then be written as:

\[ \varepsilon_{zz,x} = \frac{\sigma_{zz,x}}{E_{zz,x}} + \frac{-\nu_{rz,x} \sigma_{rr,x}}{E_{rr,x}} + \frac{-\nu_{0z,x} \sigma_{\theta\theta,x}}{E_{\theta\theta,x}} \]  \hspace{1cm} (A.4)

With \( x: i, w \) or \( h \) depending on the material that caused the reaction-force on the area \( A \). The edges are tied, therefore \( \varepsilon_{zz,x} \) is equal to 0. The radial stress is still assumed to be zero, and the tangential stress is as applied, therefore the axial stress can be written now as:

\[ \sigma_{zz,x} = \nu_{0z,x} \sigma_{\theta\theta,x} \frac{E_{zz,x}}{E_{\theta\theta,x}} \]  \hspace{1cm} (A.5)
The reaction forces acting on the area $A_x$ can be written as:

$$F_w = \sigma_{zz,w} A_{zz,w} = \nu_{0z,w} \sigma_{00,w} \frac{E_{zz,w}}{E_{00,w}} \lambda_r = E_{zz,w} \nu_{0z,w} \lambda_r = E_w \nu_{0z,w} \nu_{w} \lambda_r \quad A.6$$

$$F_i = E_i \nu_{0z,i} \nu_{i} (1-\lambda_r) \quad A.7$$

$$F_h = E_{zz,h} \nu_{0z,h} \nu_{z,h} \quad A.8$$

The axial modulus $E_{zz,h}$ of the homogeneous material can be found analogous to (2.3b):

$$E_{zz,h} = E_w \lambda_r + E_i (1-\lambda_r) \quad A.9$$

Balance of the forces: (A.6) and (A.7) equals (A.8) using (A.9) and assuming that the tangential strains in all materials are equal to the tangential composite strain, results in $\nu_{0z,h}$.

$$\nu_{0z,h} = \frac{\lambda_r \nu_w E_w + (1-\lambda_r) \nu_i E_i}{\lambda_r E_w + (1-\lambda_r) E_i} \quad A.10$$

Now the axial strain in rectangle2 can be calculated using:

$$\varepsilon_{zz,h} = \frac{-\nu_{0z,h}}{E_{00,h}} \sigma_{00,h} \quad A.11$$

Instead of calculating the combined axial strain in the unit cell, it is now easier to use formula (2.3c), because the Poisson's ratios in both rectangles are known.

$$\nu_{0z,c} = (1-\lambda_z) \nu_i + \lambda_z \nu_{0z,h} \quad A.12$$

Using A.10 and A.12, $\nu_{0z,c}$ can finally be written as:

$$\nu_{0z,c} = (1-\lambda_z) \nu_i + \lambda_z \left[ \frac{\lambda_r \nu_w E_w + (1-\lambda_r) \nu_i E_i}{\lambda_r E_w + (1-\lambda_r) E_i} \right] \quad A.13$$

Based on similar assumptions $\nu_{0r,c}$ can be found by splitting the unit cell into two rectangles with sides $(1-\lambda_z)$ and $\lambda_r$ resulting into:

$$\nu_{0r,c} = (1-\lambda_r) \nu_i + \lambda_r \left[ \frac{\lambda_z \nu_w E_w + (1-\lambda_z) \nu_i E_i}{\lambda_z E_w + (1-\lambda_z) E_i} \right] \quad A.14$$

III
For the derivation of $v_{rz,c}$ in the composite material, a (tensile) radial stress is applied to the unit cell. This stress enlarges the material in the radial direction, and due to the Poisson’s ratios the material also deforms in the axial and tangential directions. It is assumed that the axial and tangential stresses remain zero, therefore the radial and axial strains caused by the radial stress are given (see formula (2.1)) by:

\[
\varepsilon_{rr,c} = \frac{1}{E_{rr,c}} \sigma_{rr}, \quad \varepsilon_{zz,c} = \frac{-v_{rz,c}}{E_{rr,c}} \sigma_{rr}
\]

The Poisson’s ratio $v_{rz,c}$ is then found from:

\[
v_{rz,c} = \frac{-\varepsilon_{zz,c}}{\varepsilon_{rr,c}}
\]

The following indices are used: $i$ for the insulation material, $w$ for the winding material, $c$ for the composite and $h$ for the homogeneous material which will be defined later on. In formula (A.16) both the axial and the radial strains in the composite are hard to calculate without any further assumptions.

\[\text{Figure A.3: Unit cell divided into two rectangles to derive } v_{rz,c}. \lambda_r = \text{radial winding fraction}, \lambda_z = \text{axial winding fraction.}\]

Figure A.3 shows that the unit cell is divided (step 1) again into two rectangles. This rectangle division is chosen, because the small part of insulation material in rectangle 2 is expected to sustain the same radial strain that is governed by the relatively stiff winding material. For the edges of both rectangles it is assumed that these edges remain straight if the radial stress $\sigma_{rr}$ is applied.

The radial strain $\varepsilon_{rr,c}$ has to be divided in $\varepsilon_{rr,i}$ and $\varepsilon_{rr,w}$, depending on the radial moduli of the two rectangles, to be able to calculate the axial strain in both rectangles. These radial strains are calculated considering that the composite’s elongation consists of the elongation of both rectangles.
A.17

\[ \varepsilon_{rr,w} \lambda_r + \varepsilon_{rr,j} (1 - \lambda_r) = \varepsilon_{rr,c} \]

It is also considered that the radial stress is continuous in both rectangles.

A.18

\[ \varepsilon_{rr,w} E_{r,r2} = \varepsilon_{rr,j} E_i \]

With \( E_{r,r2} \) the radial modulus of rectangle 2:

A.19

\[ E_{r,r2} = \lambda_z E_w + (1 - \lambda_z) E_i \]

Expressed in \( \varepsilon_{rr,c}, \varepsilon_{rr,i} \) and \( \varepsilon_{rr,w} \) can now be written as:

A.20

\[ \varepsilon_{rr,i} = \varepsilon_{rr,c} \frac{E_{r,r2}}{E_i \lambda_r + E_{r,r2} (1 - \lambda_r)} \quad \varepsilon_{rr,w} = \varepsilon_{rr,c} \frac{E_i}{E_i \lambda_r + E_{r,r2} (1 - \lambda_r)} \]

In rectangle 1 the radial strain \( \varepsilon_{rr,i} \) causes a homogeneous strain in the axial direction in the insulation material. In rectangle 2 the radial strain \( \varepsilon_{rr,w} \) causes an axial strain, that depends on the combined Poisson’s ratio \( v_{rz,r2} \) of rectangle 2, that can be calculated with (2.3c).

A.21

\[ v_{rz,r2} = \lambda_z v_w + (1 - \lambda_z) v_i \]

\[ \begin{align*}
\text{Fr}_2 & \quad \uparrow & \quad \text{Fi} & \quad \uparrow & \quad \text{Ph} \\
\downarrow & \quad \text{Fr}_2 & \quad \downarrow & \quad \text{Fi} & \quad \downarrow \\
\text{rectangle 2} & \quad \text{rectangle 1} & \quad \text{homogeneous}
\end{align*} \]

Figure A.4: Reaction-forces of the material in rectangle 2 and 1, and in the homogeneous material, due to the same applied radial load.

The axial strains will be different in both rectangles, therefore it is assumed that an equivalent axial strain can be determined by considering the situations in figure A.4. The edges with a normal in the axial direction are bounded by a zero axial displacement. The radial strain will cause reaction-forces in both rectangles due to the Poisson’s ratios. The reaction-forces \( F_{r2} \) and \( F_i \) caused by the (different) Poisson’s ratios of both material pieces should be equivalent with \( F_h \) caused by the Poisson’s ratio \( v_{rz,c} \) for a unit cell filled with a homogeneous composite material (step 2 in figure A.3).

The reaction-forces are calculated by a multiplication of the axial stresses in each material and the area the stress is acting on. The axial stress is calculated by considering that the winding and insulation material are isotropic, and that the ‘homogeneous’ composite material is anisotropic. The strain-stress relationship can then be written as:
\[ e_{zz,x} = \frac{\sigma_{zz,x}}{E_{zz,x}} + \frac{-\nu_{rz,x}\sigma_{rr,x}}{E_{rr,x}} + \frac{-\nu_{\theta z,x}\sigma_{\theta \theta,x}}{E_{\theta \theta,x}} \] \quad A.22

With \( x: i, r2 \) or \( c \) depending on the material that caused the reaction-force on the area \( A \). The edges are tied, therefore \( e_{zz,x} \) is equal to 0. The tangential stress is still assumed to be zero, and the radial stress is as applied, therefore the axial stress can be written now as:

\[ \sigma_{zz,x} = \nu_{rz,x}\sigma_{rr,x} \frac{E_{zz,x}}{E_{rr,x}} \] \quad A.23

The reaction forces acting on the area \( A_x \) can be written as:

\[ F_r = \sigma_{zz,r2} A_{zz,r2} = \nu_{rz,r2}\sigma_{rr,r2} \frac{E_{zz,r2}}{E_{rr,r2}} \lambda_r = E_{zz,r2} e_{rr,w} \nu_{rz,r2} \lambda_r \] \quad A.24

\[ F_i = E_i e_{rr,i} \nu_i (1-\lambda_r) \] \quad A.25

\[ F_h = (\lambda_r E_{zz,r2} + (1-\lambda_r)E_i) e_{rr,c} \nu_{rz,c} = E_{zz,c} e_{rr,c} \nu_{rz,c} \] \quad A.26

The modulus \( E_{zz,c} \) is according to (2.4b). \( E_{zz,r2} \) is the axial modulus of the combined material in \( rectangle 2 \), and can be found using (2.3a):

\[ E_{zz,r2} = \frac{(E_w + \xi_E E_i) + \xi_E \lambda_z (E_w - E_i)}{(E_w + \xi_E E_i) - \lambda_z (E_w - E_i)} E_i = \mu E_i \] \quad A.27

Balance of the axial forces: (A.24) and (A.25) equals (A.26) using (A20), (A21) and (A.27) leads to \( \nu_{rz,c} \) with \( \mu \) as in (A.27).

\[ \nu_{rz,c} = \frac{\nu_i (1-\lambda_r) \{ \lambda_z E_w + (1-\lambda_z)E_i \} + \mu \lambda_r E_i \{ \lambda_z \nu_w + (1-\lambda_z)\nu_i \}} {\{ \mu \lambda_r + (1-\lambda_r) \} \{ \lambda_r E_i + (1-\lambda_r) \{ \lambda_z E_w + (1-\lambda_z)E_i \} \} } \] \quad A.28
Appendix B

Analytical model for the calculation of the mid-plane stresses

In this appendix the stress relationships are derived for the calculation of the mid-plane stresses of a magnet coil, according to Gersdorf's analytical model. At the end of this appendix the distinction between the elastic modulus and the modulus for a hydrostatically loaded material is explained.

Magnet coils can be considered as axisymmetric, when the slope and radius changes of the windings are neglected. Coils also show symmetry in the $r$-mid-plane, for the geometry as well as for the loading condition. Using cylindrical co-ordinates, the equilibrium conditions in the radial and axial direction on a unit-cell (with a body-force $F$) can be written respectively as:

$$\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + F_r = 0 \quad \text{B.1}$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + F_z = 0 \quad \text{B.2}$$

The windings in the coil can only transmit compressive forces into the radial and axial direction, because tensile forces will separate the layers of windings. In the tangential direction the windings can withstand compressive and tensile forces. The shear stiffness in the coil is only governed by the weak insulation material, because it is assumed that the windings will slide along each other without shear deformations, when a shear stress is applied. The shear stress in the windings, therefore, can be neglected, because this stress is much smaller than the other stress components. With the shear stresses omitted in the whole coil volume also the gradients in the shear stress are neglected. Summarising:

$$\sigma_{rr} \leq 0 \quad \sigma_{zz} \leq 0 \quad \tau_{rz} = 0 \quad \frac{\partial \tau_{rz}}{\partial r} = \frac{\partial \tau_{rz}}{\partial z} = 0 \quad \text{B.3}$$

The strains can be written as function of the displacement.

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \quad \varepsilon_{\theta\theta} = \frac{u_r}{r} \quad \text{B.4}$$

The compatibility relationship for the strains:

$$\varepsilon_{rr} = \frac{\partial}{\partial r} \left( r \varepsilon_{\theta\theta} \right) \quad \text{B.5}$$

The coil consists of windings and insulation material, with an axial winding fraction $\lambda_a$ and a radial winding fraction $\lambda_r$. A schematic representation of a unit cell of winding material embedded in insulation material is presented in figure B.1.a. As the insulation...
material is less strong and stiff as the winding material, the insulation is assumed to be, more or less, enclosed between the windings and it is under hydrostatic pressure. For the insulation material, separating two layers of windings in the radial direction, the assumption of hydrostatic pressure results in:

\[ \sigma_{rr,j} = \sigma_{zz,j} = \sigma_{\theta\theta,j} = \sigma_{rr,w} \]  
B.6

For the insulation material, separating two layers of windings in the axial direction:

\[ \sigma_{rr,j} = \sigma_{zz,j} = \sigma_{\theta\theta,j} = \sigma_{zz,w} \]  
B.7

Formulae (B.8) and (B.9) are the equilibrium relationships for the windings of the coil in the radial and the axial direction respectively. Hereby, it is assumed that only the tangential stress of the winding material contributes to the total tangential stress. This is because the tangential stress in the insulation material is much lower than the stress in the winding material and due to the small fraction of insulation material. In both directions, radially and axially, only the slice of material containing the winding fraction is considered. The Lorentz-forces \( F_r \) in the radial and \( F_z \) in the axial direction act on the winding material only.

\[
\left( \frac{\sigma_{rr,w} - \lambda_r \sigma_{\theta\theta,w}}{r} + \frac{\partial \sigma_{rr,w}}{\partial r} + \lambda_r F_r \right) \lambda_z = 0 
\]  
B.8

\[
\left( \frac{\partial \sigma_{zz,w}}{\partial z} + \lambda_z F_z \right) \lambda_r = 0 
\]  
B.9

\[ \begin{align*}
\lambda_r &= \text{winding fraction} \\
\lambda_z &= \text{insulation fraction} \\
\lambda_z &= (1-\lambda_r)\lambda_z
\end{align*} \]

Figure B.1:  
\text{a. Schematic representation of a unit cell with winding material embedded in insulation material.}  
\text{b. Top view of unit cell in the r\(\theta\)-plane: (1-\(\lambda_r\))\(\sigma_{\theta\theta,w}\) \(\ll\) \(\lambda_r\)\(\sigma_{\theta\theta,w}\).}

For the strain-stress relationship Hooke’s law is used:

\[ \varepsilon_{\theta\theta} = \frac{1}{E_w} \left( \sigma_{\theta\theta,w} - \nu_w \sigma_{rr,w} - \nu_w \sigma_{zz,w} \right) \]  
B.10

VIII
In (B.11) the total radial strain in the coil is partly the radial strain in the winding material and partly the radial strain in the insulation material. The radial strain in the insulation material is chosen to be one third of the volume-strain for this hydrostatically loaded material, with $B_i$ the bulk modulus. Substitution of (B.10) and (B.11) into the compatibility relationship (B.5) leads to:

$$
\begin{align*}
\frac{\lambda_r}{E_w} \left( \sigma_{rr, w} - \nu_w \sigma_{\theta\theta, w} - \nu_w \sigma_{zz, w} \right) + \frac{(1 - \lambda_r)}{3B_i} \sigma_{rr, w} &= \frac{1}{r} \left( \frac{\partial \sigma_{\theta\theta, w}}{\partial r} - \nu_w \frac{\partial \sigma_{rr, w}}{\partial r} - \nu_w \frac{\partial \sigma_{zz, w}}{\partial r} \right) \\
= r \frac{\partial \sigma_{\theta\theta, w}}{\partial r} - \nu_w \frac{\partial \sigma_{rr, w}}{\partial r} - \nu_w \frac{\partial \sigma_{zz, w}}{\partial r} \\
\end{align*}
$$

Using (B.8) in the form:

$$
\frac{\partial \sigma_{rr, w}}{\partial r} = -\lambda_r F_r - \frac{\sigma_{rr, w} - \lambda_r \sigma_{\theta\theta, w}}{r}
$$

Then (B.12) simplifies to:

$$
\begin{align*}
\frac{\partial \sigma_{\theta\theta, w}}{\partial r} &= \frac{1}{r} \left( \frac{\lambda_r}{E_w} \sigma_{rr, w} - \lambda_r \nu_w \sigma_{zz, w} - r \lambda_r \nu_w F_r \right) + \\
&+ \nu_w \frac{\partial \sigma_{zz, w}}{\partial r}
\end{align*}
$$

Formula (B.14), based on Hooke's law through the compatibility, no longer holds when the windings deform plastically. Instead of (B.14) a condition for plasticity should be used. For negligible shear stresses the von Mises criterion can be written as:

$$
\left( \sigma_{rr, w} - \sigma_{\theta\theta, w} \right)^2 + \left( \sigma_{\theta\theta, w} - \sigma_{zz, w} \right)^2 + \left( \sigma_{zz, w} - \sigma_{rr, w} \right)^2 = 2\sigma^2_{c, w}
$$

Provided that $\sigma_{rr, w}$ and $\sigma_{zz, w}$ are known $\sigma_{\theta\theta, w}$ can be solved from (B.15).

$$
\sigma_{\theta\theta, w} = \frac{1}{2} \left( \sigma_{rr, w} + \sigma_{zz, w} \right) \pm \sqrt{\sigma^2_{c, w} - \frac{3}{4} \left( \sigma_{rr, w} - \sigma_{zz, w} \right)^2}
$$

In the case of an energised magnet coil, in formula (B.16) always the '+' sign must be used.

With (B.8), (B.9), (B.14) and (B.16) the stresses in the winding material in the mid-plane of the coil can be calculated.
In section 3 the solution of the analytical model is compared with a finite element solution. In the analytical model it is assumed that the insulation material is hydrostatically loaded as can be observed in formula (B.11). The contribution of the insulation material to the radial strain is given there by the fraction:

\[ \varepsilon_{rr,i} = \frac{\sigma_{rr,iv}}{3B_i} \quad \text{B.17} \]

In the FEM the elastic insulation material's behaviour is mixed up in the composite model, and the information concerning hydrostatic deformations is lost. To create, for the reference situation of the coils, the same situation as in the analytical model the modulus of the insulation material has to be adapted, resulting into \( E_{i,FEM} = \frac{1}{1-2\nu}E_i \). This adaptation factor \( \frac{1}{1-2\nu} \) can be found by considering the situations in figure B.2, where the distinction between the elastic modulus and the modulus of a hydrostatically loaded isotropic material is explained.

![Figure B.2: Deformation of material. Situation a. free elastically loaded, b hydrostatically loaded.](image)

In situation A, there is only one stress component, therefore, the strain can be calculated as:

\[ \varepsilon = \frac{\Delta L}{L} = \frac{\sigma}{E_A} = \frac{\sigma}{3(1-2\nu)B_A} \quad \text{B.18} \]

In situation B, there is a triaxial hydrostatic pressure, the strain can now be calculated as:

\[ \varepsilon = \frac{\Delta L}{L} = \frac{\sigma}{E_B} (1-2\nu) = \frac{\sigma}{3B_B} \quad \text{B.19} \]

If in both situations the strain should be the same, for an equal stress \( \sigma \), then \( E_A \) in situation A (comparable with the FE model) has to be multiplied with \( \frac{1}{1-2\nu} \) compared to \( E_B \) in situation B (comparable with the analytical model).
Appendix C

**Measurements data fatigue tests CuAg and Cu wires**

CuAg two types of specimen made by a lathe (l) and by a milling machine (m).

<table>
<thead>
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<th>+800/-300 MPa</th>
<th>+800/0 MPa</th>
</tr>
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<tr>
<td>l p</td>
<td>824</td>
<td>1 p</td>
</tr>
<tr>
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<td>3118</td>
<td>1 e</td>
</tr>
<tr>
<td>l p</td>
<td>1036</td>
<td>1 p</td>
</tr>
<tr>
<td></td>
<td>3766</td>
<td>1 e</td>
</tr>
<tr>
<td>l p</td>
<td>1124</td>
<td>m p</td>
</tr>
<tr>
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<tr>
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<td>m p</td>
</tr>
<tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>m p</td>
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<td>m e</td>
</tr>
<tr>
<td></td>
<td>3923</td>
<td></td>
</tr>
<tr>
<td>m p</td>
<td>1936</td>
<td></td>
</tr>
</tbody>
</table>

*Table C.1:* Number of cycles and type of specimen (l or m) at the applied load; p indicates plastically pre-strained, e indicates only elastically loaded.

Hard-drawn Cu one type of specimen made by a lathe (l).

<table>
<thead>
<tr>
<th>+360/-300 MPa</th>
<th>+356/-222 MPa</th>
<th>+356/-135 MPa</th>
<th>+356/0 MPa</th>
<th>+320/-320 MPa</th>
<th>+320/-200 MPa</th>
</tr>
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<tbody>
<tr>
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<td>l p</td>
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<td>716</td>
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<td>784</td>
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<td>190</td>
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<tr>
<td>l p</td>
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<td>l p</td>
<td>817</td>
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</tr>
</tbody>
</table>

*Table C.2:* Number of cycles and type of specimen (l) at the applied load; p indicates plastically pre-strained, e indicates only elastically loaded.