Harmonic and rectangular pulse reproduction through current transformers

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Harmonic and Rectangular Pulse Reproduction through Current Transformers

by

Wang Jingshan

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THROUGH CURRENT TRANSFORMERS

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Abstract

Current transformer measuring properties for harmonic reproduction and rectangular pulse reproduction were investigated. The experimental results show that harmonics are reproduced through the conventional current transformers with less error than the fundamental frequency component when the secondary load is purely resistive. With an inductive secondary load, the current transformer accuracy deteriorates as the frequency increases. A current transformer is saturated at higher voltage levels with harmonics than it is with the fundamental frequency.

The secondary current distortion, when a current transformer is subjected to a series of rectangular pulses, is mainly determined by the ratio of the pulse duration to the CT's time constant. The rectangular pulses caused by travelling wave reflections on a power transmission line can be reproduced through the conventional current transformers with negligible distortion due to the short pulse duration, the large CT time constant and the negligible influence of the secondary winding to ground capacitance.

The maximum pulse amplitude and duration permitted to avoid the occurrence of saturation were found. Analysis indicates that the rectangular pulses caused by travelling waves along the transmission line can be reproduced through the conventional current transformers without saturation as long as the maximum pulse amplitude is less than or equal to the maximum R.M.S. value of the AC current rated for the current transformer accuracy.

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List of Symbols

\( B \) - The flux density (R.M.S.)
\( B_0 \) - The flux density at the central line of a lamination
\( C_1 \) - The coefficient of induced voltage
\( C_2 \) - The coefficient of eddy power loss
\( C_3 \) - The coefficient of hysteresis power loss
\( C_g \) - The secondary to ground capacitance
\( E_2 \) - The secondary voltage (R.M.S.)
\( E_{2p} \) - The peak value of the secondary voltage
\( f \) - The frequency
\( i'_{o(t)} \) - The instantaneous magnetizing current referred to the secondary
\( i'_{op(t)} \) - The instantaneous magnetizing current for positive pulses
\( i'_{on(t)} \) - The instantaneous magnetizing current for negative pulses
\( I'_o \) - The magnetizing current referred to the secondary (R.M.S.)
\( I'_i(t) \) - The instantaneous primary current referred to the secondary
\( I'_{iD} \) - The amplitude of the primary current pulse referred to the secondary
\( I'_{iDm} \) - The maximum primary current amplitude permitted to avoid saturation
\( I_1 \) - The primary current (R.M.S.)
\( i_2(t) \) - The instantaneous secondary current
\( i_{2p}(t) \) - The instantaneous positive secondary current
\( i_{2N}(t) \) - The instantaneous negative secondary current
\( \Delta I_2 \) - The overshoot amplitude of secondary current
\( \Delta I_{2p} \) - The overshoot amplitude in positive pulses
\( \Delta I_{2min} \) - The minimum overshoot amplitude
\( \Delta I_{2max} \) - The maximum overshoot amplitude
\( \Delta I_{2f} \) - The final overshoot amplitude
The secondary current (R.M.S.)

The rated secondary current (R.M.S.)

Any positive integer

The even positive integer

The odd positive integer

The current accuracy limit factor

The magnetizing inductance referred to the secondary

The secondary load (series) inductance

The number of primary turns

The number of secondary turns

The eddy power loss

The hysteresis power loss

The rated secondary resistance

The core loss equivalent resistance

The secondary winding resistance

The duration of rectangular pulses

The period of 50 Hz frequency

The maximum pulse duration permitted to avoid saturation

The secondary impedance

The steady-state transformation error

The power angle of secondary load

The core permibility

The core conductivity

The current transformer time constant

The current transformer time constant with purely resistive load

The secondary load time constant

The maximum AC flux with purely resistive load

The maximum AC flux
- vii -

$\phi_{Dm}^*$ - The maximum DC flux with purely resistive load

$\phi_{Dm}$ - The maximum DC flux

$\phi_R$ - The remanent magnetism

$\omega$ - The angular velocity of power frequency
1. Introduction

With the development of modern protective systems, the signals used for protective discrimination have been extended from the fundamental frequency current and voltage to much more complex forms, such as derivatives, integrals, harmonics and travelling waves. The protective relays have been transistorized, digitalized or computerized. All these factors have placed emphasis on the fidelity of primary current reproduction through current transformers during transient conditions.

This report tries to add some new aspects to the work on the current transformer measuring properties and it contains information about both harmonic reproduction and rectangular pulse reproduction. The experiments are performed on a conventional current transformer with data: \( I_1/I_2 = 10^3\text{A}/5\text{A}, \)
\( K_C = 20 \) and \( Z_2 = 1.2\Omega. \)
2. Harmonic Reproduction

In modern protective systems, harmonics are becoming more and more important. This is true not only because they may cause malfunction of the protective relays based on phase discriminants, but also due to fact that some of the computerized relays have to allow the presence of harmonics in the signals for discrimination in order to reduce the time for filtering lower frequency components. On the other hand, new protective relays, purely using harmonics, are also under development, such as the sensitive ground fault detector, fault versus load discriminator, etc.

Figure 1 presents a typical diagram of a current transformer for harmonic reproduction, in which the core loss and the secondary to ground capacitance are taken into account. Theoretically, the secondary leakage inductance is also relevant, but it is often neglected in practice because the flux linkage is enhanced sufficiently with the usual toroid core construction.

2.1. Transformation Error Versus Frequency

Simple as it is, the well-known equivalent circuit in Figure 1 can only be used for qualitative analysis, because some of its parameters are difficult to determine exactly, such as the core loss equivalent resistance. In order to determine the steady-state transformation error versus frequency, an indirect experimental method was used, with which the difficulty of producing a large primary current over a wide range of frequencies was avoided. In this experiment, the current transformer was supplied by a power amplifier with variable frequencies up to 10 kHz at its secondary terminals, while the primary terminals were left open. Each voltage supplied to the current transformer secondary terminals was carefully calculated to simulate the real secondary voltage excited at the corresponding primary current. The formula used in this calculation is:

\[ E_2 = K_c I_2 R \left( \frac{R}{W} + R + jwL_2 \right) \]  

(1)

The currents flowing through the current transformer were measured as the magnetizing currents. Both voltage and current were measured in R.M.S. values. The transformation error was defined by equation (2), which is
the ratio of the referred magnetizing current to the secondary current.

\[ \delta = \frac{I_1/I_2 - N_2/N_1}{N_2/N_1} = I'_0/I_2 \]  

Figure 2 shows the curves obtained with purely resistive secondary load, which indicates that the transformation error reduces as the frequency increases up to 10 kHz.

It is well-known that both eddy loss and hysteresis loss increase as the frequency increases. Since the error current is composed of both core power loss current and magnetizing current, the above effect seems to cause the error increase as the frequency increases. But it can be further argued that for a given secondary voltage, which corresponds to a certain secondary current with the rated secondary load, the flux density required to induce the given secondary voltage reduces linearly as the frequency increases. It is due to the reduced flux density that the power loss decreases slightly instead of increasing as the frequency increases when the secondary voltage remains at a given level. This argument can be indicated by equation (3), in which the expressions for eddy loss and hysteresis loss can be found in most textbooks on transformer theory.

\[ R_c = \frac{E_2^2}{P_e + P_h} = \frac{C_1 B_{2f}^2}{C_2 B_{2f}^2 + C_3 B_{1.6}^2 \tilde{f}^2} = \frac{C_1/\tilde{f}}{C_2 + C_3/\tilde{f}} \]  

Theoretically, the secondary to ground capacitance will increase the transformation error as the frequency increases, but this influence can be reduced by keeping the capacitance as small as possible so that it is suppressed by the other factors within a certain frequency limit. The curves in Figure 2 can be understood as the resultant effect of the factors discussed above.

Figure 3 and Figure 4 indicate the influence of the secondary power factor, which was chosen as \( \cos \theta = 0.5 \) and \( \cos \theta = 0.8 \) in accordance with the usual secondary load types. In the calculation the apparent secondary...
impedance was dominated by the reactance from $10^2$ to $10^3$ Hz. Because of the higher impedance the secondary voltage and the transformation error increases with increasing frequency. Unfortunately, the power amplifier output voltage was limited to 300 Volts, which made it impossible to increase the frequency further than 400 Hz for $K_c = 10$ and 200 Hz for $K_c = 20$. However, it can still be seen from the curves in Figure 3 and Figure 4 that the transformation error begins to exceed the error specification (1%) from 1 kHz for $\cos \theta = 0.5$ and from 2 kHz for $\cos \theta = 0.8$ at the rated secondary current.

2.2 Harmonic Saturation

The influence of eddy current does not only cause power loss in the magnetic core, but also makes the magnetic field unevenly distributed over the cross section of laminations. As the frequency increases, the distribution of flux density becomes more and more concentrated towards the surface of laminations. The effect brought about by this uneven distribution is that the equivalent area of a core becomes reduced. Figure 5 shows the flux distribution across the width of one lamination as a function of frequency, which corresponds to equation (4). The theory of flux distribution and equation (4) can be found in many textbooks specializing in electric and magnetic field analysis.

\[ |B| = B_o \sqrt{\frac{1}{2} \left( \cosh \frac{\mu \omega x}{2} + \cos \frac{\mu \omega x}{2} \right)} \quad (4) \]

where: $B_o$ = the flux density at the central line of lamination
$x$ = the distance along the lamination thickness.

As the frequency increases, another factor which should be taken into account, is that the flux density required to excite a certain secondary voltage is reduced linearly. As long as the lamination thickness is thin enough, the influence of flux reduction is much stronger than the influence of uneven flux distribution. In other words, a current transformer will become saturated at higher voltage levels as the frequency increases.

Figure 6 shows the harmonic magnetizing curves measured during the experiment. From these curves, it can be seen that the influence of uneven flux distribution is suppressed by the influence of flux reduction.
When the frequency increases, the current transformer becomes saturated at higher secondary voltages when it is excited harmonically.

3. Rectangular Pulse Reproduction

For those protective relays based on wave discriminants, signal transducing is required to reproduce rectangular pulses. The question concerned here is whether or not the conventional current transformers can be used on this application.

3.1 The Reproduction of a Steep Front

In order to evaluate the reproduction of a sudden change of primary current, the diagram in Figure 1 is used to represent the current transformer. The primary current is considered to be a step function with its amplitude \( I'_0 \). When the secondary load is purely resistive, the secondary current can be expressed as equation (5), where the core loss and the winding resistance is omitted.

\[
I_2(t) = \frac{I'_0}{\sqrt{1 - \frac{4R^2C^2}{L_0}}} \left[ e^{-\frac{t}{2R_0C}} \sqrt{1 - \frac{4R^2C^2}{L_0}} - e^{-\frac{t}{2R_0C}} \sqrt{1 + \frac{4R^2C^2}{L_0}} \right] \quad (5)
\]

Usually, the secondary winding to ground capacitance is in the order of \( 10^{-9} \) F, see Reference [2], the magnetizing inductance can be as large as several H and the resistance of the secondary load is only several Ohms. Considering the value of the above parameters, a reasonable approximation is permitted:

\[
\sqrt{1 - \frac{4R^2C^2}{L_0}} \approx 1
\]

Using the above approximation, equation (5) can be simplified as:

\[
i_2(t) = I'_0 \left(1 - e^{-\frac{t}{R_0C}}\right) \quad (6)
\]

Equation (6) indicates that a step function will be reproduced through the current transformer with a raising time which is determined by the product of the secondary resistance and the secondary winding to ground capacitance. Since this product is only in the order of \( 10^{-8} \), the time needed to raise the secondary current to its peak can be expected within one microsecond.
Compared with the operation time of the Ultra-high speed relay, which usually takes several milliseconds, the raising time caused by the secondary winding to ground capacitance is negligible.

To confirm the above conclusion, the experimental set-up in Figure 7 was used to generate steep current front. After charging the capacitor bank $C_1$ and opening $S_1$, breaker $B_0$ is closed. Then $C_1$ discharges in a 50 Hz oscillatory current through $B_0$ and $B_1$. When the 50 Hz discharge current is near its peak value, breaker $B_1$ is opened to commutate the current into the current transformer. After 2 milliseconds breaker $B_2$ was closed to shunt the current flowing in the current transformer. In this way, the primary current produced was about 1.1 kA and $di/dt$ was approx. $4 \times 10^5$ A/s. Both the primary current front and the secondary current front were recorded by a computerized data system with a sampling rate of $10^5$ s$^{-1}$. The results are presented in Figure 8. No significant time delay can be found during the time for the secondary current to reach its peak value.

3.2 The Secondary Current Response to a Series of Rectangular Pulses

In travelling wave analysis, a series of rectangular pulses are often used to represent the wave reflections. They are expressed in equation (7) and presented in Figure 10 as the dotted line.

$$i'_2(t) = i'_1 + \sum_{k=1}^{\infty} 2 \frac{L_1}{L_1 + L_2} (t - kT) \cdot (-1)^k$$

(7)

In order to find the secondary response to a series of rectangular pulses, the principle of superposition can be used, in which each step excitation is applied to the current transformer with a time interval $T$. The secondary current response during the time period $kT < t < (k + 1)T$ is obtained by summing up all the responses to each excitation applied before and at the point of time $kT$. Since it has been concluded that the secondary winding to ground capacitance has an negligible influence, the current transformer is simplified as the diagram in Figure 9. With this diagram, the secondary current response during the time period $kT < t < (k + 1)T$, where $k = 0, 1, 2, 3...$ etc., is expressed as equation (8). The derivation of equation (8) and the other equations in this section are presented in Appendix A of this report.

During the time: $kT < t < (k + 1)T$, $k = 0, 1, 2, 3...$ etc.

$$i_2(t) = i'_1 \frac{L_0}{L_0 + L_2} e^{-(t-kT)/\tau} \left[ e^{-kT/\tau} + \sum_{i=1}^{k} (-1)^i \cdot 2 \cdot e^{-(k-i)T/\tau} \right]$$

(8)

where $\tau = (L_0 + L_2)/R_2$. 
From equation (8) two equations can be derived, equation (9) for all the positive pulses and equation (10) for all the negative pulses.

During the time: \( k_1 T < t < (k_1 + 1)T \), \( k_1 = 0, 2, 4 \ldots \) etc.,

\[
i_{2p}(t) = [ I_{1D}' + \Delta I_{2p}(k_1 T)] e^{-(t - k_1 T)/\tau} \quad (9)
\]

where:

\[
\Delta I_{2p}(k_1 T) = I_{1D}' \left[ \frac{L_o}{L_o + L_2} \frac{(1-e^{-T/\tau})(1-e^{-k_1 T/\tau})}{1+e^{-T/\tau}} - \frac{L_2}{L_o + L_2} \right]
\]

During the time: \( k_2 T < t < (k_2 + 1)T \), \( k_2 = 1, 3, 5 \ldots \) etc.,

\[
i_{2N}(t) = - [ I_{1D}' + \Delta I_{2n}(k_2 T)] e^{-(t - k_2 T)/\tau} \quad (10)
\]

where:

\[
\Delta I_{2n}(k_2 T) = I_{1D}' \left[ \frac{L_o}{L_o + L_2} \frac{(1-e^{-T/\tau})(1+e^{-k_2 T/\tau})}{1+e^{-T/\tau}} - \frac{L_2}{L_o + L_2} \right]
\]

The secondary current waveform corresponding to the above equations is presented in Figure 10, in which the magnetizing current is also presented for reference. Compared with the primary current waveform in the same figure, the secondary current is distorted in two forms. a) The flat tops of the primary current are distorted as the damping waveforms with the same damping time constant \( \tau \). b) The pulse front of the primary current at each discontinuity is transformed with either reduced or increased amplitude.

If the difference between the absolute value of the primary current and the secondary current at each discontinuity is defined as the overshoot amplitude \( \Delta I_2 \), the overshoot amplitudes for all the positive pulses \( \Delta I_{2p} \) increase from the minimum value to the final value \( \Delta I_{2f} \), while the overshoot amplitudes for all the negative pulses \( \Delta I_{2n} \) decrease from the maximum value to the final value \( \Delta I_{2f} \). The changing rate of either \( \Delta I_{2p} \) or \( \Delta I_{2n} \) is determined by the current transformer time constant and the pulse duration. The minimum, maximum and final overshoot amplitudes are expressed in equations (11), (12), and (13).

\[
\Delta I_{2min} = \lim_{t \to 0} \frac{\left| i_{2p}(t) \right|}{\left| i_{1D}'(t) \right|} \quad (11)
\]

\[
= - I_{1D}' \frac{L_2}{L_o + L_2}
\]
\[
\Delta I_{2\text{max}} = \lim_{t \to T} \left[ |i_{2N}(t)| - |i_1(t)| \right]
\]

\[
= I_1' \left[ \frac{L_0}{L_0 + L_2} \left( 1 - e^{-T/\tau} \right) \right] - \frac{L_2}{L_0 + L_2}
\]

\[
\Delta I_{2f} = \lim_{k_1 \to 0} \left[ |i_{2p}(t)| - |i_1(t)| \right]
\]

\[
= t \to k_2 \left[ |i_{2N}(t)| - |i_1(t)| \right]
\]

\[
= I_1' \left[ \frac{1 - e^{-T/\tau}}{1 + e^{-T/\tau}} \right] - \frac{L_2}{L_0 + L_2}
\]

From the above equations, it can be seen that either the damping distortion or the overshoot amplitude is determined by the ratio of the pulse duration to the current transformer time constant. In practice, the ratio of a pulse duration, caused by travelling wave reflection, to a current transformer time constant is normally in the order of \(10^{-3}\), so is the ratio of the secondary inductance to the magnetizing inductance. From this fact it can be expected that both the damping distortion and the overshoot amplitude are so small as to be neglected in the travelling wave pulse reproduction.

To check the above statement experimentally, an approximately rectangular pulse was produced by using the same experimental set-up presented in Figure 7. The pulse duration was about 2 milliseconds and the amplitude was 1.7 kA. With either resistive load or inductive load, no apparent damping distortion or overshoot amplitude was found in the secondary response. The experimental result is presented in Figure 11 in order to give an impression of the way that a rectangular pulse is reproduced.

3.3 The Transient Magnetizing Current and the Maximum Flux.

In order to study the saturation problem, the transient magnetizing current waveform is first observed, which can be derived by directly subtracting the transient secondary current from the primary current.
During the time: \( k_1 T < t < (k_1 + 1)T, \ k_1 = 0, 2, 4 \ldots \) etc.,

\[
i_{op}'(t) = I_{1D}' \left( 1 - \frac{L_o}{L_o + L_2} e^{-\frac{(t - k_1 T)}{\tau}} \left[ 1 + \frac{(1 - e^{-T/\tau})(1 - e^{-k_1 T/\tau})}{1 + e^{-T/\tau}} \right] \right)
\]  

(14)

At the discontinuities of \( t = k_1 T \):

\[
i_{op}'(k_1 T^+) = I_{1D}' \left[ \frac{L_2}{L_o + L_2} - \frac{L_o}{L_o + L_2} \frac{(1 - e^{-T/\tau})(1 - e^{-k_1 T/\tau})}{1 + e^{-T/\tau}} \right]
\]  

(15)

At the discontinuities of \( t = (k_1 + 1)T \):

\[
i_{op}'((k_1 + 1)T^-) = I_{1D}' \left[ \frac{L_2}{L_o + L_2} + \frac{L_o}{L_o + L_2} \frac{(1 - e^{-T/\tau})(1 - e^{-(k_1 + 1)T/\tau})}{1 + e^{-T/\tau}} \right]
\]  

(16)

During the time: \( k_2 T < t < (k_2 + 1)T, \ k_2 = 1, 3, 5 \ldots \) etc.,

\[
i_{on}'(t) = -I_{1D}' \left( 1 - \frac{L_o}{L_o + L_2} e^{-\frac{(t - k_2 T)}{\tau}} \left[ 1 + \frac{(1 - e^{-T/\tau})(1 + e^{-k_2 T/\tau})}{1 + e^{-T/\tau}} \right] \right)
\]  

(17)

At the discontinuities of \( t = k_2 T \):

\[
i_{on}'(k_2 T^+) = -I_{1D}' \left[ \frac{L_2}{L_o + L_2} - \frac{L_o}{L_o + L_2} \frac{(1 - e^{-T/\tau})(1 - e^{-k_2 T/\tau})}{1 + e^{-T/\tau}} \right]
\]  

(18)

At the discontinuities of \( t = (k_2 + 1)T \):

\[
i_{on}'((k_2 + 1)T^-) = -I_{1D}' \left[ \frac{L_2}{L_o + L_2} + \frac{L_o}{L_o + L_2} \frac{(1 - e^{-T/\tau})(1 - e^{-(k_2 + 1)T/\tau})}{1 + e^{-T/\tau}} \right]
\]  

(19)

Here the plus sign or the minus sign indicates that the limit is taken at the righthand or lefthand of the discontinuities respectively.

The transient magnetizing current waveform is presented in Figure 12 from the above equations. For the purely resistive secondary load, the transient magnetizing current can be derived by simply letting \( L_2 = 0 \) in the equations above, which is presented in Figure 13. From the above equations, it is easy to see that the maximum magnetizing current occurs at the end of the first positive pulse. For the inductive secondary load, it is expressed in equation (20).
For the purely resistive load, it is expressed in equation (21). Both of them are mainly determined by the ratio of the pulse duration to the current transformer time constant.

\[ i_{o \text{ max}} = I_{1D}' \frac{L_2}{L_o + L_2} + \frac{L_o}{L_o + L_2} (1-e^{-T/T}) \]  
\[ i'_{o \text{ max}} = I_{1D}' (1-e^{-T/T}) \]

Before the current transformer is saturated, the magnetizing inductance can be considered as linear, so the flux is linear to the magnetizing current. The maximum flux for the inductive load can be expressed as equation (22) from equation (20) and the maximum flux for the purely resistive load can be expressed as equation (23) from equation (21).

\[ \Phi_{Dm} = \frac{L_o}{N_2} I_{1D}' \left[ \frac{L_2}{L_o + L_2} + \frac{L_o}{L_o + L_2} (1-e^{-T/T}) \right] \]
\[ \Phi'_{Dm} = \frac{L_o}{N_2} I_{1D}' (1-e^{-T/T}) \]

3.4 The Maximum Pulse Amplitude and Duration Permitted to Avoid Saturation.

Another problem which should be determined for the travelling wave reproduction is the limits of the pulse amplitude and duration permitted to avoid saturation. Since current transformers are normally specified for the fundamental frequency it is convenient to express these limits with the fundamental frequency specifications.

When a current transformer is excited by a fundamental frequency current, the maximum flux allowed to avoid saturation can be expressed by equation (24), where a purely resistive load is assumed.

\[ \Phi'_{Am} = \frac{\sqrt{2} K C I_{2R} R_2}{\omega N_2} \]

When the same current transformer with the same load operates with a series of rectangular pulses, the flux density reaches its maximum value at the end of the first positive pulse if no remanence is assumed, which has been derived in equation (23) as \( \Phi'_{Dm} \). If the current transformer is considered to be saturated when \( \Phi'_{Dm} \) reaches the value of \( \Phi'_{Am} \), the maximum pulse amplitude
of the primary current permitted to avoid saturation can be expressed by equation (25)

\[ I_{1Dm}^{'} = \frac{\sqrt{2} K_c I_{2R} R_2}{\omega L_c (1-e^{-t/\tau})} \leq \frac{K_c I_{2R} T_{50}}{\pi \sqrt{2}} \tag{25} \]

Where the term \((1-e^{-t/\tau})\) is approximated as \(T/\tau\) when \(T \ll \tau\).

If \(I_{1Dm}^{'}\) is required to be equal to or larger than \(K_c I_{2R}\), the restriction of pulse duration can be expressed as

\[ T_m \leq \frac{T_{50}}{\pi \sqrt{2}} = 4.5 \text{ ms} \tag{26} \]

When the current transformer secondary load is inductive, the maximum AC flux allowed to avoid saturation can be expressed by

\[ \Phi_{Am} = \frac{\sqrt{2} K_c I_{2R} R_2}{\omega N_2} \tag{27} \]

Considering equation (22), the maximum pulse amplitude permitted to avoid saturation can be expressed by

\[ I_{1Dm}^{'} = \frac{K_c I_{2R} T_{50}}{\pi \cos \theta \left(\tau_2 + \sqrt{\frac{R_2^2}{2} + (\omega L_2)^2}\right)} \tag{28} \]

Where: \(\tau_2 = L_2/R_2\), \(\cos \theta = R_2/\sqrt{R_2^2 + (\omega L_2)^2}\).

The derivation of equation (28) and equation (29) is presented in Appendix (B).

If \(I_{1Dm}^{'}\) is required to be equal to or larger than \(K_c I_{2R}\), the restriction of pulse duration is given by

\[ T_m \leq \frac{\left(\sqrt{2} - \sin \theta\right)}{2 \pi \cos \theta} T_{50} \tag{29} \]

According to equation (29), various secondary power factors have been used to calculate the restricted pulse durations. The result from the calculation is presented in Figure 14. The minimum value is 3.18 ms when \(\cos \theta = \frac{1}{2}\). For the usual secondary power factor, \(\cos \theta = 0.5\) or \(\cos \theta = 0.8\), the pulse duration restricted is 3.5 ms or 3.2 ms respectively.
Compared with the pulse durations produced by travelling wave reflections, a typical value of which is 1.3 ms for a distance of 200 km if the speed of light is assumed to be the travelling speed, the pulse durations restricted for different secondary power factors are still long enough for a rectangular pulse to be reproduced through a conventional current transformer without saturation as long as the pulse amplitude is equal to or lower than the maximum R.M.S. value of the maximum AC current. For those pulses with amplitudes higher than the maximum R.M.S. value of the rated AC current, whether or not a current transformer will be saturated depends on the product of the current amplitude and the pulse duration. The restricted IT product can be expressed as equation (30) for a purely resistive load or equation (31) for an inductive load.

\[
I_{1Dm}' .2\pi .T_m \leq \sqrt{2}Kc'I_{2R}^-T_50
\]  

\[
I_{1Dm}' .2\pi .\cos \theta (T_2 + T_m') \leq \sqrt{2}Kc'2R^-T_50
\]

Figure 15 shows the distorted secondary current waveform when a current transformer is saturated with a rectangular pulse. In order to get rid of the difficulty of producing a rectangular pulse with extremely high amplitude, this amplitude, in the experiment, was reached by raising the secondary resistance sufficiently. In this way the secondary voltage was raised and the current transformer time constant was reduced. The primary pulse amplitude was about 1.5 kA with a time duration of 2 ms. The raised secondary resistance was 30 Ω. The maximum primary AC current rated for accuracy was 20 kA at the rated secondary load 1.2Ω. From Figure 15 it can be seen that when the current transformer is saturated, the secondary current reduces almost to zero and a large overshoot amplitude occurs at the discontinuity.
4. Conclusions

A. With the rated load, the conventional current transformers can reproduce harmonics up to several kilohertz at least within the specified accuracy limit.

B. Rectangular pulses caused by travelling waves along a power transmission line can be reproduced through the conventional current transformers with negligible error.

C. In the development of protective systems, the conventional current transformer can be allowed to operate with relays which include harmonics in their signals or take the travelling waves as their protective discriminants.

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Appendix A

The Expressions for Secondary Transient Response to a Series of (A1) Rectangular Pulses

The primary current under consideration is expressed as equation (A1), which includes a number of step excitations with the time interval \( T \).

\[
i_1'(t) = I_{1D}' + \sum_{i=1}^{k} (-1)^i \cdot 2I_{1D}'(t - iT)
\]  
(A1)

The operational form of equation (A1) can be shown as equation (A2).

\[
i_1'(s) = \frac{I_{1D}'}{s} + \sum_{i=1}^{k} \left( \frac{I_{1D}'}{s} \cdot i \cdot 2e^{-isT} \right)
\]  
(A2)

From Figure 9, the transformation function for the secondary response is expressed as equation (A3).

\[
H(s) = \frac{sL_0}{s(L_0 + L_2) + R_2}
\]  
(A3)

The secondary response in time domain is the inverse transformation of the product of equation (A2) and equation (A3).

\[
i_2(t) = L^{-1} \left[ \frac{sL_0}{s(L_0 + L_2) + R_2} \left( \frac{I_{1D}'}{s} + \sum_{i=1}^{k} (-1)^i \cdot 2e^{-isT} \right) \right]
\]  
(A4)

where \( \tau = (L_0 + L_2)/R_2 \).

In equation (A4), let \( k = k_1 \) where \( k_1 = 2, 4, \ldots \) etc. The secondary current response during the time period \( k_1T < t < (k_1+1)T \) can be found as equation (A5), which applies to all the positive pulses.

\[
i_2p(t) = I_{1D}' \frac{L_0}{L_0 + L_2} e^{-\frac{(t-k_1T)/\tau}{1}} \left[ e^{-k_1T/\tau} - 2e^{-\frac{(k_1-1)T/\tau}{1}} - 2e^{-\frac{T/\tau}{2}} \right]
\]  
(A5)

\[
i_2p(t) = I_{1D}' \frac{L_0}{L_0 + L_2} e^{-\frac{(t-k_1T)/\tau}{1}.F(k_1)}
\]
Let \( a = e^{-T/\tau} \)

\[
F(k_1) = a^{k_1} - 2a(k_1 - 1) + 2a(k_1 - 2) - \ldots - 2a + 2
\]

\[
= 1 + (1-a) - a(1-a) + a^2(1-a) - \ldots - a^{k_1-1} (1-a)
\]

\[
= 1 + (1-a)(1-a + a^2 + \ldots + a^{k_1-2} - a^{k_1-1})
\]

\[
= 1 + (1-a)^2 (1 + a + a^2 + \ldots + a^{k_1-2})
\]

\[
= 1 + \frac{(1-a)(1-a^{k_1})}{1+a}
\]

Therefore:

\[
i_2 p(t) = I'_1 D \frac{L_0}{L_0 + L_2} e^{-(t-k_1 T)/T} \left[ \frac{1 + (1-e^{-T/\tau})}{1 + e^{-T/\tau}} \right]
\]

(A5)

In equation (A4), let \( k = k_2 \) where \( k_2 = 1, 3, 5 \ldots \) etc., the secondary current response during the time period \( k_2 T < t < (k_2 + 1)T \) can be found as equation (A6), which applies to all the negative pulses.

\[
i_2 N(t) = I'_1 D \frac{L_0}{L_0 + L_2} e^{-(t-k_2 T)/T} \left[ e^{-k_2 T/\tau} - e^{-(k_2-1)T/\tau} + \ldots + 2e^{-T/\tau} \right]
\]

\[
= I'_1 D \frac{L_0}{L_0 + L_2} e^{-(t-k_2 T)/T} F(k_2)
\]

Let \( a = e^{-T/\tau} \)

\[
F(k_2) = a^{k_2} - 2a(k_2 - 1) + 2a(k_2 - 2) - \ldots + 2a - 2
\]

\[
= [1 + (1-a) - a(1-a) + a^2(1-a) - \ldots + a^{k_2-1} (1-a)]
\]

\[
= [1 + (1-a)(1-a + a^2 + \ldots + a^{k_2-1})]
\]

\[
= [1 + (1-a)^2 (1 + a^2 + \ldots + a^{k_2-2} + \frac{a^{k_2-1}}{1-a})]
\]

\[
= [1 + \frac{(1-a)(1+a^{k_2})}{1+a}]
\]

Therefore:

\[
i_2 N(t) = - I'_1 D \frac{L_0}{L_0 + L_2} e^{-(t-k_2 T)/T} \left[ 1 + \frac{(1-e^{-k_2 T/\tau})}{1 + e^{-k_2 T/\tau}} \right]
\]

(A6)
Appendix B

The Maximum Pulse Amplitude and Duration Permitted to Avoid Saturation

The maximum AC flux can be expressed as

$$\phi_{Am} = \frac{\sqrt{2} \cdot K \cdot L_2 \cdot Z}{N_2 \cdot 2\pi f}$$  \hspace{1cm} (B1)

The maximum DC flux at the end of the first positive pulse can be expressed as

$$\phi_{Dm} = \frac{L_O}{N_2} I'_D \left[ \frac{L_2}{L_O + L_2} + \frac{L_O}{L_O + L_2} \right] \left( 1 - e^{-T/\tau} \right)$$  \hspace{1cm} (B2)

When $\phi_{Dm}$ reaches the value of $\phi_{Am}$, the current transformer is saturated.

The maximum pulse amplitude can be expressed as

$$I'_{Dm} = \frac{\sqrt{2} \cdot K \cdot I'_D \cdot R \cdot T_{50}}{2\pi \cos \theta L_O \left[ \frac{L_2}{L_O + L_2} + \frac{L_O}{L_O + L_2} \left( 1 - e^{-T/\tau} \right) \right]}$$

$$= \frac{\sqrt{2} \cdot K \cdot I'_D \cdot T_{50}}{2\pi \cos \theta (T_{2} + T)}$$  \hspace{1cm} (B3)

Where: $L_O / (L_O + L_2)$ is approximated as 1 and $(1 - e^{-T/\tau}) \approx T/\tau$.

In order to evaluate the maximum pulse duration, let $I'_{Dm} / K \cdot I'_L \approx 1$.

The maximum pulse duration can be expressed as equation (B4), from equation (B3)

$$T_m \leq \frac{(\sqrt{2} - \sin \theta)}{2\pi \cos \theta} \cdot T_{50}$$  \hspace{1cm} (B4)
Figure 1. HF equivalent circuit

Figure 2. Current error versus frequency
\[ \cos \theta = 1 \]
Figure 3. Current error versus frequency
$\cos \theta = 0.5$

Figure 4. Current error versus frequency
$\cos \theta = 0.8$
Figure 5. Steady-state flux distribution

\[ \mu = 10^3 \times 4\pi \times 10^{-7} \text{ H/m} \]

\[ \rho = 10^7 \text{ 1/m} \]

Figure 6. Harmonic magnetizing curves
Figure 7. Rectangular pulse generation circuit

\[ R_2: 0.6, 1.0, 1.2 \ \Omega \]
\[ L_2: 3.3, 2.3, 0 \ \text{mH} \]
\[ \cos\phi: 0.5, 0.8, 1.0 \]

Figure 8. (a) Primary step front  (b) Secondary step front
Figure 9. Current transformer equivalent circuit

Figure 10. The secondary current response to a series of rectangular pulses

mT ≥ (3 & 4) τ
Figure 11. Step pulse reproduction \( \cos \theta = 1 \)
Upper part: Primary current 1.1 kA/div.
Time scale 5 ms/div.
Figure 12. The magnetizing current for inductive load
Figure 13. The magnetizing current for resistive load

Figure 14. The maximum pulse duration as a function of \( \cos \theta \)
Figure 15. Step pulse saturation $R_2=30\Omega$
Upper part: Primary current 1.1 kA/div.
Time scale 2 ms/div.